Final Project

Rules

- You can submit this assignment in groups of up to four people.
- You are free to use the programming language of your liking.
- This project will also be taken into account for the course of numerical analysis.
- All answers must be thoroughly justified and must be submitted in a single .pdf file.
- Due date: Last week of the semester. Specific dates are not yet available.

The goal of this project is to gain some potentially life-saving insight into the dynamics of a tsunami and the mathematical modeling behind it.
The main reference for this project is the following article [1] which you must thoroughly read. Aside from this reference you are encouraged to consult other sources specially for the numerical simulations.

Propagation of a tsunami in the open ocean

The first part of this project consists in simulating the propagation of a tsunami in the open ocean. You must present a short animation showing the propagation of the wave from the initial surface displacement. The mathematical model under consideration is equation (1.2) of [1] which is a two dimensional, variable coefficient linear wave equation:

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{\partial}{\partial x}\left(gd(x,y)\frac{\partial \eta}{\partial x}\right) + \frac{\partial}{\partial y}\left(gd(x,y)\frac{\partial \eta}{\partial y}\right). \tag{1}$$

This equation needs initial conditions. You need to take an initial surface displacement $\eta(x, y, 0) = \phi(x, y)$ and initial velocity $\eta_t(x, y, 0) = 0$. The domain where you will solve the equation is a square of the form $[0, L_1] \times [0, L_2]$ and you can assume periodic boundary conditions.

In order to make the simulation you need to find the relevant parameters of the model. For L_1 take the distance between Tokyo and Los Angeles and for L_2 take the distance between Tokyo and New Guinea. Remember that $g = 9.81ms^{-2}$. Be careful with the units.

The average depth in the pacific ocean is 4Km. Choose a depth bounded function $d(x, y)$ that satisfies that $0 < d(x, y)$ and that typically takes the value of 4 Km.

For the initial displacement you need to choose a function that fits the description given in the paper.

Once all the data is set and the initial condition is chosen, you can run your simulation. The video should show the propagation of the surface displacement i.e. the tsunami. Run your simulation until it reaches one of the ends of the domain.

Physical description of the model

The second part of this project consists in a 10-15 min presentation regarding the physical laws taken in to account for the derivation of the model as well as a concise description of the mathematical model itself. One member of the groups will be chosen randomly and that person only will be responsible for the oral presentation. The relevant material can be found in the paper and the appendix.

References

[1] Arcas, D., & Segur, H. (2012). Seismically generated tsunamis. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 370(1964), 1505-1542.