Exam 2

Rules

- This is an individual written examination.
- You have 2 hours to complete this assigment.
- The use of handwritten notes, books, calculators is prohibited.
- **Any form of plagiarism will be reported.**

Question 1: A tuning fork may be regarded as a pair of vibrating flexible bars with a certain degree of stiffness. Each such bar is clamped at one end and is approximately modeled by the fourth-order PDE $u_{tt} + c^2 u_{xxxx} = 0$. It has initial conditions as for the wave equation. Let's say that on the end $x = 0$ it is clamped (fixed), meaning that it satisfies $u(0,t) = u_x(0,t) = 0$. On the other end $x = l$ it is free, meaning that it satisfies $u_{xx}(l,t) = u_{xxx}(l,t) = 0$. Thus there are a total of four boundary conditions, two at each end.

1. Using the separation of variables method get to the ODE $d^4X/dx^4 = \lambda X$.
2. Show that the only solution for $\lambda = 0$ is $X = 0$.
3. Assuming that $\lambda = \beta^4$, find an equation for β.
4. Compute the first two positive values of $l\beta$, say $l\beta_1$ and $l\beta_2$ and the quotient $(l\beta_2/l\beta_1)^2$.

Question 2: A quantum-mechanical particle on the line with an infinite potential outside the interval $(0,l)$ ("particle in a box") is given by Schrödinger's equation $u_t = iu_{xx}$ on $(0,l)$ with Dirichlet conditions at the ends. Separate the variables and find its representation as a series.

Question 3: Find the full Fourier series of for $g(x) = e^x$, on the interval $(-1,1)$, it is easier to find the complex Fourier series first. Use this result to solve the wave equation on $[-1,1] \times (0,\infty)$

$$
\begin{aligned}
u_{tt} &= c^2 u_{xx}, \\
u(x,0) &= g(x), \;\; u_t(x,0) = 0, \;\; \forall x \in [-1,1].
\end{aligned}
$$