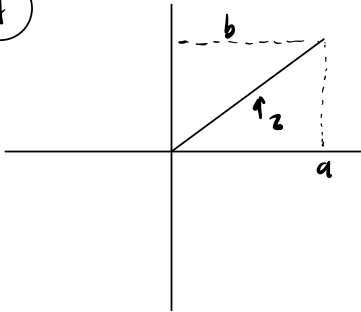


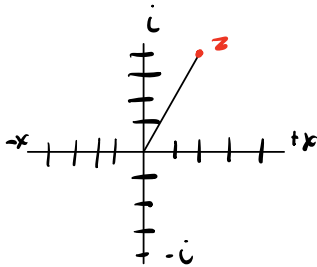
1: the value of the modulus of $z = 2 + 4i = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

(A)



The geometric interpretation of z when $z = 2 + 4i$ of the form $z = a + bi$ is that since $a = 2$ this shows the complex number's relation to the real numbers axis and then the $b=4$ shows it's relation to the complex numbers axis. In this it is 2 in the real direction and 4 in the complex. This leaves us with a magnitude of the $\sqrt{20}$

(B)



(C)

$$z = r \cdot e^{i\theta}$$

$$e^{i\theta}$$

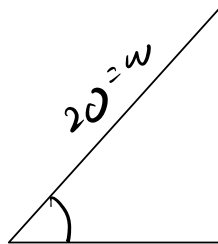
$$r = \sqrt{20}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(2) = 63.435^\circ$$

$$z = \sqrt{20} e^{i(63.435)}$$

(D)

$$w = 20 e^{i\frac{(63.435)}{2}}$$



$$10.515$$

$$b = r \sin \theta = 20 \sin(31.7175)$$

$$a = r \cos \theta = 17.013$$

$$10.515^2 = 2400$$

$$z = 17.013 + 10.515i$$

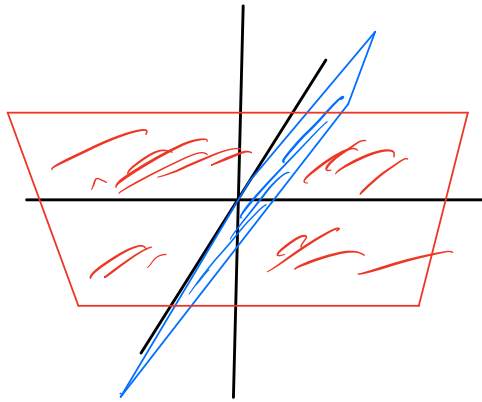
(2)

reduced matrix

$$\begin{bmatrix} 1 & 0 & -1/5 & 3/5 \\ 0 & 1 & 3/5 & 11/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x = 3/5 - 1/5 z \quad y =$$

$$x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1/5 \\ 3/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/5 \\ 11/5 \\ 0 \end{bmatrix}$$



$\bullet = x$
 $\bullet = y$

The solution set is two planes in \mathbb{R}^3

3

(A) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ (B) is where $C = C^T$

$$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad B^T = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$$

A matrix is, B is not as $B \neq B^T$

(C) $AB = \begin{bmatrix} 1 & 2 \\ 4 & 10 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 4 \\ 4 & 10 \end{bmatrix}$ no they are not equal

BA is symmetric, AB is not
the order of matrix can change a lot
as is shown here, they are not the same

(D) $(AB)^T = \begin{bmatrix} 1 & 4 \\ 2 & 10 \end{bmatrix}$ $B^T A^T = \begin{bmatrix} 1 & 8 \\ 2 & 10 \end{bmatrix}$ they are equal =

(E)

$$\begin{bmatrix} A_{11}b_{11} + A_{12}b_{21} & A_{11}b_{12} + A_{12}b_{22} \\ A_{21}b_{11} + A_{22}b_{21} & A_{21}b_{12} + A_{22}b_{22} \end{bmatrix}$$

4

A $\det \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 4 & 6 & 5 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 6 & 5 \end{bmatrix} - 3 \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 \\ 4 & 6 \end{bmatrix}$

$$2(5-6) - 3(5-4) + 2(6-4)$$

$$-2 - 3 + 4 = \boxed{1} \quad \text{since it is not } 0$$

the matrix is invertible

B

$$m = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 4 & 6 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{RREF}(m) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{thus the solutions are only } x=0, y=0, z=0$$

C

$$m = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 4 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \text{RREF}(m) = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} x_1 = 4 \\ x_2 = -3 \\ x_3 = 1 \end{matrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

D

$$\begin{bmatrix} 2-\lambda & 3 & 2 \\ 1 & 1-\lambda & 1 \\ 4 & 6 & 5-\lambda \end{bmatrix} = (2-\lambda)(1-\lambda)(5-\lambda)$$

$$(2-3\lambda+\lambda^2)(5-\lambda) = (10-15\lambda+\lambda^2) - \lambda^2 + 3\lambda^2 - \lambda^3$$

$$= 10 - 17\lambda + 8\lambda^2 - \lambda^3$$

$$\boxed{\lambda: 2, 1, 5}$$

these are the eigen values

B

The characteristic polynomial gives the eigen values by its roots. With this information information about the matrix and eigen vectors can be gathered.

5

S is a matrix of the linearly independent eigen vectors

A

D is the diagonal matrix that contains the eigen values on its diagonal

The result of SDS^{-1} is the original matrix.

SDS are a decomposition of the original matrix A

B

$$A = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3-\lambda & 2 \\ 3 & 4-\lambda \end{bmatrix}$$

$$(3-\lambda)(4-\lambda) - 6 = 0$$

$$12 - 7\lambda + \lambda^2 - 6 = 0$$

$$6 - 7\lambda + \lambda^2 = 0 \quad \lambda = 6, 1$$

$$\begin{bmatrix} 3-6 & 2 \\ 3 & 4-6 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix} \quad x = 2/3$$

$$x_2 \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2/3 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3/5 & 2/5 \\ 2/5 & -2/5 \end{bmatrix}$$

C

$$(-s-\lambda)(1-\lambda)+a \geq 0 \quad \lambda = -2$$

$$\begin{bmatrix} -s-2 & 3 \\ -3 & 1-2 \end{bmatrix} \Rightarrow \begin{bmatrix} -7 & 3 \\ -3 & -1 \end{bmatrix} \rightarrow \text{no it is not}$$

D

$$(-\lambda)$$

6

A

Two vectors that are orthogonal are two vectors that are perpendicular.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

B

A set of vectors that are linearly independent are ones that no combination of the vectors creates a 0 vector and that there is no vector that can be removed and complete the r^m . No you can not be linearly independent in R^3 with 4 vectors as the number of vectors, in order to be linearly independent, must = n in R^n

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

C

A set of vectors are orthonormal if they are all orthogonal and they are unit vectors meaning that they can be dotted with themselves and = 1.

$$(1, 1, 0), (1, 1, 0), (1, 0, 1)$$

D

I am close here but ran out of time to get the right solution

No you can not. As with $r(3)$ in order to be linearly independent you can not have more than 3 vectors. In order to be orthonormal or orthogonal you must also be linearly independent.

yes this is true since in order to be orthogonal they have be linear independent. This because in order to be orthogonal they can not be written in such a way that they result in the 0 vector.