1

A $\hat{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\hat{x}^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\hat{x}^+ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\hat{x}^{-1} = \hat{x}^+$ thus unitary

$\hat{y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\hat{y}^* = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ $\hat{y}^+ = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$\hat{y}^{-1} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ thus both

$\hat{z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\hat{x}^* = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\hat{x}^+ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\hat{z}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ thus both

B $\hat{x}$ $x^2 - 1 = 0$ values $\boxed{\lambda = -1, 1}$

$\hat{X}$ $x = \pm 1$

$\lambda = 1$ $\begin{bmatrix} 0-1 & 1 \\ 1 & 0-1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $x_1 - t = 0$ $x_2 = t$ $x_1 = t$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -1$ $\begin{bmatrix} 0+1 & 1 \\ 1 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} =$ $x_1 + t = 0$ $x_1 = -t$ $x_2 = t$ $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$x = x_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\hat{Y}$ $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\boxed{\lambda = -1, 1}$

$\lambda = 1$ $\begin{bmatrix} -1 & -i \\ i & -1 \end{bmatrix} \to \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$ $x_1 + i x_2 = 0$ $x_1 = -i x_2$ $x_2 = x_2$ $\begin{pmatrix} -i \\ 1 \end{pmatrix}$

$\lambda = -1$ $\begin{bmatrix} 1 & -i \\ i & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -i \\ 0 & 0 \end{bmatrix}$ $x_1 - i x_2 = 0$ $x_1 = i x_2$ $x_2 = x_2$ $\begin{pmatrix} i \\ 1 \end{pmatrix}$

$\boxed{x_1 \begin{pmatrix} -i \\ i \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

b)

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \boxed{\lambda = -1, 1}$

$\lambda = -1 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad x_1 = 0$, $x_2 = x_2$ $= x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\lambda = 1 = \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} =$ $\quad x_1 = x_1$, $x_2 = 0$ $\quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\boxed{x_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

c) $|z_i\rangle\langle z_i| =$

z_i where $i = -1$ or 1

z_{-1} $\lambda \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $[0\ 1]$

$\begin{bmatrix} 0 \\ -1 \end{bmatrix} [0\ 1] = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

$z_1 = \lambda \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1\ 0]$ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\sum$ of $z_i = z_{-1} + z_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

D)

it is called a phase flip because it changes the base state of |1> to -1|1> and leaves |0> the same

2

A

i

$$|x\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \qquad |y\rangle = \begin{bmatrix}\sqrt{2/3}\\ \sqrt{1/3}\end{bmatrix} \qquad |z\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$|x\rangle \otimes |y\rangle$

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2/3}\\ \sqrt{1/3}\end{pmatrix} \\ \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2/3}\\ \sqrt{1/3}\end{pmatrix}\end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2/3}}{\sqrt{2}} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \\ \frac{\sqrt{2/3}}{\sqrt{2}} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \end{pmatrix}$$

4 dimension

ii

ii

$$\begin{pmatrix} \frac{\sqrt{2/3}}{\sqrt{2}} \begin{bmatrix}0\\1\end{bmatrix} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \begin{bmatrix}0\\1\end{bmatrix} \\ \frac{\sqrt{2/3}}{\sqrt{2}} \begin{bmatrix}0\\1\end{bmatrix} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \begin{bmatrix}0\\1\end{bmatrix} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\sqrt{2/3}}{\sqrt{2}} \\ 0 \\ \frac{\sqrt{1/3}}{\sqrt{2}} \\ 0 \\ \frac{\sqrt{2/3}}{\sqrt{2}} \\ 0 \\ \frac{\sqrt{1/3}}{\sqrt{2}} \end{pmatrix}$$

8 = dimension

iii

$$\begin{pmatrix} 0 \\ \sqrt{2/3} \\ 0 \\ \sqrt{1/3} \end{pmatrix} = \left(|y\rangle \otimes |z\rangle\right)$$

$$|x\rangle \otimes (|x\rangle \otimes |z\rangle) \quad \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \otimes \begin{pmatrix}0\\ \sqrt{2/3}\\ 0\\ \sqrt{1/3}\end{pmatrix} = \begin{pmatrix}0\\ \frac{\sqrt{2/3}}{\sqrt{2}}\\ 0\\ \frac{\sqrt{1/3}}{\sqrt{2}}\\ 0\\ \frac{\sqrt{2/3}}{\sqrt{2}}\\ 0\\ \frac{\sqrt{1/3}}{\sqrt{2}}\end{pmatrix}$$

8 dimension

(b) $\hat{Y} = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix} \qquad \hat{X} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

(i)

$$\begin{matrix} 0\,\hat{X} & -i\,\hat{X} \\ i\,\hat{X} & 0\,\hat{X} \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}$$

4 dimension

(ii) $\hat{I} \otimes \hat{X}$

$$\begin{matrix} 1\,\hat{x} & 0\,\hat{x} \\ 0\,\hat{x} & 1\,\hat{x} \end{matrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

8 dimension

(iii)

$$I \otimes I = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \otimes I \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

32 dimension

(C)

(A)(i) $\begin{pmatrix} \frac{\sqrt{2/3}}{\sqrt{2}} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \\ \frac{\sqrt{2/3}}{\sqrt{2}} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \end{pmatrix}$

B(i) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{matrix} \frac{\sqrt{1/3}}{\sqrt{2}} \\ \frac{\sqrt{2/3}}{\sqrt{2}} \\ \frac{\sqrt{1/3}}{\sqrt{2}} \\ \frac{\sqrt{2/3}}{\sqrt{2}} \end{matrix}$

$B \cdot A =$

$$Y|(x) \otimes \hat{x}|Y = \begin{matrix} -\frac{\sqrt{1/3}}{\sqrt{2}} i \\ -\frac{\sqrt{2/3}}{\sqrt{2}} i \\ \frac{\sqrt{1/3}}{\sqrt{2}} i \\ \frac{\sqrt{2/3}}{\sqrt{2}} i \end{matrix}$$

they are almost same just ~~t~~ is multiplied by i and top 2 -

(3) $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$|A\rangle = \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix}$ $|B\rangle = \begin{bmatrix} \gamma \\ \delta \end{bmatrix}$

$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = |A\rangle$

$$|A\rangle \otimes |B\rangle = \begin{pmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{pmatrix}$$

$|00\rangle = \begin{pmatrix} 1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $|11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} =$

$$\begin{pmatrix} \alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ \alpha\gamma \\ 0 \\ 0 \\ 0 \\ \alpha\delta \\ 0 \\ 0 \\ 0 \\ \beta\gamma \\ 0 \\ 0 \\ 0 \\ \beta\delta \end{pmatrix}$$

$|11\rangle + |00\rangle$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \qquad = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \alpha\gamma \\ 0 \\ 0 \\ 0 \\ \alpha\delta \\ 0 \\ 0 \\ 0 \\ \beta\gamma \\ 0 \\ 0 \\ 0 \\ \beta\delta \end{pmatrix}$$

that it is like the pauli matrices and able to show others

HW1

February 1, 2023

1 Homework 1

1.0.1 Instructions:

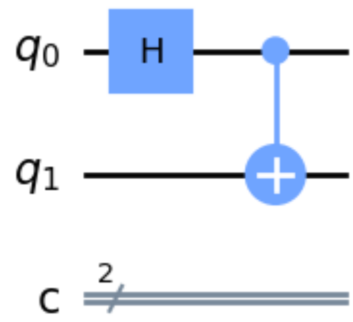
Run each block to constuct the circuit. In block 3, uncomment the final 3 lines to simulate the circuit. When you are done, attach a pdf of the completed notebook (with the circuit diagrams and histogram) to your homework submission.

[2]:
```
# Setup for quiskit

import qiskit
from qiskit import *
import numpy as np
from qiskit import QuantumCircuit, transpile, assemble, Aer, IBMQ
from qiskit.visualization import plot_histogram, plot_bloch_multivector
import matplotlib
```

In this block you will initialize the registers, create the a circuit with hadamard and CNOT gates, and print out the circuit.

[3]:
```
n=2 # number of qubits

q=QuantumRegister(2,"q") # opens two quantum registers
c=ClassicalRegister(2,"c") # opens two classical registers

circuit=QuantumCircuit(q,c) # initializes the circuit

circuit.h(q[0]) # hadamard on the first qubit
circuit.cx(q[0],q[1]) # CNOT from the first qubit to the second

circuit.draw(output="mpl") # output the circuit diagram
```

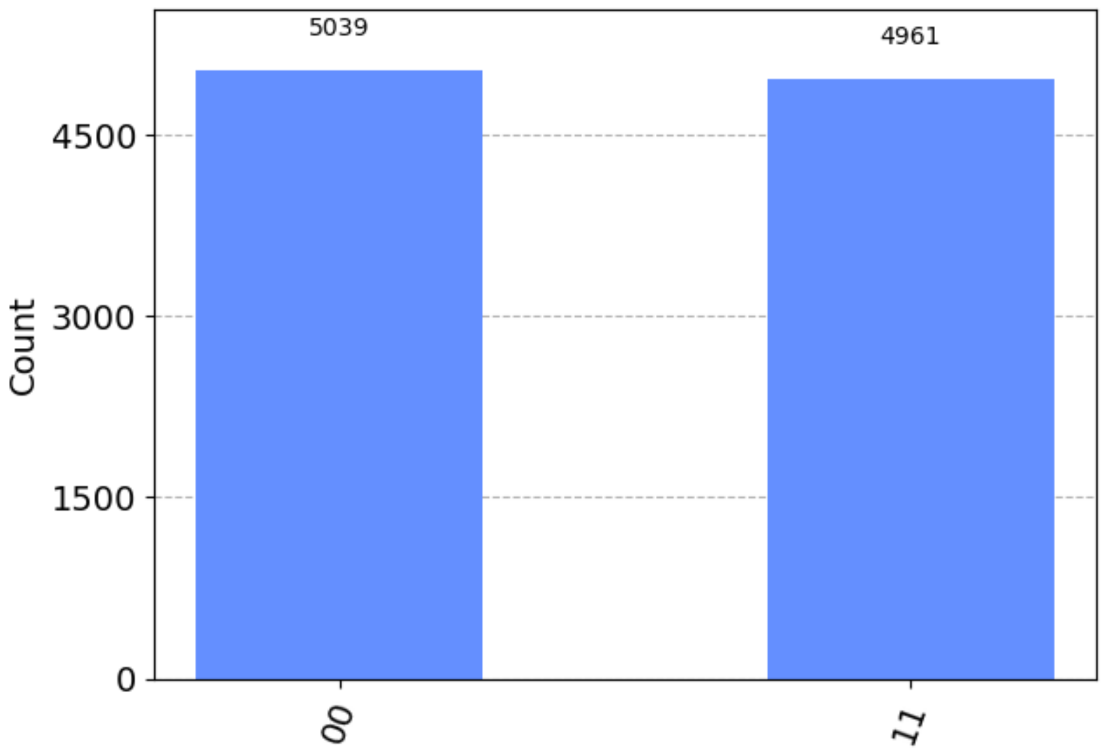
[3]:



In this block you will measure the qubits and run a simulation of the circuit.

[4]:
```
n=2

q=QuantumRegister(2,"q")
c=ClassicalRegister(2,"c")

circuit=QuantumCircuit(q,c)

circuit.h(q[0])
circuit.cx(q[0],q[1])

# above is the same as block 2

circuit.measure(q[0],c[0]) # measure first qubit and store in the first␣
 ↪classical register
circuit.measure(q[1],c[1]) # measure second qubit and store in the second␣
 ↪classical register

backend= Aer.get_backend('qasm_simulator') # initialize simulator
shots=10000 # number of trials

circuit.draw(output="mpl") # draw circuit

### UNCOMENT THE LINES BELOW ###

result=execute(circuit, backend=backend, shots=shots).result() # run simulation
counts=result.get_counts()
plot_histogram(counts) # plot histogram of results
```

[4]:


5039
4961
4500
3000
1500
0
Count
00
11

```
[ ]:
```