# Rules Reference for the Decorated Gallifrey Type System

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This is the **Decorated** version of the system.

## 1 Grammar

$$
\begin{array}{rrl}
\text{(function)} & fn \in & \mathit{FunctionNames} \\
\text{(variable)} & x \in & \mathit{VariableNames} \\
\text{(class)} & \mathcal{C} \in & \mathit{ClassNames} \\
\text{(region)} & r \in & \mathit{RegionNames} \\
\text{(location)} & l \in & \mathit{LocationNames} \\
\text{(field)} & f \in & \mathit{FieldNames} \\
\text{(type)} & \tau ::= & \mathcal{C} \mid \texttt{int} \mid \texttt{bool} \mid \texttt{unit} \mid (q_{\text{ARG}}\ \tau \to q_{\text{RET}}\ \tau)
\end{array}
$$

$$
\begin{array}{rrl}
\text{(arg qualifier)} & q_{\text{ARG}} ::= & \texttt{preserves} \mid \texttt{consumes} \\
\text{(return qualifier)} & q_{\text{RET}} ::= & \texttt{iso} \mid \texttt{bnd} \\
\text{(function definition)} & \text{FDEF} ::= & \texttt{def}\ q_{\text{RET}}\ \tau\ fn(q_{\text{ARG}}\ \tau\ x)\{e\}@\Omega \\
\text{(program)} & p ::= & \text{FDEF};\ p \mid e
\end{array}
$$

$$
\begin{array}{rrl}
\text{(virtual command)} & \text{VIR} ::= & \texttt{focus}\ x \mid \texttt{unfocus}\ x \mid \texttt{explore}\ x.f@r \mid \texttt{retract}\ x.f \mid \texttt{attach}\ \{e\}\ \texttt{to}\ \{e\} \\
& & \mid \texttt{swap}\ \{e\}\ \texttt{with}\ \{e\} \mid \texttt{drop-var}\ x \mid \texttt{drop-reg}\ \{e\} \mid \texttt{invalidate-var}\ x
\end{array}
$$

$$
\begin{array}{rrl}
\text{(expression)} & e ::= & l \mid x \mid e;e \mid e;\text{VIR} \mid e.f \mid e.f = e \mid x = e \mid fn@r \mid e(e)@\Omega \mid e \oplus_r e \mid \texttt{new-}\tau@r \\
& & \mid \texttt{declare}\ x : \tau\ \texttt{in}\ \{e\} \mid \texttt{if}(e)\{e\}\ \texttt{else}\ \{e\} \mid \texttt{while}(e)\{e\}@r \\
& & \mid \texttt{send-}\tau(e)@r \mid \texttt{recv-}\tau@r \mid \texttt{detach}\ x@r\ \texttt{in}\ \{e\}\ \texttt{else}\ \{e\} \\
\text{(evaluation context)} & E[] ::= & [];e \mid [];\text{VIR} \mid [].f \mid e.f = [] \mid [].f = l \mid x = [] \mid [](e)@\Omega \mid l([])@\Omega \mid [] \oplus_r e \mid l \oplus_r [] \\
& & \mid \texttt{if}([])\{e\}\ \texttt{else}\ \{e\} \mid \texttt{send-}\tau([])@r \mid l;\texttt{drop-reg}\ \{[]\} \\
& & \mid l;\texttt{attach}\ \{[]\}\ \texttt{to}\ \{e\} \mid l;\texttt{attach}\ \{l\}\ \texttt{to}\ \{[]\} \\
& & \mid l;\texttt{swap}\ \{[]\}\ \texttt{with}\ \{e\} \mid l;\texttt{swap}\ \{l\}\ \texttt{with}\ \{[]\}
\end{array}
$$

$$
\begin{array}{rl}
\text{(dynamic reservation)} & d ::= \ l, \ \ d \mid \cdot \\
\text{(heap)} & h ::= \ l \mapsto (\tau, v), \ \ h \mid \cdot \\
\text{(stack)} & s ::= \ x \mapsto l, \ \ s \mid \cdot \\
\text{(regionality)} & \rho ::= \ \mathrm{P}
\end{array}
$$

## 2 Final Rules

$\boxed{\mathrm{F1}^{(\mathrm{D})}}$ - Expression-Well-Typedness

$$
\frac{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}';\Gamma';\Omega' \qquad \vdash d,h,s,\rho : \mathcal{H};\Gamma;\Omega;\mathrm{P}\ \texttt{agree}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash (d,h,s,\rho,e : r\ \tau) \dashv \mathcal{H}';\Gamma';\Omega'}
$$

$\boxed{\mathrm{F2}^{(\mathrm{D})}}$ - Dynamic-Static-Agreement

$$
\frac{\begin{array}{c}\rho, s \vdash h/\mathcal{H}\ \texttt{graph-simple} \qquad \mathcal{H},\rho,h,s \vdash R_d\ \texttt{res-sufficient} \\ \rho,h,s \vdash \mathcal{H}\ \texttt{convex} \qquad \vdash h\ \texttt{heap-closed} \qquad \vdash h,\rho\ \texttt{heap-agree} \qquad \vdash \mathcal{H},\rho,h,s\ \texttt{bnd-ref-sane} \\ \vdash \mathcal{H},\Gamma\ \texttt{binding-agree} \qquad \mathcal{H},s,\rho \vdash \Gamma\ \texttt{binding-sane} \qquad s \vdash \mathcal{H}\ \texttt{non-aliasing} \\ \rho,h,s \vdash \mathcal{H}\ \texttt{target-accurate} \qquad \Omega \vdash \mathcal{H},\Gamma,\rho\ \texttt{well-bounded} \qquad \rho \vdash \mathrm{P}\ \texttt{subsumed}\end{array}}{\vdash d,h,s,\rho : \mathcal{H};\Gamma;\Omega;\mathrm{P}\ \texttt{agree}}
$$

$\boxed{\mathrm{F3}^{(\mathrm{D})}}$ - Graph-Simplicity-Enforcement

$$
\frac{G_S(\mathcal{H},\rho,h,s)\ \texttt{is a forest}}{\rho, s \vdash h/\mathcal{H}\ \texttt{graph-simple}}
$$

$\boxed{\mathrm{F4}^{(\mathrm{D})}}$ - Reservation-Sufficiency

$$
\frac{\mathit{live\text{-}set}(\mathcal{H},\rho,h,s) \subseteq d \subseteq \mathit{dom}(h)}{\mathcal{H},\rho,h,s \vdash d\ \texttt{res-sufficient}}
$$

$\boxed{\mathrm{F5}^{(\mathrm{D})}}$ - H-Convex

$$
\frac{\forall(r,r',\chi) : [((r \in \mathit{dom}(\mathcal{H})) \wedge (r' \in \mathit{dom}(\mathcal{H})) \wedge (\mathcal{H},\rho,h,s \vdash r \hookrightarrow \chi \hookrightarrow r')) \implies (\chi \in \mathit{dom}(\mathcal{H}) \cup \mathit{loc\text{-}refs}(\mathcal{H}))]}{\rho,h,s \vdash \mathcal{H}\ \texttt{convex}}
$$

$\boxed{\mathrm{F6}^{(\mathrm{D})}}$ - Heap-Closure

$$
\frac{\forall(l \in \mathit{dom}(h),\tau,v,f,l') : [((h(l) = (\tau,v) \wedge (v.f = l')) \implies (\exists q_{\mathrm{RET}},\tau_f,v_f : (q_{\mathrm{RET}}\ f\ \tau_f \in \mathit{fields}(\tau) \wedge h(l') = (\tau_f,v_f)))]}{\vdash h\ \texttt{heap-closed}}
$$

$\boxed{\mathrm{F7}^{(\mathrm{D})}}$ - Heap-Rho-Agreement

$$
\frac{\mathit{dom}(h) = \mathit{dom}(\rho) \qquad \forall(l \in \mathit{dom}(h)) : [h \restriction_\tau (l) = \rho \restriction_\tau (l)]}{\vdash h,\rho\ \texttt{heap-agree}}
$$

$\boxed{\mathrm{F8}^{(\mathrm{D})}}$ - Bounded-Ref-Sanity

$$
\frac{\forall(l,l',f) : [(l \in \mathit{live\text{-}set}(\mathcal{H},\rho,h,s) \wedge (h \restriction_v (l).f = l') \wedge (\rho \restriction_r (l) \neq \rho \restriction_r (l'))) \implies (\texttt{iso}\ f\ \tau' \in \mathit{fields}(h \restriction_\tau (l)))]}{\vdash \mathcal{H},\rho,h,s\ \texttt{bnd-ref-sane}}
$$

$\boxed{\text{F9}^{(\text{D})}}$ - H-Gamma-Agreement

$$\frac{\forall(x,r) : [(x@r \in \textit{reg-vars}(\mathcal{H})) \implies ((x \in \textit{dom}(\Gamma)) \land (\Gamma \upharpoonright_r (x) = r))]}{\vdash \mathcal{H}, \Gamma \texttt{ binding-agree}}$$

$\boxed{\text{F10}^{(\text{D})}}$ - Variable-Binding-Sanity

$$\frac{\forall(x,r,\tau) : [(\Gamma \vdash x : r\ \tau) \implies ((x \in \textit{dom}(s)) \land ((r \in \textit{dom}(\mathcal{H})) \implies (\rho(s(x)) = (r,\tau))))]}{\mathcal{H}, \rho, s \vdash \Gamma \texttt{ binding-sane}}$$

$\boxed{\text{F11}^{(\text{D})}}$ - H-Non-Aliasing

$$\frac{\forall(x,x') : [(x, x' \in \textit{vars}(\mathcal{H})) \implies ((x = x') \lor (s(x) \neq s(x')))]}{s \vdash \mathcal{H} \texttt{ non-aliasing}}$$

$\boxed{\text{F12}^{(\text{D})}}$ - H-Target-Accuracy

$$\frac{\forall(x,f,r,r_f) : [((x.f@(r \rightarrowtail r_f) \in \textit{reg-refs}(\mathcal{H})) \land (r_f \in \textit{dom}(\mathcal{H}))) \implies (\rho \upharpoonright_r (h \upharpoonright_v (s(x)).f) = r_f))]}{\rho, h, s \vdash \mathcal{H} \texttt{ target-accurate}}$$

$\boxed{\text{F13}^{(\text{D})}}$ - Omega-Bounding

$$\frac{\textit{dom}(\mathcal{H}) \cup \tau argets(\mathcal{H}) \cup \textit{range}(\rho \upharpoonright_r) \subseteq \Omega \qquad \textit{range}(\Gamma \upharpoonright_r) \subseteq \Omega \cup \{\bot\}}{\Omega \vdash \mathcal{H}, \Gamma, \mathrm{P}, \rho \texttt{ well-bounded}}$$

$\boxed{\text{F14}^{(\text{D})}}$ - Rho-Subsumption

$$\frac{\forall l \in \textit{dom}(\mathrm{P}) : l \in \textit{dom}(\rho) \land \rho(l) = \mathrm{P}(l)}{\rho \vdash \mathrm{P} \texttt{ subsumed}}$$

# 3 Meta Rules

$\boxed{\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi}$

$\boxed{\text{M1A}^{(\text{D})}}$ - Forward-Region-Reachability

$$\frac{\rho(l) = (r,\tau) \qquad \texttt{iso}\ f\ \tau_f \in \textit{fields}(\tau) \qquad \chi = r \qquad \chi' = l.f}{\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi'}$$

$\boxed{\text{M1B}^{(\text{D})}}$ - Backward-Region-Reachability

$$\frac{\begin{array}{c} h \upharpoonright_v (l).f = l' \\ \texttt{iso}\ f\ \tau_f \in \textit{fields}(h \upharpoonright_\tau (l)) \qquad \rho \upharpoonright_r (l') = r \qquad \textit{ref-valid}(\mathcal{H}, s, l, f) \qquad \chi = l.f \qquad \chi' = r \end{array}}{\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi'}$$

$\boxed{\text{M1C}^{(\text{D})}}$ - Transitive-Region-Reachability

$$\frac{\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi' \hookrightarrow \chi''}{\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi''}$$

$\boxed{\text{M1D}^{(\text{D})}}$ - Reflexive-Region-Reachability

$$\mathcal{H}, \rho, h, s \vdash \chi \hookrightarrow \chi$$

$\boxed{h \vdash l \hookrightarrow l}$

$\boxed{\text{M2A}^{(\text{D})}}$ - Forward-Location-Reachability

$$\frac{q_{\text{RET}}\ f\ \tau' \in \mathit{fields}(\tau) \qquad h(l) = (\tau, v) \qquad v.f = l'}{h \vdash l \hookrightarrow l'}$$

$\boxed{\text{M2B}^{(\text{D})}}$ - Transitive-Location-Reachability

$$\frac{h \vdash l \hookrightarrow l' \hookrightarrow l''}{h \vdash l \hookrightarrow l''}$$

$\boxed{\text{M2C}^{(\text{D})}}$ - Reflexive-Location-Reachability

$$h \vdash l \hookrightarrow l$$

$\boxed{h \vdash l \xhookrightarrow{\text{BND}} l}$

$\boxed{\text{M3A}^{(\text{D})}}$ - Forward-Location-Reachability

$$\frac{\texttt{bnd}\ f\ \tau' \in \mathit{fields}(\tau) \qquad h(l) = (\tau, v) \qquad v.f = l'}{h \vdash l \hookrightarrow l'}$$

$\boxed{\text{M3B}^{(\text{D})}}$ - Transitive-Location-Reachability

$$\frac{h \vdash l \xhookrightarrow{\text{BND}} l' \xhookrightarrow{\text{BND}} l''}{h \vdash l \xhookrightarrow{\text{BND}} l''}$$

$\boxed{\text{M3C}^{(\text{D})}}$ - Reflexive-Location-Reachability

$$h \vdash l \xhookrightarrow{\text{BND}} l$$

# 4 Typing Rules

$\boxed{\vdash p}$

$\boxed{\text{T0}^{(\text{D})}}$ - Program Typing

$$\frac{\vdash \text{FDEF}_1 \ \ldots \ \vdash \text{FDEF}_n \qquad \cdot;\cdot;\cdot;\cdot \vdash e : r\ \tau \dashv \mathcal{H}; \Gamma; \Omega}{\vdash \text{FDEF}_1; \ldots; \text{FDEF}_n; e}$$

$$\boxed{\vdash q_{\text{RET}}\ \tau\ fn(q_{\text{ARG}}\ \tau\ x)\{e\}@\Omega}$$

$\boxed{\text{T1}^{(\text{D})}}$ - Function-Definition-Typing

$$\frac{\begin{array}{c}(fn, (q_{\text{ARG}}\ \tau \rightarrow q_{\text{RET}}\ \tau')) \in \mathcal{F} \\ (r^{\dagger}\langle\rangle; x : r\ \tau; \{r\}; \cdot) \vdash e : r'\ \tau' \dashv (\mathcal{H}; x : r_{final}\ \tau; \{r\} \uplus \Omega_{out} \uplus \Omega_{extra}) \\ \vdash (q_{\text{ARG}}\ r \rightarrow q_{\text{RET}}\ r') : (r^{\circ}\langle\rangle; \{r\}) \Rightarrow (\mathcal{H}; \{r\} \uplus \Omega_{out})\end{array}}{\vdash \texttt{def}\ q_{\text{RET}}\ \tau'\ fn(q_{\text{ARG}}\ \tau\ x)\{e\}@\Omega_{out}}$$

$$\boxed{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}; \Gamma; \Omega}$$

$\boxed{\text{T2}^{(\text{D})}}$ - Variable-Ref-Typing

$$\frac{r \in dom(\mathcal{H}) \qquad x : r\ \tau \in \Gamma}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash x : r\ \tau \dashv \mathcal{H}; \Gamma; \Omega}$$

$\boxed{\text{T3}^{(\text{D})}}$ - Sequence-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}'; \Gamma'; \Omega' \qquad \mathcal{H}'; \Gamma'; \Omega'; \cdot \vdash e' : r'\ \tau' \dashv \mathcal{H}''; \Gamma''; \Omega''}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e; e' : r'\ \tau' \dashv \mathcal{H}''; \Gamma''; \Omega''}$$

$\boxed{\text{T4}^{(\text{D})}}$ - Bounded-Field-Reference-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}'; \Gamma'; \Omega' \qquad \texttt{bnd}\ f\ \tau_f \in fields(\tau)}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e.f : r\ \tau_f \dashv \mathcal{H}'; \Gamma; \Omega'}$$

$\boxed{\text{T5}^{(\text{D})}}$ - Isolated-Field-Reference-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \cdot \vdash x : r\ \tau \dashv \mathcal{H}; \Gamma; \Omega \qquad \texttt{iso}\ f\ \tau_f \in fields(\tau) \qquad \mathcal{H} = \mathcal{H}', r^{\circ}\langle x[f \rightarrowtail r_f, F], X\rangle, r_f^{\circ'}\langle X'\rangle}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash x.f : r_f\ \tau_f \dashv \mathcal{H}; \Gamma; \Omega}$$

$\boxed{\text{T6L}^{(\text{D})}}$ - Bounded-Field-Assignment-Typing–Left-Eval

$$\frac{\begin{array}{c}\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e_f : r\ \tau_f \dashv \mathcal{H}', r^{\circ}\langle X\rangle; \Gamma'; \Omega' \\ \mathcal{H}', r^{\dagger}\langle X\rangle; \Gamma'; \Omega'; \cdot \vdash e : r\ \tau \dashv \mathcal{H}'', r^{\dagger}\langle X'\rangle; \Gamma''; \Omega'' \qquad \texttt{bnd}\ f\ \tau_f \in fields(\tau)\end{array}}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e.f = e_f : r\ \tau_f \dashv \mathcal{H}'', r^{\circ}\langle X'\rangle; \Gamma''; \Omega''}$$

$\boxed{\text{T6R}^{(\text{D})}}$ - Bounded-Field-Assignment-Typing–Right-Eval

$$\frac{(l : r\ \tau_f) \in \mathrm{P}, r^{\dagger}\langle X\rangle; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}', r^{\dagger}\langle X'\rangle; \Gamma'; \Omega'}{\mathcal{H}, r^{\circ}\langle X\rangle; \Gamma; \Omega; \mathrm{P} \vdash e.f = l : r\ \tau_f \dashv \mathcal{H}', r^{\circ}\langle X'\rangle; \Gamma'; \Omega'}$$

$\boxed{\text{T7}^{(\text{D})}}$ - Isolated-Field-Assignment-Typing

$$\frac{\begin{array}{c}\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e_f : r_f\ \tau_f \dashv \mathcal{H}', r^{\circ}\langle x[f \rightarrowtail r_{old}, F], X\rangle, r_f^{\circ_f}\langle X_f\rangle; \Gamma'; \Omega' \\ (x : r\ \tau) \in \Gamma' \qquad \texttt{iso}\ f\ \tau_f \in fields(\tau)\end{array}}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash x.f = e_f : r_f\ \tau_f \dashv \mathcal{H}', r^{\circ}\langle x[f \rightarrowtail r_f, F], X\rangle, r_f^{\circ_f}\langle X_f\rangle; \Gamma'; \Omega'}$$

$\boxed{\text{T8}^{(\text{D})}}$ - Assign-Var-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}'; \Gamma', x : r_{old}\ \tau; \Omega' \qquad x \notin vars(\mathcal{H}')}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash x = e : r\ \tau \dashv \mathcal{H}'; \Gamma', x : r\ \tau; \Omega'}$$

$\boxed{\text{T9L}^{(\text{D})}}$ - Function-Application-Typing–Left-Eval

$$\frac{\begin{array}{c} \mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_f : r_f\ (q_{\text{ARG}}\ \tau \to q_{\text{RET}}\ \tau') \dashv \mathcal{H}', r_f^{\circ}\langle X\rangle;\Gamma';\Omega' \\ \mathcal{H}', r_f^{\dagger}\langle X\rangle;\Gamma';\Omega';\cdot \vdash e : r\ \tau \dashv \mathcal{H}'', r_f^{\dagger}\langle X'\rangle;\Gamma'';\Omega'' \\ \vdash (q_{\text{ARG}}\ r \to q_{\text{RET}}\ r') : (\mathcal{H}'', r_f^{\circ}\langle X'\rangle;\Omega'') \Rightarrow (\mathcal{H}''';\Omega'' \uplus \Omega_{out}) \end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_f(e)@\Omega_{out} : r'\ \tau' \dashv \mathcal{H}''';\Gamma'';\Omega'' \uplus \Omega_{out}}$$

$\boxed{\text{T9R}^{(\text{D})}}$ - Function-Application-Typing–Right-Eval

$$\frac{\begin{array}{c} (l_f : r_f\ (q_{\text{ARG}}\ \tau \to q_{\text{RET}}\ \tau')) \in \mathrm{P} \qquad \mathcal{H}, r_f^{\dagger}\langle X\rangle;\Gamma;\Omega;\mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}', r_f^{\dagger}\langle X'\rangle;\Gamma';\Omega' \\ \vdash (q_{\text{ARG}}\ r \to q_{\text{RET}}\ r') : (\mathcal{H}', r_f^{\circ}\langle X'\rangle;\Omega') \Rightarrow (\mathcal{H}'';\Omega' \uplus \Omega_{out}) \end{array}}{\mathcal{H}, r_f^{\circ}\langle X\rangle;\Gamma;\Omega;\mathrm{P} \vdash l_f(e)@\Omega_{out} : r'\ \tau' \dashv \mathcal{H}'';\Gamma';\Omega' \uplus \Omega_{out}}$$

$\boxed{\text{T10}^{(\text{D})}}$ - Function-Name-Typing

$$\frac{(fn, \tau) \in \mathcal{F} \qquad \mathcal{H};\Gamma;\Omega;\cdot \vdash \texttt{new-}\tau@r : r\ \tau \dashv \mathcal{H}';\Gamma';\Omega'}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash fn@r : r\ \tau \dashv \mathcal{H}';\Gamma';\Omega'}$$

$\boxed{\text{T11}^{(\text{D})}}$ - New-Loc-Typing

$$\frac{r \notin \Omega}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash \texttt{new-}\tau@r : r\ \tau \dashv \mathcal{H}, r^{\cdot}\langle\rangle;\Gamma;\Omega \uplus \{r\}}$$

$\boxed{\text{T12}^{(\text{D})}}$ - Declare-Var-Typing

$$\frac{\begin{array}{c} \mathcal{H};\Gamma, x : \bot\ \tau;\Omega;\cdot \vdash e : r\ \tau' \dashv \mathcal{H}';\Gamma', x : r_{final}\ \tau;\Omega' \\ x \notin vars(\Gamma) \cup vars(\Gamma') \cup vars(\mathcal{H}) \cup vars(\mathcal{H}') \end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash \texttt{declare}\ x : \tau\ \texttt{in}\ \{e\} : r\ \tau' \dashv \mathcal{H}';\Gamma';\Omega'}$$

$\boxed{\text{T13L}^{(\text{D})}}$ - OPlus-Typing–Left-Eval

$$\frac{\begin{array}{c} \mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_1 : r_1\ \tau \dashv \mathcal{H}', r_1^{\circ}\langle X\rangle;\Gamma';\Omega' \qquad \mathcal{H}', r_1^{\dagger}\langle X\rangle;\Gamma';\Omega';\cdot \vdash e_2 : r_2\ \tau \dashv \mathcal{H}'', r_1^{\dagger}\langle X'\rangle;\Gamma'';\Omega'' \\ \mathcal{H}'', r_1^{\circ}\langle X'\rangle;\Gamma'';\Omega'';\cdot \vdash \texttt{new-}\tau'@r : r\ \tau' \dashv \mathcal{H}''';\Gamma''';\Omega''' \qquad \vdash \tau \oplus \tau : \tau' \end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_1 \oplus_r e_2 : r\ \tau' \dashv \mathcal{H}''';\Gamma''';\Omega'''}$$

$\boxed{\text{T13R}^{(\text{D})}}$ - OPlus-Typing–Right-Eval

$$\frac{\begin{array}{c} (l_1 : r_1\ \tau) \in \mathrm{P} \qquad \mathcal{H}, r_1^{\dagger}\langle X\rangle;\Gamma;\Omega;\mathrm{P} \vdash e_2 : r_2\ \tau \dashv \mathcal{H}', r_1^{\dagger}\langle X'\rangle;\Gamma';\Omega' \\ \mathcal{H}', r_1^{\circ}\langle X'\rangle;\Gamma';\Omega';\cdot \vdash \texttt{new-}\tau'@r : r\ \tau' \dashv \mathcal{H}'';\Gamma'';\Omega'' \qquad \vdash \tau \oplus \tau : \tau' \end{array}}{\mathcal{H}, r_1^{\circ}\langle X\rangle;\Gamma;\Omega;\mathrm{P} \vdash l_1 \oplus_r e_2 : r\ \tau' \dashv \mathcal{H}'';\Gamma'';\Omega''}$$

$\boxed{\text{T14}^{(\text{D})}}$ - If-Statement-Typing

$$\frac{\begin{array}{c} \mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_b : r_b\ \texttt{bool} \dashv \mathcal{H}';\Gamma';\Omega' \\ \mathcal{H}';\Gamma';\Omega';\cdot \vdash e_t : r\ \tau \dashv \mathcal{H}'';\Gamma'';\Omega_t \qquad \mathcal{H}';\Gamma';\Omega';\cdot \vdash e_f : r\ \tau \dashv \mathcal{H}'';\Gamma'';\Omega_f \end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash \texttt{if}(e_b)\{e_t\}\ \texttt{else}\ \{e_f\} : r\ \tau \dashv \mathcal{H}'';\Gamma'';\Omega_t \cup \Omega_f}$$

$\boxed{\text{T15}^{(\text{D})}}$ - While-Statement-Typing

$$\frac{\begin{array}{c} \mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash e_b : r_b\ \texttt{bool} \dashv \mathcal{H};\Gamma;\Omega' \\ \mathcal{H};\Gamma;\Omega';\cdot \vdash e : r\ \tau \dashv \mathcal{H};\Gamma;\Omega'' \qquad \mathcal{H};\Gamma;\Omega'';\cdot \vdash \texttt{new-unit}@r_u : r_u\ \texttt{unit} \dashv \mathcal{H}';\Gamma';\Omega''' \end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P} \vdash \texttt{while}(e_b)\{e\}@r_u : r_u\ \texttt{unit} \dashv \mathcal{H}';\Gamma';\Omega'''}$$

$\boxed{\text{T16}^{(\mathrm{D})}}$ - Focus-Typing

$$\frac{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle\rangle;\Gamma';\Omega' \qquad (x : r\ \tau \in \Gamma')}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{focus}\ x : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle x[]\rangle;\Gamma';\Omega'}$$

$\boxed{\text{T17}^{(\mathrm{D})}}$ - Explore-Typing

$$\frac{\begin{array}{c}\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle x[F], X\rangle;\Gamma';\Omega' \\ (x : r\ \tau)\in\Gamma' \qquad \texttt{iso}\ f\ \tau' \in \mathit{fields}(\tau) \qquad r_{new}\notin\Omega'\end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{explore}\ x.f@r_{new} : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle x[f\rightarrowtail r_{new}, F], X\rangle, r^{\cdot}_{new}\langle\rangle;\Gamma';\Omega'\uplus\{r_{new}\}}$$

$\boxed{\text{T18}^{(\mathrm{D})}}$ - Retract-Typing

$$\frac{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle x[f\rightarrowtail r_{old}, F], X\rangle, r^{\circ_{old}}_{old}\langle\rangle;\Gamma';\Omega' \qquad r_e\neq r_{old}}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{retract}\ x.f : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}\langle x[F], X\rangle;\Gamma';\Omega'}$$

$\boxed{\text{T19}^{(\mathrm{D})}}$ - Unfocus-Typing

$$\frac{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r\langle x[], X\rangle;\Gamma';\Omega' \qquad (x : r\ \tau)\in\Gamma'}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{unfocus}\ x : r_e\ \tau_e \dashv \mathcal{H}', r\langle X\rangle;\Gamma';\Omega'}$$

$\boxed{\text{T20L}^{(\mathrm{D})}}$ - Attach-Typing–Left-Eval

$$\frac{\begin{array}{c}\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ_e}_e\langle X_e\rangle;\Gamma';\Omega' \\ \mathcal{H}', r^{\dagger}_e\langle X_e\rangle;\Gamma';\Omega';\cdot\vdash e_1 : r_1\ \tau_1 \dashv \mathcal{H}'', r^{\dagger}_e\langle X'_e\rangle, r^{\cdot}_1\langle X_1\rangle;\Gamma'';\Omega'' \\ \mathcal{H}'', r^{\dagger}_e\langle X'_e\rangle, r^{\dagger}_1\langle X_1\rangle;\Gamma'';\Omega'';\cdot\vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}''', r^{\dagger}_e\langle X''_e\rangle, r^{\dagger}_1\langle X'_1\rangle, r^{\circ_2}_2\langle X_2\rangle;\Gamma''';\Omega''' \\ \mathcal{H}_{out} = \mathcal{H}'''[r_1\mapsto r_2], r^{\circ_e}_e\langle X''_e[r_1\mapsto r_2]\rangle, r^{\circ_2}_2\langle X'_1[r_1\mapsto r_2], X_2[r_1\mapsto r_2]\rangle \qquad r_e\neq r_1\end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{attach}\ \{e_1\}\ \texttt{to}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out};\Gamma'''[r_1\mapsto r_2];\Omega'''}$$

$\boxed{\text{T20M}^{(\mathrm{D})}}$ - Attach-Typing–Middle-Eval

$$\frac{\begin{array}{c}(l : r_e\ \tau_e)\in\mathrm{P} \qquad \mathcal{H}, r^{\dagger}_e\langle X_e\rangle;\Gamma;\Omega;\mathrm{P}\vdash e_1 : r_1\ \tau_1 \dashv \mathcal{H}', r^{\dagger}_e\langle X'_e\rangle, r^{\cdot}_1\langle X_1\rangle;\Gamma';\Omega' \\ \mathcal{H}', r^{\dagger}_e\langle X'_e\rangle, r^{\dagger}_1\langle X_1\rangle;\Gamma';\Omega';\cdot\vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}'', r^{\dagger}_e\langle X''_e\rangle, r^{\dagger}_1\langle X'_1\rangle, r^{\circ_2}_2\langle X_2\rangle;\Gamma'';\Omega'' \\ \mathcal{H}_{out} = \mathcal{H}''[r_1\mapsto r_2], r^{\circ_e}_e\langle X''_e[r_1\mapsto r_2]\rangle, r^{\circ_2}_2\langle X'_1[r_1\mapsto r_2], X_2[r_1\mapsto r_2]\rangle \qquad r_e\neq r_1\end{array}}{\mathcal{H}, r^{\circ_e}_e\langle X_e\rangle;\Gamma;\Omega;\mathrm{P}\vdash l;\texttt{attach}\ \{e_1\}\ \texttt{to}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out};\Gamma''[r_1\mapsto r_2];\Omega''}$$

$\boxed{\text{T20R}^{(\mathrm{D})}}$ - Attach-Typing–Right-Eval

$$\frac{\begin{array}{c}(l : r_e\ \tau_e)\in\mathrm{P} \qquad (l_1 : r_1\ \tau_1)\in\mathrm{P} \qquad \mathcal{H}, r^{\dagger}_e\langle X_e\rangle,\ r^{\dagger}_1\langle X_1\rangle;\Gamma;\Omega;\mathrm{P}\vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}', r^{\dagger}_e\langle X'_e\rangle, r^{\dagger}_1\langle X'_1\rangle, r^{\circ_2}_2\langle X_2\rangle;\Gamma';\Omega' \\ \mathcal{H}_{out} = \mathcal{H}'[r_1\mapsto r_2], r^{\circ_e}_e\langle X'_e[r_1\mapsto r_2]\rangle, r^{\circ_2}_2\langle X'_1[r_1\mapsto r_2], X_2[r_1\mapsto r_2]\rangle \qquad r_e\neq r_1\end{array}}{\mathcal{H}, r^{\circ_e}_e\langle X_e\rangle, r^{\cdot}_1\langle X_1\rangle;\Gamma;\Omega;\mathrm{P}\vdash l;\texttt{attach}\ \{l_1\}\ \texttt{to}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out};\Gamma'[r_1\mapsto r_2];\Omega'}$$

$\boxed{\text{T21L}^{(\mathrm{D})}}$ - Swap-Typing–Left-Eval

$$\frac{\begin{array}{c}\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e : r_e\ \tau_e \dashv \mathcal{H}', r^{\circ}_e\langle X_e\rangle;\Gamma';\Omega' \\ \mathcal{H}', r^{\dagger}_e\langle X_e\rangle;\Gamma';\Omega';\cdot\vdash e_1 : r_1\ \tau_1 \dashv \mathcal{H}'', r^{\dagger}_e\langle X'_e\rangle, r^{\cdot}_1\langle X_1\rangle;\Gamma'';\Omega'' \\ \mathcal{H}'', r^{\dagger}_e\langle X'_e\rangle, r^{\dagger}_1\langle X_1\rangle;\Gamma'';\Omega'';\cdot\vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}''', r^{\dagger}_e\langle X''_e\rangle, r^{\dagger}_1\langle X'_1\rangle, r^{\cdot}_2\langle X_2\rangle;\Gamma''';\Omega''' \\ \mathcal{H}_{out} = \mathcal{H}'''[r_1\mapsto r_2, r_2\mapsto r_1], r^{\circ}_e\langle X''_e[r_1\mapsto r_2, r_2\mapsto r_1]\rangle, r^{\cdot}_1\langle X_2[r_1\mapsto r_2, r_2\mapsto r_1]\rangle, r^{\cdot}_2\langle X'_1[r_1\mapsto r_2, r_2\mapsto r_1]\rangle \\ r_e\neq r_1 \qquad r_e\neq r_2\end{array}}{\mathcal{H};\Gamma;\Omega;\mathrm{P}\vdash e;\texttt{swap}\ \{e_1\}\ \texttt{with}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out};\Gamma'''[r_1\mapsto r_2, r_2\mapsto r_1];\Omega'''}$$

$\boxed{\text{T21M}^{(\text{D})}}$ - Swap-Typing–Middle-Eval

$$\frac{\begin{array}{c}(l : r_e\ \tau_e) \in \mathrm{P} \qquad \mathcal{H}, r_e^{\dagger}\langle X_e\rangle; \Gamma; \Omega; \mathrm{P} \vdash e_1 : r_1\ \tau_1 \dashv \mathcal{H}', r_e^{\dagger}\langle X_e'\rangle, r_1^{\cdot}\langle X_1\rangle; \Gamma'; \Omega' \\ \mathcal{H}', r_e^{\dagger}\langle X_e'\rangle, r_1^{\dagger}\langle X_1\rangle; \Gamma'; \Omega'; \cdot \vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}'', r_e^{\dagger}\langle X_e''\rangle, r_1^{\dagger}\langle X_1'\rangle, r_2^{\cdot}\langle X_2\rangle; \Gamma''; \Omega'' \\ \mathcal{H}_{out} = \mathcal{H}''[r_1 \mapsto r_2, r_2 \mapsto r_1], r_e^{\circ}\langle X_e''[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle, r_1^{\cdot}\langle X_2[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle, r_2^{\cdot}\langle X_1'[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle \\ r_e \neq r_1 \qquad r_e \neq r_2\end{array}}{\mathcal{H}, r_e^{\circ}\langle X_e\rangle; \Gamma; \Omega; \mathrm{P} \vdash l; \texttt{swap}\ \{l_1\}\ \texttt{with}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out}; \Gamma''[r_1 \mapsto r_2, r_2 \mapsto r_1]; \Omega''}$$

$\boxed{\text{T21R}^{(\text{D})}}$ - Swap-Typing–Right-Eval

$$\frac{\begin{array}{c}(l : r_e\ \tau_e) \in \mathrm{P} \qquad (l_1 : r_1\ \tau_1) \in \mathrm{P} \qquad \mathcal{H}, r_e^{\dagger}\langle X_e\rangle, r_1^{\dagger}\langle X_1\rangle; \Gamma; \Omega; \mathrm{P} \vdash e_2 : r_2\ \tau_2 \dashv \mathcal{H}', r_e^{\dagger}\langle X_e'\rangle, r_1^{\dagger}\langle X_1'\rangle, r_2^{\cdot}\langle X_2\rangle; \Gamma'; \Omega' \\ \mathcal{H}_{out} = \mathcal{H}'[r_1 \mapsto r_2, r_2 \mapsto r_1], r_e^{\circ}\langle X_e'[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle, r_1^{\cdot}\langle X_2[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle, r_2^{\cdot}\langle X_1'[r_1 \mapsto r_2, r_2 \mapsto r_1]\rangle \\ r_e \neq r_1 \qquad r_e \neq r_2\end{array}}{\mathcal{H}, r_e^{\circ}\langle X_e\rangle, r_1^{\cdot}\langle X_1\rangle; \Gamma; \Omega; \mathrm{P} \vdash l; \texttt{swap}\ \{l_1\}\ \texttt{with}\ \{e_2\} : r_e\ \tau_e \dashv \mathcal{H}_{out}; \Gamma'[r_1 \mapsto r_2, r_2 \mapsto r_1]; \Omega'}$$

$\boxed{\text{T22}^{(\text{D})}}$ - Location-Ref-Typing

$$\frac{(l : r\ \tau) \in \mathrm{P} \qquad r \in dom(\mathcal{H})}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash l : r\ \tau \dashv \mathcal{H}; \Gamma; \Omega}$$

$\boxed{\text{T23}^{(\text{D})}}$ - Send-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r\ \tau \dashv \mathcal{H}'; \Gamma'; \Omega' \qquad \vdash (\texttt{consumes}\ r \rightarrow \texttt{iso}\ r') : (\mathcal{H}'; \Omega') \Rightarrow (\mathcal{H}''; \Omega'')}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash \texttt{send-}\tau(e)@r' : r'\ \texttt{unit} \dashv \mathcal{H}''; \Gamma'; \Omega''}$$

$\boxed{\text{T24}^{(\text{D})}}$ - Receive-Typing

$$\frac{r \notin \Omega}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash \texttt{recv-}\tau()@r : r\ \tau \dashv \mathcal{H}, r^{\cdot}\langle\rangle; \Gamma; \Omega \uplus \{r\}}$$

$\boxed{\text{T25}^{(\text{D})}}$ - Drop-Variable-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r_e\ \tau_e \dashv \mathcal{H}'; \Gamma', x : r\ \tau; \Omega' \qquad x \notin vars(\mathcal{H}')}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e; \texttt{drop-var}\ x : r_e\ \tau_e \dashv \mathcal{H}'; \Gamma'; \Omega'}$$

$\boxed{\text{T26L}^{(\text{D})}}$ - Drop-Region-Typing–Left-Eval

$$\frac{\begin{array}{c}\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r_e\ \tau_e \dashv \mathcal{H}', r_e^{\circ_e}\langle X_e\rangle; \Gamma'; \Omega' \\ \mathcal{H}', r_e^{\dagger}\langle X_e\rangle; \Gamma'; \Omega'; \cdot \vdash e_d : r\ \tau \dashv \mathcal{H}'', r_e^{\dagger}\langle X_e'\rangle, r^{\circ}\langle X'\rangle; \Gamma''; \Omega'' \qquad r \neq r_e\end{array}}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e; \texttt{drop-reg}\ \{e_d\} : r_e\ \tau_e \dashv \mathcal{H}'', r_e^{\circ_e}\langle X_e'\rangle; \Gamma''; \Omega''}$$

$\boxed{\text{T26R}^{(\text{D})}}$ - Drop-Region-Typing–Right-Eval

$$\frac{(l : r_e\ \tau_e) \in \mathrm{P} \qquad \mathcal{H}, r_e^{\dagger}\langle X_e\rangle; \Gamma; \Omega; \mathrm{P} \vdash e_d : r\ \tau \dashv \mathcal{H}', r_e^{\dagger}\langle X_e'\rangle, r^{\circ}\langle X'\rangle; \Gamma'; \Omega' \qquad r \neq r_e}{\mathcal{H}, r_e^{\circ_e}\langle X_e\rangle; \Gamma; \Omega; \mathrm{P} \vdash l; \texttt{drop-reg}\ \{e_d\} : r_e\ \tau_e \dashv \mathcal{H}', r_e^{\circ_e}\langle X_e'\rangle; \Gamma'; \Omega'}$$

$\boxed{\text{T27}^{(\text{D})}}$ - Detach-Typing

$$\frac{\begin{array}{c}x \notin vars(X) \qquad \mathcal{H}, r^{\circ}\langle X\rangle, r_{new}^{\cdot}\langle\rangle; \Gamma, x : r_{new}\ \tau; \Omega; \cdot \vdash e_{succ} : r_{out}\ \tau_{out} \dashv \mathcal{H}'; \Gamma', x : r_{final}\ \tau; \Omega_{succ} \\ \mathcal{H}, r^{\circ}\langle X\rangle; \Gamma, x : r\ \tau; \Omega; \cdot \vdash e_{fail} : r_{out}\ \tau_{out} \dashv \mathcal{H}'; \Gamma', x : r_{final}'\ \tau; \Omega_{fail}\end{array}}{\mathcal{H}, r^{\circ}\langle X\rangle; \Gamma, x : r\ \tau; \Omega; \mathrm{P} \vdash \texttt{detach}\ x@r_{new}\ \texttt{in}\ \{e_{succ}\}\ \texttt{else}\ \{e_{fail}\} : r_{out}\ \tau_{out} \dashv \mathcal{H}'; \Gamma', x : \bot\ \tau; \Omega_{succ} \cup \Omega_{fail}}$$

T28[(D)] - Invalidate-Variable-Typing

$$\frac{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e : r_{out}\ \tau_{out} \dashv \mathcal{H}'; \Gamma', x : r\ \tau; \Omega'}{\mathcal{H}; \Gamma; \Omega; \mathrm{P} \vdash e; \texttt{invalidate-var}\ x : r_{out}\ \tau_{out} \dashv \mathcal{H}'; \Gamma', x : \bot\ \tau; \Omega'}$$

# 5 Heap Rules

$\boxed{\vdash q_{\text{ARG}}\ r : \mathcal{H} \Rightarrow \mathcal{H}}$

H1[(D)] - Consumes-Heap-Effect

$$\vdash \texttt{consumes}\ r : \mathcal{H}, r^{\circ}\langle\rangle \Rightarrow \mathcal{H}$$

H2[(D)] - Preserves-Heap-Effect

$$\vdash \texttt{preserves}\ r : \mathcal{H}, r^{\circ}\langle\rangle \Rightarrow \mathcal{H}, r^{\circ}\langle\rangle$$

$\boxed{\vdash (q_{\text{ARG}}\ r \rightarrow q_{\text{RET}}\ r) : (\mathcal{H}; \Omega) \Rightarrow (\mathcal{H}; \Omega)}$

H3[(D)] - Isolated-Func-Heap-Effect

$$\frac{\vdash q_{\text{ARG}}\ r : \mathcal{H} \Rightarrow \mathcal{H}' \qquad r_{new} \notin \Omega}{\vdash (q_{\text{ARG}}\ r \rightarrow \texttt{iso}\ r_{new}) : (\mathcal{H}; \Omega) \Rightarrow (\mathcal{H}', r^{\cdot}_{new}\langle\rangle; \Omega \uplus \{r_{new}\})}$$

H4[(D)] - Consumes-Bounded-Func-Heap-Effect

$$\frac{\vdash (\texttt{consumes}\ r \rightarrow \texttt{iso}\ r') : (\mathcal{H}; \Omega) \Rightarrow (\mathcal{H}'; \Omega')}{\vdash (\texttt{consumes}\ r \rightarrow \texttt{bnd}\ r') : (\mathcal{H}; \Omega) \Rightarrow (\mathcal{H}'; \Omega')}$$

H5[(D)] - Preserves-Bounded-Func-Heap-Effect

$$\frac{\vdash \texttt{preserves}\ r : \mathcal{H} \Rightarrow \mathcal{H}}{\vdash (\texttt{preserves}\ r \rightarrow \texttt{bnd}\ r) : (\mathcal{H}; \Omega) \Rightarrow (\mathcal{H}; \Omega)}$$

# 6 Evaluation Rules

E1A[(D)] - Common-Context-Step

$$\frac{(d, h, s, \rho, \Omega, e) \xrightarrow{\text{eval}} (d', h', s', \rho', \Omega', e') \qquad e \notin \mathit{VariableNames} \qquad e\ \mathit{non\text{-}detaching}}{(d, h, s, \rho, \Omega, E[e]) \xrightarrow{\text{eval}} (d', h', s', \rho', \Omega', E[e'])}$$

E1B[(D)] - Var-Resolve-Context-Step

$$\frac{\begin{array}{c}(d, h, s, \rho, \Omega, x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l) \\ \mathit{matches\text{-}field\text{-}access}(E) \implies \mathit{matches\text{-}}\textsc{bnd}\mathit{\text{-}fld\text{-}access}(E, x, h, s)\end{array}}{(d, h, s, \rho, \Omega, E[x]) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, E[l])}$$

$\boxed{\text{E2}^{\text{(D)}}}$ - Variable-Ref-Step

$$\frac{s(x) = l \qquad l \in d}{(d, h, s, \rho, \Omega, x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)}$$

$\boxed{\text{E3}^{\text{(D)}}}$ - New-Loc-Step

$$\frac{\texttt{extracts-fresh-heap}(\Omega; \rho, r, \tau; \rho_{new}, h_{new}, l) \qquad d_{new} = dom(h_{new})}{(d, h, s, \rho, \Omega, \texttt{new-}\tau@r) \xrightarrow{\text{eval}} (d \uplus d_{new}, h \uplus h_{new}, s, \rho \uplus \rho_{new}, \Omega \uplus (regs(\rho_{new}) - \{r\}), l)}$$

$\boxed{\text{E4}^{\text{(D)}}}$ - Sequence-Step

$$(d, h, s, \rho, \Omega, l; e) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, e)$$

$\boxed{\text{E5}^{\text{(D)}}}$ - OPlus-Step

$$\frac{l_1, l_2 \in d \qquad l_3 \notin dom(h) \qquad [[\oplus]](h \restriction_v (l_1), h \restriction_v (l_2)) = v_3 \qquad \vdash h \restriction_\tau (l_1) \oplus h \restriction_\tau (l_2) : \tau'}{(d, h, s, \rho, \Omega, l_1 \oplus_r l_2) \xrightarrow{\text{eval}} (d \uplus \{l_3\}, h \uplus (l_3 \mapsto (\tau', v_3)), s, \rho \uplus (l_3 \mapsto (r, \tau')), \Omega, l_3)}$$

$\boxed{\text{E6}^{\text{(D)}}}$ - If-True-Step

$$\frac{h \restriction_v (l) = \texttt{true} \qquad l \in d}{(d, h, s, \rho, \Omega, \texttt{if}(l)\{e_t\}\ \texttt{else}\ \{e_f\}) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, e_t)}$$

$\boxed{\text{E7}^{\text{(D)}}}$ - If-False-Step

$$\frac{h \restriction_v (l) = \texttt{false} \qquad l \in d}{(d, h, s, \rho, \Omega, \texttt{if}(l)\{e_t\}\ \texttt{else}\ \{e_f\}) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, e_f)}$$

$\boxed{\text{E8}^{\text{(D)}}}$ - While-Step

$$\frac{\begin{array}{c}\Omega_{new} \cap \Omega = \emptyset \qquad \phi \in bijections(NR(e_{body}) \uplus NR(e_{bool}), \Omega_{new}) \\ e = \texttt{while}(e_{bool})\{e_{body}\}@r_u \qquad e' = \texttt{if}(e_{bool})\{e_{body}; \phi(e)\}\ \texttt{else}\ \{\texttt{new-unit}@r_u\}\end{array}}{(d, h, s, \rho, \Omega, e) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega \uplus \Omega_{new}, e')}$$

$\boxed{\text{E9}^{\text{(D)}}}$ - Declare-Var-Step

$$(d, h, s, \rho, \Omega, \texttt{declare}\ x : \tau\ \texttt{in}\ \{e\}) \xrightarrow{\text{eval}} (d, h, s[x \mapsto \bot], \rho, \Omega, e; \texttt{drop-var}\ x)$$

$\boxed{\text{E10}^{\text{(D)}}}$ - Assign-Var-Step

$$\frac{l \in d}{(d, h, s \uplus (x \mapsto l_{old}), \rho, \Omega, x = l) \xrightarrow{\text{eval}} (d, h, s \uplus (x \mapsto l), \rho, \Omega, l)}$$

$\boxed{\text{E11}^{\text{(D)}}}$ - Function-Application-Step

$$\frac{\begin{array}{c}\Omega_{new} \cap \Omega \subseteq \Omega'_{out} \\ \phi \in bijections(NR(e), \Omega_{new}) \qquad \phi(\Omega_{out}) = \Omega'_{out} \qquad l_f, l \in d \qquad h(l_f) = ((q_{\text{ARG}}\ \tau \to q_{\text{RET}}\ \tau'), v_f) \\ F_d(v_f) = \lambda x.e@\Omega_{out} \qquad e \equiv_\alpha e' \qquad FV(e') = \{x\} \qquad vars(e') \cap dom(s) = \emptyset\end{array}}{(d, h, s, \rho, \Omega, l_f(l)@\Omega'_{out}) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega \uplus (\Omega_{new} - \Omega'_{out}), \texttt{declare}\ x : \tau\ \texttt{in}\ \{x = l; \phi(e')\})}$$

$\boxed{\text{E14}^{(\text{D})}}$ - Bounded-Reference-Step

$$\frac{l, l_f \in d \qquad h \restriction_v (l).f = l_f \qquad \texttt{bnd}\ f\ \tau \in \mathit{fields}(h \restriction_\tau (l))}{(d, h, s, \rho, \Omega, l.f) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l_f)}$$

$\boxed{\text{E15}^{(\text{D})}}$ - Isolated-Reference-Step

$$\frac{l, l_f \in d \qquad s(x) = l \qquad h \restriction_v (l).f = l_f \qquad \texttt{iso}\ f\ \tau \in \mathit{fields}(h \restriction_\tau (l))}{(d, h, s, \rho, \Omega, x.f) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l_f)}$$

$\boxed{\text{E16}^{(\text{D})}}$ - Bounded-Assignment-Step

$$\frac{l, l_f \in d \qquad \texttt{bnd}\ f\ \tau_f \in \mathit{fields}(\tau)}{(d, h \uplus (l \mapsto (\tau, v)), s, \rho, \Omega, l.f = l_f) \xrightarrow{\text{eval}} (d, h \uplus (l \mapsto (\tau, v[f \mapsto l_f])), s, \rho, \Omega, l_f)}$$

$\boxed{\text{E17}^{(\text{D})}}$ - Isolated-Assignment-Step

$$\frac{s(x) = l \qquad l, l_f \in d \qquad \texttt{iso}\ f\ \tau_f \in \mathit{fields}(\tau)}{(d, h \uplus (l \mapsto (\tau, v)), s, \rho, \Omega, x.f = l_f) \xrightarrow{\text{eval}} (d, h \uplus (l \mapsto (\tau, v[f \mapsto l_f])), s, \rho, \Omega, l_f)}$$

$\boxed{\text{E18A}^{(\text{D})}}$ - Focus-Step

$$(d, h, s, \rho, \Omega, l; \texttt{focus}\ x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E18B}^{(\text{D})}}$ - Unfocus-Step

$$(d, h, s, \rho, \Omega, l; \texttt{unfocus}\ x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E18C}^{(\text{D})}}$ - Explore-Step

$$\frac{r_{old} = \rho \restriction_r (h \restriction_v (s(x)).f)}{(d, h, s, \rho, \Omega, l; \texttt{explore}\ x.f@r_{new}) \xrightarrow{\text{eval}} (d, h, s, \rho[r_{old} \mapsto r_{new}], \Omega, l)}$$

$\boxed{\text{E18D}^{(\text{D})}}$ - Retract-Step

$$(d, h, s, \rho, \Omega, l; \texttt{retract}\ x.f) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E18E}^{(\text{D})}}$ - Attach-Step

$$\frac{r_1 = \rho \restriction_r (l_1) \qquad r_2 = \rho \restriction_r (l_2)}{(d, h, s, \rho, \Omega, l; \texttt{attach}\ l_1\ \texttt{to}\ l_2) \xrightarrow{\text{eval}} (d, h, s, \rho[r_1 \mapsto r_2], l)}$$

$\boxed{\text{E18F}^{(\text{D})}}$ - Drop-Variable-Step

$$(d, h, s, \rho, \Omega, l; \texttt{drop-var}\ x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E18G}^{(\text{D})}}$ - Drop-Region-Step

$$(d, h, s, \rho, \Omega, l; \texttt{drop-reg}\ l_d) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E18H}^{(\text{D})}}$ - Swap-Step

$$\frac{r_1 = \rho \restriction_r (l_1) \qquad r_2 = \rho \restriction_r (l_2)}{(d, h, s, \rho, \Omega, l; \texttt{swap } l_1 \texttt{ with } l_2) \xrightarrow{\text{eval}} (d, h, s, \rho[r_1 \mapsto r_2, r_2 \mapsto r_1], \Omega, l)}$$

$\boxed{\text{E18I}^{(\text{D})}}$ - Invalidate-Variable-Step

$$(d, h, s, \rho, \Omega, l; \texttt{invalidate-var } x) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, l)$$

$\boxed{\text{E19}^{(\text{D})}}$ - Function-Name-Step

$$\frac{(fn, \tau) \in \mathcal{F} \qquad v_f = F_v(fn) \qquad l \notin dom(h)}{(d, h, s, \rho, \Omega, fn@r) \xrightarrow{\text{eval}} (d \uplus \{l\}, h \uplus (l \mapsto (\tau, v_f)), s, \rho \uplus (l \mapsto (r, \tau)), \Omega, l)}$$

$\boxed{\text{E20A}^{(\text{D})}}$ - Detach-Step–Success

$$\frac{\begin{array}{c} \textit{heap-separable}(h, s, E^*[], x) \\ \rho = \bar{\rho} \uplus \rho_{sep} \qquad dom(\rho_{sep}) = \textit{min-reg}(h, s(x)) \cup \{l \in dom(h) \mid h \vdash s(x) \hookrightarrow l\} \\ \phi \in \textit{bijections}(range(\rho_{sep} \restriction_r), r_{new} \uplus \Omega_{new}) \qquad \phi(\rho_{sep} \restriction_r (s(x))) = r_{new} \qquad \rho' = \bar{\rho} \uplus \phi(\rho_{sep}) \end{array}}{(d, h, s, \rho, \Omega, E^*[\texttt{detach } x@r_{new} \texttt{ in } \{e_{succ}\} \texttt{ else } \{e_{fail}\}]) \xrightarrow{\text{eval}} (d, h, s, \rho', \Omega \uplus \Omega_{new}, E^*[e_{succ}; \texttt{invalidate-var } x])}$$

$\boxed{\text{E20B}^{(\text{D})}}$ - Detach-Step–Failure

$$\frac{\neg \textit{heap-separable}(h, s, E^*[], x)}{(d, h, s, \rho, \Omega, E^*[\texttt{detach } x@r_{new} \texttt{ in } \{e_{succ}\} \texttt{ else } \{e_{fail}\}]) \xrightarrow{\text{eval}} (d, h, s, \rho, \Omega, E^*[e_{fail}; \texttt{invalidate-var } x])}$$

# 7 Concurrency Rules

$\boxed{\vdash \ (h, \overline{\langle d, s, e \rangle})}$

$\boxed{\text{TC1}^{(\text{D})}}$ - Concurrent-Well-Typedness

$$\frac{\begin{array}{c} \forall i \in \{1..n\} : (\mathcal{H}_i; \Gamma_i; \Omega_i; \text{P} \vdash e_i : r_i \ \tau_i \dashv \mathcal{H}'_i; \Gamma'_i; \Omega'_i) \wedge (d_i, h, s_i : \mathcal{H}_i; \Gamma_i; \Omega_i; \text{P } \texttt{agree}) \\ \forall i, j \in \{1..n\} : (d_i \cap d_j \neq \emptyset \implies i = j) \end{array}}{\vdash \ (h, \overline{\langle d_n, s_n, e_n \rangle}) \texttt{ well-typed}}$$

$\boxed{(d, s, e; d, s, e) \xrightarrow{\text{comm-eval}} (d, s, e; d, s, e)}$

$\boxed{\text{EC1}^{(\text{D})}}$ - Concurrent-Single-Step

$$\frac{j \in \{1..n\} \qquad (d_j, h, s_j, e_j) \xrightarrow{\text{eval}} (d'_j, h', s'_j, e'_j) \qquad \forall i \in \{1..n\} - \{j\} : (d'_i, s'_i, e'_i) = (d_i, s_i, e_i)}{(h, \overline{\langle d_n, s_n, e_n \rangle}) \xrightarrow{\text{concur-eval}} (h', \overline{\langle d'_n, s'_n, e'_n \rangle})}$$

$\boxed{\text{EC2}^{(\text{D})}}$ - Concurrent-Paired-Step

$$\frac{a, b \in \{1..n\} \qquad h \vdash (d_a, s_a, e_a; d_b, s_b, e_b) \xrightarrow{\text{comm-eval}} (d'_a, s'_a, e'_a; d'_b, s'_b, e'_b) \quad \forall n \in \{1..n\} - \{a, b\} : (d'_n, s'_n, e'_n) = (d_n, s_n, e_n)}{(h, \overline{\langle d_n, s_n, e_n \rangle}) \xrightarrow{\text{concur-eval}} (h, \overline{\langle d'_n, s'_n, e'_n \rangle})}$$

$$\boxed{(d, s, e; d, s, e) \xrightarrow{\text{comm-eval}} (d, s, e; d, s, e)}$$

$\boxed{\text{EC3}^{(\text{D})}}$ - Communication-Paired-Step

$$\frac{d_{sep} = \{l \in dom(h) : h \vdash l_{root} \hookrightarrow l\}}{h \vdash (d_a \uplus d_{sep}, s_a, E^*_a[\texttt{send-}\tau(l_{root})]; d_b, s_b, E^*_b[\texttt{recv-}\tau()]) \xrightarrow{\text{comm-eval}} (d_a, s_a, E^*_a[\texttt{new-unit}]; d_b \uplus d_{sep}, s_b, E^*_b[l_{root}])}$$