

Lesson 6

(1) Let $A = \{1, 2, 3, 4, 5, 6\}$. In each of the following, give an example of a function $f : A \rightarrow A$ with the indicated properties, or explain why no such function exists.

(a) f is bijective, but is not the identity function $f(x) = x$.

(b) f is neither one-to-one nor onto.

(c) f is one-to-one, but not onto.

(d) f is onto, but not one-to-one.

(2) In each of the following, give an example of a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ with the indicated properties, or explain why no such function exists.

(a) f is bijective, but is not the identity function $f(x) = x$.

(b) f is neither surjective nor injective.

(c) f is surjective, but not injective.

(d) f is injective, but not surjective.

(3) Write each of the following as a single logarithm.

(a) $\log(1) + \log(2) + \log(3) + \log(4) + \log(5)$

(b) $\sum_{k=1}^n \log(k)$

Hint: Observe $\sum_{k=1}^n \log(k) = \log(1) + \log(2) + \log(3) + \cdots + \log(n)$ and then reduce to a single logarithm.

(4) Compute the following values:

(a) $\lfloor \pi + e \rfloor$

(b) $\lceil \sqrt{11} - e \rceil$

(c) $\text{int}(-7.04)$

(d) $\text{frac}(-7.04)$

- (5) (bonus) Suppose $g : A \rightarrow B$ and $f : B \rightarrow C$ are both surjective (i.e. onto) functions. Prove that the function $f \circ g$ is surjective.