Math 208: Discrete Mathematics
Lesson 9: Lecture Video Notes

Topics

15. Recursively defined sets
    (a) recursive definition of sets
    (b) sets of strings

Readings: Chapter 15

§15. Recursively defined sets

As we saw for recursively defined sequences, a recursive definition has two parts:

1. initial conditions
2. recursive formula

Ex. The Fibonacci sequence is given by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$.

15a. Recursive definition of sets

Recursive definitions can also be handy for defining certain sets of numbers.

Ex. Define the set S recursively by the initial condition: $2 \in S$, and recursive formula: if $x \in S$, then $x + 2 \in S$. A more common description of S is the set of positive even numbers.

Remark. To mathematically show the the recursive definition of S is the set of positive even numbers, two things must be established. (1) S contains all positive even numbers E, and (2) S only contains positive even numbers E. That is, to prove $S = E$ we must show $S \supseteq E$ and $S \subseteq E$.

Ex. Give a recursive definition of the set T of all nonnegative integer powers of 4.

Ex. Describe the integers in the set A defined recursively by initial conditions $1 \in A$ and $2 \in A$ and recursive rule: if $x \in A$, the $x + 4 \in A$.

15b. Sets of strings

Recursively defined sets are often used in certain computer science courses to describe sets of strings over an alphabet Σ.

Ex. Let $\Sigma = \{a, b, c, d\}$ be an alphabet of four symbols.

	string over Σ?	length
$abdcd$		
caa		
$baxx$		
λ		

Defn. The set of symbols used to form strings is called an *alphabet* and denoted Σ. A *string* of length n is any finite sequence of length n of symbols from the alphabet Σ. The string of length 0 is called the *empty string* and denoted by λ.

Ex. Define a set S of strings over the alphabet $\Sigma = \{a, b\}$ recursively by (1) $\lambda \in S$, and (2) if $x \in S$, then $axbb \in S$. Describe the strings in S.

Ex. A palindrome is a string that reads the same in both direction. For example $aabaa$ and $babccbab$ are palindromes but $abbaa$ is not. The empty string is considered a palindrome. Give a recursive definition of the set of palindromes over the alphabet $\Sigma = \{a, b, c\}$.