# Lesson 18

(1) Suppose on December 31, 2000, a deposit of \$100 is made in a savings account that pays 10% annual interest (Ah, those were the days!). So one year after the initial deposit, on December 31, 2001, the account will be credited with \$10, and have a value of \$110. On December 31, 2002 that account will be credited with an additional \$11, and have value \$121. Find a **recursive relation** that gives the value of the account $n$ years after the initial deposit.

(2) Sal climbs stairs by taking either one, two, or three steps at a time.

(a) Determine a recursive formula for the number of different ways Sal can climb a flight of $n$ steps. Don't forget to include the initial conditions.

(b) In how many different ways can Sal climb a flight of ten steps?

(3) Passwords for a certain computer system are strings of uppercase letters. A valid password must contain an even number of $X$'s. Determine a recurrence relation for the number of valid passwords of length $n$. Note: 0 is an even number, so $ABBC$ is a valid password. This counting problem is pretty tricky. Here's a good way to think about it: to make a good password of length $n$ you can either (a) add any non-X to the end of a good password of length $n-1$, or (b) add an X to the end of a *bad* password of length $n-1$. For (b) you can use the *Good* = *Total-Bad* trick to count the number of bad passwords of length $n-1$.

(4) Solve by unfolding: $a_0 = 3$, and, for $n \geq 1$, $a_n = 5a_{n-1}$.

(5) Solve by unfolding: $a_0 = 3$, and, for $n \geq 1$, $a_n = 5a_{n-1} + 3$. Hint: This one will involve applying the geometric sum formula.

(6) (bonus) Suppose the Tower of Hanoi rules are changed so that stones may only be transferred to an adjacent clearing in one move. Let $I_n$ be the minimum number of moves required to transfer tower from clearing $A$ to clearing $C$? For example, it takes two moves to move a one stone tower from $A$ to $C$: One move from $A$ to $B$, then a second move from $B$ to $C$. So $I_1 = 2$

(a) By brute force, determine $I_2$, and $I_3$.

(b) Find a recursive relation for $I_n$.

(c) Guess a formula for $I_n$.