Lesson 19

Warning: These problems are going to severely tax your algebra skills and maybe also your patience since they are likely to require a lot of time and paper. Be prepared to review some college algebra techniques if necessary.

(1) Solve $a_0 = 1$, and $a_n = 3a_{n-1}$, for $n \geq 1$ using the characteristic equation method.

(2) Solve $a_0 = 1$, and $a_n = 3a_{n-1} + 1$ for $n \geq 1$ using the characteristic equation method. Hint: Try $a_n = A$, a constant, to find a particular solution.

(3) Solve $a_0 = 1, a_1 = 3$ and $a_n = a_{n-1} + 6a_{n-2}$, for $n \geq 2$.

(4) $a_0 = 1, a_1 = 3$ and $a_n = a_{n-1} + 6a_{n-2} + 1$, for $n \geq 2$. Hint: Use the general solution to the homogeneous problem above, and then try $a_n = A$, a constant, to find particular solution.

(5) Solve $a_0 = 1, a_1 = 6$ and $a_n = 6a_{n-1} - 9a_{n-2} + n$, for $n \geq 2$.
Hints: (1) Be sure to remember what to do when there are repeated characteristic roots. (2) Try $a_n = An + B$ for a particular solution.

(6) (bonus) Solve $a_0 = 0, a_1 = 1$ and $a_n = a_{n-1} + a_{n-2} + 2^n$, for $n \geq 2$.
Hints: This one involves a lot more algebra than the problems above. Solving the homogeneous problem will involve using the quadratic formula. The characteristic roots turn out to be $\dfrac{1 \pm \sqrt{5}}{2}$ (but show the work!). For a particular solution, try $a_n = A2^n$. You should find $A = 4$ (but show the work!). Put those two pieces together to write down the general solution

$$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n + 4(2^n)$$

and the determine the values for A and B by using the two initial conditions, $a_0 = 0$ and $a_1 = 1$. The necessary arithmetic will be somewhat complicated, but not impossible.