COSC1125/1127 Artificial Intelligence

School of Computer Science and IT
RMIT University
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Tutorial Sheet 4
Propositional Logic

We use $\Rightarrow$ to denote logical implication and $\Leftrightarrow$ to denote logical equivalence. Other symbols typically used for these logical connectors are $\rightarrow$ and $\supset$ for logical implication, and $\equiv$ for logical equivalence.

1. Use truth tables to show that the following are valid (i.e. that the equivalences hold).

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	Distribution of $\wedge$
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	de Morgan's Law
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	de Morgan's Law
$P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P$	Contraposition
$P \Rightarrow Q \Leftrightarrow \neg P \vee Q$	

2. For each of the following sentences, decide whether it is **valid**, **unsatisfiable**, or **neither**. Firstly, trying "guessing" the answer; then evaluate each properly (e.g. using truth tables). How did your guesses match up?

   (a) $Smoke \Rightarrow Smoke$

   (b) $Smoke \Rightarrow Fire$

   (c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

   (d) $Smoke \vee Fire \vee \neg Fire$

   (e) $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$

   (f) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

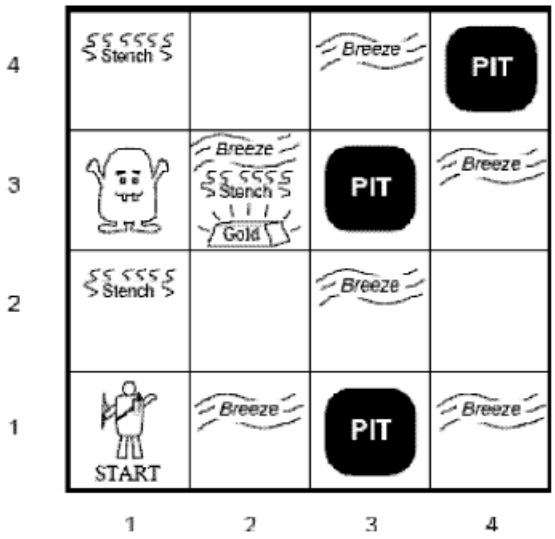
   (g) $Big \vee Dumb \vee (Big \Rightarrow Dumb)$

   (h) $(Big \wedge Dumb) \vee \neg Dumb$

3. Represent the following sentences in propositional logic. Can you prove that the unicorn is mythical? What about magical? Horned?

   > If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

4. For the following Wumpus world:



(a) Develop a notation capturing the important propositions.

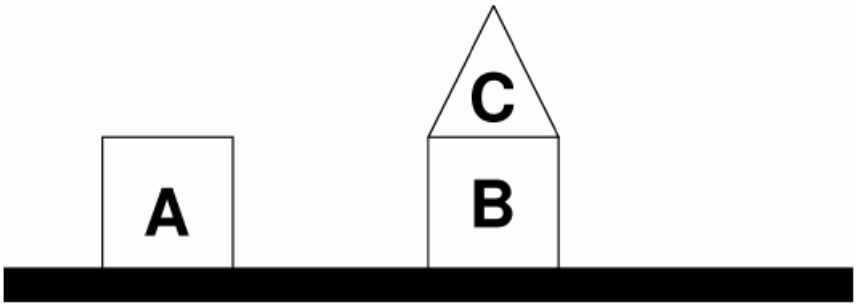
(b) How would you express in a propositional logic sentence:

i. If square $[2, 2]$ has no smell then the Wumpus is not in this square or any of the adjacent squares?

ii. If there is stench in square $[1, 2]$ there must be a Wumpus in this square or any of the adjacent squares?

(c) How can the agent deduce that the Wumpus is in square $[1, 3]$ using the laws of inference in propositional logic.

5. Represent the following scene in propositional calculus.



6. Consider a knowledge base build of just these three weird implications:

$$\neg A \Rightarrow B$$
$$B \Rightarrow A$$
$$A \Rightarrow (C \wedge D)$$

(a) Prove formula $A \wedge C \wedge D$ using Modus Ponens only, or explain why this is not possible.

(b) Prove formula $A \wedge C \wedge D$ using resolution.

7. Given the following symbols and sentences:

C to indicate that Gianni is a climber;
F to indicate that Gianni is fit; L to indicate that Gianni is lucky;
E to indicate that Gianni climbs mount Everest.

(a) Formalize the above sentences in propositional logic:

If Gianni is a climber and he is fit, he climbs mount Everest.
If Gianni is not lucky and he is not fit, he does not climb mount Everest.
Gianni is fit.

(b) Tell if the KB buit in above is consistent, and tell if some of the following sets are models for the above sentences:
$\{\}$; $\{C, L\}$; $\{L, E\}$; $\{F, C, E\}$; $\{L, F, E\}$.

8. Tell whether the propositional formula $[(A \Rightarrow C) \vee (B \Rightarrow C)] \Rightarrow [(A \wedge B) \Rightarrow C]$ is:

   (a) satisfiable;

   (b) valid;

   (c) a contradiction.

9. Do (and test online!) Exercises 5.1, 5.2, and 5.3 in http://intrologic.stanford.edu/notes/chapter_05.html

10. Let A, B, C be propositional symbols. Given $KB = \{A \Rightarrow C, B \Rightarrow C, A \vee B\}$, tell whether C can be derived from KB or not. Use resolution.

11. Heads, I win. Tails, you lose. Use propositional resolution to prove that I always win.

12. Do (and test online!) Exercises 5.4 in http://intrologic.stanford.edu/notes/chapter_05.html

13. There are three suspects for a murder: Adams, Brown, and Clark. Adams saysI didn't do it. The victim was old acquaintance of Brown's. But Clark hated him. Brown states I didn't do it. I didn't know the guy. Besides I was out of town all the week. Clark saysI didn't do it. I saw both Adams and Brown downtown with the victim that day; one of them must have done it. Assume that the two innocent men are telling the truth, but that the guilty man might not be. Write out the facts as sentences in Propositional Logic, and use propositional resolution to solve the crime.