Newton Method

$$\boxed{X_{k+1} = X_k - H_k^{-1} g_k}$$

$$\begin{array}{r} 1 \\ 0.5 \\ \times\ 2 \\ \hline 1.0 \end{array}$$

using Newton's method

$$f(x) = 0.5x_1^2 + 2.5x_2^2 \qquad 2.5$$

Sol:-

$$H_{f(x)} = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 5x_2 \end{bmatrix}$$

$$\cancel{H} \; [$$

$$\frac{\partial f}{\partial x_1} = x_1 \qquad \frac{\partial^2 f}{\partial x_1^2} = 1$$

$$\frac{\partial f}{\partial x_2} = 5x_2 \Rightarrow \frac{\partial^2 f}{\partial x_2^2} = 5$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial}{\partial x_1}(5x_2) = 0$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial}{\partial x_2}(x_1) = 0$$

$H'4 H = 0$

$$H_K = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\nabla f_{x_0} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$g_K = -\nabla f_{\cancel{x_0}} = \begin{bmatrix} -5 \\ -5 \end{bmatrix}$$

$$X_2 = X_1 - H_K^{-1} g_K$$

$$\cancel{H}\; X_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \frac{1}{5} \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ -5 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} -5 \\ -5 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} -5+0 \\ 0-1 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} -5 \\ -1 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$