

$$1) \quad \text{assume:} \quad m = 2k+1$$

$$k, j \in \mathbb{Z}$$

$$\text{even} = 2j$$

$$n = 2j$$

$$\text{hypothesis} \quad m+n = 2k+1+2j$$

$$\text{even} + \text{even} = \text{even}$$

$$m+n = (\text{even}) + (\text{even}) + 1$$

$$\text{thus odd by odd} = 2k+1$$

$$\uparrow \text{even}$$

**answer:**

A number  $m$  is odd if  $m = 2k+1$  and a number  $n$  is even if  $n = 2j$ . The sum of  $m+n = 2k+2j+1$ . This is the sum of 2 even numbers and 1. An even number plus another even number is even. So this now follows the form of  $m = 2k+1$  as it is an even number to which 1 is added to making it odd.

$$2) \quad m^3 = \text{even} \quad ?$$

$$p \rightarrow q$$

$$m = \text{even}$$

$$\neg q \rightarrow \neg p$$

Proof: where  $k$  is an Int

If  $m$  is not even, odd, then  $m^3$  is odd.  $m$  is then equal to  $m = 2k+1$  for some  $k$ . Cubing the equation gets  $m^3 = (2k+1)^3$ . Simplified to  $8k^3 + 12k^2 + 6k + 1 = m^3 = 2(k^3 + 6k^2 + 3k) + 1$ . This shows  $m^3$  is odd, not even, through the use of the contrapositive

$$\textcircled{3} \quad S_{n-4} \text{ odd} \\ n \text{ is odd}$$

$$\text{If } S_{n-4} \text{ is odd} \rightarrow n \text{ is odd}$$

Scratch

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Suppose  $S_{n-4}$  is odd and  $n$  is even,

$$S_{n-4} = S(2k)-4 \quad n = 2k$$

$$S_{n-4} \geq 10k-4$$

$$S_n \geq 10k$$

$$\underline{n \geq 2k} \quad \text{scribble}$$

Proof:

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Suppose  $S_{n-4}$  is odd and  $n$  is even in a proof by contradiction. The integer  $n$  to be even must

equal  $2k$  for some  $k$ . Thus  $S_{n-4} = S(2k)-4$  leading to  $n = 2k$ .

This shows  $S_{n-4}$  is even when  $n$  is even and vice versa, a contradiction to the original assumption

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predicate  $P(n)$

true 1- billion every

false all

$$P(n) : n < (\text{billion} + 1)$$

$$\forall n, P(n) = \{n \mid n < \text{billion} + 1\}$$

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213 is not prime, ~~3421~~  $213 = 3 \times 71$

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$$s, t \in \mathbb{R}$$

$$\min(s, t) + \max(s, t) = s + t$$

case 1:  $\min(s, t) = s$  thus  $\max(s, t) = t$

taking  $\min(s, t) = s$  plus  $\max(s, t) = t$  gives  $s + t$

$$s + t = s + t$$

case 2:  $\min(s, t) = t$  thus  $\max(s, t) = s$   
taking  $\min(s, t) = t$  plus  $\max(s, t) = s$  gives  $s + t$

$$t + s = s + t$$

If  $s = t$  then  $s + s = t + t$  as  $s$  and  $t$  are interchangeable