1

A $P(5) \quad 5^2 \not> 4$ F

B $\neg \forall x \, P(x) = \exists x \neg P(x)$ T

there exists positive integers that when squared are larger than 4. Also could be said numbers that are not less than or equal to 4

$4^2 > 4 = 16 > 4$

2

x has read y

x: class
y: books by mark Twain

A $\forall x P(x, H.F) =$ **all of the students in the class have read Huck Finn**

B $\exists x \forall y P(x,y) =$ **there is a student in the class who has read all of the books by Mark Twain**

C $\forall y \exists x P(x,y) =$ **all of the books by Mark Twain have been read by the class**

3

A

i) $(\neg\forall x)\, P(x, \text{ralph})$

ii) $\forall x\, P(x, \text{Fred})$

iii) $\exists x \forall y\, P(x, y)$

B

i) $\neg\forall x\, P(x, \text{ralph}) = \exists x\, (P(x, \text{ralph}))$

negating $= (\neg\exists x \neg P(x, \text{ralph}))$

$= \boxed{\exists x\, P(x, \text{ralph})}$

ii) $\neg\forall x\, P(x, \text{Fred}) = \boxed{\exists x \neg P(x, \text{Fred})}$

iii) $\exists x \forall y\, P(x, y)$

negated $= \forall x \exists y \neg P(x, y)$

C

i) everybody trusts ralph

ii) somebody doesn't trust Fred

iii) Everybody doesn't trust somebody

4

$p \rightarrow q$

$\neg p$

p	q	$p \rightarrow q \wedge \neg p$			$\neg q$
T	T	T	F	F	F
T	T	F	F	F	F
F	T	F	F	T	F
F	F	T	T	T	T

values in boxes are =

valid

(5) s = ralph has a sore shoulder p = buys popcorn

f = ralph feels sick

b = " goes bowling

m = goes to the movie

$$\neg s \vee \neg f \rightarrow b \wedge m$$

$$m \rightarrow p$$

$$\neg p$$

$$\therefore s$$

(1)	$(\neg s) \vee (\neg f) \rightarrow (b \wedge m)$	hypoth
(2)	$m \rightarrow p$	hyp
(3)	$\neg p$	hyp
(4)	$\neg m$	modus tollens
(5)	$\neg(b \wedge m) \rightarrow \neg(\neg s \vee \neg f)$	contrapositive

- (6) $(\neg b \vee \neg m) \rightarrow (s \wedge f)$ double negation

(7) $\neg m \vee \neg b$ Community

(8) $s \wedge f$ modus ponens

(9) $\therefore s$ simplification

Bonus:

$\exists x (A(x) \wedge \neg B(x))$

$\forall (A(x) \rightarrow C(x))$

$\therefore \exists x (C(x) \wedge \neg B(x))$

① $\exists x (A(x) \wedge \neg B(x))$ — hypothesis

② $\forall (A(x) \rightarrow C(x))$

③ $A(m) \wedge \neg B(m)$ — existential instantiation

④ $A(m) \rightarrow C(m)$ — universal instantiation

⑤ $A(m)$ — simplification

⑥ $\neg B(m)$

⑦ $C(m)$ — modus ponens of ④

⑧ $C(m) \wedge B(m)$ — addition ⑥, ⑦

⑨ $\exists (m)((C(m) \wedge B(m))$ — ex- generalization

$\exists (m) (C(m) \wedge \neg B(m))$