CLT without Characteristic Functions

Fill in the gaps in Le Cam (1986, pp. 82–83)'s description of Lindeberg's CLT argument. In particular find out how to use the "standard truncation argument" mentioned in Le Cam (1986, p. 83), to obtain a CLT of Lindeberg type.

By this means you will derive a central limit theorem theorem of Berry-Essen type. The multiplicative constant of this bound can be substantially improved. (See Wikipedia en.wikipedia, https://en.wikipedia.org/wiki/Berry%E2%80%93Esseen_theorem, or the discussion at Le Cam, 1986, p. 79). Use simulation methods to investigate how good the best bounds (upper and lower) in the literature are, compared with simulation computation in which the distribution function of the sum $X_1 + \ldots + X_n$ is replaced by an empirical distribution function derived by simulation.

If you are feeling ambitious, investigate martingale central limit theorems.

Construct an interactive animation to illustrate your findings using Shiny.

This project will benefit from close cooperation between theoreticians and computational people, in order to determine how to choose challenging sums $X_1 + \ldots + X_n$!

References:

Le Cam, L. (1986). The Central Limit Theorem Around 1935. *Statistical Science*, 1(1), 78-91.

Shiny (2018) https://shiny.rstudio.com/.