5

Sets: Basic Definitions

A **set** IS A COLLECTION OF OBJECTS. Often, but not always, sets are denoted by capital letters such as $A, B, \cdots$ and the objects that make up a set, called its **elements**, are denoted by lowercase letters. Write $x \in A$ to mean that the object x is an element of A. If the object x is not an element of A, write $x \notin A$.

Two sets A and B are **equal**, written $A = B$ provided A and B comprise exactly the same elements. Another way to say the same thing: $A = B$ provided $\forall x \, (x \in A \longleftrightarrow x \in B)$.

5.1 *Specifying sets*

There are a number of ways to specify a given set. We consider two of them.

5.1.1 *Roster method*

One way to describe a set is to list its elements. This is called the **roster method**. Braces are used to signify when the list begins and where it ends, and commas are used to separate elements. For instance, $A = \{1, 2, 3, 4, 5\}$ is the set of positive whole numbers between 1 and 5 inclusive. It is important to note that the order in which elements are listed is immaterial. For example, $\{1, 2\} = \{2, 1\}$ since $x \in \{1, 2\}$ and $x \in \{2, 1\}$ are both true for $x = 1$ and $x = 2$ and false for all other choices of x. Thus $x \in \{1, 2\}$ and $x \in \{2, 1\}$ always have the same truth value, and that means $\forall x \, (x \in \{1, 2\} \longleftrightarrow x \in \{2, 1\})$

is true. According to the definition of equality given above, it follows that $\{1,2\} = \{2,1\}$. The same sort of reasoning shows that repetitions in the list of elements of a set can be ignored. For example $\{1,2,3,2,4,1,2,3,2\} = \{1,2,3,4\}$. There is no point in listing an element of a set more than once.

The roster method has certain drawbacks. For example we probably don't want to list all of the elements in the set of positive integers between 1 and 99 inclusive. One option is to use an **ellipsis**. The idea is that we list elements until a pattern is established, and then replace the missing elements with ... (which is the ellipsis). So $\{1,2,3,4,\ldots,99\}$ would describe our set.

The use of an ellipsis has one pitfall. It is **hoped** that whoever is reading the list will be able guess the proper pattern and apply it to fill in the gap.

5.1.2 *Set-builder notation*

Another method to specify a set is via the use of **set-builder notation**. A set can be described in set-builder notation as $A = \{x|p(x)\}$. Here we read A is the set of all objects x for which the predicate $p(x)$ is true. So $\{1,2,3,4,\ldots,99\}$ becomes $\{x|x$ is a whole number and $1 \le x \le 99\}$.

5.2 *Special standard sets*

Certain sets occur often enough that we have special notation for them.

$\mathbb{N} = \{x|x \text{ is a non-negative whole number}\} = \{0,1,2,\ldots\}$, **the natural numbers.**

$\mathbb{Z} = \{x|x \text{ is a whole number}\} = \{\ldots,-2,-1,0,1,2,\ldots\}$, **the integers.**

$\mathbb{Q} = \{x|x = \dfrac{p}{q}, p \text{ and } q \text{ integers} \quad \text{with } q \neq 0\}$, **the rational numbers.**

$\mathbb{R} = \{x|x \text{ is a real number}\}$, **the real numbers.**

$\mathbb{C} = \{x|x = a + ib, a, b \in \mathbb{R}, i^2 = -1\}$, **the complex numbers**.

5.3 *Empty and universal sets*

In addition to the above sets, there is a set with no elements, written as $\emptyset$ (also written using the roster style as $\{\ \}$), and called the **empty set**. This set can be described using set builder style in many different ways. For example, $\{x \in \mathbb{R} | x^2 = -2\} = \emptyset$. In fact, if $P(x)$ is any predicate which is always false, then $\{x \mid P(x)\} = \emptyset$. There are two easy slips to make involving the empty set. First, don't write $\emptyset = 0$ (the idea being that both $\emptyset$ and 0 represent *nothing*[1]). That is not correct since $\emptyset$ is a set, and 0 is a number, and it's not fair to compare two different types of objects. The other error is thinking $\emptyset = \{\emptyset\}$. This cannot be correct since the right-hand set has an element, but the left-hand set does not.

[1] One is empty the other is something, namely zero.

At the other extreme from the empty set is the **universal set**, denoted $\mathcal{U}$. The universal set consists of all objects under consideration in any particular discussion. For example, if the topic du jour is basic arithmetic then the universal set would be the set of all integers. Usually the universal set is left for the reader to guess. If the choice of the universal set is not an obvious one, it will be pointed out explicitly.

5.4 *Subset and equality relations*

The set A is a **subset** of the set B, written as $A \subseteq B$, in case $\forall x(x \in A \longrightarrow x \in B)$ is true. In plain English, $A \subseteq B$ if every element of A also is an element of B. For example, $\{1,2,3\} \subseteq \{1,2,3,4,5\}$. On the other hand, $\{0,1,2,3\} \not\subseteq \{1,2,3,4,5\}$ since 0 is an element of the left-hand set but not of the right-hand set. The meaning of $A \not\subseteq B$ can

be expressed in symbols using De Morgan's law:

$$\begin{aligned} A \not\subseteq B &\longleftrightarrow \neg(\forall x(x \in A \to x \in B)) \\ &\equiv \exists x \neg(x \in A \to x \in B) \\ &\equiv \exists x \neg(\neg(x \in A) \vee x \in B) \\ &\equiv \exists x(x \in A \wedge x \notin B) \end{aligned}$$

and that last line says $A \not\subseteq B$ provided there is at least one element of A that is not an element of B.

The empty set is a subset of every set. To check that, suppose A is any set, and let's check to make sure $\forall x(x \in \emptyset \longrightarrow x \in A)$ is true. But it is since for any x, the hypothesis of $x \in \emptyset \longrightarrow x \in A$ is F, and so the implication is T. So $\emptyset \subseteq A$. Another way to same the same thing is to notice that to claim $\emptyset \not\subseteq A$ is the same as claiming there is at least one element of $\emptyset$ that is not an element of A, but that is ridiculous, since $\emptyset$ has no elements at all.

To say that $A = B$ is the same as saying every element of A is also an element of B and every element of B is also an element of A. In other words, $A = B \longleftrightarrow (A \subseteq B \wedge B \subseteq A)$, and this indicates the method by which the common task of showing two sets are equal is carried out: to show two sets are equal, show that each is a subset of the other.

If $A \subseteq B$, and $A \neq B$, A is a **proper subset** of B, denoted by $A \subset B$, or $A \subsetneq B$. In words, $A \subset B$ means every element of A is also an element of B and there is at least one element of B that is not an element of A. For example $\{1,2\} \subset \{1,2,3,4,5\}$, and $\emptyset \subset \{1\}$.

5.5 *Cardinality*

A set is **finite** if the number of distinct elements in the set is a non-negative integer. In this case we call the number of distinct elements in the set its **cardinality** and denote this natural number by $|A|$. For example, $|\{1,3,5\}| = 3$ and $|\emptyset| = 0$, $|\{\emptyset\}| = 1$, and $|\{\emptyset, \{a,b,c\}, \{X,Y\}\}| = 3$. A set, such as $\mathbb{Z}$, which is not finite, is **infinite**.

5.6 *Power set*

Given a set A the **power set of** A, denoted $\mathcal{P}(A)$, is the set of all subsets of A. For example if $A = \{1,2\}$, then $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$. For a more confusing example, the power set of $\{\emptyset, \{\emptyset\}\}$[2] is

$$\mathcal{P}\left(\{\emptyset, \{\emptyset\}\}\right) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}.$$

It is not hard to see that if $|A| = n$, then $|\mathcal{P}(A)| = 2^n$.

[2] Try finding the power set of the empty set: $\mathcal{P}(\emptyset)$.

5.7 Exercises

Exercise 5.1. *List the members of the following sets.*

a) $\{x \in \mathbb{Z} | 3 \leq x^3 < 100\}$ *b)* $\{x \in \mathbb{R} | 2x^2 = 50\}$

c) $\{x \in \mathbb{N} | 7 > x \geq 4\}$

Exercise 5.2. *Use set-builder notation to give a description of each set.*

a) $\{-5, 0, 5, 10, 15\}$ *b)* $\{0, 1, 2, 3, 4\}$

c) *The interval of real numbers:* $[\pi, 4)$

Exercise 5.3. *Determine the cardinality of the sets in exercises 5.1 and 5.2.*

Exercise 5.4. *Is the proposition Every element of the empty set has three toes true or false? Explain your answer!*

Exercise 5.5. *Determine the power set of* $\{1, \{2\}\}$.

Exercise 5.6. *True or False:* $\{1, 2, 3, 4, 5\} = \{5, 2, 3, 1, 2, 1, 5, 4, 3, 2, 1\}$.

Exercise 5.7. *True or False: The set of even integers is a subset of the set of integers that are multiples of four.*

5.8 Problems

Problem 5.1. *List the members of the following sets.*

a) $\{x \in \mathbb{N} | 3 \leq x^2 < 100\}$

b) $\{x \in \mathbb{Z} | 3 \leq x^2 < 100\}$

c) $\{x \in \mathbb{R} | 0 < x \leq 5\}$

d) $\{x \in \mathbb{N} | x^2 < 0\}$

e) $\{1, \{1\}, \{1, 2\}\}$

Problem 5.2. *Determine the power set of* $\{\emptyset, 1, \{1\}\}$.

Problem 5.3. *Determine the truth value of the following propositions:*

a) *Every element of the empty set is positive.*

b) *Some element of the empty set is positive.*

c) $0 \in \emptyset$.

d) $0 = \emptyset$.

e) *The empty set is nothing.*

f) *If x is a real number and $x^2 < 0$, then $x \in \emptyset$.*

Problem 5.4. *Determine the truth value of the following propositions:*

a) *For any set A, $A \subseteq \mathcal{P}(A)$.*

b) *For any set A, $A \in \mathcal{P}(A)$.*

c) *For any finite set A, the cardinality of $\mathcal{P}(A)$ is greater than the cardinality of A.*

Problem 5.5. *Determine the cardinality of the following sets:*

a) *The empty set.*

b) *The power set of the empty set.*

c) *The power set of the power set of the empty set.*

d) *The power set of* $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

e) $\{1, \{1\}, \{1, 2\}\}$

Problem 5.6. *True or False:*

a) *If* $n \in \mathbb{N}$*, then* $n + 1 \in \mathbb{N}$.

b) *If* $n \in \mathbb{N}$*, then* $n - 1 \in \mathbb{N}$.

Problem 5.7. *True or False:*

a) *If* $n \in \mathbb{Z}$*, then* $n + 1 \in \mathbb{Z}$.

b) *If* $n \in \mathbb{Z}$*, then* $n - 1 \in \mathbb{Z}$.

Problem 5.8. *True or False:* $\mathbb{Z} \subseteq \mathbb{Q}$.

Problem 5.9. *What is wrong with the following expressions:*

a) $\{x < 3\}$.

b) $\{x \mid x \text{ is even }\}$.

c) $\{x, m \in \mathbb{N} \mid x = 2m\}$.

d) $[1, \infty]$.

e) $\{x \in \mathbb{N}\}$.

f) $A = \{x \mid x \in A \to x \text{ is an even integer }\}$.

6

Set Operations

There are several ways of combining sets to produce new sets.

6.1 Intersection

The **intersection** of A with B denoted $A \cap B$ is defined as $\{x|x \in A \land x \in B\}$. For example $\{1,2,3,4,5\} \cap \{1,3,5,7,9\} = \{1,3,5\}$. So the intersection of two sets consists of the objects which are in both sets simultaneously. Two sets are **disjoint** if $A \cap B = \emptyset$.

6.2 Venn diagrams

Set operations can be visualized using **Venn diagrams**. A circle (or other closed curve) is drawn to represent a set. The points inside the circle are used to stand for the elements of the set. To represent the set operation of intersection, two such circles are drawn with an overlap to indicate the two sets may share some elements. In the Venn diagram, figure 6.1, the shaded area represents the intersection of A and B.

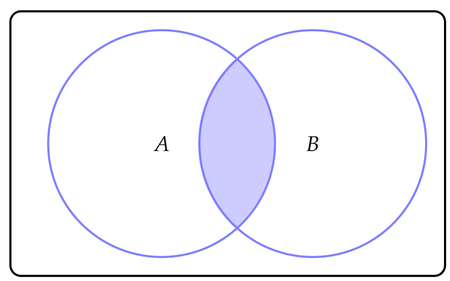

Figure 6.1: Venn diagram for $A \cap B$

6.3 Union

The **union** of A with B denoted $A \cup B$ is $\{x|x \in A \lor x \in B\}$. In words, $A \cup B$ consists of those elements that appear in at least one of A and B. So for example $\{1,2,3,4,5\} \cup \{1,3,5,7,9\} = \{1,2,3,4,5,7,9\}$. The Venn Diagram 6.2 represents the union of A and B.

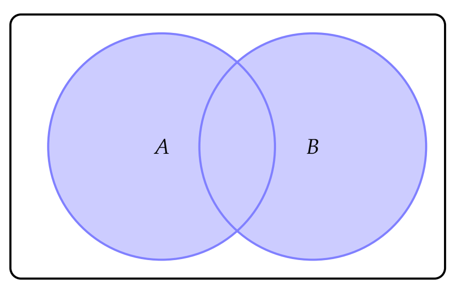

Figure 6.2: Venn diagram for $A \cup B$

6.4 *Symmetric difference*

The **symmetric difference** of A and B is defined to be $A \oplus B = \{x | x \in A \oplus x \in B\}$. So $A \oplus B$ consists of those elements which appear in exactly one of A and B. For example $\{1,2,3,4,5\} \oplus \{1,3,5,7,9\} = \{2,4,7,9\}$. The corresponding Venn diagram for the symmetric difference is figure 6.3.

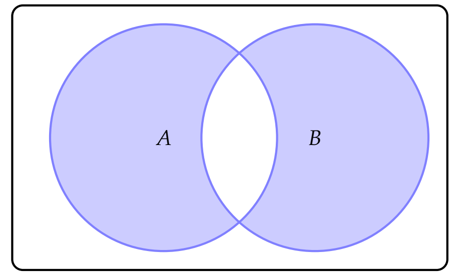

Figure 6.3: Venn diagram for $A \oplus B$

6.5 *Complement*

The **complement of** B **relative to** A, denoted $A - B$ is $\{x | x \in A \wedge x \notin B\}$. So $\{1,2,3,4,5\} - \{1,3,5,7,9\} = \{2,4\}$. Figure 6.4 is the correponding Venn diagram.

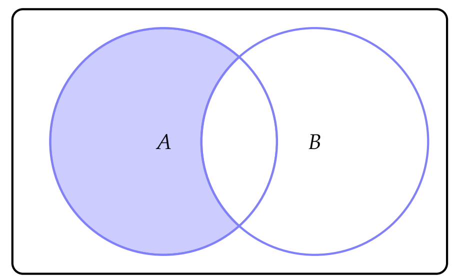

Figure 6.4: Venn diagram for $A - B$

When $\mathcal{U}$ is a universal set, we denote $\mathcal{U} - A$ by $\overline{A}$ and call it the **complement** of A. See figure 6.5 for the Venn diagram. If $\mathcal{U} = \{0,1,2,3,4,5,6,7,8,9\}$, then $\overline{\{0,1,2,3,4\}} = \{5,6,7,8,9\}$. The universal set matters here. If $\mathcal{U} = \{x \in N | x \leq 100\}$, then $\overline{\{0,1,2,3,4\}} = \{5,6,7,8,...,100\}$.

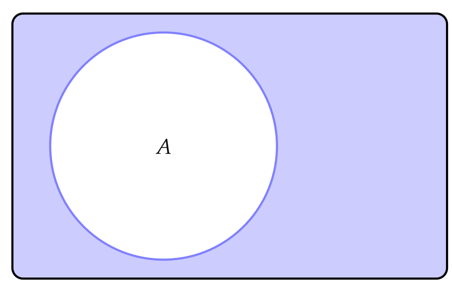

Figure 6.5: Venn diagram for $\overline{A} = \mathcal{U} - A$

6.6 *Ordered lists*

The order in which elements of a set are listed does not matter. But there are times when order is important. For example, in a horse race, knowing the order in which the horses cross the finish line is more interesting than simply knowing which horses were in the race. There is a familiar way, introduced in algebra, of indicating order is important: ordered pairs. Ordered pairs of numbers are used to specify points in the Euclidean plane when graphing functions. For instance, when graphing $y = 2x + 1$, setting $x = 3$ gives $y = 7$, and so the ordered pair $(3,7)$ will indicate one of the points on the graph.

In this course, ordered pairs of any sorts of objects, not just numbers, will be of interest. An **ordered pair** is a collection of two objects (which might both be the same) with one specified as first (the first coordinate) and the other as second (the second coordinate). The ordered pair with a specified as first and b as second is written (as usual) (a,b). The most important feature of ordered pairs is that

$(a, b) = (c, d) \longleftrightarrow a = c \textit{ and } b = d$. In words, two ordered pairs are equal provided they match in both coordinates. So $(1, 2) \neq (2, 1)$.

More generally, an **ordered n-tuple** $(a_1, a_2, ..., a_n)$ is the ordered collection with a_1 as its first coordinate, a_2 as its second coordinate, and so on. Two ordered n-tuples are equal provided they match in every coordinate.

6.7 Cartesian product

The last operation to be considered for combining sets is the **Cartesian product** of two sets A and B. It is defined by $A \times B = \{(a, b) | a \in A \wedge b \in B\}$. In other words, $A \times B$ comprises all ordered pairs that can be formed taking the first coordinate from A and the second coordinate from B. For example if $A = \{1, 2\}$, and $B = \{\alpha, \beta\}$, then $A \times B = \{(1, \alpha), (2, \alpha), (1, \beta), (2, \beta)\}$. Notice that in this case $A \times B \neq B \times A$ since, for example, $(1, \alpha) \in A \times B$, but $(1, \alpha) \notin B \times A$.

A special case occurs when $A = B$. In this case we denote the Cartesian product of A with itself by A^2. The familiar example $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$ is called the Euclidean plane or the Cartesian plane.

More generally given sets $A_1, ..., A_n$ the Cartesian product of these sets is written as $A_1 \times A_2 \times ... \times A_n = \{(a_1, a_2, ..., a_n) | a_i \in A_i, 1 \leq i \leq n\}$. Also A^n denotes the Cartesian product of A with itself n times.

In order to avoid the use of an ellipsis we also denote the Cartesian product of $A_1, ..., A_n$ as $\displaystyle\prod_{k=1}^{n} A_k$. The variable k is called the **index** of the product. Most often the index is a whole number. Unless we are told otherwise we start with $k = 1$ and increment k by 1 successively until we reach n. So if we are given A_1, A_2, A_3, A_4, and A_5, $\displaystyle\prod_{k=1}^{5} A_k = A_1 \times A_2 \times A_3 \times A_4 \times A_5$.

6.8 Laws of set theory

There is a close connection between many set operations and the logical connectives of Chapter 1. The intersection operation is related to conjunction, union is related to disjunction, and complementation is related to negation. It is not surprising then that the various laws of logic, such as the associative, commutative, and distributive laws carry over to analogous laws for the set operations. Table 6.1 exhibits some of these properties of these set operations.

Table 6.1: Laws of Set Theory

Identity	**Name**
$\overline{(\overline{A})} = A$	Double Negation
$A \cap \mathcal{U} = A$ $A \cup \emptyset = A$	Identity laws
$A \cup \mathcal{U} = \mathcal{U}$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{(A \cap B)} = (\overline{A} \cup \overline{B})$ $\overline{(A \cup B)} = (\overline{A} \cap \overline{B})$	De Morgan's laws
$A \cup \overline{A} = \mathcal{U}$ $A \cap \overline{A} = \emptyset$	Law of Excluded Middle Law of Contradiction

These can be verified by using **membership tables** which are the analogs of truth tables used to verify the logical equivalence of propositions. For a set A either an element under consideration is in A or it is not. These binary possibilities are kept track of using 1 if $x \in A$ and 0 if $x \notin A$, and then performing related bit string operations.

Example 6.1. *Verify the De Morgan's law given by $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$.*

A	B	$A \cap B$	$\overline{(A \cap B)}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cup \overline{B}$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

The meaning of the first row of the table is that if $x \in A$ and $x \in B$, then $x \notin \overline{A \cap B}$, as indicated by the 0 in the first row, fourth column, and also *not in $\overline{A} \cup \overline{B}$ as indicated by the 0 in the first row, last column. Since the columns for $\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$ are identical, it follows that $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ as promised.*

6.9 Proving set identities

Just as compound propositions can be analyzed using truth tables, more complicated combinations of sets can be handled using membership tables. For example, using a membership table, it is easy to verify that $\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B} \cup \overline{C})$. But, just as with propositions, it is usually more enlightening to verify such equalities by applying the few basic laws of set theory listed above.

Example 6.2. *Let's prove* $\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B} \cup \overline{C})$

Proof. *The proof is just two applications of De Morgan's laws:*

$$\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B \cap C}) = \overline{A} \cap (\overline{B} \cup \overline{C}).$$

♣

6.10 Bit string operations

There is a correspondence between set operations of finite sets and bit string operations. Let $\mathcal{U} = \{u_1, u_2, ..., u_n\}$ be a finite universal set with distinct elements listed in a specific order[1] For a set A under consideration, we have $A \subseteq \mathcal{U}$. By the law of excluded middle, for each $u_j \in \mathcal{U}$, either $u_j \in A$ or $u_j \notin A$. We define a binary string of length n, called the **characteristic vector** of A, denoted $\chi(A)$, by setting the jth bit of $\chi(A)$ to be 1 if $u_j \in A$ and 0 if $u_j \notin A$. For example if $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $A = \{1, 3, 4, 5, 8\}$, then $\chi(A) = 101110010$.

An interesting side-effect is that for example $\chi(A \cap B) = \chi(A) \wedge \chi(B)$ [2], $\chi(A \cup B) = \chi(A) \vee \chi(B)$ and $\chi(\overline{A}) = \neg\chi(A)$. Since every proposition can be expressed using $\wedge, \vee$ and $\neg$, if we represent sets by their characteristic vectors, we can get a machine to perform set operations as logical operations on bit strings. This is the method programmers use to manipulate sets in computer memory.

[1] Notice the universal set is **ordered**. We may write it as and n-tuple: $\mathcal{U} = (u_1, u_2, ..., u_n)$.

[2] As a function, we say that χ maps intersection to conjunction

6.11 Exercises

Exercise 6.1. *Let* $A = \{2, 3, 4, 5, 6, 7, 8\}$ *and* $B = \{1, 2, 4, 6, 7, 8, 9\}$. *Find*

a) $A \cap B$ *b)* $A \cup B$ *c)* $A - B$

d) $B - A$

Exercise 6.2. *Determine the sets* A *and* B, *if* $A - B = \{1, 2, 7, 8\}$, $B - A = \{3, 4, 10\}$ *and* $A \cap B = \{5, 6, 9\}$.

Exercise 6.3. *Use membership tables to show that* $A \oplus B = (A \cup B) - (A \cap B)$.

Exercise 6.4. *Verify* $A \oplus B = (A \cup B) - (A \cap B)$ *using Venn diagrams.*

Exercise 6.5. *Verify* $A \cup (A \cap B) = A$ *using the rules of set algebra.*

Exercise 6.6. *Let* $A = \{1, 2, 3, 4\}, B = \{a, b, c\}, C = \{\alpha, \beta\}$, *and* $D = \{7, 8, 9\}$. *Write out the following Cartesian products.*

a) $A \times B$ *b)* $B \times A$ *c)* $C \times B \times D$

Exercise 6.7. *What can you conclude about* A *and* B *if* $A \times B = B \times A$.

Exercise 6.8. *If* $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, *determine* $\chi(\{1, 2, 4, 8\})$.

Exercise 6.9. *Let* $A = \{1, 2, 3\} \times \{1, 2, 3, 4\}$. *List the elements in the set* $B = \{(s, t) \in A \mid s \geq t\}$.

6.12 Problems

Problem 6.1. *Let* $A = \{1,2,3,5,6,7,9\}$ *and* $B = \{1,3,4,6,8,9\}$. *Find*

a) $A \cap B$

b) $A \cup B$

c) $A - B$

d) $B - A$

Problem 6.2. *Use the rules of set algebra to verify* $A \oplus B = (A \cup B) - (A \cap B)$.

Problem 6.3. *Let* $A = \{1,2,3\} \times \{1,2,3,4\}$. *List the elements of the set* $B = \{(s,t) \in A \mid s < t\}$.

Problem 6.4. *Let* $A = \{0,1,2,3,4,5,6,7,8,9\} \times \{0,1,2,3,4,5,6,7,8,9\}$.

a) *What is the cardinality of* A?

b) *What is the cardinality of* $A \times A$?

c) *What is the cardinality of* $B = \{s \mid (s, s^2) \in A\}$?

Problem 6.5. *True or False:*

a) *For all sets* A, B, C, $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

b) *For all sets* A, B, C, $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

c) *For all sets* A, B, C, D, $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

d) *For all sets* A, B, C, D, $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.

Problem 6.6. *Let* $\mathcal{U} = \{1,2,3,4,5,6,7,8\}$. *Find* $\chi(\{1,2,3,4\})$.