Lesson 16

(1) (a) (a permutation problem, order matters) An ID code consists of 4 different letters and 3 different digits. How many codes are possible if the letters must be kept together? For example, $34ABCD9$ and $WAXT604$ are good, but $TR67YZ3$ is bad.

(b) (a combination problem, order does not matter) A lottery ticket consists of four different integers between 1 and 99 (inclusive). How many different lottery tickets are there?

(2) Determine the coefficient of x^4y^5 in the expansion of $(3x-2y)^9$. Be sure to take the 3 and the -2 coefficients into account.

(3) Give an algebraic proof that $\binom{2n}{2} = 2\binom{n}{2} + n^2$.

Hint: The idea is to expand and simplify each side of the equation, then compare the two results and make sure they are the same. This problem seems to cause a lot of anguish. It might help if the two sides are computed for a few specific values of n to see how the arithmetic works out. Try $n = 3$ and $n = 4$ for example.

(4) A poker hand consists of five cards selected from a 52 card deck. The order of the cards in a poker hand does not matter. A poker hand is called a *full house* if it has two cards of one rank and three cards of a second rank. For example, a hand consisting of two 7's and three *queens* is a full house. How many different full house hands are there?

(5) How many permutations of the letters *a,b,c,d,e,f,g* either begin with an *a* or end with an *a*?

(6) (bonus) A committee of size five is selected from a group of ten clowns and twelve lion tamers.

(a) How many different committees are possible?

(b) How many committees are possible if there must be exactly two clowns on the committee?

(c) How many committees are possible if lion tamers must outnumber clowns on the committee?