Lesson 15

(1) (a) Suppose we have a 52 card deck with the cards in order ace $, 2, 3, \ldots,$ queen, king top to bottom, for clubs, then diamonds, then hearts, then spades. A step consists to taking the top card and moving it to the bottom of the deck. We start with the ace of clubs as the top card. After two steps, the top card is the 3 of clubs. What is the top card after 735 steps?

(b) The marks on a combination lock are numbered 0 to 39. If the lock is at mark 19, and the dial is turned one mark clockwise, it will be at mark 18. If the lock is at mark 19 and turned 137 marks clockwise, at what mark will it be?

(2) (a) Arrange the numbers $-39, -27, -8, 11, 37, 68, 91$ so they are in the order $0, 1, 2, 3, 4, 5, 6$ modulo 7.

(b) Determine n between 0 and 16 such that $311 + 891 \equiv n \pmod{17}$.

(c) Determine n between 0 and 16 such that $(405)(777) \equiv n \pmod{17}$.

(d) Determine n between 0 and 16 such that $710^{447} \equiv n \pmod{17}$.

(3) Find all solutions: $33x \equiv 183 \pmod{753}$.

(4) A multiple choice test contains 10 questions. There are four possible answers for each question.

(a) How many ways can a student complete the test if every question must be answered?

(b) How many ways can a student complete the test if questions can be left unanswered?

(5) Here are two questions most easily done using the $Good = Total - Bad$ method.

(a) How many seven-letter words contain at least one X?

(b) How many seven-letter words contain at least two X's? Hint: The *Bad* ones are those with no X's and those with exactly one X. Think carefully about counting the number of words with exactly one X.

(6) (bonus) A code word is either a sequence of three letters followed by two digits or two letters followed by three digits. (Unless otherwise indicated, *letters* will means letters chosen from the usual 26-letter alphabet and *digits* are selected from $\{0, 1, 2, 3, \ldots, 8, 9\}$. Also, unless it is stated that letters have to be different, you should assume repeats are allowed. Likewise for digits.) How many different code words are possible?