1

A $|\psi\rangle\ \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{2}|01\rangle - \frac{i}{2}|11\rangle$

| Result | prob | |
|---|---|---|
| 00 | $\frac{1}{2}$ | left: $\|00\rangle$ |
| 01 | $1/4$ | left: $\|01\rangle$ |
| 11 | $1/4$ | left: $\|11\rangle$ |

B

| | prob | |
|---|---|---|
| 00 | $\frac{27}{100}$ | $\|00\rangle$ |
| 01 | $\frac{48}{100}$ | $\|01\rangle$ |
| 10 | $\frac{9}{100}$ | $\|10\rangle$ |
| 11 | $\frac{16}{100}$ | $\|11\rangle$ |

C

0 prob: $1/4$ $|00\rangle$

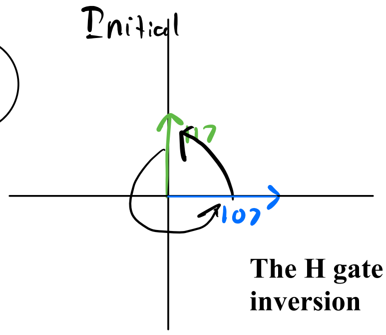
1 prob: $1/2 + \frac{1}{4} = 3/4$ $\frac{-1/\sqrt{2}|10\rangle - i/2|11\rangle}{\sqrt{3/4}}$

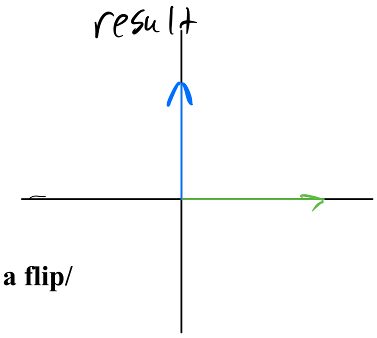
D

0 $1/4 + 1/2 = 3/4$ $\frac{1/2|00\rangle - 1/\sqrt{2}|10\rangle}{\sqrt{3/4}}$

1 $1/4 = 1/4$ $|11\rangle$

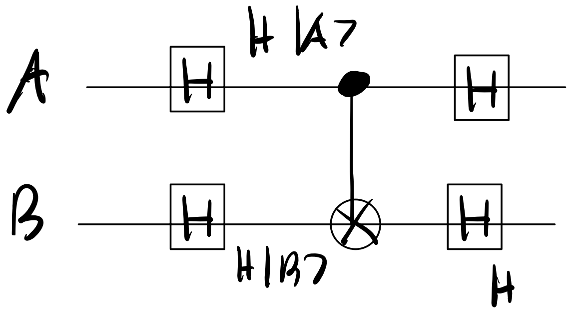
2 A




**The H gate cause a flip/ inversion**

B




$HH|A\rangle$

$H \times H|B\rangle, \quad HH|A\rangle$

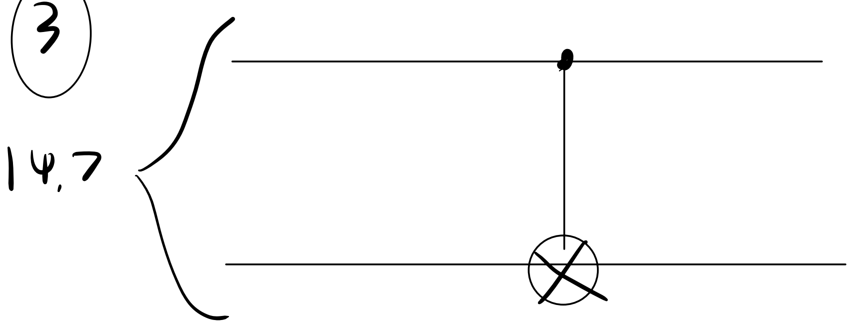
$Z|B\rangle, \quad |A\rangle$

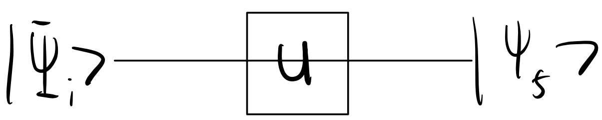
**I don't really think i did this right but what was i was trying to represent was that since A and B are arbitrary vectors we do the H operators and not operations on them. Since the question had two H gates on A and B before and after the CNOT gate i have them set up this way. I might have misinterpreted what the question was asking. Since also H*H creates the identity matrix they are removed from effect. I could be wrong, but I believe that this might make a CZ gate**

C We would then have a result of $Z|A\rangle$

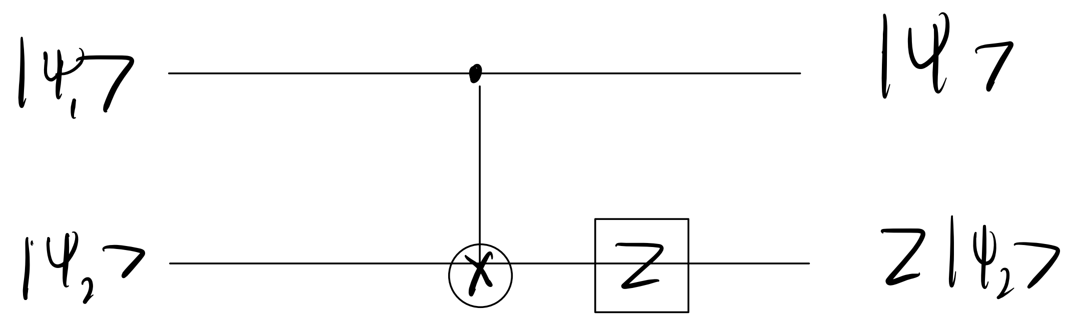
$|B\rangle$

3




A $|\psi_f\rangle = U|\Psi_i\rangle$




B

not opperation

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} |\psi_2\rangle$

$[x][z] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \longrightarrow |\psi_2\rangle$

$z \cdot x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $\quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$xz|\psi_2\rangle = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} |\psi_2\rangle$

$|\psi_2\rangle$

$0 \to |0\rangle$

$1 \to -|1\rangle$

$|\psi_1\rangle \longrightarrow |\psi_1\rangle$

$\psi_1 = 1 \longrightarrow$ cnot happens both $= |\psi_1\rangle$

$= 0 \longrightarrow$ nothing

**My idea is that this is a CNOT gate with a Z gate attached to it. The Z gate acts similar to the not gate which CNOT uses hut it also negates the result. For when the first is 1 the second is inverted by the CNOT and then agin with the Z grate. This means that if 0 it will be -1 if 1 then it will be 0.**

# HW2

February 8, 2023

# 1 Homework 2: The Bell Basis

### 1.0.1 Instructions:

Just like last week, run each block sequentially from the top to load the libraries and constuct the first circuit. After the Problem 3 header below, you have some of your own coding to do to construct three more circuits, which is the actual assignment task. When you are done, attach a pdf of the entire completed notebook to your homework submission.

[1]:
```
#Setup for Qiskit

import qiskit
from qiskit import *
import numpy as np
from qiskit import QuantumCircuit, transpile, assemble, Aer, IBMQ, execute
from qiskit.visualization import plot_histogram, plot_bloch_vector,␣
↪plot_bloch_multivector, array_to_latex
from qiskit.quantum_info import Statevector
import matplotlib
```

Recall that the Hadamard gate maps the computational basis to

$$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

Also, remember from last week that to generate $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ from the computational basis we apply the following gates to the $|00\rangle$ state;

$$(CNOT_{01})(\hat{H} \otimes \hat{I})|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Let's construct the circuit. In circuit form, we have

[2]:
```
n=2 # define the number of qubits

q=QuantumRegister(2,"q") # initialize a circuit with two qubits in the |0> state
```

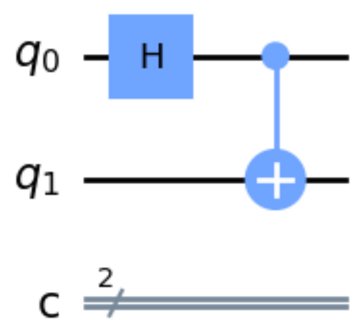
```python
c=ClassicalRegister(2,"c") # create two classical registers in case we want to measure our qubits

circuit=QuantumCircuit(q,c) # define the quantum circuit

circuit.h(q[0]) # apply a Hadamard operation to the zeroth qubit
circuit.cx(q[0],q[1]) # apply a controlled-NOT operation using the zeroth qubit as the control

circuit.draw(output="mpl") # draw circuit
```

[2]:




[9]:
```python
n=2

q=QuantumRegister(2,"q")
c=ClassicalRegister(2,"c")

circuit=QuantumCircuit(q,c)

circuit.h(q[0])
circuit.cx(q[0],q[1])

backend = Aer.get_backend('statevector_simulator') # exactly simulates the evolution of the state starting in |00>
shots = 1 # simulate once
result=execute(circuit, backend=backend, shots=shots).result() # run simulation
statevector=result.get_statevector() # store the output statevector
print(statevector)
```

```
Statevector([0.70710678+0.j, 0.        +0.j, 0.        +0.j,
             0.70710678+0.j],
            dims=(2, 2))
```

By inspecting the statevector, we see that this circuit indeed created

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

## 2 Problem 3

Your task is to modify the code above to create the circuits which generate the remaining three states of the Bell basis. The Bell basis is a set of four maximally entangled states of two qubits given as

$$\begin{aligned}
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\
|\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\
|\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).
\end{aligned}$$

The Bell states are discussed on Page 136 and 137 of the textbook, where example circuits are given.

Your code goes below:

[18]:
```
q=QuantumRegister(2,"q")
c=ClassicalRegister(2,"c")

circuit=QuantumCircuit(q,c)
circuit.x(q[0])
circuit.h(q[0])
circuit.cx(q[0],q[1])
print(circuit)
backend = Aer.get_backend('statevector_simulator') # exactly simulates the␣
 ↪evolution of the state starting in |00>
shots = 1 # simulate once
result=execute(circuit, backend=backend, shots=shots).result() # run simulation
statevector=result.get_statevector() # store the output statevector
print(statevector)
```

```
q_0:  X   H

q_1:        X
```

```
c: 2/

Statevector([ 0.70710678+0.00000000e+00j,  0.        +0.00000000e+00j,
              0.        +0.00000000e+00j, -0.70710678-8.65956056e-17j],
            dims=(2, 2))
```

[17]:
```
q=QuantumRegister(2,"q")
c=ClassicalRegister(2,"c")

circuit=QuantumCircuit(q,c)
circuit.x(q[1])
circuit.h(q[0])
circuit.cx(q[0],q[1])
print(circuit)
backend = Aer.get_backend('statevector_simulator') # exactly simulates the␣
 ↪evolution of the state starting in |00>
shots = 1 # simulate once
result=execute(circuit, backend=backend, shots=shots).result() # run simulation
statevector=result.get_statevector() # store the output statevector
print(statevector)
```

```
q_0:  H

q_1:  X   X

c: 2/

Statevector([0.        +0.j, 0.70710678+0.j, 0.70710678+0.j,
             0.        +0.j],
            dims=(2, 2))
```

[16]:
```
q=QuantumRegister(2,"q")
c=ClassicalRegister(2,"c")

circuit=QuantumCircuit(q,c)
circuit.x(q[1])
circuit.h(q[0])
circuit.z(q[0])
circuit.z(q[1])
circuit.cx(q[0],q[1])
print(circuit)
backend = Aer.get_backend('statevector_simulator') # exactly simulates the␣
 ↪evolution of the state starting in |00>
shots = 1 # simulate once
result=execute(circuit, backend=backend, shots=shots).result() # run simulation
statevector=result.get_statevector() # store the output statevector
```

```python
print(statevector)
```

```
q_0:   H   Z

q_1:   X   Z   X

c: 2/

Statevector([ 4.32978028e-17+0.00000000e+00j,
              7.07106781e-01+1.73191211e-16j,
             -7.07106781e-01-8.65956056e-17j,
             -4.32978028e-17-5.30245156e-33j],
            dims=(2, 2))
```

[ ]:

```python

```

[ ]:

```python

```