$U_f(|x\rangle|y\rangle) = |x\rangle\, |f(x) \otimes y\rangle$

1

A $f(x) = 0$ for all x or thus 2 constant

$f(x) = 1$ for all x

also 2 balanced values as $2^{(2-1)} = 2$

B

still 2 constant functions

C $\{$array has; $q+1)$ with 2^n values thus $(q+1)^{(2^n)}$

constant 0 or 1 thus 2^1 total $= \boxed{2^{q+1}}$

D n^m = total functions

2 $|\psi\rangle_{ab}\, |\psi\rangle_{ac}\, |\phi\rangle_{\alpha\beta} = \frac{1}{\sqrt{2}}\left(\left(|00\rangle_{(ab)} + |11\rangle_{(ab)}\right)\otimes\left(|00\rangle_{(ac)} + |11\rangle_{ac}\right)\otimes\left(|00\rangle_{\alpha\beta} + |11\rangle_{\alpha\beta}\right)\right)$

Observed and C-Not operation:

$$\frac{1}{2}\left(|00\rangle_{(abc)} + |11\rangle_{(abc)}\right)\otimes\left(|00\rangle_{(\alpha\beta)} + |11\rangle_{(\alpha\beta)}\right)$$

$$1/2\left(\left(|00\rangle_{bc} + |11\rangle_{bc}\right)\otimes\left(|00\rangle_a + X_b|1\rangle_a + Z_c|1\rangle + Z_c X_b |1\rangle_a\right)\otimes\left(|00\rangle_{\alpha\beta} + |11\rangle_{\alpha\beta}\right)\right)$$

$$|\psi\rangle_{abcd\alpha\beta} = \frac{1}{2}\left(|00\rangle_{ac}\otimes|0\rangle_b + |11\rangle_{ac}\otimes|1\rangle_b\right)\otimes\frac{1}{2}\left(|00\rangle_{\alpha\beta} + |11\rangle_{\alpha\beta}\right)\otimes|+\rangle$$

If 00: I

01: X

10: Z

11: ZX

$$= \frac{1}{2}\left(|00\rangle_{ac} + |11\rangle_{ac}\right) \otimes \frac{1}{2}\left(|00\rangle_{aB} + |11\rangle_{aB}\right) \otimes$$

$$\left(|0\rangle_b + |1\rangle_b\right) \otimes |\psi\rangle_c)$$

$$|\psi\rangle_{bc} = \frac{1}{\sqrt{2}}\left(|00\rangle_{bc} + |11\rangle_{bc}\right)$$

I got a bit lost while doing this problem but I believe that the end result does show that the two are an entangled pair as the end result os the bell state: 00 which is entangled.

3) A

$$\frac{1}{\sqrt[3]{2}}(|00\rangle_{AA'} + |11\rangle_{AA'}) \otimes (\frac{1}{\sqrt[3]{2}}(|00\rangle_{BB'} + |11\rangle_{BB'})) \otimes (\frac{1}{\sqrt[3]{2}}(|00\rangle_{CC'} + |11\rangle_{CC'}))$$

leads to → $\frac{1}{\sqrt[3]{2}}\left(|0\rangle_a \otimes |0\rangle_b \otimes |0\rangle_c \text{ .... for } a'b'c'\right) +$

$$\frac{1}{\sqrt[3]{2}}\left(|1\rangle_a \otimes |1\rangle_b \otimes |1\rangle_c \text{ .... for } a'b'c'\right)$$

$$|000\rangle_{abc} \otimes \frac{1}{\sqrt[3]{2}}\left(|00\rangle_{a'b'c'} + |11\rangle_{a'b'c'}\right)$$

in $|000\rangle_{abc}$

result

$$\boxed{\frac{1}{\sqrt[3]{2}}\left(|00\rangle_{a'b'c'} + |11\rangle_{a'b'c'}\right) = \text{entangled}}$$

B) GHZ_+

$$\frac{1}{2}\left(|000\rangle_{a'b'c'} + |011\rangle_{a'b'c'} + |101\rangle_{a'b'c'} + |110\rangle_{a'b'c'}\right.$$

$$+ 1/2\left(|111\rangle + |100\rangle + |010\rangle + |001\rangle\right)_{A'B'C'}$$

Entangled