Northeastern University

College of Engineering

Department of Mechanical and Industrial Engineering

Northeastern University: 3470 Take-Home Final

Submitted by	Jack Ryan
Date Submitted	4/26/2025
Course Instructor	Mehdi Abedi

1. Introduction

The purpose of this report is to outline the design of a jet engine in its entirety. The engine that will be outlined in this report is designed to follow the following parameters: 75lb of thrust, an outlet velocity of Mach 1.5, use methane fuel, use airfoil design NACA 65210, operate at sea level conditions, and have materials that are capable of withstanding temperatures of 1200k.

A typical jet engine contains 5 stages, the inlet, compressor, combustor, turbine, and outlet. The engine begins with a diverging area inlet called a bellmouth inlet. The reason for this is because the flow enters inlet at extremely low speeds, but the compressor requires higher speeds typically at least Mach 0.5 to operate. In a typical jet engine air flows directly from the compressor to the combustor, and then directly from the combustor to the turbine. The jet engine outlined in this report contains a single stage compressor and a single stage turbine, a single stage of a turbine or compressor does not decrease the flow speed significantly, so for a compressor with relatively high speeds it would not be feasible for the air to flow directly into the combustion chamber. The combustion chamber requires lower speeds in order to have successful chemical exothermal reactions, therefore this engine design contains a diverging area duct in between the compressor outlet and combustor inlet to sufficiently reduce the speed of the flow. The turbine also requires high speed, and although the combustor does cause some increase in flow speed it is not significant enough to allow the turbine to properly operate. For this reason there is a diverging area duct after the combustor and before the turbine. Following the turbine the flow passes through the nozzle which follows a converging diverging path in order to sufficiently accelerate the flow to meet the design requirements.

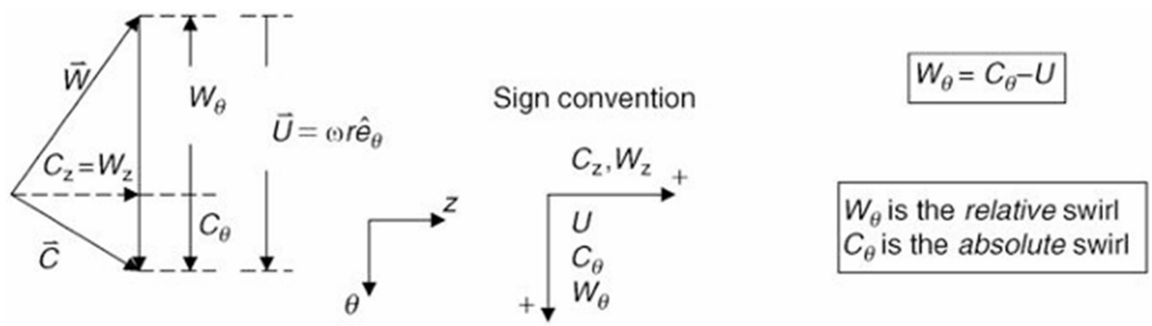
2. Inlet

The engine is said to operate on a lab table in Boston, implying sea-level conditions. These conditions are as follows: T_0=288.15k, P_0=101.325kPa, ρ_0=1.225kg/m^3 these conditions will be assumed for the inlet flow of the air. The first step done in design the engine was to calculate a mass flow rate. Due to the fact that the engine is stationary for this design there is no ram drag to be accounted for, this means that the thrust of the engine can be represented by: $F_{Thru} \quad = \dot{m}_9 * v_9$. The issue that arises from this is that without a known outlet temperature the speed of sound cannot be calculated. To solve this issue, we take the restrain of having materials capable of withstanding 1200k and assume that to be the outlet temperature of the combustor. Through this the temperature of the turbine outlet/nozzle inlet can be found. This is done by using this equation: $T_{t5} = T_{t4}(\frac{P_{t5}}{P_{t4}})^{\frac{\gamma-1}{\gamma}}$; The total temperature at the combustor outlet, state 4, is assumed to be the same as the static temperature at that state due to the low speeds. The total temperature across the nozzle stays constant, and the equation determining the outlet temperature is shown as $T_9 = T_{t9}(1 + \frac{\gamma-1}{2}M^2{}_9)^{-1}$. This yields an estimated outlet temperature of 813.922k. Applying that to the speed of sound equation $a_9 = \sqrt{\gamma RT_9} = 571.869m/s$. Then with $v_9 = M_9a_9 = 857.80m/s$. After finding the outlet velocity the inlet velocity can be found when the inlet velocity is set to 0.1 m/s. The inlet mass flow rate is then defined as $\dot{m}_1 = 0.390kg/s$, that number is taken from the thrust equation. The numbers that have now been defined allow for the inlet capture area to be found from the mass flow rate equation $\dot{m}_0 = A_0\rho_0v_0$, rearranged this gives $A_0 = \frac{\dot{m}_0}{v_0\rho_0} = 3.111m^2$.

The inlet area converges as the flow pass through it in order to get the air to a speed of Mach 0.5 to ensure functionality of the compressor. At the end of the inlet the shaft that connected the compressor to the turbine begins. This is accounted for in the area calculations at state 2. The first step in finding this final area is to find the static temperature of the flow, this done with this equation here $T_2 = T_0(1 + \frac{\gamma-1}{2}M^2{}_9)^{-1} = 274.429k$. This is used with the isentropic pressure-temperature relationship to find the static pressure $P_2 = P_0(\frac{T_2}{T_0})^{\frac{\gamma}{\gamma-1}} = 85.419kPa$. Then using the ideal gas law, the density at state 2 can be found $\rho_0 = \frac{P_2}{RT_2} = 1.085\frac{kg}{m^3}$. The velocity at state 2 is then also found $v_2 = M_2\sqrt{\gamma RT_2} = 166.031m/s$. With all of that the final flow area, total area-shaft area, is found to be $A_2 = \frac{\dot{m}_0}{\rho_0v_2} = 0.00216m^2$. The total cross-sectional area of the engine at that point is defined as 0.002473 m^2.

3. Compressor

The compressor design of this engine was built with one stage consisting of an inlet guide vane (IGV), a rotor, and a stator. The shaft that delivers the work from the turbine to power the compressor has a radius of 1 cm. The axial velocity denoted with C_x is considered constant through each part of the compressor, because the mass flow rate must be maintained. The different parts of the compressor are denoted with the subsection number 2.1 for before the IGV, 2.2 for before the rotor, 2.3 for before the stator, and then 2.4 for after the stator. The IGV is a stationary section that changes the flow direction so that the relative radial velocity of flow to the spinning rotor blades is more aligned. The three sections of the compressor will all be designed with 20 blades each. The flow angle relative to the rotor is defined as β. The beta angle should be near 0 degrees with a good IGV. The absolute angle of the flow is defined as α, alpha enters the compressor as 0, is then changed to be non-zero with the IGV, and then after the stator it returns to 0 degrees to create completely axial flow out of the compressor. The relative velocities in the compressor are shown as W. The velocity triangle is used to find all the velocity and angle components in the compressor, and also in the turbine later on, and is displayed below:



In this image the U is the radial velocity of the rotor blade at the pitch line radius of the compressor defined as $r_{pitchline} = \frac{r_h - r_t}{2}$, in this equation the r_h, r_t refer to the hub radius and tip radius of the compressor, both defined from the centerline of the compressor. The velocity triangle for state 2.2, after the IGV before the rotor, now found. With a U of 50m/s at the pitchline radius the RMP of the compressor rotor is 25,090. Then with a $C_x = 166.031\, m/s$, $\alpha_{2.2} = 20^o$, the radial velocity $C_{\theta 2.2} = C_x * \tan(\alpha_{2.2}) = 60.43 m/s$. The relative radial velocity can be found through the following calculation $W_{\theta 2.2} = C_{\theta 2.2} - U = 10.43\, m/s$. From this the relative velocity angle can be calculated with $\beta_{2.2} = tan^{-1}\left(\frac{W_{\theta 2.2}}{C_x}\right) = 3.59^0$. The magnitude of the C velocity vector for the post IGV state can also be found as $\left|\vec{C}_{2.2}\right| = \sqrt{{C_{\theta 2.2}}^2 + {C_x}^2} = 176.686 m/s$. Similarly, the relative velocity vector magnitude is found as $\left|\overrightarrow{W}_{2.2}\right| = \sqrt{{W_{\theta 2.2}}^2 + {C_x}^2} = 166.358 m/s$. The total temperature and pressure are known to be the same from state 2.1→ 2.2 as the compressor is assumed to be ideal and there is no work or energy change through the IGV. The total temperature for state 2.3 can be found from the following relationship $T_{2.2} = T_{t2.2} - \frac{|\vec{C_2}|}{2*cp} = 272.619k$. From this the isentropic temperature pressure relationship was used to find the static pressure at state 2.2 as shown here $P_{2.2} = P_{t2.2}(\frac{T_{2.2}}{T_{t2.2}})^{\frac{\gamma}{\gamma-1}} = 83.463kPa$. After all of that the flow is now ready to pass through the rotor.

The next step is doing a similar process to find the velocity triangle and flow conditions for after the rotor stage of the compressor. The first step here is using the compressor pressure ratio defined as $\pi_c = 1.1$, to find the increase in total temperature from the rotation of the blades, the equation and result is shown as $T_{2.3} = T_{t2.2}(\pi_c)^{\frac{\gamma-1}{\gamma}} = 296.105k$. The total temperature increase is used to find the specific enthalpy increase through the rotor. This is found as $\Delta h = cp(T_{t2.3} - T_{t2.2}) = 7994.34\,J/kg$. This value is then used in finding the absolute radial velocity through $\Delta h = U(C_{\theta 2.3} - C_{\theta 2.2})$, this equation is rearranged to $C_{\theta 2.3} = \frac{\Delta h}{U} + C_{\theta 2.2} = 220.317m/s$. The relative radial velocity is then found by $W_{\theta 2.3} = C_{\theta 2.3} - U = 170.317m/s$. The relative and absolute velocity magnitudes are then found $|\vec{W}_{2.3}| = \sqrt{{W_{\theta 2.3}}^2 + {C_x}^2} = 237.833m/s$, and $|\vec{C}_{2.3}| = \sqrt{{C_{\theta 2.3}}^2 + {C_x}^2} = 275.871m/s$. The total pressure increase is found using the compressor ratio $P_{t2.3} = P_{t2.2} * \pi_c = 111.458kPa$. The static pressure is then found with the same isentropic relationship as was used in the previous state $P_{2.3} = P_{t2.3}(\frac{T_{2.3}}{T_{t2.3}})^{\frac{\gamma}{\gamma-1}} = 69.046kPa$. The relative flow angle is found with $\beta_{2.3} = tan^{-1}\left(\frac{W_{\theta 2.3}}{C_x}\right) = 45.73^0$. $|\vec{C}_{2.3}| = 166.031m/s$. Then from those definitions we can find the static temperature similarly to how the 2.4 static temperature was found

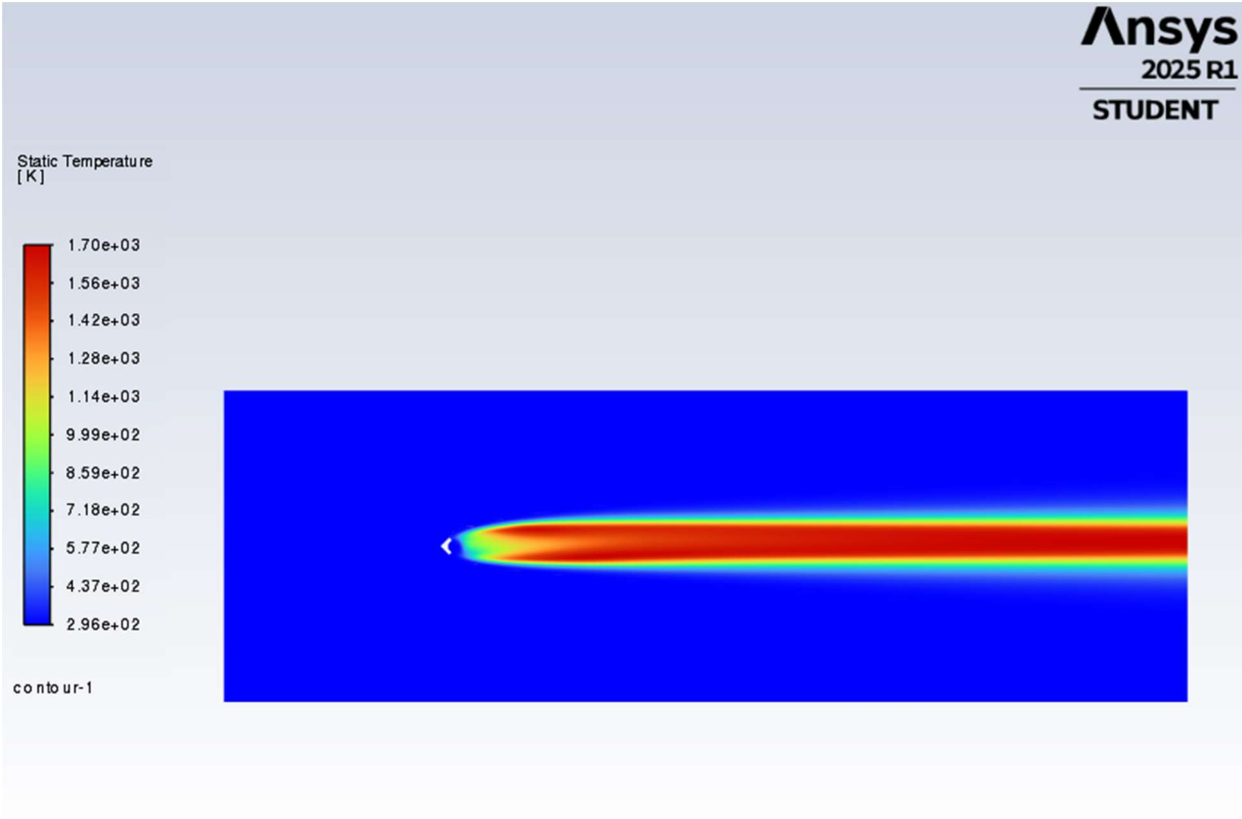
We can then move onto state 2.4 which is at the outlet of the compressor. The 4.2 stage calculation start with defining $T_{t4.3} = T_{t2.4}$, $P_{t4.3} = P_{t2.4}$, $C_{\theta 2.4} = 0$, $|\vec{C}_{2.3}| = 166.031m/s$. Then from those definitions we can find the static temperature similarly to how the 2.4 static temperature was found

$T_{2.2} = T_{t2.2} - \frac{|\overrightarrow{C_2}|}{2*cp} = 272.619k$. Again the static pressure can be found with the isentropic relationship $P_{2.4} = P_{t2.4}(\frac{T_{2.4}}{T_{t2.4}})^{\frac{\gamma}{\gamma-1}} = 94.411kPa$.

The flow does not directly flow into the combustor from the outlet of the compressor. The flow speed is far too high to achieve adequate combustion. The flow comes out of the compressor and then passes through a diverging area duct to slow the flow down. The total area of the compressor, A_{T2}=0.00247m^2 and the flow area $A_{f2} = 0.00216m^2$. The flow area increases to $A_{f3} = 1.3514m^2$ and the total area increase to $A_{T3} = 1.3517m^2$. This causes the flow to decrease the amount of the decrease is shown through the following: $A_{flow\ ratio} = \frac{A_{f3}}{A_{f2}} = 625.984$, and $v_3 = \frac{v_2}{A_{ratio}} = 0.242m/s$. That equation is derived from the Quasi-1Dimentional flow conditions. The total temperature and pressures once again remain constant through this duct. The static pressure then going into the combustor is found with $T_3 = T_{t3} - \frac{|\overrightarrow{v_3}|}{2*cp} = 296.10497k$. The low speed causes the total and static temperature to be approximately the same and for the calculations they will be assumed as equal.

4. Combustion

The combustion is broken down into two segments, the ANSYS simulation portion relating to the properties of the flow coming out of the combustion chamber. With the other portion being the chemical reaction in the combustion. The ANSYS simulation was done with the parameters found from the calculations that got the state 4 conditions. The temperature graph of the ANSYS simulation is shown below:



The maximum flame temperature is 1702.753 kelvin, the surface integral of the outlet of the combustion chamber with respect to temperature give it has 619.374k which is defined as T_4. The outlet velocity is also found through the surface integral at the outlet, it is found as $v_4 = 0.5 m/s$. The pressure at the outlet is $P_4 = 104.018 kPa$.

The actual chemical reaction that occurs in the combustion chamber is shown here: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. This is known as the stoichiometric reaction; in a perfect reaction this would be the balance. It looks more like:

$$CH_4 + aO_2 + bN_2 \rightarrow cCO_2 + dH_2O + eO_2 + fN_2 + gNO + hNO_2 + iN_2O$$

With a fuel air ratio of $f = 0.13\overline{3}$, the reaction is fuel rich which is why O_2 exists as one of the products of the reaction. The conditions for the design of this reaction dictate that dissociation of the N_2 & O_2 molecules were possible, however, O_2 dissociation does not begin occurring until flame temperatures reach around 2000k, and N_2 dissociation does not begin until flame temperatures reach around 3000k. Therefore, the actual chemical equation of the reaction can be shown by $CH_4 + aO_2 + bN_2 \rightarrow cCH_4 + dO_2 + eCO_2 + fH_2O + gN_2$. Then from the ANSYS simulation the mole fraction for all of the reactions found in the simulation area as follows $x_{N_2} = 0.7221, x_{H_2O} = 0.04990, x_{CO_2} = 0.02495, x_{O_2} = 0.13883, x_{CH_4} = 0.06460$.

5. Turbine

Moving on, the turbine is the next segment within the jet engine, however, the issue of flow speed that existed before the compressor exists here again before the turbine. The same idea of decreasing the flow cross sectional area will be used. The shaft CS area remains constant at $A_S = 3.1459 * 10^{-4} m^2$. The total pressure and total temperature are assumed to be the static values due to the flow speeds coming out of the combustor. The total temperature and pressure through the duct leading to the IGV of the turbine remain constant. Speed of the flow will again increase to Mach 0.5, the speed of sound at this point is $a_{4.1} = \sqrt{\gamma R T_{4.1}} = 486.841 m/s$. Therefore $v_{4.1} = 243.420 m/s$, this is also the value that will be set for C_{x4} *and* W_{x4}. The flow area is found using the same mass flow equation used earlier in this paper with updated values, which yields $A_{f4.1} = 0.00351 m^2$. The total area of the engine at state 4.1 is defined as $A_{f4.1} + A_S = A_{T4.1} = 0.00376 m^2$. The total number of blades for the turbine is the same as the compressor, 20 blades for each stage. The pitchline radius of the turbine is given by $r_{pitchline4} = \frac{r_{T4}+r_S}{2} = 0.01842 m$.

The first condition established for state 4.2, the state after the IGV, is an absolute flow angle of $\alpha_{4.2} = 20^0$, from this we can follow the same method of solving as was used in the compressor. The next step is finding the absolute radial velocity of the flow: $C_{\theta 4.2} = C_{x4} \tan(\alpha_{4.2}) = 88.598\, m/s$. The rotor speed at the pitchline radius for the turbine is set at $U_4 = 65 m/s$, from this the relative radial velocity is found with $W_{\theta 4.2} = C_{\theta 4.2} - U = 23.598 m/s$. The RPM of the turbine rotor can also be found from the rotor speed at the pitchline radius and is found to be 23,160 RMP. The magnitude of the absolute and relative velocity vectors is then found as follows $|\vec{C}_{4.2}| = \sqrt{{C_{\theta 4.2}}^2 + {C_x}^2} = 259.042 m/s$, $|\overrightarrow{W}_{4.2}| = \sqrt{{W_{\theta 4.2}}^2 + {C_x}^2} = 244.561 m/s$. The relative flow angle can also be found as $\beta_{4.2} = tan^{-1}\left(\frac{W_{\theta 4.2}}{C_{x4}}\right) = 5.537^0$. The static temperature and pressure can both also be calculated in a similar fashion to the way done in the corresponding state of the compressor $T_{4.2} = T_{t4.2} - \frac{|\overrightarrow{C_{4.2}}|}{2*cp} = 585.990 k$, $P_{4.2} = P_{t4.2}(\frac{T_{4.2}}{T_{t4.2}})^{\frac{\gamma}{\gamma-1}} = 85.682 kPa$.

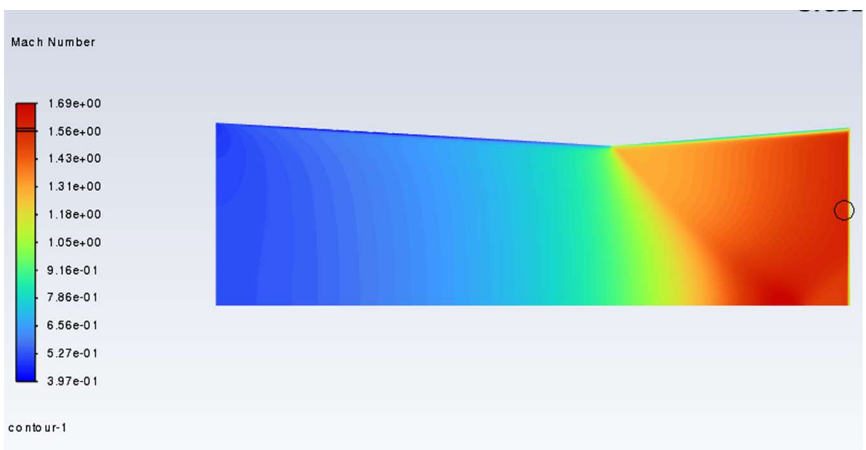
The next stage of the turbine is the post rotor state, this state is denoted as 4.3. The pressure ratio of the turbine is designed to be $\pi_t = 0.85$, from this the total pressure and temperature can be found as $T_{4.3} = T_{t4.2}(\pi_c)^{\frac{\gamma-1}{\gamma}} = 591.272k$, $P_{t4.3} = P_{t4.2} * \pi_t = 88.415kPa$. Using the difference in the total temperatures the energy extracted from the turbine can be found the specific enthalpy change across the stage, shown as, $\Delta h = cp(T_{t4.3} - T_{t4.2}) = -28.243\ kJ/kg$, the negative symbol on the enthalpy change denotes that it is energy taken out of the system, this is required in order for the compressor to be calculated. The turbine generates an excess 20kJ/kg approximately that is not needed by the compressor which can be used for other electrical systems or could enable the engine to be used as a power generator. Moving on, the velocity triangle after the rotor is solved for by using the same methods as were used in the compressor. First up is the radial velocity at state 4.3, $C_{\theta 4.3} = \frac{\Delta h}{U_4} + C_{\theta 4.2} = -345.910m/s$, the negative denotes that the flow changed direction after passing through the turbine rotor, the relative velocity is found as $W_{\theta 4.3} = C_{\theta 4.3} - U_4 = -410.910m/s$. The magnitude of both relative and absolute velocity vectors are as follows $|\vec{C}_{4.3}| = \sqrt{{C_{\theta 4.3}}^2 + {C_x}^2} = 422.974m/s$, $|\vec{W}_{4.3}| = \sqrt{{W_{\theta 4.3}}^2 + {C_x}^2} = 477.598m/s$. The relative flow angle is then $\beta_{4.3} = tan^{-1}\left(\frac{W_{\theta 4.3}}{C_{x4}}\right) = 59.339^0$.

Moving on to state 4.4, the post-stator state, the flow angle is returned to zero degrees, which therefore means that the radial velocity must be $C_{\theta 4.4} = 0m/s$. This definition makes the axial velocity of both the relative and absolute flow velocities equal to magnitudes of both velocities. The total pressure and temperature remains constant from state 4.3 to 4.4. The static temperature at 4.4 is found with $T_{4.4} = T_{t4.4} - \frac{|\vec{C_4}|}{2*cp} = 561.793k$. The pressure is then found $P_{4.4} = P_{t4.4}(\frac{T_{4.4}}{T_{t4.4}})^{\frac{\gamma}{\gamma-1}} = 73.924kPa$.

6. CD Nozzle

The CD nozzle works by having the air flow through a converging area duct until the flow reaches Mach 1.0, at which point the duct diverges causing a continuation in the acceleration of the flow. This works because of the differences in flow conditions for subsonic and supersonic flow, subsonic flow accelerates when in a converging area duct, and it decelerates when passing through a diverging area duct, the opposite is true for supersonic flow. The state defined as 4.4 in the previous section is also known as state 7, which denotes the entrance to the CD nozzle. State 8 is the sonic throat of the nozzle (where flow reaches Mach 1). Finally, state 9 is where the outlet of the engine exists. The total pressure and temperature at state 4.4 is the same as state 7, and the total pressure and temperatures remain constant through the nozzle.

The areas of the sonic throat and outlet of a CD nozzle are very specifically designed. One important note is that the shaft ends before the nozzle, so the total radius of the engine decreases, however the total area occupied by the flow remains the same from state 4.4 to state 7. It starts with the sonic throat area given by $A^* = \frac{\dot{m}_9}{P_{t7}}\sqrt{\frac{RT_{t7}}{\gamma}}(\frac{\gamma+1}{2})^{\frac{\gamma+1}{2(\gamma-1)}} = 0.002822m^2$, the throat radius is given by $r^* = 0.02997m$. The inlet radius is defined as $r_7 = 0.0331m$. The outlet area is governed by the following equation when $M_9 = 1.5$: $A_9 = \frac{A^*}{M_9}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_9^2\right)\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.00319m^2$. From that the outlet radius can be determined to be $r_9 = 0.0325m$. These radii along with the inlet conditions are enough to create and run an ANSYS model of the CD nozzle. That simulation is shown here:



The outlet Mach number from this is found to be 1.62 on average. The pressure at the outlet of the nozzle can be found to then find temperature and subsequently speed of sound. The pressure is given by $P_9 = P_{t9}\left(1+\frac{\gamma-1}{2}M_9^2\right)^{\frac{-\gamma}{\gamma-1}} = 24.084kPa$. The temperature is found as $T_9 = T_{t9}\left(1+\frac{\gamma-1}{2}M_9^2\right)^{-1} = 407.774k$. The speed of sound at the outlet is $a_9 = \sqrt{\gamma RT_9} = 404.776m/s$, then $v_9 = a_9M_9 = 655.737m/s$. With an outlet mass flow rate of $\dot{m}_9 =$

$0.450\ kg/s$, the thrust of the engine is given as $F_{Thrus} = \dot{m}_9 v_9 = 295.082N$. Converting to lb where 1 newton=0.225lb, the thrust of the engine is found to 66.39lb. This number comes in below what the design parameters were set up for, the reason for this slight shortcoming is the overestimation of outlet temperature from the combustor in the very first steps of the design. The lower temperatures caused a lower speed of sound, and consequently lower velocity at the outlet which caused the thrust to come in lower than the anticipated value.

Citations:

S. Farokhi, *Aircraft Propulsion*, 2nd ed. Hoboken, NJ, USA: Wiley, 2014.