Lesson 17

(1) Inclusion-exclusion counting: Among the members of a book club, 20 people have read *War and Peace (WP)*, 14 have read *Moby Dick (MD)* and 27 have read *The Great Gatsby (GG)*, 5 have read WP and GG, 3 have read MD and GG, 6 have read WP and MD, and 2 have read all three books. Assuming everyone in the club has read at least one of those three books, how many members does the book club have?

(2) More inclusion-exclusion counting: How many bit strings of length 15 have bits $1, 2$, and 3 equal to 101, or have bits $12, 13, 14$, and 15 equal to 1001 or have bits $3, 4, 5$, and 6 equal to 1010? (Number bits from left to right. In other words, bit #1 is the left most bit and bit #15 is the right most bit.)

Hint: The fact that the third bit appears in two of the required patterns means some special care will be needed to get the count correct.

(3) Show that there are two different positive powers of 5 (in other words, 5^n, for positive integers n), that differ by a multiple of 742361. (Hint: Using the Pigeon Hole Principle, no arithmetic is needed to do this problem.)

(4) Plutonians have three feet. Suppose the Plutonian Vancleef has a box with one hundreds each of red, blue, yellow, green, and white socks. Plutonian etiquette requires that Vancleef wear three socks of the same color. It's dark, and Vancleef can't see. What is the minimum number of socks Vancleef needs to take from the box to be sure there will be a suitable triplet of socks?

(5) Rocky has 31 days to prepare for his Discrete Math final. He has decided to do at least one tough problem each day, but no more than 50 problems total. Show there must be a sequence of consecutive days in which he does exactly 11 problems.

(6) (bonus) Here's a pretty hard looking (but really not too hard if done carefully) inclusion-exclusion problem. The correct answer is 76. You job is to show the work that leads to that answer.

How many permutations of the digits $1, 2, 3, 4, 5$, have at least one digit in *its own spot*? In other words, a 1 in the first spot, or a 2 in the second, etc. For example, 35241 is OK since it has a 4 in the fourth spot, and 14235 is OK, since it has a 1 in the first spot (and also a 5 in the fifth spot). But 31452 in no good. Hint: Let A_1 be the set of permutations that have 1 in the first spot, let A_2 be the set of permutations that 2 in the second spot, and so on. As a start, notice that $|A_1| = |A_2| = |A_3| = |A_4| = |A_5| = 4!$