Lesson 8

(1) What is the 50^{th} term of the arithmetic sequence with initial term 4 and common difference 3?

(2) Evaluate $\displaystyle\sum_{k=-3}^{4}(2k+5)$. (Hint: Since there are only eight terms in the sum, you can just write them all out and add.)

(3) Evaluate $\displaystyle\sum_{i=0}^{99}\left(-\frac{2}{3}\right)^i$. (Hint: Since there are 100 terms in the sum, it isn't a good idea to write them all out and add. Use the formula for the sum of terms of a geometric sequence. Leave the answer with exponents rather than using a calculator to try to get a decimal approximation of the answer.)

(4) (a) List the first four terms of the sequence defined recursively by $a_0 = 2$, and, for $n \geq 1$, $a_n = 2a_{n-1}^2 - 1$.

(b) List the first five terms of the sequence with initial terms $u_1 = 1$ and $u_2 = 5$, and, for $n \geq 3$, $u_n = 5u_{n-1} - 6u_{n-2}$. Guess a closed form formula for the sequence. Hint: The terms are simple combinations of powers of 2 and powers of 3.

(5) Give a **recursive** definition of the geometric sequence with initial term a and common ratio r.

Hint: $a_n = ar^{n-1}$ isn't a correct answer since this formula isn't recursive. Make sure you write down a recursive formula: (1) give the initial term, and (2) give the rule for building new terms from previous terms.

(6) (bonus) Express in summation notation: $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}$, the sum of the reciprocals of the first n odd positive integers. (Note that there are n terms in the sum.)

Hint: A common mistake on this question is using the symbol n both as an index for summation and to indicate the last term to be added in. To make sure you haven't fallen into that trap, replace every n in your formula by a specific value, say 5. The result should be a sum $\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}$.