(1) $a_0 = 1 \quad a_n = 3a_{n-1} \quad n \geq 1$

$r^n = 3^{n-1}$

$r = 3$ (mult 1)

$a_n = \alpha \cdot 3^n$

$1 = \alpha \cdot 3$

$\alpha = 1/3$

$\boxed{a_n = 1/3\, 3^n}$

(2) $\alpha \cdot 3^n + 1$

$\alpha \cdot 3^n - 1/2$

$a_n = B \quad B = 3B + 1$

$-2B = 1$

$B = -1/2$

$a_n = \alpha \cdot 3^n - 1/2$

$1 = \alpha - 1/2$

$\alpha = 1.5$

$\boxed{a_n = 1.5 \cdot 3^n - 1/2}$

(3) $x^2 = 1x + 6$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2, \; x = 3$

$a_n = \alpha (-2)^n + \alpha_2 (3)^n$

$1 = \alpha + \alpha_2$

$3 = \alpha \cdot -2 + \alpha_2 3$

I used a matrix and then reduced it to get solution to system

$\alpha = 0 \quad \alpha_2 = 1$

$a_n = 0 \cdot (-2)^n + 3^n$ test $n=0, = 1$

$a_n = 3^n$ $n=1, = 3$

(4) $a_n = \alpha (-2)^n + \alpha_2 3^n + 1$

$B = 1B + 6B + 1$

$B = 7B + 1$

$B = -1/6$

$a_n = \alpha_1 (-2)^n + \alpha_2 3^n - 1/6$

$1 = \alpha_1 + \alpha_2 - 1/6$

$3 = -2\alpha_1 + 3\alpha_2 - 1/6$

$\alpha_1 = 5/6 - \alpha_2$

$3 = -2(5/6 - \alpha_2) + 3\alpha_2 - 1/6$

$3 = -10/6 - 2\alpha_2 + 3\alpha_2 - 1/6$

$3 = -11/6 + \alpha_2$

$\frac{29}{6} = \alpha_2$

$1 = \alpha_1 + \frac{29}{6} - 1/6$

$\alpha_1 = -11/3$

$$a_n = -11/3 (-2)^n + \frac{29}{6} 3^n - 1/6$$

$1 = 9/1 - 9/29 + 9/22 -$

5

$a_0 = 1 \quad a_1 = 6 \quad a_n = 6a_{n-1} - 9a_{n-2} + n \quad n \geq 2$

$x^2 = 6x - 9 \qquad a_n = \alpha 3^n + \alpha_2 \cdot n \cdot 3^n$

$x^2 - 6x + 9 = 0$

$(x-3)^2 \qquad An + B$

$x = 3 \quad \text{mult } 1$

$a_n = 6a_{n-1} - 9a_{n-2} + n$

$An + B = 6(A(n-1) + B) - 9(A(n-2) + B) + n$

$An + B = 6A_{n-1} + 3B + 9A_{n-2} + n$

$6A_n - 6A + 9A_n + 9A_2 + 3B + n$

$-3A_n - 6A + 9A_2 + 3B + n$

$= n(-3A + 1) - 6A + 9A_2 + 3B$

$A = -3A + 1 \qquad B = -6A + 9A_2 + 3B$

$4A = 1 \qquad B = -6(\tfrac{1}{4}) + 9(\tfrac{1}{4}) + 3B$

$B = \tfrac{3}{4} + 3B$

$-2B = \tfrac{3}{4} \qquad B = -\tfrac{3}{8}$

$h(n) = \tfrac{3}{4}n - \tfrac{3}{8}$

$a_n = \alpha_1 3^n + \alpha_2 n \cdot 3^n + \tfrac{3}{4}n - \tfrac{3}{8}$

$1 = \alpha - \tfrac{3}{8} \qquad \boxed{\tfrac{11}{8} = \alpha}$

$6 = \alpha 3 + \alpha_2 3 + \tfrac{3}{8}$

$\tfrac{45}{8} = \tfrac{33}{8} + \alpha_2 3$

$\tfrac{3}{2} = \alpha_2 3$

$\boxed{\alpha_2 = \tfrac{1}{2}}$

$\boxed{a_n = \tfrac{11}{8} 3^n + \tfrac{n}{2} 3^n + \tfrac{3}{4}n - \tfrac{3}{8}}$