1) $m = 2k+1$

$n = 2j$

even = $2g$

$m+n = 2k+1+2g$

$m+n = (\text{even}) + (\text{even}) + 1$

thus odd by odd $= \underset{\text{even}}{2k}+1$

answer:

A number m in odd if m = 2k+1 and a number n is even if n= 2j. The sum of m + n = 2k + 2j +1. This is the sum of 2 even numbers and 1. An even number plus another even number is even. So this now follows the form of m = 2k+1 as it is an even number to which 1 is added to making it odd.

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2) $m^3 = \text{even}$?

$m = \text{even}$

$p \to q$

$\neg q \to \neg p$

If m is not even, odd, then m^3 is odd. m is then equal to $m = 2k+1$ for some k. Cubing the equation gets $m^3 = (2k+1)^3$. simplified to $8k^3 + 12k^2 + 6k + 1 = m^3 = 2(k^3 + 6k^2 + 3k) + 1$. this shows m^3 is odd, not even through the use of the contrapositive

3

$5n-4 =$ odd

$n =$ odd

If $5n-4 = \text{odd} \rightarrow n = \text{odd}$

Suppose $5n-4$ is odd and n is even,

$5n-4 = 5(2k)-4$ $\quad n = 2k$

$5n-4 = 10k-4$

$5n = 10k$

$n = 2k$

Suppose $5n-4$ is odd and n is even in a proof by contradiction. The integer n to be even must equal $2k$ for some k. Thus $5n-4 = 5(2k)-4$ leading to $n = 2k$. This shows $5n-4$ is even when n is even and vice versa, a contradiction to the original assumption

④ predicate $P(n)$

true 1- 1bill every

false all

$P(n): n < (1\text{billion}+1)$

$\forall n P(n) = \{n \mid n < 1\text{billion}+1\}$

⑤ 213 is not prime.

⑥ $\min(s,t) + \max(s,t) = s+t$

case 1: $\min(s,t) = s$ thus $\max(s,t) = t$

$s + t = s+t$

case 2: $\min(s,t) = t$ thus $\max(s,t) = s$

$t + s = s+t$

If $s=t$ then $s+s = t+t$ as s and t are interchangeable