Computational Physics II 5640, Spring 2017

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1 Critical Exponents from Landau's Theory

An essential part of the study of phase transitions are the Landau Theory critical exponents. These can be solved for, and then tested in the next section. In Landau theory we begin with the Gibb's free energy, expanded in the order parameter, m, with only even powers appearing.

$$G(m,T) = a(T) + \frac{1}{2}b(T)m^2 + \frac{1}{4}c(T)m^4. \tag{1}$$

From here we find β, the critical exponent of the order parameter, by noting that magnetization is given by,

$$\frac{\partial G}{\partial m}\Big|_{m=m_0} = 0, \tag{2}$$

and,

$$b(T) = b_0(T - T_c), \tag{3}$$

so,

$$\begin{aligned} 2b_0(T-T_c)m_0 &= -4cm_0^3, \\ m_0 &\propto (T_c - T)^{\frac{1}{2}}, \end{aligned} \tag{4}$$

therefore $\beta = \frac{1}{2}$.

Next, α is the critical exponent to do with the specific heat, but because c would have a discontinuity if α were non-zero in the square Ising lattice, we set it to zero by convention. With two exponents we can get the other two, ν, the exponent of correlation length, and γ, the exponent of susceptibility, via the scaling relation,

$$\begin{aligned} \nu d = 2 - \alpha &= 2\beta + \gamma, \\ \nu &= \frac{2}{d}, \\ \gamma &= 2(1-\beta). \end{aligned} \tag{5}$$

With d, the number of spacial dimensions equal to two, we get $\nu = \gamma = 1$.

2 Critical exponents from finite size scaling of Monte Carlo simulations

To see the critical exponents in action we can use them to construct "universal curves" of the different parameters of the Ising lattice for multiple lattice sizes, L. For our demonstration we will look at:

Specific Heat,

$$C = \frac{\langle H^2\rangle - \langle H\rangle^2}{T^2L^2}, \tag{6}$$

where,

$$H = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j. \tag{7}$$

Magnetization,

$$m = \langle |M| \rangle / L^2 = \langle | \sum_i \sigma_i | \rangle / L^2 \tag{8}$$

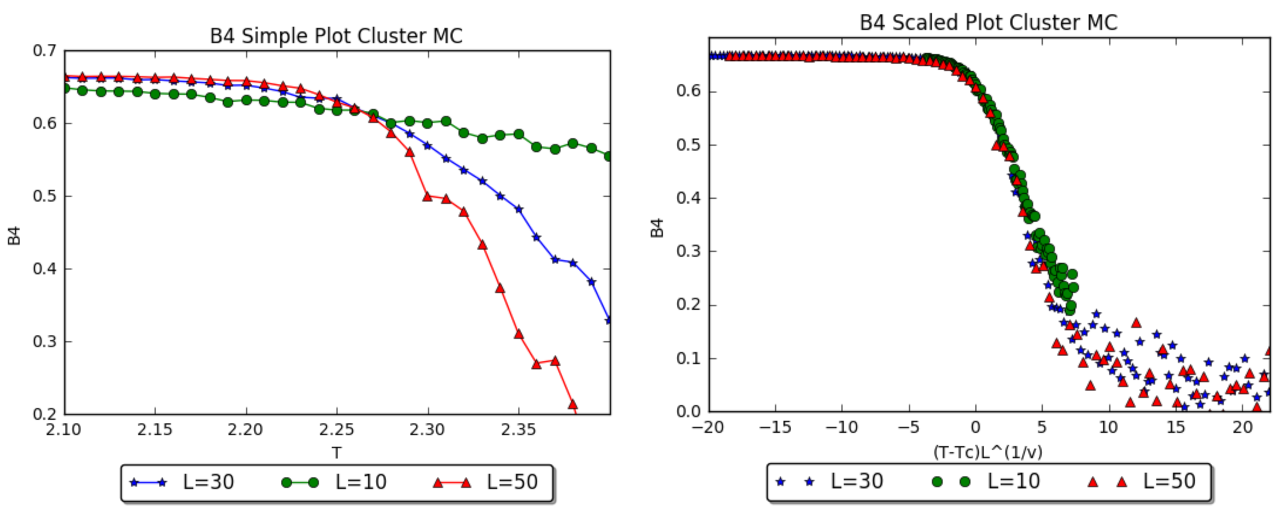
Magnetic susceptibility,

$$\chi = \frac{\langle M^2 \rangle - \langle |M| \rangle^2}{T L^2}, \tag{9}$$

And the forth-order Binder cumulant,

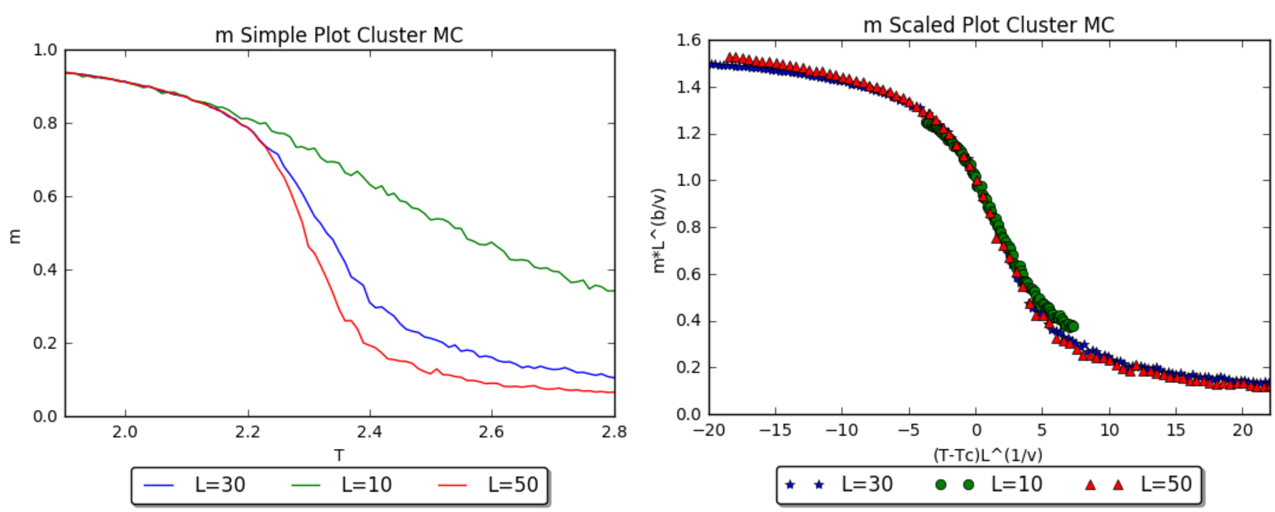
$$B_4 = 1 - \frac{\langle M^4 \rangle}{3 \langle M^2 \rangle^2}, \tag{10}$$

By plotting, not simply these quantities against temperature, but the scaled versions against the scaled temperature, using $T_c = 2.269$, we can find the universal curves for all L. Remember $\beta = 1/2$, $\alpha = 0$, and $\nu = \gamma = 1$. By finding the universal curves we show evidence to verify the critical exponents we found.



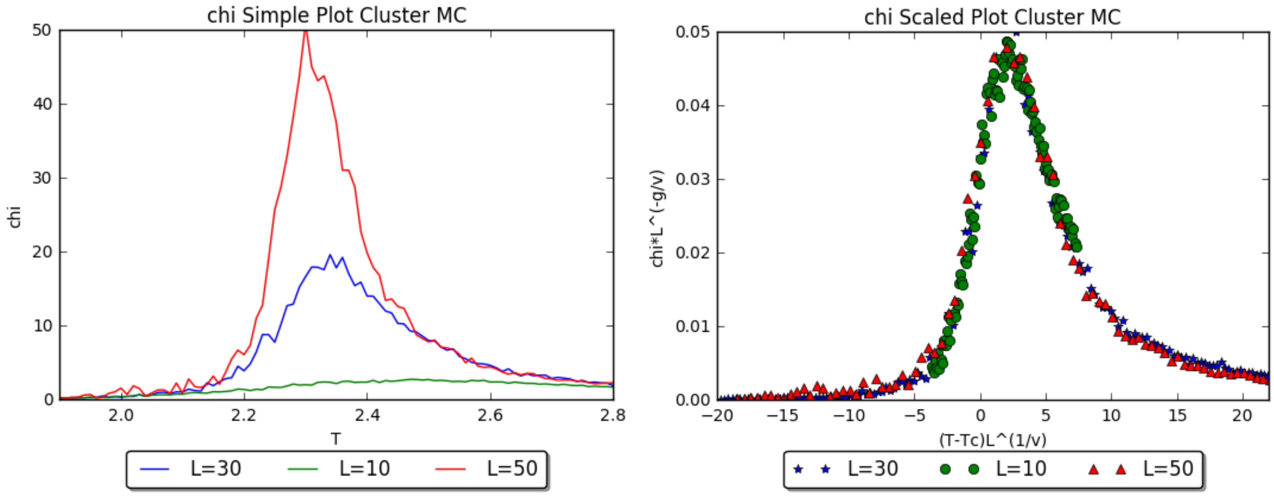
(a) (b)

Figure 1:



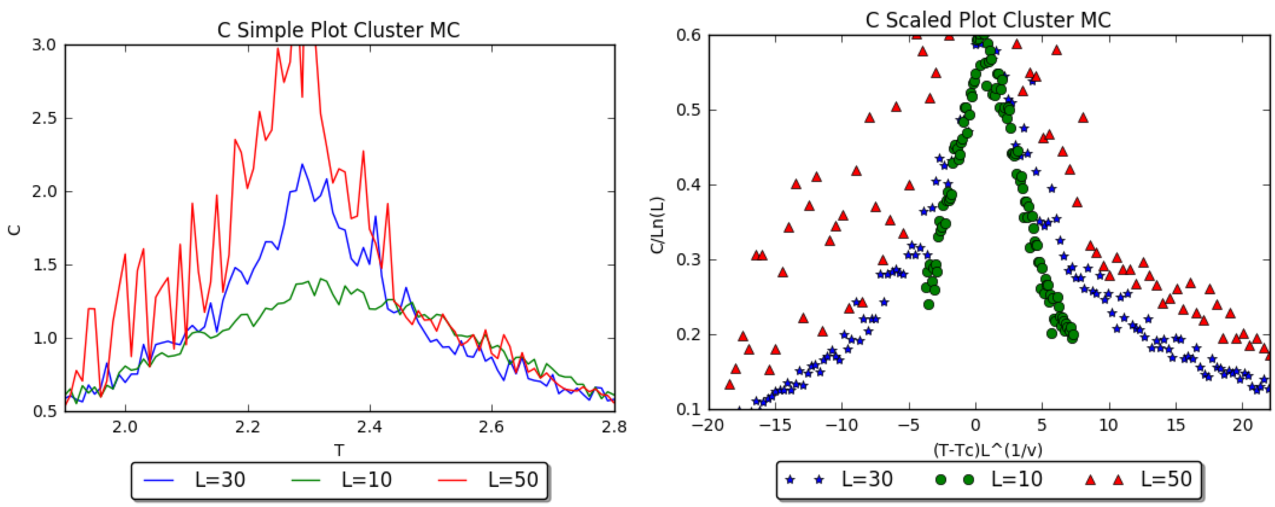
(a) (b)

Figure 2:



(a) (b)

Figure 3:



(a) (b)

Figure 4: Note, c requires a logarithmic correction in this case, as $\alpha = 0$.