Lesson 2

(1) Let $P(x) : x^2 \leq 4$. The domain for x is all positive integers $(1, 2, 3, \ldots)$. Determine the truth values of the following propositions.
  (a) $P(5)$
  (b) $\neg \forall x \, P(x)$

(2) Let $P(x, y)$ be x *has read* y, where the domain of discourse for x is all students in this class, and the domain of discourse for y is all books written by Mark Twain. Express the following propositions in English.
  (a) $\forall x \, P(x, \text{Huckleberry Finn})$.
  (b) $\exists x \, \forall y \, P(x, y)$.
  (c) $\forall y \, \exists x \, P(x, y)$.

(3) Let $F(x, y)$ be the statement x *trusts* y, where the domain of discourse for both x and y is all people.
  (a) Use quantifiers to express each of the following propositions in symbols.
    (i) Nobody trusts Ralph.
    (ii) Everybody trusts Fred.
    (iii) Somebody trusts everybody.
  (b) Now write the negation of those propositions in symbols. The cheap way would be to simply write $\neg$ in front of each of the answers to part (a). Don't do that. Use the rules discussed in the text so your answer does not have $\neg$ occurring to the left of any quantifiers.
  (c) Express each of those negations in an English sentence.

(4) Show that $p \longrightarrow q$ *and* $\neg p, \quad \therefore \neg q$ is not a valid rule of inference. It is called the fallacy of denying the hypothesis. To expand a bit on the reading for this lesson, here is how to show a proposed rule of inference is valid using a truth table: Construct a truth table with columns for the hypotheses and a column for the conclusion. Check that in every row where the hypotheses all have truth value **T**, the conclusion also has truth value **T**. In plain English, check that if we agree the hypotheses are all true, then the conclusion is true as well. Notice that in rows where the hypotheses do not all have truth value **T**, the truth value of the conclusion does not matter. Flipping that *validity test* around, an argument is not valid if there is a row in the truth table where the hypotheses are all **T**, but the conclusion is **F**. Such a row shows that it is possible to agree that the hypotheses are all true, and yet still have the conclusion false. That means the argument is not valid.

(5) Express the following argument symbolically, and then prove, using the style of proof shown in table 4.2 of the text, that the argument is valid. *If Ralph doesn't have a sore shoulder or he doesn't feel sick, then he will go bowling and he will go to the movie. If he goes to the movie, he will buy popcorn. He didn't buy popcorn. So Ralph has a sore shoulder.*

(Use s for Ralph has a sore shoulder. f for Ralph feels sick. b for Ralph goes bowling. m for Ralph goes to the movie. p for Ralph buys popcorn.

Warning: You will probably want to use the rule of Simplification at some point in your proof. You need to be careful with that rule! In plain English, the rule says that if you know p and q is true, then you can conclude p is true. But watch out: if $p \wedge q$ is part of a larger proposition, you can not replace $p \wedge q$ with p. For example, consider the proposition *If I bet $1000 on My Pony and My Pony wins the race, then I will be rich.* Obviously it is not valid to say, by Simplification, *If I bet $1000 on My Pony, then I will be rich.* So, be careful using Simplification.

(6) (bonus) Prove

$$\begin{array}{l} \exists x(A(x) \wedge \neg B(x)) \\ \forall x(A(x) \longrightarrow C(x)) \\ \hline \therefore \exists x(C(x) \wedge \neg B(x)) \end{array}$$

Hint: This is very similar to example 4.2 in the text.