Lesson 10

Warning: It's not unusual to find these problems really tough. One difficulty is that these problems will make some demands on your algebra skills. Another difficulty is just getting the format of an inductive proof correct. Here are a few suggestions concerning that: (1) Be sure to check the basis case; (2) At the start of the inductive step, clearly state the inductive hypothesis; (3) After stating the inductive hypothesis, write down what you need to prove; (4) Point out clearly where you apply the inductive hypothesis in the proof of the inductive step.

(1) Use induction to prove: For every integer $n \geq 1$,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

(2) Use the first form of induction (see example 16.5) to show that using only 4¢ stamps and 7¢ stamps, any postage amount 18¢ or greater can be formed.

(3) Redo the previous problem using the second form of induction (see example in the text). The basis step is just a bit messier this time, but the inductive step is much easier.

(4) A sequence is defined recursively by the rules: (1) $a_0 = 0$, and (2) for $n \geq 1$, $a_n = 2a_{n-1} + 2$. Use induction to prove $a_n = 2^{n+1} - 2$ for all $n \geq 0$.

(5) Use induction to prove: For every integer $n \geq 1$, the number $n^5 - n$ is a multiple of 5.
Hint: An integer is a multiple of 5 if it is 5 times some integer. For example, $165 = (5)(33)$ so 165 is a multiple of 5.

(6) (bonus) Here is a *proof* that for $n \geq 0$, $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1}$.

Proof. Suppose $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1}$ for some $n \geq 0$. Then

$$\begin{aligned} 1 + 2 + 2^2 + \cdots + 2^n + 2^{n+1} &= 2^{n+1} + 2^{n+1} \qquad \text{using the inductive hypothesis} \\ &= 2(2^{n+1}) = 2^{n+2} = 2^{(n+1)+1}, \end{aligned}$$

as we needed to show. □

Now, obviously there is something wrong with this proof by induction since, for example, $1 + 2 + 2^2 = 7$, but $2^{2+1} = 2^3 = 8$. What specifically is wrong with the proof?