Heavy Traffic in Queues

In the standard queue (often denoted by M/M/1), customers arrive according to a Poisson process. Each customer carries a random workload, Exponentially distributed with fixed mean, independent of other customers and of arrival process. Arrivals are served in order of arrival. There is just one server, dealing with workload at unit rate. The queue is stable when traffic is sub-critical:

(mean customer workload) times (rate of arrival) < 1.

Things get more complicated when the workloads are i.i.d. but non-Exponential, but the same rule for stability applies.

1. Things get even more complicated when the intervals between customer arrivals are i.i.d. but have a non-Exponential distribution. Find out about the beautiful and beautifully simple method of Lindley, which relates the equilibrium of such a queue to the maximum of a certain random walk.
2. Find out about Kingman's (1961) celebrated heavy traffic idea, which shows that when traffic gets heavy (nearly critical) then the details of workload and inter-arrival distribution cease to matter very much. (Kingman's paper may not be the easiest place to learn about this: Google is your friend!)
3. (For those who want a challenge) In case there is more than one server, find out about the Kiefer and Wolfowitz (1955) recursion, which generalized Lindley's work.
4. Construct simulations, or interactive animations using Shiny, to illustrate your findings.

References:

Asmussen, S. (2003). Applied probability and queues (Second Edition). New York; Berlin; Heidelberg: Springer.

Kiefer, J., & Wolfowitz, J. (1955). On the theory of queues with many servers. *Transactions of the American Mathematical Society*, 18(1), 1–18. Retrieved from http://www.jstor.org/stable/10.2307/1992945.

Kingman, J. F. C. (1961). The single server queue in heavy traffic. *Mathematical Proceedings of the Cambridge Philosophical Society*, 3(iii), 902–904. Retrieved from http://journals.cambridge.org/abstract_S0305004100036094.

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