Steepest descent

$$f(\underbrace{x_1}_{x}, \underbrace{x_2}_{y}) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$$

Starting from the point

$$X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \end{matrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:-

Iteration 1 :-

$$\nabla f = \begin{Bmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \end{Bmatrix} = \begin{Bmatrix} 1 + 4x_1 + 2x_2 \\ -1 + 2x_1 + 2x_2 \end{Bmatrix}$$

$$\nabla f = \nabla f(X_1) = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underline{S_1} = -\nabla f_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

* ~~Now~~ determine the optimal step length λ^* ~~in~~

$$X_2 = X_1 + \lambda_1^* S_1$$

$$X_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times 1} + \lambda_1^* \begin{bmatrix} -1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$X_2 = \begin{bmatrix} -\lambda_1^* \\ \lambda_1^* \end{bmatrix} \begin{matrix} \rightarrow x_1 \\ \rightarrow x_2 \end{matrix}$$

$$f(X_1 + \lambda_1^* S) = f(-\lambda_1, \lambda_1)$$

$$\cancel{f(\lambda_1} = -\lambda_1 - \lambda_1 + 2(-\lambda_1)^2 + 2(-\lambda_1)(\lambda_1) + 2(\lambda_1)^2$$

$$= -2\lambda_1 + \cancel{2\lambda_1^2} - \cancel{2\lambda_1^2} + \lambda_1^2$$

$$f(-\lambda_1, \lambda_1) = \lambda_1^2 - 2\lambda_1$$

$$\cancel{\partial f} \quad \frac{df}{d\lambda_1} = 2\lambda_1 - 2$$

$$\frac{df}{d\lambda_1} = 0$$

$$2\lambda_1 - 2 = 0$$

$$\boxed{\lambda_1 = 1}$$

$$X_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\boxed{X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{array}{l} \to x_1 \\ \to x_2 \end{array}}$$