Lesson 11

For the proofs requested below, use facts and theorem given in this lesson, including results given in the exercises, as justifications. Assume all letters represent integers.

(1) Mimic the proof given in the sample solutions for the proposition *if $a > 0$ and $b > 0$, then* $ab > 0$ to prove:

(a) If $a < 0$ and $b < 0$, then $ab > 0$.

(b) If $a < 0$ and $b > 0$, then $ab < 0$.

(2) Prove that is $m^2 = n^2$, then $m = n$ or $m = -n$.
(Hints: (1) From algebra: $a^2 - b^2 = (a + b)(a - b)$, and
(2) From exercises: If $ab = 0$, then either $a = 0$ or $b = 0$.)

(3) Determine all the integers that 0 divides.
Hint: Think carefully about the **definition** of the *divides* relation! This question is about the divides relation, not about the arithmetic operation of division (a maybe subtle distinction). The correct answer is probably not what you might first think it is.

(4) Prove: For integers r, s, t,and u, if $r|t$ and $s|u$, then $rs|tu$.

(5) Determine the quotient and remainder when 117653 is divided by 27869. (Finally, an easy problem.)

(6) (bonus) Prove or give a counterexample: If p is a prime, then $6p + 1$ is a prime.