An R Implementation for Correlated Pseudo Marginal Monte Carlo Methods

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Abstract

We present the R package 'testpkg', which does some useful things.

1 Introduction

Here is a nice description of the background.

2 Methodology

Let $p(y|\theta)$ denote the likelihood for the observations y, and let $p(\theta)$ be the prior density for the parameter $\theta \in \Theta \subseteq \mathbb{R}^d$. Then for a Bayesian model, the posterior density can be written as $\pi(\theta) \propto p(y|\theta)p(\theta)$.

The Metropolis-Hastings (MH) algorithm can be used to compute expectation with respect to $\pi(\theta)$, the implementation of this algorithm involves evaluating the likelihood ratio $\frac{p(y|\theta')}{p(y|\theta)}$ when the acceptance probability is calculated for a proposed candidate θ'.

For may latent variable models, the likelihood is often intractable and there may be strong correlation between the parameter and the latent varibles under the joint posterior density. The MH algorithm is impossible to implement under these circumstances and Markov Chain Monte Carlo (MCMC) schemes targeting the joint posterior density can be inefficient.

On the other hand, the pseudo-marginal (PM) algorithm introduces a nonnegative unbiased estimator for the intractable likelihood to replace the true likelihood ratio required for the acceptance probability in the MH algorithm. Furthermore, the correlated pseudo-marginal (CPM) algorithm, with details given in later sections, correlates the estimators for the likelihood to reduce the variance of the resulting likelihood ratio in order to improve efficency in implementing the scheme.

2.1 Correlated Pseudo-Marginal Algorithm

Let $U \sim m$ be the $\mathcal{U}$-valued auxiliary random variables used to obtain the non-negative unbiased estimator $\hat{p}(y|\theta, U)$ for the likelihood $p(y|\theta)$. The joint density $\bar{\pi}(\theta, u)$ on $\Theta \times \mathcal{U}$ can then be given as

$$\bar{\pi}(\theta, u) = \pi(\theta)m(u)\frac{\hat{p}(y|\theta, u)}{p(y|\theta, u)}$$

under the assumption that $m(du) = m(u)du$. Given that $\hat{p}(y|\theta, U)$ is unbiased, we have that $\pi(\theta)$ is a marginal density of $\bar{\pi}(\theta, u)$.

2.2 Random Effect Model

2.3 Discussion

– Theory weak points – Implementation weak points. inefficiencies

3 Application

3.1 Simulation Study

4 Conclusion

References

Nicholas G Polson, James G Scott, and Jesse Windle. Bayesian inference for logistic models using pólya–gamma latent variables. *Journal of the American Statistical Association*, 108(504):1339–1349, 2013.