ELSEVIER European Journal of Operational Research 90 (1996) 487–513

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

Theory and Methodology

A survey of business failures with an emphasis on prediction methods and industrial applications

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Received January 1995

Abstract

The considerable interest in the prediction of business failures is reflected in the large number of studies presented in the literature. Various methods have been used to construct prediction models. This paper provides a review of the literature and a framework for the presentation of this information. Articles can be classified according to the country, industrial sector and period of data, as well as the financial ratios and models or methods employed. Relationships and research trends in the prediction of business failure are discussed.

Keywords: Business failures; Statistical analysis

1. Introduction

In recent years a large number of researchers and practitioners (credit managers, financial analysts, etc.) have worked on the prediction of business failures. Failure can be defined in many ways, depending on the specific interest or condition of the firms under examination. This accounts for the many definitions of failure in the literature. According to a general definition, failure is the situation that a firm cannot pay lenders, preferred stock shareholders, suppliers, etc., or a bill is overdrawn, or the firm is bankrupt according to law. All these situations result in a discontinuity of the firm's operations.

A unifying theory of business failure has not been developed, in spite of a few notable efforts. These include Wilcox's (1971) ruin model equating failure to zero net worth and Scapens et al. (1981) catastrophic theory approaches. In several cases, alternative options to declared failure are considered. Examples are liquidation or reorganization of bankrupt firms (Casey et al., 1986) and acquisition by or merger with a healthy company (Bulow and Shoven, 1978). These alternatives will not be considered in our survey.

Clearly, failure affects a firm's entire existence and it has a high cost to the firm, the collaborators (firms and organizations), the society and the country's economy (Warner, 1977). Beaver (1968) showed the decline of stock market price as a firm was coming closer to failure. Altman (1969) provided similar conclusions. Consequently, the

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0377-2217/96/$15.00
SSDI 0377-2217(95)00070-4

prediction of failure is important for all those involved: owners or shareholders, managers, workers, lenders, suppliers, clients, the community and the government. The development and use of models able to predict failure can be very important for them in two different ways. First, as 'early warning systems', such models are very useful to those (managers, authorities, etc.) that can take action to prevent failure. These actions include the decision about merger of the distressed firm, liquidation or reorganization type and associated costs (Casey et al., 1986).

Second, such models can also be useful in aiding decision makers of financial institutions in evaluating and selecting firms to collaborate with or to invest in. Such decisions have to take into account the opportunity cost and the risk of failure. In that regard, these models are close to those developed for the credit granting problem. Credit granting problem is different from the failure problem, but lenders of a firm can use the failure models to assess the probability of failure for that firm. This probability can be an important factor or even the main criterion in credit granting decisions for financially distressed firms. Furthermore, the idea of classifying a firm as healthy or not is common to both the failure and credit granting problems. Credit applicants are classified into groups according to the acceptance or not of their credit application.

Since business failure prediction became a field of study, researchers introduced a plethora of methods for the classification and selection of firms. Different views, requirements and reliability needs have led researchers in using more sophisticated methods, already applied to other scientific fields. This diversity and large interest on this subject have been addressed partially in a few review articles. Vernimmen (1978) examined various failure models and criticized their contribution and limits. Scott (1981) investigated the empirical models developed as well as the bankruptcy theories presented to identify the overlap between them, focusing mainly on US studies. Zavgren (1983) surveyed different methods and empirical models developed for the prediction of corporate failure in the USA. Altman (1984) presented a review of models developed in several countries for the prediction of business failure. Jones (1987) examined the techniques used for bankruptcy prediction in USA. Keasey and Watson (1991) explored the limitations and usefulness of methods used for the prediction of firm financial distress.

These reviews do not include recent methods or applications in the field of business failure. There is clearly a need for a comprehensive survey of literature on business failures including world-wide applications, models/methods and characteristics (e.g. financial ratios). The aim of this paper is to reduce this need and provide a discussion of findings based on a new framework, which includes classification of the studies by country, method and industrial sector.

2. Financial ratios and other variables

Financial ratios were introduced early as characteristics able to predict the failure of a firm. The early studies were using only the ratios from specific year(s) to make predictions. However, failure is a continuous process. This means that although the appraisal of failure happens at a certain time, it is the result of a specific policy of the firm for a number of years. Therefore, the values of the ratios should be inspected over time to provide full information about the progress of a firm. To get this information over time researchers used the time trend, the coefficient of variation and shift away from the trend in the period(s) prior to failure (e.g. Meyer and Pifer, 1970; Dambolena and Khoury, 1980; Falbo, 1991). In a comparison work Collins (1983) concluded that Meyer and Pifer's variables did not improve the predictive ability compared to a simpler model.

A large number of ratios has been proposed in the literature. Courtis (1978) made an attempt to identify the variables useful in predictive studies. In his survey 79 financial ratios were identified from various studies and were grouped in three main categories: (a) profitability ratios; (b) managerial performance ratios and (c) solvency ratios. Later studies proposed additional financial ratios for inclusion in failure prediction analysis.

A firm is not standing alone. Its performance and survival are influenced by several factors; e.g. the environment and its changes, as well as national and international economic conditions. In turn these can affect any model built at that time, possibly producing occasionally good predictions. Therefore a good model, constructed under normal circumstances, may be unable to predict failure successfully during periods of difficulties. Macro-economic variables for failure prediction were proposed by Foster (1986) and Rose et al. (1982). Mensah (1984) noted that different economic environments as well as different sectors lead to different models for the prediction of failure. Such a conclusion is also derived from Tables A.4a and A.4b in Appendix A, where not even two models are identical for those variables found significant. Among different countries, sectors and periods of time the financial ratios able to predict failure differ from study to study.

Furthermore, there are studies proposing models for specific sectors. A cluster analysis by Gupta and Huefner (1972) showed that firms in different sectors have different characteristics. Additionally, in different sectors, there are various adapted characteristics able to provide predictions about firms. The failure of firms in each sector can be better predicted by consideration of these characteristics. So, there are special studies about financial institution failure prediction (i.e. Booth et al., 1989; Lane et al., 1986; Martin, 1977; Santomero and Vinso, 1977; Zanakis and Walter; 1994) or hospital closure (Wertheim and Lynn, 1993).

On the other hand, a company's performance and future may be influenced by characteristics other than financial, some of them qualitative, such as management, personnel, products, equipment, etc. Zopounidis (1987) employs a set of 'strategic criteria' to assess the risk of failure of French firms. These were: quality of management, research and development level, diversification stage, market trend, market niche/position, cash out method and world market share. Similar propositions were taken by Shaw and Gentry (1988), while Peel et al. (1986) proposed other qualitative variables, such as the changes in the lag in reporting accounts of a firm, the number of director resignations and appointments and the changes in directors' shareholdings. The social importance of the firm and the strength of its bank relationship (Suzuki and Wright, 1985) could be also critical. The narrative portion of financial statements and the information it contains were proposed by Tennyson et al. (1990) to explain bankruptcy. The analysts' forecasts on earnings per share were proposed by Moses (1990) and decomposition measures analysis were suggested by Booth and Hutchinson (1989).

The lack of a normal distribution fit to most financial ratios has been noticed for different types of firms by several authors (Eisenbeis, 1977; Karels and Prakash, 1987; Ezzamel and Molinero, 1990; Zanakis and Walter, 1994). This may create poor model fits and/or predictions for statistical methods, such as discriminant analysis (more so in quadratic than linear).

3. Framework

After an extensive but not necessarily exhaustive review of journal articles written in English or French, we identified a list of 158 articles on the subject. These papers were published in the period 1932–1994 in various journals specialized in accounting, finance, operations research and decision science. Working papers and dissertations on the subject were not included; books and monographs were not considered, unless they were dedicated to the subject of this survey. All publications are listed in the References for information purposes. However, in order to keep this review manageable our analysis was restricted only to: (a) journal articles presenting models and (b) pertaining to industrial and retail application. The former condition (a) excluded from the analysis all methodological and expository articles without any models. The latter condition (b) excluded essentially only a few banking applications; this was deemed necessary to avoid adding a large number of financial ratios specific only to bank operations. Inclusion of such diverse entities would have affected negatively the depth and readability of this survey. This selectivity produced 47 journal articles, identified in Appendix A, written mainly by scholars and only a few by practitioners.

The tables in Appendix A depict the information contained in the above 47 articles, based on a proposed framework; namely according to *country*, *year of publication*, *industry type*, *sample periods* and *method* (Table A.3). A detailed listing of the financial ratios included in the models presented in these articles is summarized in Tables A.4a and A.4b. The models are identified by the author(s) name(s) and year of publication cited in the reference list. If a paper contains more than one model, these are characterized by the method used, thus producing a total of 59 models presented in these 47 articles. Tables A.1 and A.2 in Appendix A define the abbreviations used for methods and ratios respectively. Summarizing and coding this information in Appendix A was a difficult task due to lack of uniformity on common language and ratio definitions between authors.

An examination of the articles in our survey revealed the most frequent journal outlets for papers on business failures. Three accounting/finance journals (*Journal of Banking and Finance*, *Journal of Business Finance and Accounting* and *Journal of Accounting Research*) are the most frequent sources of such articles with 15, 14 and 13 articles respectively. These are followed by *OMEGA* (8 publications), *Decision Sciences* and *The Journal of Finance* (7 each) and *European Journal of Operational Research* (6 articles).

4. Survey of methods and models

A variety of methodologies have appeared in the literature for modeling business failures. Each method has its own assumptions and different contributions in the field of business failure. The basic assumption on which most of the failure prediction methods are based, is that firms can generally be split into groups, usually two; the group of healthy and the group of failing firms. Accordingly, firms are characterized by a variable y such that

$$y_i = \begin{cases} 0 & \text{if the } i\text{-th firm is bankrupt,} \\ 1 & \text{if the } i\text{-th firm is not bankrupt.} \end{cases} \qquad (1)$$

Suggestions have also been made for defining more than two groups according to the associated level of risk (Zopounidis, 1987). Because of the general acceptance of the two group classification, the interest has been mainly focused on dichotomous classification methods, being referred to as discriminating approaches. Methods in this category include discriminant analysis and its alternatives, logit/probit analysis and linear probability models. Recursive partitioning algorithms employ a Bayesian scheme for group discrimination. Prediction of time to failure, instead of group membership, was the aim of survival analysis adopted for this problem.

Statistical methods were the earliest employed by researchers for the prediction of business failures. The increasing progress on such methods has led researchers to apply and modify these methods to financial problems with numerous financial characteristics. An interesting such alternative is the use of mathematical programming for solving the discriminant analysis problem. The ambiguity of measuring some qualitative characteristics has led to different approaches such as individual or group judgments (Libby, 1975b; Houghton, 1984; Houghton and Senagupta, 1984), multicriteria decision aid and expert systems. These methods are reviewed in Appendix B and the next sections, and illustrated with typical literature examples.

5. Summary of methods, sectors and country applications

The techniques for business failure prediction, as presented earlier in this article, consist mainly of three parts:

(i) sample selection and collection of data (variables and sample sizes);

(ii) selection of method and specific variables (ratios) to develop a predictive model;

(iii) model validation, i.e. statistical significance and accuracy of results.

For each one of these parts, considerable discussion exists in the literature. The selection of the method can be the most important part. This selection depends on the data to be analyzed and

Table 1
Number of studies by country

Country	No. of studies
Australia	2
Canada	1
Finland	4
France	5
Greece	7
Israel	1
Italy	2
Japan	1
Sweden	1
Netherlands	1
UK	9
USA	13

the objectives of the study. The data selection is influenced by their availability or reliability, the definition of failure or underlying failure theory and the study objectives.

The prediction of business failures, being an area of high interest, was faced in many countries by various authors. This implies that failure prediction is a field of world-wide interest. Table 1 summarizes by country the number of studies contained in Table A.3, Appendix A, of 47 industrial and retail applications. This table reflects the world-wide interest and the relative frequency by country.

One may naturally wonder whether different methods are more frequently used in some countries. Table 2 summarizes this frequency by country. Some studies employed more than one method to develop failure prediction models; so the total frequency of methods is larger than the number of 47 studies considered in this analysis.

Some countries (i.e. Finland, Greece) present only a few case study applications, because the subject of business failure has been analyzed only recently and by a handful of authors. From the countries for which more than two methods are used, the primarily employed methods are discriminant and logit analysis. One would suspect that the special interest in specific industries in different countries would favor different models. This does not seem to be the case here. Most studies used three types of firms in their samples: manufacturing firms, combination of manufacturing and retailing firms or firms from several industrial sectors. Only two studies focused on a specific industrial sector, the textile industry (Altman et al., 1974; Michalopoulos et al., 1993). These two studies (one in France and one in Greece) probably indicate that the textile industry is of high interest in the specific countries. In general we found no relation between the industrial sectors and the methods used for the prediction of failure.

6. Summary of ratios by country application

Business failure prediction models are based on the financial characteristics of firms in the

Table 2
Use of methods [a] by country

COUNTRY	DA	LPM	Logit	Probit	RPA	Electre	ES	SA	UA	Other
Australia	1						1			
Canada	1									
Finland	3	1						1		
France	4					1				
Greece	3	5	3	1	1					
Israel										1
Italy	2									
Japan	1									
Sweden				1						
Netherlands	1									
UK	6		4							
USA	4		8		1				1	
Total	26	6	15	2	2	1	1	1	1	1

[a] See Table A.1, Appendix A, for method definitions.

form of financial ratios. Most of these models included entirely or mainly financial characteristics (financial ratios) of the firms available. Tables A.4a and A.4b present a large number of ratios used in predictive studies of industrial failures.

As can be easily seen in these tables, there are no financial characteristics common to all predictive studies. The ratios found useful in earlier studies (e.g. Altman, 1968) were the first under consideration by many researchers and subsequently used in later studies.

The availability of data was a major limitation in the selection of the variables. As the number of variables available was rather high in many cases, some further selection was feasible. Too many ratios in a model may cause multicollinearity, which of course can be reduced or eliminated by factor analysis. This approach has been overlooked by many authors. Another favored approach involves stepwise procedures, which reduce the number of variables in the model but pay no attention to multicollinearity. Additionally a large number of characteristics in a model may cause difficulties in data collection for real cases. Furthermore, as Hamer (1983) mentioned, the variable set should be constructed on the basis of (a) minimizing the cost of data collection and (b) maximizing the model applicability.

The financial ratios employed in the 47 industrial applications we reviewed are presented in Tables A.4a, b. Table 3 summarizes by country the most commonly used ratios (those with total frequency less than 4 were not included). It should be noted that one ratio, (S-TP)/TC, was used in 5 UK models, four of which are from one study referring to different years before failure.

The most frequently used financial ratios are WC/TA (16 times in 5 countries), TD/TA (15 models in 4 countries), and CA/CL, EBIT/TA and NI/TA with 12, 12 and 11 models respectively. Ratios preferred or found more significant in different country studies can easily be identified in Table 3. For example, TD/TA was encountered more frequently in US studies (but was never used in the UK), while CF/S prevailed in Finnish studies. Financial ratios often encountered in the studies of one country sometimes were ignored in the other countries. For example the ratios TD/TA and NI/TA found large application in US studies but they were not used at all in UK studies.

7. Concluding remarks

This paper presents a comprehensive survey of literature on business failures, summarized according to a new framework. We first identified a

Table 3
Use of financial ratios [a] by country

	WC/TA	TD/TA	CA/CL	EBIT/TA	NI/TA	CF/TD	QA/CL	CF/S	RE/TA	S/TA	GP/TA	NI/SE	Cash/TA	PBT/S	S-TP/TC	Inv./S	QA/TA	TA/GNP
Australia	–	–	2	1	1	–	–	–	1	–	–	–	–	–	–	–	–	–
Canada		1	1							1								
Finland	–	1	1	3	1	2	1	6	–	2	–	–	–	–	–	–	–	–
France	1	–	–	–	–	1	–	1	–	1	–	–	–	–	–	–	–	–
Greece	5	5	2	1	2	–	–	–	1	–	6	1	–	–	–	–	–	–
Israel	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
Italy	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–
Japan	1	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–
Sweden	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	1	–	–
Netherlands	–	–	–	–	–	–	–	–	1	1	–	1	–	–	–	–	–	–
UK	5	–	–	1	–	–	5	–	1	–	–	3	2	5	5	2	1	2
USA	4	8	5	5	7	6	3	1	3	2	–	–	2	–	–	1	3	2
Total	16	15	12	12	11	9	9	8	7	7	6	6	5	5	5	4	4	4

[a] See Table A.2, Appendix A, for ratio definitions.

list of 158 articles on this topic and then proceed to analyze those articles that contained predictive models for mainly industrial failures.

The major conclusions of this analysis can be summarized as follows:

– There is a world-wide interest for business failure studies. Such studies were made in industrial countries (USA, UK, France) as well as in countries under development (Greece).

– The discriminant analysis method was the most frequently used in business failure studies. Logit analysis ranks second among the methods used. Recently, some multicriteria methods have been employed in business failure studies, primarily to take into account qualitative variables. The question remaining is whether these methods can respond efficiently to the problem.

– Many methods appeared mainly after the 80's for the prediction of business failure in order to overcome the limitations of DA.

– The most important financial ratios came from the solvency category (e.g. WC/TA, TD/TA). The profitability ratios were also important, indicating that the viability of a firm depends to a large extent on profit making.

Although the framework of this study involved only financial ratios as predictors of failure, the research trend is on using non-financial and/or qualitative variables (which of course can be accommodated by our framework).

The survey of methods, bibliography and analysis of articles on business failures presented in this paper are hoped to be useful to teachers, researchers and practitioners in this important area.

Appendix A

Table A.1
Code names for methods

DA	Discriminant Analysis
LPM	Linear Probability Model
Probit	Probit Analysis
Logit	Logit Analysis
RPA	Recursive Partitioning Algorithm
SA	Survival Analysis
UA	Univariate Analysis
ES	Expert Systems

Table A.2
Code names for financial quantities and ratios

AE	Administrative Expenses
AP	Accounts Payable
APP	Average Payment Period (for accounts payable)
AV	Added Value
CA	Current Assets
Ca.	Cash
CF	Cash Flow
CL	Current Liabilities
D	Depreciation
EBIT	Earnings Before Interest & Taxes
FA	Fixed Assets
FAP	Free Assets Percentage (= non-collateralized tangible assets divided by the total tangible assets)
GFA	Gross Fixed Assets
GNP	Gross National Product
GP	Gross Profit
IE	Interest Expenses
Inv.	Inventory
LA	Liquid Assets
LTD	Long Term Debt
MVE	Market Value of Equity
CE	Capital Employed
NCI	No Credit Interval
NI	Net Income
NP	Notes Payable
NW	Net Worth
OE	Operating Expenses
PBD	Profit Before Depreciation
PBT	Profit Before Taxes
Prod	Production (= Sales + Inventory)
QA	Quick Assets
R	Receivables
RE	Retained Earnings
ROI	Return on Investment
S	Sales
SC	Shareholders' Capital
SE	Shareholders' Equity
SOP	Stock Option Percentage (= number of shares of common stock potentially issued from outstanding stock options as a percentage of the number of outstanding shares of common stock)
SP	Stock Price
STD	Short Term Debt
T	Taxes
TCr	Total Creditors
TP	Trading Profit
TA	Total Assets
TC	Total Capital
TD	Total Debt
TE	Total Expenses
TL	Total Liabilities
WC	Working Capital

Table A.3
Characteristics of industrial failures studies

No.	Country	Author (year publ.)	Industry type	Sample period	Method
1	Australia	Izan (1984)	Several	1963–74	DA
2	Australia	Messier & Hansen (1988)	Land development firms	–	ES
3	Canada	Altman & Levallee (1981)	Manufacturing/retailing	1970–79	DA
4	Finland	Laitinen (1991)	Several	–	DA
5	Finland	Laitinen (1992)	Manufacturing	1980–85	DA
6	Finland	Laitinen (1993)	Several	1986–88	LPM
7	Finland	Luoma & Laitinen (1991)	Manufacturing/retailing	–	SA
8	Finland	Luoma & Laitinen (1991)	Manufacturing/retailing	–	DA
9	Finland	Luoma & Laitinen (1991)	Manufacturing/retailing	–	Logit
10	France	Altman et al. (1974)	Textile industry	1968–69	DA
11	France	Conan & Holder (1979)	Manufacturing	1970–75	DA
12	France	Collongues (1977)	Manufacturing	–	DA (1)
13	France	Collongues (1977)	Manufacturing	–	DA (2)
14	France	Micha (1984)	Manufacturing	1975–80	DA
15	France	Zollinger (1982)	–	1975–76	Electre
16	Greece	Gloubos & Grammatikos (1988)	Manufacturing	1977–81	DA
17	Greece	Gloubos & Grammatikos (1988)	Manufacturing	1977–81	Logit, Probit, LPM
18	Greece	Grammatikos & Gloubos (1984)	Manufacturing	1977–81	DA
19	Greece	Grammatikos & Gloubos (1984)	Manufacturing	1977–81	LPM
20	Greece	Michalopoulos et al. (1993)	Textile industry	–	RPA
21	Greece	Slowinski & Zopounidis (1994)	Several	1988	RS
22	Greece	Theodossiou (1991)	Manufacturing	–	DA, Logit, LPM
23	Greece	Vranas (1991)	Manufacturing	1979–84	LPM, Logit
24	Greece	Vranas (1992)	Manufacturing	1980–84	LPM
25	Israel	Tamari (1964)	–	1956–60	
26	Italy	Appetiti (1984)	Manufacturing	1979–81	DA (1)
27	Italy	Appetiti (1984)	Manufacturing	1979–81	DA (2)
28	Italy	Falbo (1991)	Manufacturing	–	DA
29	Japan	Takahashi et al. (1984)	–	1977	DA
30	Sweden	Skogsvik (1990)	Mining/manufacturing	1966–80	Probit
31	Netherlands	Bilderbeek (1977) [a]	–	1950–79	DA
32	UK	Lis (1972) [b]	–	1964–72	DA
33	UK	Peel & Peel (1987)	Manufacturing	1982–85	DA, Logit
34	UK	Peel et al. (1986)	Manufacturing	–	Logit
35	UK	Keasey & McGuinness (1990)	Several	1976–84	Logit (1 year prior)
36	UK	Keasey & McGuinness (1990)	Several	1976–84	Logit (2 years prior)
37	UK	Keasey & McGuinness (1990)	Several	1976–84	Logit (3 years prior)
38	UK	Keasey & McGuinness (1990)	Several	1976–84	Logit (4 years prior)
39	UK	Keasey & McGuinness (1990)	Several	1976–84	Logit (5 years prior)
40	UK	Keasey et al. (1990)	Several	1976–84	Multilogit
41	UK	Taffler (1976) [b]	Manufacturing	1968–73	DA
42	UK	Taffler (1977) [b]	Manufacturing	1969–76	DA
43	UK	Taffler (1983)	–	1972–77	DA
44	UK	Tisshaw (1976) [b]	Manufacturing	1975–76	DA
45	USA	Altman (1968)	Manufacturing	1946–65	DA
46	USA	Altman et al. (1977)	Manufacturing/retailing	1969–75	DA
47	USA	Beaver (1966)	–	–	UA
48	USA	Dambolena and Khoury (1980)	Manufacturing/retailing	1969–75	DA
49	USA	Deakin (1972)	–	1965–75	DA
50	USA	Frydman et al. (1985)	Manufacturing/retailing	1971–81	DA
51	USA	Frydman et al. (1985)	Manufacturing/retailing	1971–81	RPA(1)
52	USA	Frydman et al. (1985)	Manufacturing/retailing	1971–81	RPA(2)
53	USA	Gilbert et al. (1990)	Several	1974–83	Logit
54	USA	Lau (1987)	–	1971–80	Logit
55	USA	Ohlson (1981)	Manufacturing	1970–76	Logit
56	USA	Platt & Platt (1990)	Several	1972–76	Logit
57	USA	Tennyson et al. (1990)	–	1980	Logit
58	USA	Zavgren (1985)	Several	1972–78	Logit
59	USA	Zmijhewski (1985)	–	1972–78	Probit

[a] Cited by Altman (1984).
[b] Cited by Taffler (1984).

Appendix B

B.1. Univariate statistical methods

Univariate statistical methods were the first ones used in an attempt to discriminate between healthy and failing firms. Some of the early studies were those of Fitzpatrick (1932), Smith and Winakor (1935), and Merwin (1942). Later, Beaver (1966) introduced a univariate technique for the classification of firms in two groups by using some financial ratios. The ratios were used individually and a cut-off score was calculated for each ratio on the basis of minimizing misclassifications. In Beaver's study, the ratios Cash Flow/Total Debts, Net Income/Total Assets and Total Debts/Total Assets were the ones providing the highest discrimination capability. A detailed description of the univariate method is given by Beaver (1968) and Elam (1975). The univariate method was later criticized, in spite of its good predictions, because of the correlation among ratios and the associated difficulty in getting clear signals when different variables provide conflicting predictions for a firm. Also, there is a large number of factors that describe efficiently the financial status of a firm, so that a single ratio cannot include the full information (see Edmister, 1972). Even though univariate methods received considerable criticism, they have paved the road for subsequent research on business failures. Researchers recognized that business failure can be caused and affected by many different factors at the same time. The idea of adopting a multivariate index of risk was rather old. Tamari (1964) presented such a model for Israeli firms, using six variables weighed according to the opinion of financial analysts, economists and credit analysts. Different multivariate statistical methods were used in an attempt to relate the predictions to more than one variable, in order to try to improve classification results.

B.2. Survival analysis

There were some methods, earlier used in medicine, biochemistry or other scientific fields, that were applied to discriminate between failed and non-failed firms.

Earlier methods tried to predict failure by using data from one to five years prior failure; they were not able to estimate really the time of failure. Lane et al. (1986) employed a proportional hazard model, proposed by Cox, for the prediction of bank failure, called survival analysis (SA). A full description of the method is provided by Cox and Oakes (1984).

SA assumes that both failed and non-failed firms in the sample are from the same population. Non-failed firms are considered as some kind of censored observations. The risk of failure is measured by the survival time that is calculated for each firm. Assuming that T is the time on which a firm will fail, the survivor function $S(t)$ is the probability that the time T is greater than t. The probability $F(t)$ for a firm to fail before t is

$$F(t) = 1 - S(t). \tag{B.1}$$

The hazard function $h(t)$ is given by

$$h(t) = f(t)/S(t) = -S'(t)/S(t), \tag{B.2}$$

and

$$h(t \mid x) = h_0 \exp(x'\hat{a}), \tag{B.3}$$

where x is the variable vector (vector of characteristics) and $\hat{a}$ is the coefficient vector.

Vector $\hat{a}$, which is the parametric part of the function, is estimated by a technique similar to that of maximum likelihood; $h_0(t)$, the non-parametric part, is calculated by setting $x = 0$.

The survivor function $S(t \mid x)$ is then

$$S(t \mid x) = S_0(t) \exp(\hat{a}'x), \tag{B.4}$$

where $S_0(t)$ is given by

$$S_0(t) = \exp\left(-\int_0^t h_0(u)\, \mathrm{d}u \right). \tag{B.5}$$

As $h_0(t)$ is distribution free, SA is a 'semiparametric' methodology, and it partially skips the criticisms against parametric techniques. Furthermore, the method appears to be more natural in dealing with the failure problem.

SA was employed by Luoma and Laitinen (1991) for the prediction of business failures. Their sample consisted of 36 industrial and retailing failed Finnish companies, matched by size

Table A.4a
Financial ratios included in industrial failure models

No.	WC/TA	TD/TA	CA/CL	EBIT/TA	NI/TA	CF/TD	QA/CL	CF/S	RE/TA	S/TA	GP/TA	NI/SE	Cash/TA	PBT/S	S-TP/TC	Inv./S	QA/TA	TA/GNP
1			+ [a]	+														
2			+		+				+									
3		+	+							+								
4		+	+		+			+		+								
5						+		+		+								
6				+		+	+	+										
7				+				+										
8								+										
9				+				+										
10										+								
11																		
12	+																	
13																		
14						+												
15								+										
16	+	+	+								+							
17	+	+									+							
18	+										+							
19	+										+							
20		+	+	+								+						
21																		
22	+	+			+				+									
23											+							
24		+			+						+							
25			+															
26																		
27																		
28												+						
29	+, (+)												(+) [b]					
30				+												+		
31									+	+		+						
32	+			+					+									
33	+						+							(+)				(+)
34	+																	(+)
35														+	+			
36	+											+				+* [c]		
37							+					+		+	+			
38													+		+			
39							+								+			
40	+						+					+	+	+	+	+*		
41																	+	
42																		
43																		
44							+							+				
45	+			+					+	+								
46			+	+, (+)					+									
47		+			+	+												

CA/TA	EBIT/IE	GP/S	IE/GP	MVE/TC	MVE/TD	S/nFA	Salaries/AV	SC/TC	APP	AV/TA	CA/TD	Cash/CL	Cash/S	CL/TA	CL/TD	FA/TA	No.
	+				+												1
																	2
																	3
																	4
																	5
																	6
								+	+								7
								+	+								8
								+									9
							+									+	10
							+										11
							+										12
																	13
		+	+														14
																	15
																	16
																	17
+																	18
																	19
																	20
																	21
																	22
																	23
			+														24
																	25
																	26
																	27
		+	+*														28
										+							29
												+					30
										+							31
																	32
															+*		33
																	34
																	35
																	36
																	37
						+											38
																	39
						+											40
																	41
											+			+			42
											+			+			43
																	44
					+												45
	+			+													46
																	47

Table A.4a (continued)
Financial ratios included in industrial failure models

No.	WC/TA	TD/TA	CA/CL	EBIT/TA	NI/TA	CF/TD	QA/CL	CF/S	RE/TA	S/TA	GP/TA	NI/SE	Cash/TA	PBT/S	S-TP/TC	Inv./S	QA/TA	TA/GNP
48		+			+													
49	+	+	+		+	+	+						+				+	
50			+	+	+	+	+										+	
51						+												
52						+											+	
53				+		+												(+)
54		+																
55	+	+, (+)	+*		+													(+)
56		+, (+)						+, (+)										
57	+			+					+	+								
58		+			+		+						+			+		
59		+	+		+													

[a] + indicates that the ratio is used in the corresponding model.
[b] (+) indicates that the ratio is used in the corresponding model somehow transformed.
[c] +* indicates that the ratio is used in the corresponding model inversed.

and industry type to 36 non-failed firms. The goal of the method was to calculate the survival time starting from the end of accounting period, assuming that at this time the failure process starts. This time is not necessarily a natural start of the failure process. This model consisted of financial ratios (see Appendix A), as well as a measure of the size; it performed satisfactory compared to DA and logit analysis.

The interpretation of results, according to the expected failure time, provides decision makers with important information about a firm. The SA method, although a viable alternative to statistical methods, has not been often applied for the prediction of business failure.

B.3. Discriminant analysis

Discriminant analysis (hereafter DA) is a multivariate analytical method. DA was used in a large number of studies for the development of models able to predict business failure. First, we consider that any firm i is characterized by a vector X the elements of which are measurements of n independent variables x (predictors). For the two populations (failing and non-failing firms) it is assumed that the independent variables are distributed within each group according to a multivariate normal distribution with different means but equal dispersion matrices. The objective of this method is to obtain the linear combination of the independent variables that maximizes the variance between the populations relative to within group variance. The method estimates a discriminant function that is a coefficient vector A, $(a_1, a_2, \ldots, a_n)$, and a constant term a_0. The linear combination of the variables provides for each firm a Z-score, according to

$$Z_i = a_0 + a_1 x_{i1} + a_2 x_{i2} + a_3 x_{i3} + \cdots + a_n x_{in}, \tag{B.6}$$

where Z_i is the Z-score for firm i and $x_{i1}, x_{i2}, \ldots, x_{in}$ are the n independent variables for firm i.

A cut-off score is calculated according to the a-priori probabilities of group membership and the costs of misclassification. Based on its Z-score and the cut-off score, a firm is classified to the failure or the non-failure group. If the assumption of equality of dispersion matrices is not satisfied, then quadratic DA instead of linear DA may be advantageous. Altman et al. (1981) provided a detailed description of DA and its financial applications.

Altman first proposed DA for the failure prediction in 1968. This study was of much interest as it was the first to relate the classification of firms to more than one variable, using DA. He

CA/TA	EBIT/IE	GP/S	IE/GP	MVE/TC	MVE/TD	S/nFA	Salaries/AV	SC/TC	APP	AV/TA	CA/TD	Cash/CL	Cash/S	CL/TA	CL/TD	FA/TA	No.
																	48
+												+	+				49
+	(+)			+													50
				+									+				51
																	52
																	53
																	54
															+	+	55
					+												56
						+											57
																	58
																	59

selected 33 manufacturing firms failed in the period 1946–1965 matched by industry and asset size to 33 non-failed firms. Applying linear DA, he constructed a 5-variable model to classify bankrupt/non-bankrupt firms into the two groups. The ratios used in the model were chosen because of their popularity in the relative literature and their potential relevancy to the study. These financial ratios were calculated for up to five years prior to the failure.

The discriminant function proposed was

$$Z = 0.021X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5,$$

where:
$X_1 = \mathrm{WC/TA}$.
$X_2 = \mathrm{RE/TA}$.
$X_3 = \mathrm{EBIT/TA}$.
$X_4 = \mathrm{MVE/TD}$.
$X_5 = S/\mathrm{TA}$.
(See Table A.2 for interpretation of the used code names.)

The cut-off score minimizing the total number of misclassifications was 2.675. Firms with a Z-score less than this cut-off score were assigned as bankrupt. A test of the model was provided on a holdout sample consisting of 99 distressed firms with similar asset size to the original group. As the holdout sample's firms were really assigned to neither of the two initial subsamples of firms (bankrupt/non-bankrupt), the goal was to determine the sensitivity of the model to distinguish between firms facing permanently and temporal financial problems.

Moyer (1977) pointed out that Altman's (1968) model had poor predictive ability and he used a stepwise DA method to construct a better model. Joy and Tollefson (1975) and the relative replies by Altman and Eisenbeis (1978) and Scott (1978) discussed the applicability of DA to financial problems. Based on Altman's (1968) model, Joy and Tollefson (1975) focused their criticism on the predictive ability, the relative discriminatory power of variables and classification efficiency of DA.

Altman et al. (1977), using quadratic DA, constructed a 7-variable model called the 'Zeta® model'. This study was based on a sample of 53 manufacturing and retailing firms that went bankrupt in the period of 1969–1975. The bankrupt firms were matched by industry group and year of data to 58 non-bankrupt firms. They used log-transformations of variables to improve their normality. To overcome the assumption of equal dispersion matrices, required by linear DA, they used quadratic DA. The variables entering the model were:
$X_1 = \mathrm{EBIT/TA}$.
$X_2 = \mathrm{TA}$.
$X_3 = \mathrm{EBIT/IE}$.
$X_4 = \mathrm{CA/CL}$.
$X_5 = \mathrm{RE/TA}$.

Table A.4b
Financial ratios and other ratios included in industrial failure models

No.	Fl · F / TD	IE / S	IE / TE-De	Inv. / nWC	NCI	NI / CL	NI / NCe	NI / S	NI / TD	NW / TD	PBD / TD	PBT / CL	R / S	SC / TA	SE / TA	WC / LTD	FA / NW
1																	
2																	
3									+ [a]								
4																	
5														+			
6														+			
7																	
8																	
9																	
10													+				
11		+															
12		+															
13																	
14																	
15																	
16																	
17																	
18				+		+											
19						+											
20																	
21																	
22																	
23			+								+						
24			+								+						
25													+(1/)		+		
26																+	
27																	
28								+	+							+	
29																	
30															+		
31																	
32											+						
33																	
34	+																
35																	
36							+										
37																	
38																	
39																	
40							+										
41					+												
42												(+)					
43					+							+					
44																	
45																	
46																	
47																	

No.	Other ratios					
1	Funded Debt/Shareholders Funds					
2						
3	Rate of growth of Equity-Rate of Growth of TA					
4						
5						
6						
7	ROI·APP	Official Expenses/corrected expenses				
8						
9						
10	Av/Prod.	EBIT/SE	LTD/(LTD + SE)	(LTD + SE)/STD	SE/*S*	SE/TD
11	(CA-Inv.)/Inv.	EBIT/TD	(LTD + SE)/(TD + SE)			
12						
13	EBIT/*S*	WC/Inv.				
14	FA/AV	(LTD + SE)/CE	Trade Debt/Purchases	(WIP – customers' prepayment + trade acc. rcvble)/output		
15	SE/LTD	NW/(SE + LTD)	PBT/Prod.	WC/Needs in financing		
16	GP/CL					
17						
18	NP/TA					
19						
20	(CA-Inv)/CL	TD/SE				
21	(General & Administrative Expences)/*S*					
22	LTD/TA					
23	(R + Ca)/CA					
24	QA/(TE-D)	(*R* + LA)/CL				
25	Prod./Inv.	Prod./WC				
26	DE/GFA					
27	IE/AV					
28	DE/AV					
29	NW/TA					
30	*T*/PBT	IE/TD				
31	AP/*S*					
32						
33						
34	*S*/TD					
35						
36						
37	Capital Gearing					
38						
39						
40	Capital Gearing					
41	TD/NCe	WC/NW				
42						
43						
44	QA/NCe					
45						
46						
47						

Table A.4b (continued)
Financial ratios and other ratios included in industrial failure models

No.	Fl·F / TD	IE / S	IE / TE-De	Inv. / nWC	NCI	NI / CL	NI / NCe	NI / S	NI / TD	NW / TD	PBD / TD	PBT / CL	R / S	SC / TA	SE / TA	WC / LTD	FA / NW
48				(+)				+									+, (+)
49																	
50																	
51																	
52																	
53																	
54										(+)							
55	+																
56																	
57																	
58																	
59																	

[a] + indicates that the ratio is used in the corresponding model.
[b] (+) indicates that the ratio is used in the corresponding model somehow transformed.
[c] +* indicates that the ratio is used in the corresponding model inversed.

X_6 = MVE/TC.
X_7 = Stability of variable X_1 over a ten-year tree.

The development of the Zeta® model created an increased interest in the application of DA. The method was also employed for the classification of firms in different areas. The extensive application and study of DA pointed out rather early the problems of the method. Eisenbeis (1977) summarized 7 main problems in the application of it:

1) violation of the assumption of multivariate normal distribution of the variables;

2) use of linear instead of quadratic discriminant functions when the group dispersions are unequal;

3) unsuitable interpretation of the role of the independent variables;

4) reduction in dimensionality;

5) group definition;

6) inappropriate choice of *a priori* probabilities and/or costs of misclassifications;

7) problems in estimating classification error rates to assess the performance of the models.

Analogous problems were also reported by Malecot in his research work on the applicability of DA (Malecot, 1981, 1986, 1991).

Other researchers suggested improvements to DA to get better prediction results. Deakin (1972) modified DA to get the probability of a firm to be a member of the two classes (bankrupt or non-bankrupt). Taffler (1983) made some modifications of DA method deriving for each firm a PAS-score (Performance Analysis). The PAS-score allows the relative performance of a firm to be tracked throughout the whole performance range and within the at risk region. Taffler's (1983) PAS-score method has been improved by Giordano (1986).

Studies employing DA for the business failure prediction are presented in Appendix A. DA was also applied in other classification problems in the field of finance. Lane (1972) applied DA in the credit-scoring problem (sometimes similar to the bankruptcy problem, as mentioned before). Recently, Laitinen (1992), studying venture capital decisions, used DA for the prediction of failure of newly founded firms.

DA has been highly criticized, thus forcing researchers to suggest new methods and compare them to DA. This explains why DA is used in so many studies in the business failure literature (see Table 2 and Appendix A), although after the 80's its use has decreased.

In summary, DA provides the decision maker with a dichotomous classification of the firms. This classification, although important, does not

No.	Other ratios		
48	Funded debt/net WC		
49	CA/S	QA/S	WC/S
50			
51			
52			
53			
54	WC/TD		
55			
56			
57			
58	R/Inv.		
59			

provide any estimate of the associated risk of failure. On the basis of this idea the next step in failure prediction was to use methods and models able to provide a probability of failure. Researchers proposed linear conditional probability models and logit/probit analysis.

B.4. Linear probability model

As the probability of failure was an attractive way for dealing with failure, researchers developed models estimating the probability of failure. Linear probability models (LPM) were proposed as an alternative to DA.

The *linear probability model* is a special case of ordinary least squares (OLS) regression with a dichotomous (0–1) dependent variable. A detailed description of the method is given by Gujarati (1988). The method assumes that the dummy variable y in Eq. (1), that represents the membership of a firm i in one of the specified groups, is a linear combination of n characteristics of the firm. By transformation (see Collins and Green, 1982) the probability P_i for a firm to fail is given by

$$P_i = a_0 + a_1 x_{i1} + a_2 x_{i2} + a_3 x_{i3} + \cdots + a_n x_{in} \tag{B.7}$$

where $a_0, a_1, \ldots, a_n$ are OLS estimates and $x_{i1}, x_{i2}, \ldots, x_{in}$ are the n independent variables for firm i.

Serious statistical problems were mentioned in the application of LPM. The error terms are heteroskedastic and their distribution is not normal. Also there is a problem of interpretation, as the predicted value of probability can lie outside the (0–1) interval. Altman et al. (1981) provided some comments and solutions for these. On the other hand, while underlying assumptions of DA and LPM are not similar, the results of the methods are identical. These might explain in part why the method did not find large application for the prediction of business failure (see Table 2); however, there are studies where LPM was preferred over DA for convenience (e.g. Theodossiou, 1991).

Meyer and Pifer (1970) first employed LPM for bank bankruptcy prediction. Other studies employing LPM for business failure prediction are presented in Appendix A.

B.5. Logit and probit analyses

Multivariate conditional probability models were later introduced into the failure prediction literature. These methods, based on a cumulative probability function, provide the probability of a firm belonging to one of the prescribed classes, given the financial characteristics of the firm.

In the *logit method* the probability of a firm i to go bankrupt, given the vector of variables X_i, is

$$P(X_i, \hat{a}) = F(\tilde{a} + \hat{a} X_i), \tag{B.8}$$

where $F(\tilde{a} + \hat{a}X_i)$ is the cumulative logistic function:

$$F(\tilde{a} + \hat{a}X_i) = \frac{1}{1 + e^{-(\tilde{a} + \hat{a}X_i)}}. \quad \text{(B.9)}$$

The logit method provides the probability $P(X_i, \hat{a})$ for a firm to go bankrupt. Based on that probability a firm is classified as bankrupt or non-bankrupt, using a 'cut-off' probability, attempting to minimize the type I and type II errors. The model coefficients are obtained by maximizing the log-likelihood function.

Logit analysis was first proposed for bank failure prediction by Martin (1977) and for the prediction of business failure by Ohlson (1980). Ohlson selected 105 industrial firms failed in the period 1970–1976. All firms had to have been traded on the stock exchange during the three years before failure. The non-failed firms were selected at random. The goal was to construct three models able to predict firm failure up to three years prior to actual failure.

The variables inserted to the three models estimated by Ohlson were:

$$X_1 = \log(\mathrm{TA}/\mathrm{GNP}).$$

$$X_2 = \mathrm{TL}/\mathrm{TA}.$$

$$X_3 = \mathrm{WC}/\mathrm{TA}.$$

$$X_4 = \mathrm{CL}/\mathrm{CA}.$$

$$X_5 = \begin{cases} 1 & \text{if TL} > \text{TA}, \\ 0 & \text{if TL} < \text{TA}. \end{cases}$$

$$X_6 = \mathrm{NI}/\mathrm{TA}.$$

$$X_7 = \mathrm{FPO}/\mathrm{TL},$$

where FPO is Funds Provided by Operation.

$$X_8 = \begin{cases} 1 & \text{if NI} < 0 \text{ for the last two years}, \\ 0, & \text{otherwise}. \end{cases}$$

$$X_9 = \frac{\mathrm{NI}_t - \mathrm{NI}_{t-1}}{|\mathrm{NI}_t| + |\mathrm{NI}_{t-1}|},$$

where NI_t is net income for the most recent period.

Appendix A presents some more studies for the prediction of business failure using logit analysis. The basic techniques of logit analysis were explored and extended by other researchers to obtain better classification accuracy. Zavgren (1985) developed a measure of the information contained in a logistic function using measures of entropy to assess the uncertainty of unexpected failure. Keasey and McGuinness (1990) criticized the results of such an approach as not applicable in their application of UK firms. Keasey et al. (1990) developed a multilogit model to classify firms according to the time they are expected to fail.

While logit analysis seems preferable to DA, because of the limitations of DA, comparative studies between the two methods have not proved a higher classification accuracy for all cases and types of samples (e.g., Press and Wilson, 1978; Collins and Green, 1982). Hamer (1983) compared DA to the logit method for different data sets, concluding that the models derived are of comparable ability to assess the probability of failure.

Logit analysis has found considerable applications in failure predictions. Many models in several countries were constructed as stated in appendix A and Table 2. Studies presented after 1981 used mostly logit analysis, mainly because of the limitations of DA.

Probit models are similar to the logit ones. The main difference between them is that for the calculation of probability

$$P(X_i, \hat{a}) = F(\tilde{a} + \hat{a}X_i),$$

$F(\tilde{a} + \hat{a}X_i)$ is the cumulative standard normal distribution function:

$$F(\tilde{a} + \hat{a}X_i) = \int_{-\infty}^{\tilde{a} + \hat{a}X_i} \frac{1}{(2\pi)^{1/2}} e^{-z^2/2}\,dz. \quad \text{(B.10)}$$

Maximum likelihood estimation is employed as in logit analysis. Grablowsky and Talley (1981) compared probit analysis and DA for classifying credit applicants. They tested the uniqueness of coefficients, the multicollinearity effect and classification accuracy, concluding that probit analysis can be an alternative to DA.

As shown in Table 2, studies using probit analysis are much fewer than logit. This might be because probit analysis requires more computational effort compared to logit, as it involves non-linear estimation (Gloubos and Grammatikos, 1988).

B.6. Recursive partitioning algorithm

The *recursive partitioning algorithm* (RPA) is a non-parametric classification technique. The method starts with the sample of firms, their financial characteristics, the actual group classification, the prior probabilities and the misclassification costs. A binary classification tree is built, where a rule is associated to any node. These are, usually, univariate rules; that is a certain financial characteristic and a cut-off point that minimize the cost of misclassification for the rest of the firms. The risk of misclassification in any node t, $R(t)$, is

$$R(t) = (C_{21} + C_{12}) P_1 P_2 \frac{1}{p(t)} \frac{n_2(t)}{N_2} \frac{n_1(t)}{N_1}, \tag{B.11}$$

where:

N_1, N_2 : The total number of firms in each group (failed and non-failed firms).

$n_1(t)$, $n_2(t)$: The number of firms in each group on node t.

C_{21} : Cost of misclassifying a firm in group 1 while it is in group 2.

C_{12} : Cost of misclassifying a firm in group 2 while it is in group 1.

P_1, P_2 : Prior probability of a firm to be a member of group 1 or group 2.

$p(t)$: Probability of classifying a firm on node t.

After the classification tree is constructed, the risk of the final nodes and the risk for the entire tree is calculated. For the classification of any new object (firm), the object descends the tree and falls into a final node that identifies the group membership for the specific firm and the associated probability. Breiman et al. (1984), provided an extended description of the method, including theory of binary trees, splitting rules, etc.

Frydman et al. (1985) first employed RPA as an alternative method to study the failure problem. The purpose of this study was to introduce RPA for the prediction of business failure and to compare resulting classification trees to models derived by DA. A sample of 58 bankrupt industrial companies and of 142 non-bankrupt manufacturing and retailing ones was selected at random from the period 1971–1981. Two RPA classification trees and two DA models were constructed and compared in the study.

The RPA trees, RPA_1 and RPA_2 (see Fig. B.1a and Fig. B.1b respectively) were constructed for different costs of misclassification. For RPA_1, $c_{12} = 20$ and $c_{21} = 1$ and for RPA_2, $c_{12} = 50$ and $c_{21} = 1$. For both trees, the prior probabilities for bankrupt and non-bankrupt groups (see Eq. (B.11)) were set at $(P_1, P_2) = (0.02, 0.98)$. From the various trees constructed according to the above parameters, the trees with the smallest cross validation costs were selected.

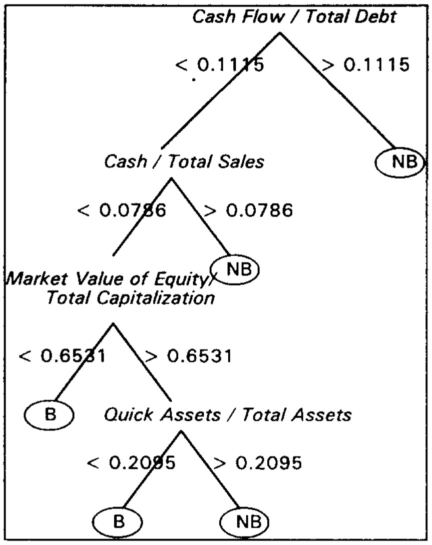

Fig. B.1a. The RPA_1 tree in Frydman, Altman and Kao (1985).

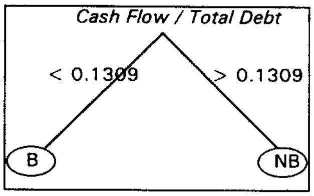

Fig. B.1b. The RPA_2 tree in Frydman, Altman and Kao (1985).

The DA models were constructed using a forward stepwise procedure. The first model, DA_1 included 10 variables, while model DA_2 included only the four most significant variables as provided by the stepwise method. These discrimination functions were

$$Z_1 = -1.761 + 5.452X_1 + 1.758X_2 + 0.505X_3 + 1.850X_4 - 6.292X_5 - 1.021X_6 + 8.970X_7 - 1.995X_8 + 3.482X_9 - 1.033X_{10},$$

$$Z_2 = -4.041 + 5.322X_1 + 0.622X_2 + 0.712X_3 + 1.149X_4,$$

where:
X_1 = NI/TA.
X_2 = CA/CL.
X_3 = log(TA).
X_4 = MVE/TC.
X_5 = CA/TA.
X_6 = CF/TD.
X_7 = QA/TA.
X_8 = QA/CL.
X_9 = EBIT/TA.
X_{10} = log(IE + 15).

The results and classification efficiency of RPA trees in this study were good; however, direct comparison with DA results was difficult, as DA provides a continuous scoring system while RPA provides categories of risk for failure.

RPA has the characteristics of both the univariate and multivariate methods. Classification on a node is made upon the rule of a single variable (although rules in the form of linear combination of financial characteristics can also be assigned to a node). On the other hand the method uses a sequence of nodes, i.e. a sequence of characteristics to classify a firm.

RPA is a forward selection method. It does not review previous classifications while it introduces new classification rules and this can result in a reappearing of the same variable to a later stage, with a different cut-off score. There is also a problem of overfitting, as continuation of partitioning process can result in a tree were each firm is classified by one terminal node. To avoid such problems different trees of various degrees of complexity are derived and tested.

One of the main advantages of RPA is that the binary tree easily explains failure for a specific firm. This simplicity is eliminated if, instead of single variable rules, rules of linear combination of characteristics are used. In such cases the resulting model can not easily explain failure. The estimation of the tree is also difficult. From a decision maker point of view, RPA just classifies firms into categories of risk. It does not permit comparisons between firms in the same category, making it difficult to assess the relative performance of firms.

Appendix A presents studies using RPA for failure prediction. In spite of its attractiveness, RPA did not find a large application for the prediction of business failure, as shown in Table 2. Nevertheless, the method was proposed for other classification problems in finance. Marais et al. (1984) and Srinivasan and Kim (1988) used RPA for commercial bank loans classifications.

B.7. Mathematical programming

Mathematical programming methods were employed early on to solve the problem of grouping items into one or more groups. Different clustering and discrimination methodologies based on mathematical programming were proposed in many studies. The basic goal of these approaches was to escape the assumptions and restrictions of DA and improve the classification accuracy. Several mathematical programming formulations and discrimination criteria were proposed. Such formulations were those of minimizing the sum of absolute deviations (Freed and Glover, 1981a), minimizing the maximum deviation (Freed and Glover, 1981b) and minimizing the number of misclassifications (Bajgier and Hill, 1982). Some studies, like the one by Glover (1990), proposed some 'hybrid' mathematical programming formulations. Koehler and Erenguc (1990) provided a review of mathematical programming discrimination methods.

Mahmood and Lawrence (1987) used mathematical programming discrimination (MPD) methods in the bankruptcy problem. More recently, Gupta et al. (1990) applied linear goal programming, as proposed by Freed and Glover

(1981a,b), for the failure problem. Formulating the intra-group and the inter-group differences between firms, this model sets a score for each firm and boundaries for group discrimination. The method generates a hyperplane of the form

$$Ax = b. \tag{B.12}$$

Given the n observations (firms) A_i and the two groups of firms G_1 and G_2, this method looks for a linear transformation of x and a 'cut-off' (boundary) point b. The goal is to maximize the weighted sum of squares and minimize the weighted sum of boundary violations. These are expressed by the following formulation:

$$\text{Min} \quad \sum_{i=1}^{n} p_i a_i - \sum_{i=1}^{n} q_i d_i \tag{B.13}$$

subject to

$$\begin{aligned} &A_i x + d_i \leq b + a_i, \quad A_i \in G_1, \\ &A_i x + d_i \geq b + a_i, \quad A_i \in G_2, \\ &b + \sum x_i = M, \end{aligned} \tag{B.14}$$

where:
the last constraint is considered as a 'normalizing constraint'.
a_i is the degree to which an A_i violates the 'cut-off' point,
d_i is the measure of the distance between A_i and the 'cut-off' point; d_i, $a_i \geq 0$,
p_i and q_i are weights for a_i and d_i respectively, and
x and b are unrestricted in sign.

Gupta et al. (1990) used the same variables as Altman (1968) in a sample consisting of firms in several industries, covering the period 1971–1986 and using data for up to five years prior to failure. The MPD model provided better results than an alternative DA model. The Mahmood and Lawrence (1987) study, however, concluded the opposite, indicating sensitivity to the nature of the sample.

The results of mathematical programming discrimination methods were usually compared to those provided by DA using either real or simulated data. Some MPD methods were criticized for providing arbitrary and not very helpful classification scores (Glorfeld and Gaither, 1982). In contrast, Bajgier and Hill (1982) and Markowski and Markowski (1987), using MPD methods including qualitative variables, and Rubin (1990), demonstrated that for some classes of problems, linear programming discrimination methods perform usually better than DA. While MPD methods are distribution free and flexible (Freed and Glover, 1981a,b) this fact makes them interesting alternatives to statistical discrimination procedures. The infrequent application of these recent and promising methods in failure prediction does not allow us to fully evaluate their usefulness.

MPD methods were not used only for business failure prediction (see Appendix A for more studies). Srinivasan and Kim (1988) used a formulation proposed by Freed and Glover (1981a) for credit granting classifications.

B.8. Multicriteria decision aid / support methods

Many real-world failure problems include non-quantitative variables, usually other than financial, e.g. managerial or marketing. Such variables are difficult but not impossible to insert in 'classical' discrimination methods; even though there are some studies on the use of qualitative variables (e.g., Gilbert, 1968; Press and Wilson, 1988; Libby, 1975a). To overcome these difficulties, multicriteria decision aid methods were proposed in order to rank companies according to the level of risk (CNME, 1973; Diakoulaki et al., 1992; Moscarola and Roy, 1977; Zopounidis, 1987; Zopounidis, 1990). These methods included both quantitative (financial ratios) and qualitative variables for the assessment of a company performance.

Later, the above tools were inserted in a decision support framework and the created system was called a multicriteria decision support system (MCDSS). The characteristic of an MCDSS is that it allows the decision maker to interact with the system at any time, setting the parameters of the problem and data to be used. A typical MCDSS is that developed by Siskos et al. (1994). Basic parts of the system are the data base and the models base. Data base includes basic financial statements (balance sheet and income statement) and also qualitative information (information security, market trend, quality of management, etc.) for firms. Model base includes financial analysis methods providing common size statements, financial ratios and graphical representation of them, as well as methods for the assessment of firm performance (DA, Principal Components Analysis, multicriteria decision methods, etc.). The user of the system can select the information useful to the analysis and methods to be employed – often interactively. The multicriteria idea for predicting failure or assessing the performance of a firm led to the development of some MCDSSs. Mareschal and Brans (1991), Massaglia and Ostanello (1991) and Zopounidis et al. (1992) propose such systems that are able to help in financial decision-making, involving financial and qualitative variables to assess a company's performance.

As an alternative to statistical approach, multicriteria methods were suggested for the prediction of business failure. The ELECTRE method (Roy, 1991), and Rough Sets method (Slowinski and Zopounidis, 1994) have been proposed to overcome the limitations of DA and other statistical methods.

The ELECTRE family of multicriteria methods (cf. Roy, 1991) belongs to the French school, which uses the majority rule within an outranking relationship, instead of the American school, which uses the unanimity rule within the concept of dominance (Pareto optimality). The particularity of the ELECTRE family (and of the French school) is to refuse the possibility of total compensation between the alternative's performance on the criteria, and then to accept incomparability and intransitivity. The ELECTRE method was employed by Zollinger (1982) to assess business failure risk, using the following criteria:

$X_1 = \mathrm{EBIT}/\mathrm{TA}$.
$X_2 = \mathrm{CF}/S$.
$X_3 = \mathrm{WC}/\mathrm{NF}$, where NF = Needs in Financing.
$X_4 = \mathrm{NW}/(\mathrm{TL} + \mathrm{SE})$
$X_5 = \mathrm{SE}/\mathrm{LTD}$.

The ELECTRE method proceeds to the sorting of firms in categories that adapts well to the purpose of risk failure. The ELECTRE family accepts incomparability and intransitivity using straightforward techniques. The methods belonging to the ELECTRE family were popular in several European studies (cf. Roy and Bouyssou, 1993; Siskos et al., 1984) but no so much in the USA.

Rough sets approach deals with vagueness, caused by granularity in the representation of a decision situation, in sorting problems. Thus, it enables one to: (a) discover minimal subsets of criteria ensuring an acceptable quality of classification of the firms, and (b) derive decision rules to be used to support decisions. Slowinski and Zopounidis (1994) used the rough sets approach for the classification of 39 Greek firms in three categories of risk (failed/uncertain/non-failed). The criteria found most important and employed in the decision rules were: general and administrative expenses/sales; manager's work experience; market niche-position; special competitive advantages of firms.

B.9. Expert systems

The development and application of artificial intelligence led some researchers to employ expert systems and neural networks in the bankruptcy problem. (For instance, neural networks are only used for bank bankruptcy prediction, see Dutta and Shekhar (1992), Tam (1991), and Tam and Kiang (1992).) Messier and Hansen (1988) made an attempt to use expert systems (ES) for the prediction of business failure. They used a 'data-driven' method developed by Quinlan (1983), which is based on the Concept Learning System proposed by Hunt et al. (1966). The objective of the method was to take firms of known classes (bankrupt/non-bankrupt) described by a fixed set of attributes (financial ratios), and then to generate a production system using the attributes which correctly classify all the firms of the sample. The rules at each stage (i.e., the variable and the cut-off score) were defined by using measures of entropy and selecting the minimum entropy rule. A decision tree was derived from the production system rules. The decision tree provided by the method is rather similar to the RPA's decision tree. So, ES's decision

IF *Current Assets/Current Liabilities* < *1.533* **and**
IF *Earnings/Total Tangible Assets* < *0.092* **and**
IF *Retained Earnings/Total Tangible Assets* < 0.064 **and**
IF *Retained Earnings/Total Tangible Assets* < -0.942
THEN the business will not fail
ELSE the firm will fail
ELSE the firm will not fail
ELSE the firm will not fail
ELSE the firm will not fail

Fig. B.2. The production system proposed by Handen and Messier (1988).

trees are also able to explain the prediction of failure or non-failure.

Messier and Hansen's study was based on a sample of 23 Australian land development firms. The sample consisted of 8 bankrupt and 15 non-bankrupt firms. The methodology develops a production system as presented in Fig. B.2. The classification accuracy of the production system was encouraging in this small case but, as the authors mentioned, there are some limitations in the use of such inductive methods; e.g., computational burden when the sample is large. A conflict arises when two firms have similar attributes but belong to different classes. Although the authors proposed some possible solutions, they pointed that more research is needed in this area.

In addition to inductive learning methods, the common knowledge engineering process of interviewing experienced decision makers and incorporation of their knowledge in production rules was also employed. The model quality is still affected by the amount and reliability of information.

Expert systems were introduced for both failure and credit granting problems. Elmer and Borowski (1988) proposed an ES approach for bankruptcy at savings & loans institutions. Credit granting studies using ES were those of Bouwman (1983), Cronan et al. (1991), Duchessi and Belardo (1987), Duchessi et al. (1988), Levine and Pomerol (1989), Pinson (1989,1991), Ruparel and Srinivasan (1992), Shaw and Gentry (1988) and Srinivasan and Ruparel (1990).

References

Altman, E.I. (1968), "Financial ratios, discriminant analysis and the prediction of corporate bankruptcy", *The Journal of Finance*, 23, 589–609.

Altman, E.I. (1969), "Corporate bankruptcy, potential stockholder returns and share valuation", *The Journal of Finance*, December, 887–900.

Altman, E.I. (1983), *Corporate Financial Distress*, Wiley, New York.

Altman, E.I. (1984), "The success of business failure prediction models. An international survey", *Journal of Banking and Finance* 8/2, 171–198.

Altman, E.I., and Eisenbeis, R.A. (1978), "Financial applications of discriminant analysis: A clarification", *Journal of Financial and Quantitative Analysis* 13, 185–195.

Altman, E.I., and Lavallee, M.Y. (1981), "Business failure classification in Canada", *Journal of Business Administration*, Summer, 147–164.

Altman, E.I., and Spivack, J. (1983), "Predicting bankruptcy: The value line relative financial strength systems vs. the zeta bankruptcy classification approach", *Financial Analysts Journal*, November–December, 60–67.

Altman, E.I., Hadelman, R.G., and Narayanan, P. (1977), "Zeta analysis, a new model to identify bankruptcy risk of corporations", *Journal of Banking and Finance* 1, 29–51.

Altman, E.I., Avery, R., Eisenbeis, R., and Stinkey, J. (1981), *Application of Classification Techniques in Business, Banking and Finance*, Contemporary Studies in Economic and Financial Analysis, Vol. 3, JAI Press, Greenwich.

Altman, E.I., Margaine, M., Schlosser, M., and Vernimmen, P. (1974), "Statistical credit analysis in the textile industry: A French experience", *Journal of Financial and Quantitative Analysis* 9, 195–211.

Appetiti, S. (1984), "Identifying unsound firms in Italy. An attempt to use trend variables", *Journal of Banking and Finance* 8/2, 269–279.

Bajgier, S.M., and Hill, A.V. (1982), "An experimental comparison of statistical and linear programming approaches to the discriminant problem", *Decision Sciences* 13, 604–617.

Beaver, W.H. (1966), "Financial ratios as predictors of failure", in: *Empirical Research in Accounting: Selected Studies*, Supplement to *Journal of Accounting Research* 5, 179–199.

Beaver, W.H. (1968), "Market prices, financial ratios and the prediction of failure", *Journal of Accounting Research*, Autumn, 179–192.

Booth, D.E., Alam, P., Ahkam, S.N., and Osyk, B. (1989), "A robust multivariate procedure for the identification of problem Saving and Loan institutions", *Decision Sciences* 20, 320–333.

Booth, P., and Hutchinson, P. (1989), "Distinguishing between failing and growing firms: A note on the use of decomposition measure analysis", *Journal of Banking and Finance* 16/2, 267–271.

Bouwman, M. (1983), "Human diagnostic reasoning by computer: An illustration from financial analysis", *Management Science* 29/6, 653–672.

Breiman, L., Friedman, J.H., Olsen, R.A., and Stone, C.J. (1984), *Classification and Regression Trees*, The Wadsworth Statistics/Probability Series, Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, CA.

Bulow, J.I., and Shoven, J.B. (1978), "The bankruptcy decision", *The Bell Journal of Economics*, Autumn, 437–456.

Caevel, J. de (1983), "Un systéme de clignotants-credits", *Revue Banque*, No. Special, 43–47.

Casey, M., McGee, V., and Stinkey, C. (1986), "Discriminating between reorganized and liquidated firms in bankruptcy", *The Accounting Review*, April, 249–262.

Chalos, P. (1985), "Financial distress: A comparative study of individual, model and committee assessments", *Journal of Accounting Research*, Autumn, 527–543.

CNME (1973), "Méthode de décision multicritère appliqée à l'évaluation de l'entréprise", No. 58, 1–27.

Collins, R.A. (1983), "An empirical comparison of bankruptcy prediction models", *Financial Management*, Summer, 52–57.

Collins, R.A., and Green, R.D. (1982), "Statistical methods for bankruptcy forecasting", *Journal of Economics and Business* 34, 349–354.

Collongues, Y. (1977), "Ratios financiers et prevision des faillites des petites et moyennes entreprises, *Revue Banque* 365, 963–970.

Courtis, J.K. (1978), "Modelling a financial ratios categoric framework", *Journal of Business Finance and Accounting* 5/4, 371–386.

Cox, D.R., and Oakes, D. (1984), *Analysis of Survival Data*, Chapman & Hall, London.

Cronan, T.P., Glorferd, L.W., and Perry, L.G. (1991), "Production system development for expert systems using a recursive partitioning induction approach: An application to mortgage, commercial, and consumer lending", *Decision Sciences* 22, 812–845.

Dambolena, I.G., and Khoury, S.J. (1980), "Ratio stability and corporate failure", *The Journal of Finance* 35, 1017–1026.

Deakin, E.B. (1972), "A discriminant analysis of predictors of business failure", *Journal of Accounting Research*, Spring, 167–179.

Diakoulaki, D., Mavrotas, G., and Papayannakis, L. (1992), "A multicriteria approach for evaluating the performance of industrial firms", *OMEGA* 20/4, 467–474.

Duchessi, P., and Belardo, S. (1987), "Lending analysis support system (LASS): An application of a knowledge-based system to support commercial loan analysis", *IEEE Transactions on Systems, Man, and Cybernetics* 17/4, 608–616.

Duchessi, P., Shawky, H., and Seagle, J.P. (1988), "A knowledge-engineered system for commercial loan decisions", *Financial Management* 17/3, 57–65.

Dutta, S., and Shekhar, S. (1992), "Generalization with neural networks: An application in the financial domain", *Journal of Information Science and Technology* 1/4, 309–330.

Edmister, R.A. (1972), "An empirical test of financial ratio analysis for small business failure prediction", *Journal of Financial and Quantitative Analysis* 7, 1477–1493.

Eisenbeis, R.A. (1977), "Pitfalls in the application discriminant analysis in business and economics", *The Journal of Finance* 32, 875–900.

Eisenbeis, R.A. (1978), "Problems in applying discriminant analysis in credit scoring models", *Journal of Banking and Finance* 2, 205–219.

Elam, E. (1975), "The effect of lease data on the predictive ability of financial ratios", *The Accounting Review*, 15–43.

Elmer, P.J., and Borowski, D.M. (1988), "An expert system approach to financial analysis: The case of S & L bankruptcy", *Financial Management*, Autumn, 66–76.

Ezzamel, M., and Molinero, C.M. (1990), "The distributional properties of financial ratios in UK manufacturing companies", *Journal of Business Finance and Accounting* 17/1, 1–29.

Falbo, P. (1991), "Credit scoring by enlarged discriminant analysis", *OMEGA* 19/4, 275–289.

Fitzpatrick, P.I. (1932), "A comparison of the ratios of the successful industrial enterprises with those of failed companies", The Accountants Publishing Company.

Foster, G. (1986), *Financial Statements Analysis*, Prentice-Hall, London.

Freed, N., and Glover, F. (1981a), "Simple but powerful goal programming models for discriminant problems", *European Journal of Operational Research* 7, 44–60.

Freed, N., and Glover, F. (1981b), "A linear programming approach to the discriminant problem", *Decision Sciences* 12, 68–74.

Frydman, H., Altman, E.I., and Kao, D.L. (1985), "Introducing recursive partitioning for financial classification: The case of financial distress", *The Journal of Finance* XL/1, 269–291.

Gilbert, E.S. (1968), "On discrimination using qualitative variables", *American Statistical Association Journal*, December, 1399–1412.

Gilbert, L.R., Menon, K., and Schwartz, K.B. (1990), "Predicting bankruptcy for firms in financial distress", *Journal of Business Finance and Accounting* 17/1, 161–171.

Giordano, Y. (1986), "Indicateur synthétique de positionnement: Un nouvel outil de gestion pour la firme", *Revue Banque* 456, 879–884.

Glorfeld, L.W., and Gaither, N. (1982), "On using linear programming in discriminant problems", *Decision Sciences* 13, 166–171.

Glover, F. (1990), "Improved linear programming models for discriminant analysis", *Decision Sciences* 21, 771–785.

Gujarati, D.N. (1988), *Basic Econometrics*, 2nd ed., McGraw-Hill, Singapore.

Gupta, Y.P., Rao, R.P., and Bagghi, P.K. (1990), "Linear goal programming as an alternative to multivariate discriminant analysis: A Note", *Journal of Business Finance and Accounting*, 17/4, 593–598.

Gloubos, G., and Grammatikos, T. (1988), "The success of bankruptcy prediction models in Greece", *Studies in Banking and Finance* 7, 37–46.

Grablowsky, B.J., and Talley, W.K. (1981), "Probit and discriminant factors for classifying credit applicants: A comparison", *Journal of Economics and Business* 33, 254–261.

Grammatikos, T., and Gloubos, G. (1984), "Predicting bankruptcy of industrial firms in Greece", *Spoudai, The University of Piraeus Journal of Economics, Business, Statistics and Operations Research* 3–4, 421–443.

Gupta, M.C., and Huefner, R.J. (1972), "A cluster analysis study of financial ratios and industry characteristics", *Journal of Accounting Research*, Spring, 77–95.

Hamer, M.M. (1983), "Failure prediction: Sensitivity of classification accuracy to alternative statistical methods and variable sets", *Journal of Accounting and Public Policy* 2, 289–307.

Houghton, K.A. (1984), "Accounting data and the prediction of business failure: The setting of priors and the age of data", *Journal of Accounting Research* 22/1, 361–368.

Houghton, K.A., and Senagupta, R. (1984), "The effect of prior probability disclosure and information set construction on bankers' ability to predict failure", *Journal of Accounting Research* 22/2, 768–775.

Hunt, E.B., Marin, J., and Stone, P.T. (1966), *Experiments in Induction*, Academic Press, New York.

Izan, H.Y. (1984), "Corporate distress in Australia", *Journal of Banking and Finance* 8, 303–320.

Jensen, R.E. (1971), "A cluster analysis study of financial performance of selected business firms", *The Accounting Review* XLVI, 36–56.

Jones, F.L. (1987), "Current techniques in bankruptcy prediction", *Journal of Accounting Literature* 6, 131–164.

Joy, O.M., and Tollefson, J.O. (1975), "On the financial applications of discriminant analysis", *Journal of Financial and Quantitative Analysis* 101, 723–739.

Karels, G.V., and Prakash, A. (1987), "Multivariate normality and forecasting of business bankruptcy", *Journal of Business Finance and Accounting* 14, 573–593.

Keasey, K., and McGuinness, P. (1990), "The failure of UK industrial firms for the period 1976–1984: Logistic analysis and entropy measures", *Journal of Business Finance and Accounting* 17/1, 119–135.

Keasey, K., and Watson, R. (1991), "Financial distress prediction models: A review of their usefulness", *British Journal of Management* 2, 89–102.

Keasey, K., McGuinness, P., and Short, H. (1990), "Multilogit approach to predicting corporate failure – Further analysis and the issue of signal consistency", *OMEGA* 18/1, 85–94.

Koehler, G.J., and Erenguc, S.S. (1990), "Minimizing misclassifications in linear discriminant analysis", *Decision Sciences* 21, 63–85.

Laitinen, E.K. (1991), "Financial ratios and different failure processes", *Journal of Business Finance and Accounting* 18/5, 649–673.

Laitinen, E.K. (1992), "Prediction of failure of a newly founded firm", *Journal of Business Venturing* 7, 323–340.

Laitinen, E.K. (1993), "Financial predictors for different phases of the failure process", *OMEGA* 21/2, 215–228.

Lane, S. (1972), "Submarginal credit risk classification", *Journal of Financial and Quantitative Analysis* 7, 1379–1386.

Lane, W.R., Looney, S.W., and Wansley, W. (1986), "An application of the Cox proportional hazards model to bank failure", *Journal of Banking and Finance* 10, 511–531.

Lau, A.H.L. (1987), "A five-state financial distress prediction model", *Journal of Accounting Research* 25/1, 127–138.

Levine, P., and Pomerol, J.-Ch. (1989), "Knowledge representation by schemata in financial expert systems", *Theory and Decision* 27, 147–161.

Libby, R. (1975a), "Accounting ratios and the prediction of failure: Some behavioral evidence", *Journal of Accounting Research* 13/1, 150–161.

Libby, R. (1975b), "The use of simulated decision makers in information evaluation", *The Accounting Review*, July, 475–489.

Luoma, M., and Laitinen, E.K. (1991), "Survival analysis as a tool for company failure prediction", *OMEGA* 19/6, 673–678.

Mahmood, M.A., and Lawrence, E.C. (1987), "A performance analysis of parametric and nonparametric discriminant approaches to business decision making", *Decision Sciences* 18, 308–326.

Malecot, J.F. (1981), "Limites des modèles de prévision de défaillances", *Finance* 2/4, 291–315.

Malecot, J.F. (1986), "Sait-on vraiment prévoir les défaillances d'entreprises?", *ISMEA*, *Revue Sciences de Gestion*, Decembre, 55–82.

Malecot, J.F. (1991), "Analyses théoriques des défaillances d'entreprises: Une revue de la littérature", Cahier de recherche No. 9108, CEREG, Universite de Paris-Dauphine.

Marais, M.L., Patell, J.M., and Wolfson, M.A. (1984), "The experimental design of classification models: An application of recursive partitioning and bootstrapping to commercial bank loan classifications", *Journal of Accounting Research* 22, 87–113.

Mareschal, B., and Brans, J.P. (1991), "BANKADVISER: An industrial evaluation system", *European Journal of Operational Research* 54, 318–324.

Markowski, C.A., and Markowski, E.P. (1987), "An experimental comparison of several approaches to the discrimination problem with both qualitative and quantitative variables", *European Journal of Operational Research* 28, 74-78.

Mar Molinero, C., and Ezzamel, M. (1991), "Multidimensional scaling applied to corporate failure", *OMEGA* 19/4, 259-274.

Martikainen, T., and Ankelo, T. (1991), "On the instability of financial patterns of failed firms and the predictability of corporate failure", *Economic Letters* 35, 209-214.

Martin, D. (1977), "Early warning of bank failure. A logit regression approach", *Journal of Banking and Finance* 1, 249-276.

Massaglia, M., and Ostanello, A. (1991), "N-TOMIC: A decision support for multicriteria segmentation problems", in: P. Korhonen (ed.), *International Workshop on Multicriteria Decision Support, Helsinki 7-11, 1989*, Lecture Notes in Economics and Mathematical Systems 356, Springer-Verlag, Berlin, 167-174.

Mensah, Y. (1984), "An examination of the stationarity of multivariate bankruptcy prediction models: A methodological study", *Journal of Accounting Research* 22/1, 380-395.

Merwin, C.L. (1942), "Financing small corporations: In five manufacturing industries, 1926-1936", National Bureau of Economic Research.

Messier, W.F., and Hansen, J.V. (1988), "Including rules for expert system development: An example using default and bankruptcy data", *Management Science* 34/12, 1403-1415.

Meyer, P.A., and Pifer, H.W. (1970), "Prediction of bank failures", *Journal of Finance* 25, 853-868.

Micha, B. (1984), "Analysis of business failures in France", *Journal of Banking and Finance* 8, 281-291.

Michalopoulos, M., Zopounidis, C., and Kouris, I. (1993), "An integrated DSS for the assessment of the risk of failure", in: *Applied Stochastic Models and Data Analysis*, World Scientific Publishing, New Jersey, 606-625.

Moscarola, J., and Roy, B. (1977), "Procédure automatique d'examen de dossiers fondée sur une segmentation trichotomique en présence de critères multiples", *RAIRO Recherche Opérationnelle* 11/2, 145-173.

Moses, O.D. (1990), "On analysts' earnings forecasts for failing firms", *Journal of Business Finance and Accounting* 17/1, 101-118.

Moyer, R.C. (1977), "Forecasting financial failure: A re-examination", *Financial Management*, Spring, 11-17.

Ohlson, J.A. (1980), "Financial ratios and the probabilistic prediction of bankruptcy", *Journal of Accounting Research*, Spring, 109-131.

O'Leary, D.E. (1993), "On bankruptcy information systems", *European Journal of Operational Research* 56, 67-79.

O'Neill, H.M., and Duker, J. (1986), "Survival and failure in small business", *Journal of Small Business Management*, 30-37.

Peel, M.J. (1987), "Timeliness of private company reports predicting corporate failure", *Investment Analysis* 83, 23-27.

Peel, M.J., and Peel, D.A. (1987), "Some further empirical evidence on predicting private company failure", *Accounting and Business Research* 18/69, 57-66.

Peel, M.J., Peel, D.A., and Pope, P.F. (1986), "Predicting corporate failure - Some results for the UK corporate sector", *OMEGA* 14/1, 5-12.

Pinson, S. (1989), "Credit risk assessment and meta-judgement", *Theory and Decision* 27, 117-133.

Pinson, S. (1991), "A multi-expert architecture for credit risk assessment: The CREDEX system", in: D.E. O'Leary and P.R. Watkins (eds.), *Expert Systems in Finance*, Elsevier Science Publishers, Amsterdam, 37-64.

Platt, H.P., and Platt, M.B. (1990), "Development of a class of stable predictive variables: The case of bankruptcy prediction", *Journal of Business Finance and Accounting* 17/1, 31-51.

Press, S.J., and Wilson, S. (1978), "Choosing between logistic regression and discriminant analysis", *Journal of the American Statistical Association* 73/364, 699-705.

Quinlan, J.R. (1983), "Learning efficient classification procedures and their application to chess and games", in: R.S. Michalski, J.G. Carbonell and T.M. Mitchell (eds.), *Machine Learning: An Artificial Intelligence Approach*, Tioga, Palo Alto, CA.

Rose, P.S., Andrews, W.T. and Giroux, G.A. (1982), "Predicting business failure: A macroeconomic perspective", *Journal of Accounting and Finance* 6/1, 20-31.

Roy, B. (1991), "The outranking approach and the foundations of ELECTRE methods", *Theory and Decision* 31, 49-73.

Roy, B., and Bouyssou, D. (1993), *Aide Multicritère à la Décision: Méthodes et Cas*, Economica, Paris.

Rubin, P.A. (1990), "A comparison of linear and parametric approaches to the two-group discriminant problem", *Decision Sciences* 21, 373-386.

Ruparel, B., and Srinivasan, V. (1992), "A dedicated shell for designing expert credit support systems", *Decision Support Systems* 8, 343-359.

Salchenberger, L.M., Cinar, E.M., and Lash, N.A. (1992), "Neural networks: A new tool for predicting thrift failures", *Decision Sciences* 23, 899-916.

Santomero, A.M., and Vinso, J.D. (1977), "Estimating the probability of failure for commercial banks and banking systems", *Journal of Banking and Finance* 1, 185-205.

Scapens R.W., Ryan, R.J., and Flecher, L. (1981), "Explaining corporate failure: A catastrophe theory approach", *Journal of Business Finance and Accounting* 8/1, 1-26.

Scott, E. (1978), "On the financial applications of discriminant analysis: Comment", *Journal of Financial and Quantitative Analysis* 13, 201-205.

Scott, J. (1981), "The probability of bankruptcy: A comparison of empirical predictions and theoretical models", *Journal of Banking and Finance* 5, 317-344.

Sharma, S., and Mahajan, V. (1980), "Early warning indicators of business failure", *Journal of Marketing* 44, 80-89.

Shaw, M.J., and Gentry, J.A. (1988), "Using an expert system with inductive learning to evaluate business loans", *Financial Management* 17/3, 45-56.

Siskos, Y, Washer, G., and Winkels, H.M. (1984), "Outranking approaches versus MAUT in MCDM", *European Journal of Operational Research* 16, 270–271.

Siskos, Y., Zopounidis, C., and Pouliezos, A. (1994), "An integrated DSS for financing firms by an industrial development bank in Greece", *Decision Support Systems* 12, 151–168.

Skogsvik, K. (1990), "Current cost accounting ratios as predictors of business failure: The Swedish case", *Journal of Business Finance and Accounting* 17/1, 137–160.

Slowinski, R., and Zopounidis, C. (1994), "Rough-set sorting of firms according to bankruptcy risk", in: M. Paruccini (ed), *Applying Multiple Criteria Aid for Decision to Environmental Management*, Kluwer Academic Publishers, Dordrecht, 339–357.

Smith, R.F., and Winakor, A.H. (1935), "Changes in the financial structure of unsuccessful corporations", Bureau of Business Research, University of Illinois, Urbana, IL.

Srinivasan, V., and Kim, Y.H. (1987), "Credit granting: A comparative analysis of classification procedures", *The Journal of Finance* XLII/3, 665–683.

Srinivasan, V., and Kim, Y.H. (1988), "Designing expert financial systems: A case study of corporate credit management", *Financial Management*, Autumn, 32–44.

Srinivasan, V., and Ruparel, B. (1990), "CGX: An expert support system for credit granting", *European Journal of Operational Research* 45, 293–308.

Stein, J.H. von, and Ziegler, W. (1984), "The prognosis and surveillance of risks from commercial credit borrowers", *The Journal of Finance* 8, 249–268.

Suzuki, S., and Wright, R.W. (1985), "Financial structure and bankruptcy risk in Japanese companies", *Journal of International Business Studies*, Spring, 97–110.

Taffler, R.J. (1982), "Forecasting company failure in the UK using discriminant analysis and financial ratio data", *Journal of the Royal Statistical Society. Series. A. General* 145/3, 342–358.

Taffler, R.J. (1983), "The *z*-score approach to measuring company solvency", *The Accountant's Magazine* 87/921, 91–96.

Taffler, R.J. (1984), "Empirical models for the monitoring of UK corporations", *Journal of Banking and Finance* 2, 199–227.

Takahashi, K., Kurokawa, Y., and Watase, K. (1984), "Corporate bankruptcy prediction in Japan", *Journal of Banking and Finance* 8, 229–247.

Tam, K.Y. (1991), "Neural network models and the prediction of bank bankruptcy", *OMEGA* 19/5, 429–445.

Tam, K.Y., and Kiang, M.Y. (1992), "Managerial applications of neural networks: The case of bank failure predictions", *Management Science* 38/7, 926–947.

Tamari, M. (1964), "Financial ratios as a means of forecasting bankruptcy", Economic Review, Bank of Israel, Jerusalem.

Tamari, M. (1984), "The use of a bankruptcy forecasting model to analyse corporate behavior in Israel", *Journal of Banking and Finance* 8, 293–302.

Tennyson, B.N., Ingram, R.W., and Dugan, M.T. (1990), "Assessing the information content of narrative disclosures in explaining bankruptcy", *Journal of Business Finance and Accounting* 17/3, 391–410.

Theodossiou, P. (1991), "Alternative models for assessing the financial condition of business in Greece", *Journal of Business Finance and Accounting* 18/5, 697–720.

Vernimmen, P. (1978), "Panorama des recherches portant sur le risque du creancier", *Analyse Financière* 1, 54–61.

Vranas, A.S. (1991), "Probability models for the forecasting of Greek industrial firms' failure" (in Greek), *Spoudai, The University of Piraeus Journal of Economics, Business, Statistics and Operations Research* 4/4, 431–448.

Vranas, A.S. (1992), "The significance of financial characteristics in predicting business failure: An analysis in the Greek context", *Foundations of Computing and Decision Sciences* 17/4, 257–275.

Warner, J.B. (1977), "Bankruptcy costs: Some evidence", *The Journal of Finance* 32/2, 337–347.

Wertheim, P., and Lynn, M.L. (1993), "Development of a prediction model for hospital closure using financial accounting data", *Decision Sciences* 24/3, 529–546.

Wilcox, J.W. (1971), "A simple theory of financial ratios as predictors of failure", *Journal of Accounting Research*, Autumn, 389–395.

Zanakis, S.H., and Walter, G.A. (1994), "Discriminant characteristics of US banks acquired with or without federal assistance", *European Journal of Operational Research* 77, 440–465.

Zavgren, C.V. (1983), "The prediction of corporate failure: The state of the art", *Journal of Financial Literature* 2, 1–37.

Zavgren, C.V. (1985), "Assessing the vulnerability to failure of American industrial firms. A logistic analysis", *Journal of Business Finance and Accounting* 12/1, 19–45.

Zmijewski, M.E. (1984), "Methodological issues related to the estimation of financial distress prediction models", *Studies on Current Econometric Issues in Accounting Research*, 59–82.

Zollinger, M. (1982), "L'analyse multicritère et le risque de crédit aux entreprises", *Revue Française de Gestion*, 56–66.

Zopounidis, C. (1987), "A multicriteria decision making methodology for the evaluation of the risk of failure and an application", *Foundations of Control Engineering* 12/1, 45–67.

Zopounidis, C. (1990), *La Gestion du Capital Risque*, Economica, Paris.

Zopounidis, C., Pouliezos, A., and Yannacopoulos, D. (1992), "Designing a DSS for the assessment of company performance and viability", *Computer Science in Economics and Management* 5, 41–56.