Lesson 9

(1) A set S of integers is defined recursively by the rules:
(1) $1 \in S$, and (2) If $n \in S$, then $2n+1 \in S$.

(a) Is $20 \in S$?

(b) Is $175 \in S$?

Explain your answers.

(2) A set, S, of strings over the alphabet $\Sigma = \{a, b, c\}$ is defined recursively by (1) $a \in S$ and (2) if $x \in S$ then $xbc \in S$. List all the strings in S of length seven or less.

(3) A set, S, of positive integers is defined recursively by the rule:
(1) $3 \in S$, and (2) If $n \in S$, then $2n - 3 \in S$. List **all** the elements in the set S.

(4) Give a recursive definition of the set of positive integers that end with the digit 7.

(5) Describe the strings in the set S of strings over the alphabet $\Sigma = \{a, b, c\}$ defined recursively by (1) $c \in S$ and (2) if $x \in S$ then $xa \in S$ and $xb \in S$ and $cx \in S$.

Hint: Your description should be a sentence that provides an easy test to check if a given string is in the set or not. An example of such a description is: *S consists of all strings of a's, b's, and c's, with more a's than* b's. That isn't a correct description since *cab* is in S and doesn't have more a's than b's, and also *baac* isn't in S, but does have more a's than b's. So that attempted description is really terrible. The best way to do this problem is to use the rules to build a bunch of strings in S until a suitable description becomes obvious.

(6) (bonus) A set S of ordered pairs of integers is defined recursively by (1) $(1, 2) \in S$ and $(2, 1) \in S$, and (2) if $(m, n) \in S$, then $(m+2, n) \in S$, and $(m, n+2) \in S$, and $(m+1, n+1) \in S$. There is a simple description of the ordered pairs in S. What is it? Your description should be good enough so that you can instantly decide if $(12236, 912242)$ is in S.