OkWasp: Exercise [redacted]
Outline [redacted]

1.1.1 Try $\bigcap_{M} \bigcup_{N}$ [redacted]
not $\bigcap_{\varepsilon>0} \ldots$ [redacted]

1.1.2 Try $\bigcap_{M} \bigcup_{N} \bigcap$ [redacted]
a limiting ran[redacted]
random sums [redacted]
the sequence([redacted]
event of w[redacted]

1.2.1 $\mathbb{P}[\bigcap_{M} \bigcup_{N} \bigcap_{n\geq N}$ [redacted]
this is non [redacted]
But
$\mathbb{P}[|X_n$ [redacted]

1.2.2 Pick $A_1, A_2, \ldots$ [redacted]
independent [redacted]
variables). [redacted]
infinitely ma[redacted]
$X_n \not\to 0$ a[redacted]
(if $\varepsilon<1$) a[redacted]

1.2.3 Suppose, for [redacted]
the same con[redacted]
shows, sinc[redacted]
often, with [redacted]
$\mathbb{P}[$ [redacted]
Now take [redacted]

1.3.1 Suppose $\mathbb{E}[|X_n-X|^p] \to 0$. Now, for $\varepsilon>0$,

$$\mathbb{E}[|X_n-X|^p ; |X_n-X|>\varepsilon] \le \mathbb{E}[|X_n-X|^p] \to 0.$$

But if $|X_n-X|>\varepsilon$ then $|X_n-X|^p \ge \varepsilon^p$.
Hence

$$\varepsilon^p \,\mathbb{P}[|X_n-X|>\varepsilon] \le \mathbb{E}[|X_n-X|^p ; |X_n-X|>\varepsilon]$$

("Markov inequality"). Convergence in probability follows.

1.3.2 Use events A_n with $\mathbb{P}[A_n]=\frac{1}{n}$. Then

$$\mathbb{E}[|X_n-0|^p] = n^p \cdot 1/n \ge 1 \quad \text{for } p \ge 1$$

but evidently $X_n \to 0$ in probability.

(For $0<p<1$, you could use $\mathbb{P}[A_n] = 1/n^p$.)

(Construct A_n with $\mathbb{P}[A_n]=1/n$ as $A_n=[U<1/n]$ for U Uniform$(0,1)$.)

1.3.3 Markov inequality works if $p_1 \ge p_2$. If $\mathbb{E}[|X_n-X|^{p_1}]\to 0$ then $X_n \to X$ in probability so

$$\mathbb{E}[|X_n-X|^{p_2} ; |X_n-X|<1] \le \varepsilon + \mathbb{P}[|X_n-X|>\varepsilon]$$

hence we can deduce $\mathbb{E}[|X_n-X|^{p_2} ; |X_n-X|<1] \to 0$.

On the other hand,

$$\mathbb{E}[|X_n-X|^{p_2} ; |X_n-X|\ge 1] \le \mathbb{E}[|X_n-X|^{p_1} ; |X_n-X|\ge 1]$$
$$\le \mathbb{E}[|X_n-X|^{p_1}] \to 0.$$

Counterexample: $X_n = n^{1/p_1}\,\mathbb{I}_{A_n}$ with A_n having probability $1/n$.

1.4.1 Pick A_n with $\mathbb{P}[A_n]=1/n$, $X_n = n\,\mathbb{I}_{A_n}$.

1.4.2 Using limits, $\mathbb{E}[|X|] \le K + \mathbb{E}[|X| ; |X|>K]$.
Also if $\mathbb{E}[|X|]<\infty$ then consider $|X| \times \mathbb{I}_{[|X|>K]}$ and use Dominated Convergence Theorem.

1-5-1. Very simple. [redacted]
whether or w[redacted]
of iid Unifo[redacted]
X_n [redacted]

Then $\mathbb{E}[$ [redacted]
converges [redacted]
$\mathbb{P}$ [redacted]
so not over [redacted]

1-5.2 Take U u[redacted]
$\mathbb{P}[F^{-1}($ [redacted]
of $0 \le x \le 1$. [redacted]
is to be ar[redacted]
There are we[redacted]

1-5.3 $Y_n =$ F[redacted]
For the distribu[redacted]
$F_n(x) \to F$[redacted]
at continui[redacted]
Then F is al[redacted]

1-5.4 Hint: con[redacted]

1-5-5 Hint: con[redacted]

1-5-6 Convergence in Dis[redacted]
for all real x ([redacted]
of 1-5.2, 1-5.3 w[redacted]
then establish [redacted]
satisfy regular[redacted]
is required to [redacted]

1.3.7 Consider r [redacted]

2.1.1 Suppose X_1, [redacted]
and cova [redacted]
Expanding [redacted]
Hence Var [redacted]
Now emplo [redacted]
to deduce [redacted]

2.2.1 Study $\mathbb{P}$ [redacted]
$= 2^{-n+1}$ fr [redacted]
Bel{±1}) . [redacted]
($x_1 \times x_2 \times \cdots \times$ [redacted]

2.2.2 A theore [redacted]
that $\frac{X_1 + \cdots}{n}$ [redacted]

2.3.1 Consider $\sum_{i=1}^{n}$ [redacted]
for $\delta > 0$ ([redacted]

Note, if $\delta =$ [redacted]
$\sum_{i=1}^{n} E$ [redacted]

but $\sum_{i=1}^{n} \mathbb{E}\left[\left|\frac{X_i}{s_n}\right|^2\right.$ [redacted]

2.3.2 Try the follow [redacted]
Now follow by [redacted]
such that [redacted]
as $(0, Y_i)$. [redacted]
$X_{n_i+1} = (Y_{n_i+1},$ [redacted]
is nearly des [redacted]

2.3.3 Study Var [redacted]