1

A $P(5) \quad 5^2 \not> 4$ F

B $\neg \forall x \, P(x) = \exists x \neg P(x)$ T

there exists positive integers that when squared are larger than 4. Also could be said numbers that are not less than or equal to 4

2

x has read y

x: class
y: books by Mark Twain

A $\forall x \, P(x, H.F) =$ **all of the students in the class have read Huck Finn**

B $\exists x \forall y \, P(x, y) =$ **there is a student in the class who has read all of the books by Mark Twain**

C $\forall y \exists x P(x, y) =$ **all of the books by Mark Twain have been read by the class**

3

A

i) $\neg \forall x\, P(x, \text{ralph})$

ii) $\forall x\, P(x, \text{Fred})$

iii) $\exists x \forall y\, P(x, y)$

B

i) $\neg \forall x\, P(x, \text{ralph}) \equiv \exists x \neg P(x, \text{ralph})$

negating $\equiv (\neg \exists x \neg P(x, \text{ralph}))$

$\equiv \boxed{\forall x\, P(x, \text{ralph})}$

ii) $\neg \forall x\, P(x, \text{Fred}) \equiv \boxed{\exists x \neg P(x, \text{Fred})}$

iii) $\exists x \forall y\, P(x, y)$

$\boxed{\text{negated} \equiv \forall x \exists y \neg P(x, y)}$

C

i) everybody trusts ralph

ii) nobody trusts Fred

iii) Everybody doesn't trust somebody

4

$p \rightarrow q$

$\neg p$

p	q	$p \rightarrow q$	$\neg p$	$\neg q$	none of truth values line up
T	T	T	F	F	
T	F	F	F	T	
F	T	T	T	F	
F	F	T	T	T	

not valid

(5) s = ralph has a sore shoulder p = buys popcorn

f = ralph feels sick

b = " goes bowling

m = goes to the movie

$$\neg s \vee \neg f \rightarrow b \wedge m$$

$$m \rightarrow p$$

$$\neg p$$

$$\therefore s$$

(1)	$\neg s \vee \neg f \rightarrow b \wedge m$	hypoth
(2)	$m \rightarrow p$	hyp
(3)	$\neg p$	hyp
(4)	$\neg m$	modus tollens
(5)	$\neg(b \vee m) \rightarrow \neg(\neg s \wedge \neg f)$	contrapositive

6) $(\neg b \vee \neg m) \rightarrow (s \wedge f)$ — double negation

7) $\neg m \vee \neg b$ — addition

8) $s \wedge f$ — modus ponens

9) $\therefore s$ — simplification

Bonus:

$\exists x (A(x) \wedge \neg B(x))$

$\forall (A(x) \rightarrow (x))$

$\therefore \exists x (C(x) \wedge \neg B(x))$

1) $\exists x (A(x) \wedge \neg B(x))$ — hypothesis

2) $\forall (A(x) \rightarrow (x))$ ↓ existential instantion

3) $A(x) \wedge \neg B(x)$

4) $A(x) \rightarrow C(x)$ — universal instant

(5) $A(x)$

(6) $\neg B(x)$

(7) $C(x)$

simplification

↓

modus ponens of (4)

thus by addition and instantiation

$\exists(x)\,(C(x) \wedge \neg B(x))$