DiceFall

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December 23, 2024

Statement

You are playing a dice game where you start with a set of dice of different sizes. Each die is represented by the notation `XdY`, where `X` is the quantity of dice, and `Y` is the number of faces on each die.

In each round, all of the dice are rolled. If none of the dice show a `1`, the game proceeds to the next round. However, if any die rolls a `1`, that die is removed from the set, and you gain an additional roll (of all remaining dice) for that round. You can only gain one additional round per roll.

The game continues until there are no dice remaining.

Find the expected number of rounds (excluding the first one) required to finish the game, given the initial set of dice.

Input

The first line contains an expression in the form "Ad4 Bd6 Cd8 Dd10 Ed12 Fd20" where A, B, C, D, E and F ($0 \leq$ A, B, C, D, E, F, A+B+C+D+E+F ≤ 18) represent the quantities of dice with 4, 6, 8, 10, 12, and 20 faces respectively in the initial set of dice.

Output

Print the expected number of rounds (excluding the first one) required to finish the game, given the initial set of dice.

The answer will be accepted if the absolute or relative error does not exceed 10^5 .

Sample

Input 1	Output 1	Input 2	Output 2
1d4 0d6 0d8 0d10 0d12 0d20	3.0000000	1d4 2d6 0d8 3d10 1d12 0d20	16.518454

Input 3	Output 1	Input 2	Output 2
3d4 3d6 4d8 4d10 2d12 1d20	22.740551	2d4 2d6 4d8 4d10 4d12 2d20	27.882591

Solution and comments

This problem solution is straight-forward but the implementantion needs to take care of some details. Let's solve it for a list containing 2 elements (1d4, 1d6), and then generalize it.

Particular solution for [4, 6]: Given the initial list [1d4, 1d6] let $E_{4,6}$ denote the expected number of rounds (excluding the first one) required to win the game starting with 1d4 and 1d6. Then:

$$E_{4,6} = p_{4,6} \cdot (E_{4,6} + 1) + p_4 \cdot E_4 + p_6 \cdot E_6 + p \cdot E$$

where the probabilities are given by:

$$p_{4,6} = \frac{3 \cdot 5}{4 \cdot 6} \quad p_4 = \frac{3 \cdot 1}{4 \cdot 6} \quad p_6 = \frac{1 \cdot 5}{4 \cdot 6} \quad p = \frac{1 \cdot 1}{4 \cdot 6}$$

And the expected values for individual dice are:

$$E_4 = 3 \quad E_6 = 5 \quad E = 0$$

Substituting these values, the solution can be expressed as:

$$E_{4,6} = \frac{p_{4,6} + p_4 \cdot E_4 + p_6 \cdot E_6}{1 - p_{4,6}} = 5.\hat{4}$$

To simplify the calculations, note that all p-coefficients share the same denominator. We can define p' as the numerator of each p-coefficient:

$$E_{4,6} = \frac{p'_{4,6} + p'_4 \cdot E_4 + p'_6 \cdot E_6}{\prod\limits_{i \in \{4,6\}} i - p'_{4,6}} = 5.\hat{4}$$

General solution: Generalizing the solution for a list L, let S represent the set of all possible subsets of L. The expected value E_L can then be calculated as:

$$E_L = \frac{p'_L + \sum\limits_{\substack{s \in S \\ s \neq L}} p'_s \cdot E_s}{\prod\limits_{i \in L} i - p'_L} \qquad p'_s = \prod_{i \in s} (i-1) \quad p'_L = \prod_{i \in L} (i-1)$$

Optimizations: First of all, note that we can calculate $p'_s \cdot E_s$ instead of E_s and save the whole result for all expected values except the initial one. Also, if we save the states sorted, there will be few of them because we only have six possible different dice sizes. By the Pigeonhole Principle, any initial list with more than six dice will have duplicates, leading to a dense graph with relatively few unique states.

Time Complexity: The final time complexity of the algorithm is $O(2^n \cdot n)$ where n is the total number of dice in the initial round. It is recommended to set the time limit to 5 seconds.

Testing

- 0 dice input.
- All dice of the same size with different quantities.
- 3 times each dice size (I estimate this should be the most complex test case in time limit).

Difficulty

Easy, 3 out of 10. I estimate that approximately 15% (±5%) of the teams won't solve this problem.