1. $n \geq 1$

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

$$1\cdot 2 = \frac{1(1+1)(1+2)}{3} = \frac{1(2)(3)}{3} = \frac{6}{3} = 2$$

$\boxed{1\cdot 2 = 2}$ base

if $p(1)$ and $p(n)$ is true then $p(n+1)$ will also be true

$$1\cdot 2 + \dots n = \frac{n(n+1)(n+2)}{3}$$

$$(n+1)((n+1)+1) = \frac{n(n+1)(n+2)}{3} + (n+1)((n+1)+1) = \text{''} + \frac{3(n+1)(n+2)}{3}$$

$(n+1)(n+2)$

$$= \frac{n(n+1)(n+2) + 3(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$(n+1)$ is then $\dfrac{(n+1)((n+1)+1)((n+1)+2)}{3} = \dfrac{(n+1)(n+2)(n+3)}{3}$

(2)

$4\cdot(1)+7\cdot(2)=18$

$4\cdot 3+7\cdot 1=19$

$4\cdot 0+7\cdot 3=21$

$4\cdot 2+7\cdot 2=22$

$4\cdot 4+7\cdot 1=23$

$4\cdot 6+7\cdot 0=24$

Base:

Inductive step

case 1: if in sum of of 18 ¢ or more, if 1 or moe 7 ¢ used (n+1) can be shown by remoing one 7 ¢ and replacing it with 2, 4 ¢ stamps

case 2: 4 or more 4 ¢ stamps used can be replaced with 3 7 ¢ stamps for n replaced with (n+1)

(3) Using information above ↑

Inductive step

Assume we can make postage ≥ 18 ¢ .... n

$n \geq 24$, By induction we can make $(n+1)-2\cdot 4$ ¢ $= n-7$

just add 2 4 ¢ stamps to get (n+1) ¢ postage

④ $\frac{\text{Base}}{n=0}$ $a_0 = 2\cdot(0)+2 = 2$ get to $a_n = 2^{n+1}-2$

$a_1 = 2\cdot 1 + 2 = \boxed{4}$

have $= 0$

$a_{n \geq 1} = 2(a_{n-1})+2$

$2(2^n)-2$

$\boxed{= 2^{n+1}-2}$

⑤ $\boxed{5 \mid n^5 - n}$ = hypothesis

Basis: $1^5 - 1 = 0$ which is a multiple of 5

$(n+1)^5 - (n+1)$

$(n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1) - (n+1)$

$n^5 + \underbrace{5n^4 + 10n^3 + 10n^2 + 5n}_{\text{divisible by } 5} - n$

$5 \mid n^5 - n$

is the initial given hypothesis