Lesson 5

(1) Let $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, c), (b, b), (b, d), (c, a), (c, c)\}$ be a relation on A. Draw a digraph which represents R. You might want to review the definition of digraph!

(2) Find the composition, $R \circ S$, where $S = \{\, (1, a), (4, a), (5, b), (2, c), (3, c), (3, d)\, \}$ with $R = \{(a, x), (a, y), (b, x), (c, z), (d, z)\}$ as a set of ordered pairs.

(3) Let $R_1 = \{(1, 2), (1, 3), (1, 5), (2, 1), (5, 6), (6, 6)\}$ and $R_2 = \{(1, 2), (1, 6), (3, 6), (4, 2), (5, 6), (6, 2), (6, 3)\}$. Find $R_1 \cup R_2$ and $R_1 \cap R_2$.

(4) Define a relation on $\{1, 2, 3, 4, 5\}$ by

$$R = \{(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 1), (1, 5)\}.$$

For each of the five properties of a relation defined in this chapter (reflexive, irreflexive, symmetric, antisymmetric, and transitive) either show R satisfies the property, or explain why it does not.

(5) Let A be the set of people alive on earth. For each relation defined below, determine if it is an equivalence relation on A. **If it is, describe the equivalence classes.** If it is not, determine which properties of an equivalence relation fail.

(a) $a\,H\,b \iff a$ and b are the same age in (in years).

(b) $a\,G\,b \iff a$ and b have at least one grandparent in common.

(6) Consider the relation $S(x, y)$: *x is a brother or sister of y* on the set, H, of living humans. (For the purposes of this problem, *a sibling* of a person means another person with the same two parents, so don't consider *half siblings*.) Determine which of the three properties, reflexive, symmetric, transitive, hold for the relation S (explain your three answers). (Hint: Think carefully about transitive! Almost everyone gets this part wrong.) Is S an equivalence relation on H?

(7) There are many different equivalence relation possible on the set $A = \{a, b, c, d\}$. For example, here are just three different ones:

(a) $E_1 = \{(a, a), (b, b), (c, c), (d, d), (a, c), (c, a), (b, d), (d, b)\}$.

(b) $E_2 = \{(a, a), (b, b), (c, c), (d, d), (a, c), (c, a), (a, b), (b, a), (b, c), (c, b)\}$.

(c) $E_3 = \{(a, a), (b, b), (c, c), (d, d)\}$.

E_1 has 8 ordered pairs while E_2 has 10 and E_3 has 4. Question: Of all the possible equivalence relations on A, what is the largest number of ordered pairs possible in the relation?

(8) Let $A = \{1, 2, 3, 4, 5, 6\}$. The sets $\{1, 2\}, \{3, 4, 5\}$, and $\{6\}$ form a partition of A. These are the equivalence classes for an equivalence relation, E, on A. Draw the **digraph** of E.

(9) (bonus) Let $A = \{1, 2, 3\}$. The relation $E = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$ is an equivalence relation on A. $F = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ is another equivalence relation on A. Compute the composition $F \circ E$. Is $F \circ E$ and equivalence relation on A?