Optimizing the Energy Output of a Solar Panel

An Extended Essay in Physics

How does the tilt angle and time of day affect the energy output of a solar panel?

Word Count: 3,864

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INTRODUCTION

Energy in the form of electricity in today's world is of crucial importance, and its demand is only rising. I was with my parents when they were at a company that installs solar panels, and the clerk was explaining how much money we would save on the long run and how environment-friendly, compared to the burning of fossil fuels, it is. Since then, my main interest in science is the harnessing of energy in the most efficient and non-polluting way. It is for this reason why I chose a specific topic within physics and to investigate one of the cleanest forms of electrical energy production: the use of photovoltaic solar panels.

Utilization of the photovoltaic cell is one of the few solutions that stands a chance of reducing the additional warming of the Earth caused by the enhanced greenhouse effect. The solar panel allows for instant electrical energy without the emission of any greenhouse gas. Therefore, we must find a way to maximize the efficiency of these panels in order to create a cleaner future for the environment and humanity. This can be done by one of two ways: the first is to design the panel to ensure maximum energy output, and the second is to orient the panel itself in the best possible way.

This investigation revolves around the latter since it is the more manageable portion for a student of my level. My experiment is also narrowed down to focus on only two factors – the tilt angle relative to the ground and the time of day – shown in the following research question:

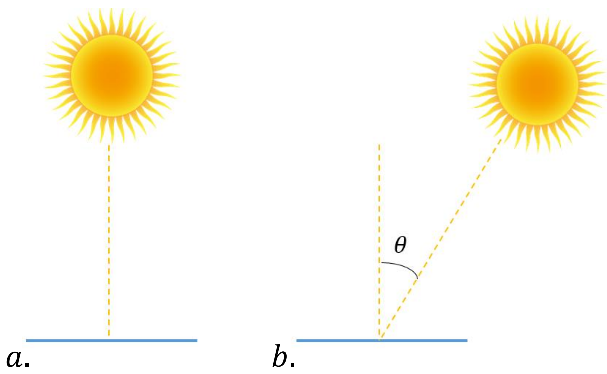
- How does the tilt angle and time of day affect the energy output of a solar panel?

My hypothesis is that there will be an optimum tilt angle in which every angle before it and after it will have an energy output less than it. This is because there will only be *one* angle with a maximum output. I hypothesize this angle to be the $45°$ angle the solar panel makes with the floor as this is the midpoint between $0°$ and $90°$. I also predict the time of day that gives the maximum output to be 12PM as this is when the sun's intensity is usually highest.

BACKGROUND INFORMATION

A photovoltaic solar panel is a device that "converts sunlight into electricity" (Dhar). The amount of energy converted depends to a large extent on the tilt angle of the panel and the intensity of sunlight received. Looking at Figure 1, there are two diagrams, a and b, both showing a solar panel placed horizontally on the ground with its normal.

Figure 1: Diagrams showing the sun's insolation when it is perpendicular to a horizontal solar panel (a) and forms an angle θ with the normal of the horizontal solar panel (b)



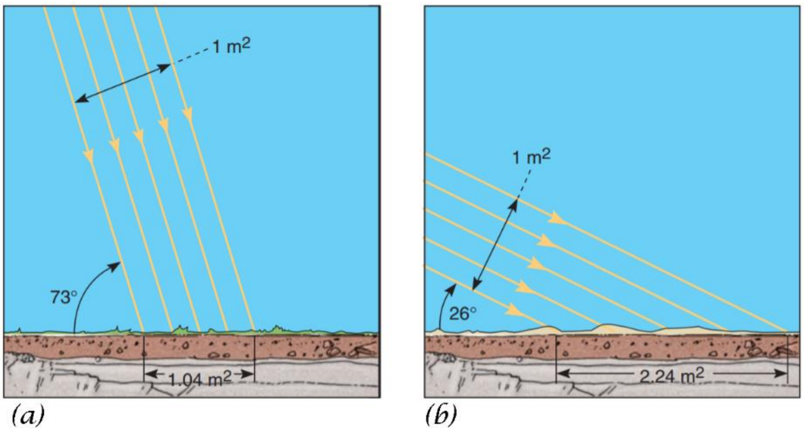
Insolation means "*in*coming *sol*ar radi*ation*" (Earth-Sun Relationships) In Figure $1.a$, the insolation forms an angle of $0°$ with the normal of the solar panel, while in Figure $1.b$, it forms an angle of θ. θ has no scientific name, but the angle by which the sun deviates from the normal of the Earth is called the "sun's zenith angle." (Earth-Sun Relationships) Note that in Figure 1, the normal of the Earth and of the solar panel are the same. Now, we can use Lambert's Law which "relates [θ] to the irradiance of the surface." (Mortimer) Lambert's Law is given by:

$$I = I_{insolation} \cos \theta$$

In this case, I is the irradiance of the solar panel. In Figure $1.a$, theta is equal to zero, therefore $\cos\theta = 1$. Here the solar panel receives the maximum intensity from insolation, thereby producing the most power. In Figure $1.b$, the angle theta is no longer zero and will cause a decrease in the irradiance of the solar panel, so the power output is smaller. Therefore, to have a maximum power output, the angle between the insolation and the normal of the solar panel must be zero, which means that the angle of tilt of the panel must be equal to the sun's zenith angle.

The time of day also has a major impact on the power output of the solar panel. To explain this, we must first understand that at different times during the day, the sun's rays form different angles with respect to the ground (this is shown in the Figure below).

Figure 2: Petersen, James F., et al. The geometric relationships between Earth and the sun during the June and December solstices. Cengage Learning, 1 Aug. 2012



In Figure $2a$, the rays form a larger angle with the ground than in Figure $2b$. Therefore, in Figure $2b$, the "rays spread out over a much wider area" (Earth-Sun Relationships) due to the smaller angle. Now, the formula for intensity is shown below, where P is the power of radiation received, in W, and A is the area over which it was received, in m^2:

$$I = \frac{P}{A}$$

From this formula, area and intensity are inversely proportional; increasing the area will decrease the intensity of the insolation. Referring back to Figure $2a$, the larger angle formed a smaller area than Figure $2b$, therefore the intensity is higher. This means that there will be an optimum time of day when the sun rays form the largest angle with respect to the ground.

VARIABLES

Independent Variables:

- The tilt angle of the solar panel with respect to the ground.
- The time of day between $9\mathrm{AM}$ and $3\mathrm{PM}$ with one-hour intervals. I chose these times in specific because during these times the insolation will be most intense.

Dependent Variable:

- The voltage and current reading on the multimeter will increase/decrease according to the variation of the independent variables.

Controlled Variables & Assumptions:

- The same position will be used for all trials (school's playground) to ensure that no changes in altitude will affect the power output.
- The solar panel will be directed towards the South for all trials because this is the standard direction for all solar panels in the Northern Hemisphere
- It is assumed that the experiment happens under Standard Test Conditions, which means that the solar panel receives $1{,}000W \cdot m^{-2}$ of sunlight and that the temperature is $25°\mathrm{C}$ for all trials.
- It is assumed that no other factors, such as clouds entering the sky for example, will contribute to a change in power output.

EQUIPMENT

Table 1: Equipment with their range and uncertainty values if applicable

Equipment	Range	Uncertainty
Protractor	$180°$	$\pm 1°$
Multimeter on DC Voltage	$500V$	$\pm 0.01V$
Multimeter on Current	$10A$	$\pm 0.01A$
iPhone Compass	$360°$	$\pm 2°$
$1m^2$ solar photovoltaic panel		
2 alligator clips		

Note that the actual uncertainty of the protractor is $\pm 0.25°$ as the smallest graduation is $0.5°$, but I increased it to $1°$ to compensate for human error. Also, the uncertainty of the compass should be $1°$ as the smallest graduation is $2°$; I kept it at $2°$ because while tilting the panel, it deviated slightly from true south, so this error must be taken into account.

PROCEDURE

1. On the junction box on the back of the solar panel, connect the red alligator clip to the positive terminal and the black alligator clip to the negative terminal.
2. Connect the other sides of the clips to the multimeter.
3. Use the compass to find the direction of the South.
4. Place the solar panel on the floor ($0°$ tilt angle) and directed South.
5. To find the voltage from the multimeter, set it to DC $20V$. If the voltage is greater than $20V$, set the multimeter to DC $200V$. Wait until the reading becomes almost the same and record the voltage.
6. To find the current from the multimeter, set it to $10A$. Wait until the reading becomes almost the same and record the current.
7. Repeat steps 1 to 7 for angles $10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, \& \ 90°$, and repeat them for 3 days every hour from $9{:}00\text{AM}$ to $3{:}00\text{PM}$.

SAFETY

Photovoltaic cells are made mostly from silicone, therefore “there are no dangers of leaking toxins or fumes.” (Are Solar Panels Safe?) Also, the experiment does not involve the use of a solar battery, so there are no safety issues with explosions or acid leakage.

RAW DATA

Tables 2,3 and 4 below show the data that was collected from the trials. Note that V is the voltage measured in Volts, I is the current measured in Amperes, and the uncertainty of both is ± 0.01.

Furthermore, while conducting the experiment, I noticed that the voltage was approximately the same with little fluctuations, but it was the current that would vary substantially. I also noticed that for the majority of the experiment, the sun was on the East while the panel was directed South. As for the general weather, Day 1 was slightly cloudy while Day 2 and Day 3 were very sunny.

Tables 2,3,4: Voltage & current values for the varying tilt angles from 9AM to 3PM on three different days

Time	Day 1																			
	0°		10°		20°		30°		40°		50°		60°		70°		80°		90°	
	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I
9AM	18.95	1.16	19.00	1.20	19.06	1.24	19.10	1.28	19.11	1.28	18.93	1.20	18.75	1.17	18.67	1.11	18.57	1.00	18.43	0.97
10AM	19.13	1.39	19.24	1.50	19.41	1.57	19.45	1.59	19.46	1.59	19.27	1.53	19.06	1.47	18.99	1.41	18.92	1.27	18.47	1.21
11AM	19.38	1.65	19.52	1.84	19.61	1.90	19.65	1.95	19.67	1.96	19.54	1.94	19.35	1.83	19.23	1.65	19.17	1.47	18.55	1.23
12PM	19.61	1.95	19.89	2.08	20.01	2.13	20.02	2.15	20.10	2.18	19.77	2.11	19.58	2.00	19.40	1.83	19.33	1.61	18.67	1.30
1PM	19.31	1.89	19.39	1.95	19.47	2.05	19.50	2.06	19.49	2.06	19.32	2.00	19.23	1.87	19.16	1.69	19.12	1.49	18.63	1.24
2PM	19.15	1.56	19.21	1.67	19.28	1.75	19.30	1.78	19.31	1.78	19.17	1.72	19.13	1.69	19.04	1.54	18.96	1.33	18.54	1.22
3PM	19.07	1.40	19.13	1.45	19.15	1.50	19.19	1.56	19.19	1.58	19.16	1.50	19.12	1.41	18.93	1.35	18.79	1.26	18.39	1.18

Time	Day 2																			
	0°		10°		20°		30°		40°		50°		60°		70°		80°		90°	
	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I	V	I
9AM	18.97	1.46	19.05	1.53	19.27	1.59	19.41	1.65	19.45	1.63	19.34	1.57	19.23	1.52	19.14	1.41	19.10	1.26	18.97	1.13
10AM	19.22	1.75	19.31	1.83	19.58	1.90	19.65	1.95	19.68	1.95	19.57	1.88	19.44	1.77	19.29	1.68	19.17	1.54	19.02	1.25
11AM	19.66	2.09	19.79	2.16	19.91	2.24	19.97	2.31	20.02	2.30	19.93	2.25	19.85	2.16	19.62	1.95	19.42	1.62	19.24	1.36

Time	Day 2																			
	0°		10°		20°		30°		40°		50°		60°		70°		80°		90°	
	V	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***
12PM	19.99	2.37	20.07	2.45	20.28	2.50	20.35	2.52	20.40	2.57	20.35	2.67	20.23	2.48	19.90	2.06	19.63	1.78	19.43	1.57
1PM	19.71	2.03	19.90	2.19	20.09	2.28	20.25	2.38	20.28	2.37	20.13	2.22	20.09	2.13	19.71	1.92	19.48	1.70	19.19	1.33
2PM	19.50	1.78	19.58	1.88	19.82	1.99	19.88	2.09	19.88	2.10	19.77	2.03	19.66	1.92	19.43	1.70	19.32	1.45	19.14	1.35
3PM	19.17	1.61	19.29	1.72	19.46	1.80	19.71	1.85	19.71	1.86	19.62	1.74	19.53	1.65	19.18	1.58	19.23	1.36	18.99	1.22

Time	Day 3																			
	0°		10°		20°		30°		40°		50°		60°		70°		80°		90°	
	V	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***	***V***	***I***
9AM	18.93	1.34	19.10	1.41	19.20	1.52	19.32	1.62	19.33	1.58	19.38	1.59	19.22	1.49	19.11	1.37	18.95	1.22	18.85	1.12
10AM	19.14	1.59	19.31	1.66	19.39	1.77	19.51	1.82	19.48	1.79	19.51	1.73	19.40	1.68	19.28	1.55	19.11	1.36	18.99	1.26
11AM	19.48	1.78	19.53	1.92	19.65	2.00	19.79	2.09	19.79	2.1	19.67	1.88	19.65	1.84	19.41	1.73	19.27	1.51	19.12	1.33
12PM	19.71	2.11	19.84	2.18	19.92	2.23	19.96	2.28	19.97	2.32	19.96	2.18	19.88	2.07	19.69	1.88	19.50	1.78	19.33	1.55
1PM	19.37	1.94	19.62	2.05	19.77	2.12	19.81	2.16	19.75	2.17	19.59	2.05	19.57	1.88	19.56	1.81	19.36	1.67	19.20	1.32
2PM	19.20	1.70	19.37	1.83	19.53	1.90	19.57	1.93	19.52	1.93	19.54	1.91	19.33	1.79	19.50	1.58	19.16	1.48	19.08	1.17
3PM	19.06	1.43	19.13	1.59	19.26	1.69	19.35	1.74	19.33	1.73	19.43	1.70	19.12	1.62	19.30	1.44	19.03	1.32	18.91	1.10

PROCESSING DATA

Using $P = V \times I$ will allow us to find the power output of the solar panel. For the Day 1, 0°, 9AM trial:

$$P = 19.26V \times 0.72A = 13.87W$$

The uncertainty of both the voltage and current is ± 0.01, therefore the uncertainty of the power for the trial take above is:

$$\frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta I}{I} \quad \rightarrow \quad \Delta P = (VI)\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) = I\Delta V + V\Delta I$$

Since $\Delta V = \Delta I = 0.01$, then:

$$\Delta P = 0.01 \times (V + I) = 0.01 \times (19.26 + 0.72) = 0.20W$$

Now, we can assume that from 9AM to 10AM, the total energy output is:

$$P = \frac{13.87J}{1\ s} \times \frac{60\ s}{1\ min} \times \frac{60\ min}{1\ hour} \times \frac{1\ kJ}{1000\ J} = 49.92kJ \cdot hr^{-1}$$

The same calculation can be done for ΔP:

$$\Delta P = \frac{0.20J}{1\ s} \times \frac{60\ s}{1\ min} \times \frac{60\ min}{1\ hour} \times \frac{1\ kJ}{1000\ J} = 0.72kJ \cdot hr^{-1}$$

Now, if the solar panel were kept at *one* angle from 9AM to 3PM, the total power output would be the sum of outputs of each hour. An example is given for the Day 1, 0° trial.

$$P_{Total} = P_{9AM} + P_{10AM} + P_{11AM} + P_{12PM} + P_{1PM} + P_{2PM} + P_{3PM}$$

$$P_{Total} = 49.92 + 136.76 + 268.46 + 298.17 + 203.48 + 150.05 + 147.42 = 1254.25kJ \cdot hr^{-1}$$

The uncertainty of P_{Total} is the sum of all uncertainties:

$$\Delta P_{Total} = 0.72 + 0.79 + 0.87 + 0.86 + 0.81 + 0.78 + 0.78 = 5.61kJ \cdot hr^{-1}$$

This tells us that if the solar panel were kept flat on the ground from 9AM to 3PM, the total power it would generate would be $1254.25 \pm 5.61kJ$. The tables below summarize the calculations done above for all three days.

Tables 5,6,7: Power output and uncertainty values for days 1, 2, and 3, and the total power output for each angle

Time	Day 1 – Power ($kJ \cdot hr^{-1}$)									
	0°	**10°**	**20°**	**30°**	**40°**	**50°**	**60°**	**70°**	**80°**	**90°**
9AM	79.14 ± 0.72	82.08 ± 0.73	85.08 ± 0.73	88.01 ± 0.73	88.06 ± 0.73	81.78 ± 0.72	78.98 ± 0.72	74.61 ± 0.71	66.85 ± 0.70	64.36 ± 0.70
10AM	95.73 ± 0.74	103.90 ± 0.75	109.71 ± 0.76	111.33 ± 0.76	111.39 ± 0.76	106.14 ± 0.75	100.87 ± 0.74	96.39 ± 0.73	86.50 ± 0.73	80.46 ± 0.71
11AM	115.12 ± 0.76	129.30 ± 0.77	134.13 ± 0.77	137.94 ± 0.78	138.79 ± 0.78	136.47 ± 0.77	127.48 ± 0.76	114.23 ± 0.75	101.45±0.74	82.14 ± 0.71
12PM	137.66 ± 0.78	148.94 ± 0.79	153.44 ± 0.80	154.95 ± 0.80	157.74 ± 0.80	150.17 ± 0.79	140.98 ± 0.78	127.81 ± 0.76	112.04±0.75	87.38 ± 0.72
1PM	131.39 ± 0.76	136.12 ± 0.77	143.69 ± 0.77	144.61 ± 0.78	144.54 ± 0.78	139.10 ± 0.77	129.46 ± 0.76	116.57 ± 0.75	102.56±0.74	83.16 ± 0.72
2PM	107.55 ± 0.75	115.49 ± 0.75	121.46 ± 0.76	123.67 ± 0.7	123.74 ± 0.76	118.70 ± 0.75	116.39 ± 0.75	105.56 ± 0.74	90.78 ± 0.73	81.43 ± 0.71
3PM	96.11 ± 0.74	99.86 ± 0.74	103.41 ±0.74	107.77 ± 0.75	109.15 ± 0.75	103.46 ± 0.74	97.05 ± 0.74	92.00 ± 0.73	85.23 ± 0.72	78.12 ± 0.70
P_{Total}	**762.69 ± 5.24**	**815.68 ±5.29**	**850.92 ± 5.33**	**868.30 ± 5.35**	**873.41 ± 5.36**	**835.83 ± 5.30**	**791.19 ± 5.24**	**727.16 ± 5.18**	**645.41 ± 5.12**	**557.04 ± 4.97**

Time	Day 2 – Power ($kJ \cdot hr^{-1}$)									
	0°	**10°**	**20°**	**30°**	**40°**	**50°**	**60°**	**70°**	**80°**	**90°**
9AM	99.71 ± 0.74	104.93 ± 0.74	110.30 ± 0.75	115.30 ± 0.76	114.13 ± 0.76	109.31 ± 0.75	105.23 ± 0.75	97.15 ± 0.74	86.64 ± 0.73	77.17 ± 0.72
10AM	121.09 ± 0.75	127.21 ± 0.76	133.93 ± 0.77	137.94 ± 0.78	138.15 ± 0.78	132.45 ± 0.77	123.87 ± 0.76	116.67 ± 0.75	106.28±0.75	85.59 ± 0.73
11AM	147.92 ± 0.78	153.89 ± 0.79	160.55 ± 0.80	166.07 ± 0.80	165.77 ± 0.80	161.43 ± 0.80	154.35 ± 0.79	137.73 ± 0.78	113.26±0.76	94.20 ± 0.74
12PM	170.55 ± 0.80	177.02 ± 0.81	182.52 ± 0.82	184.62 ± 0.82	188.74 ± 0.83	195.60 ± 0.83	180.61 ± 0.82	147.58 ± 0.79	125.79±0.77	109.82±0.76

Time	Day 2 – Power ($kJ \cdot hr^{-1}$)									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
1PM	144.04 ± 0.78	156.89 ± 0.80	164.90 ± 0.81	173.50 ± 0.81	173.03 ± 0.82	160.88 ± 0.80	154.05 ± 0.80	136.24 ± 0.78	119.22±0.76	91.88 ± 0.74
2PM	124.96 ± 0.77	132.52 ± 0.77	141.99 ± 0.79	149.58 ± 0.79	150.29 ± 0.79	144.48 ± 0.78	135.89 ± 0.78	118.91 ± 0.76	100.85±0.75	93.02 ± 0.74
3PM	111.11 ± 0.75	119.44 ± 0.76	126.10 ± 0.77	131.27 ± 0.78	131.98 ± 0.78	122.90 ± 0.77	116.01 ± 0.76	109.10 ± 0.75	94.15 ± 0.74	83.40 ± 0.73
P_{Total}	**919.37 ± 5.38**	**971.90 ± 5.43**	**1020.29 ± 5.50**	**1058.27 ± 5.54**	**1062.09 ± 5.55**	**1027.05 ± 5.51**	**970.01 ± 5.46**	**863.37 ± 5.35**	**746.18 ± 5.26**	**635.08 ± 5.15**

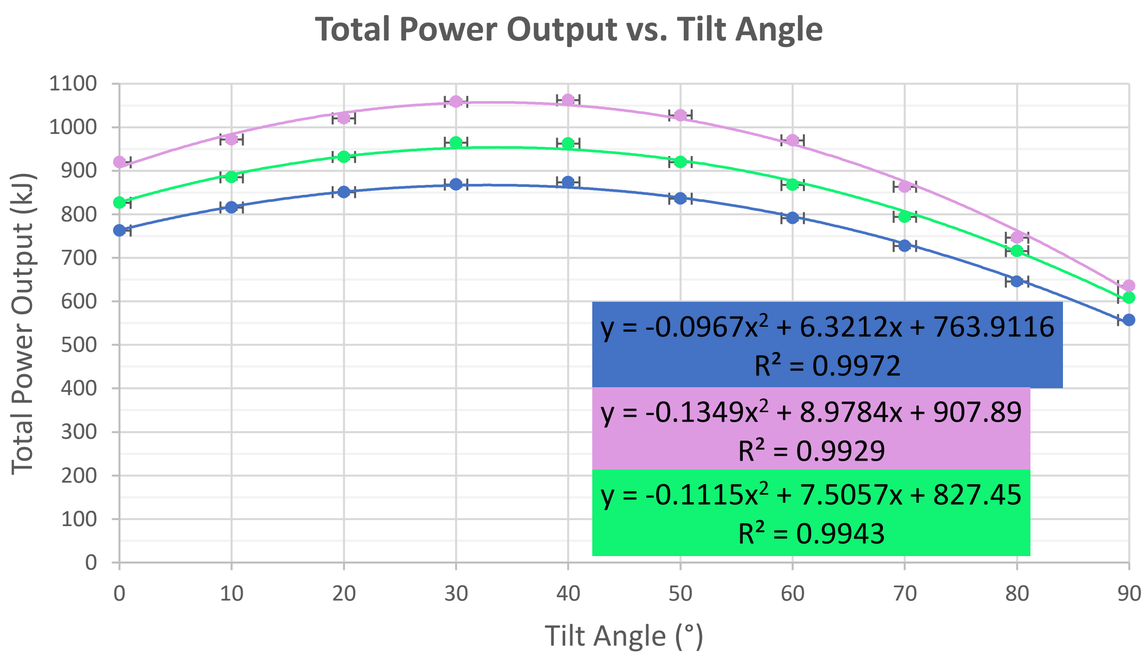
Time	Day 3 – Power ($kJ \cdot hr^{-1}$)									
	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
9AM	91.32 ± 0.73	96.95 ± 0.74	105.06 ± 0.75	112.67 ± 0.75	109.95 ± 0.75	110.93 ± 0.75	103.10 ± 0.75	94.25 ± 0.74	83.23 ± 0.73	76.00 ± 0.72
10AM	109.56 ± 0.75	115.40 ± 0.75	123.55 ± 0.76	127.83 ± 0.77	125.53 ± 0.77	121.51 ± 0.76	117.33 ± 0.76	107.58 ± 0.75	93.56 ± 0.74	86.14 ± 0.73
11AM	124.83 ± 0.77	134.99 ± 0.77	141.48 ± 0.78	148.90 ± 0.79	149.61 ± 0.79	133.13 ± 0.78	130.16 ± 0.77	120.89 ± 0.76	104.75±0.75	91.55 ± 0.74
12PM	149.72 ± 0.79	155.70 ± 0.79	159.92 ± 0.80	163.83 ± 0.80	166.79 ± 0.80	156.65 ± 0.80	148.15 ± 0.79	133.26 ± 0.78	124.96±0.77	107.86±0.75
1PM	135.28 ± 0.77	144.80 ± 0.78	150.88 ± 0.79	154.04 ± 0.79	154.29 ± 0.79	144.57 ± 0.78	132.45 ± 0.77	127.45 ± 0.77	116.39±0.76	91.2 ± 0.74
2PM	117.50 ± 0.75	127.61 ± 0.76	133.59 ± 0.77	135.97 ± 0.77	135.62 ± 0.77	134.36 ± 0.77	124.56 ± 0.76	110.92 ± 0.76	102.08±0.74	80.36 ± 0.73
3PM	98.12 ± 0.74	109.50 ± 0.75	117.18 ± 0.75	121.21 ± 0.76	120.39 ± 0.76	118.91 ± 0.76	111.51 ± 0.75	100.05 ± 0.75	90.43±0.73	74.88 ± 0.72
P_{Total}	**826.33 ± 5.28**	**884.95 ± 5.35**	**931.66 ± 5.40**	**964.46 ± 5.43**	**962.18 ± 5.43**	**920.05 ± 5.40**	**867.25 ± 5.35**	**794.40 ± 5.30**	**715.41 ± 5.21**	**608.04 ± 5.12**

THE TILT ANGLE FACTOR

The graph below shows the relationship between the total power output and the tilt angle.

The graph can tell us that if the solar panel was kept at $10°$ from 9AM to 3PM during Day 1, the total power output would be $1116.68 \pm 5.53kJ$.

Graph 1: Total power output versus the tilt angle for days 1, 2, and 3



In Graph 1, the blue curve corresponds to the data values of Day 1, the purple to Day 2, and the green to Day 3. Also, in the equation for the line of best fit, x is the tilt angle in degrees and y is the total power output in kJ.All three curves have a best-fit line of the same shape – a negative parabolic curve – which means that the experiment yielded accurate data. From $0°$ to $40°$, the power output increases until there is a maximum output at somewhere between $30°$ and $40°$. Then, after this maximum, the power output decreases until the minimum power output, which occurs at $90°$. Note that the horizontal error bars are for the uncertainty values of the angle, which is $\pm 1°$. The vertical error bars are for the uncertainty values of the power output, but as they are very small compared to the power output itself, they do not show on the graph.

Now, from Graph 1, it is evident that the optimum tilt angle is at the vertex of the parabola, which can be found by deriving the equation of the best-fit line and equating it to zero:

Day 1	Day 2	Day 3
$\frac{dy}{dx} = -0.1946x + 6.3638$ & $\frac{dy}{dx} = 0$ ➔ $x = 32.70°$	$\frac{dy}{dx} = -0.2694x + 8.9522$ & $\frac{dy}{dx} = 0$ ➔ $x = 33.23°$	$\frac{dy}{dx} = -0.2218x + 7.4483$ & $\frac{dy}{dx} = 0$ ➔ $x = 33.58°$

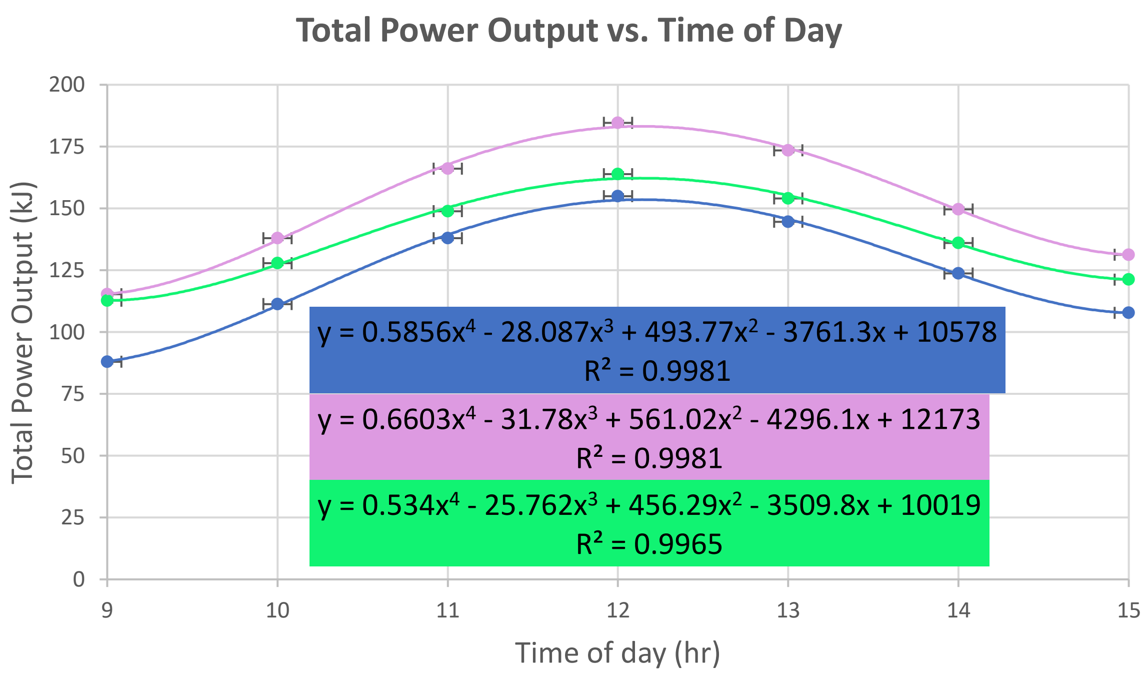
$$\bar{x} = \frac{32.70 + 33.23 + 33.58}{3} = 33.17°$$

The physics behind this optimum angle will be explained in the Analysis section.

THE TIME OF DAY FACTOR

Now that the optimum angle has been found, it is possible to find the time during the day that produces the most energy. Knowing that the optimum tilt angle is around 33.17°, we can take the 30° trial (because this is the angle that I experimented with closest to 33.17°) and graph what the energy output was during each hour. This is shown in the graph below:

Graph 2: Total power output of the 30° tilt angle versus the time of day



Note that on the $x - axis$, the numbers 9 to 15 indicate the hours of the day from 9AM to 3PM, respectively. Also, from the equations of the best fit lines, x is the time of day and y is the total power output. From Graph 2, the best-fit line is a quartic curve that has a global maximum somewhere around the vertical axis $x = 12$. To find it, we can do the same process that was done to find the experimental optimum tilt angle: derive the equation of the best-fit line and equate to 0. The process will be shown for the Day 1 trial only:

$$\frac{dy}{dx} = 2.3424x^3 - 84.261x^2 + 987.54x - 3761.3 = 0$$

Solving this on a Graphic Display Calculator gives three values:

$x = 8.80$ *Local Maximum*	$x = 12.14$ *Global Maximum*	$x = 15.03$ *Local Maximum*

From Graph 2, the global maximum is shown to be somewhere around $x = 12$, therefore the root of the function's derivative closest to 12 is the value of the time that gives the maximum power output. The same process is done for Day 2 and Day 3 yielding $x = 12.13$ and $x = 12.15$, respectfully. The average value of x is then:

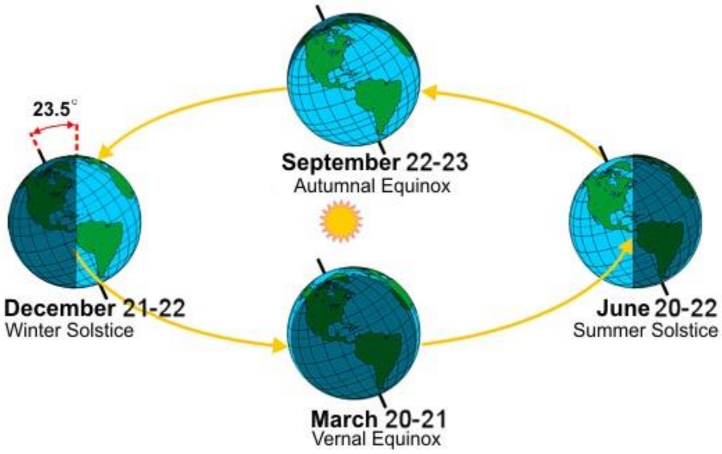
$$\bar{x} = \frac{12.14 + 12.13 + 12.15}{3} = 12.14$$

This means that the time around $12{:}08\text{PM}$ gives the peak power output. Now, it is possible to analyze and to understand the reasons that these results were obtained.

ANALYSIS

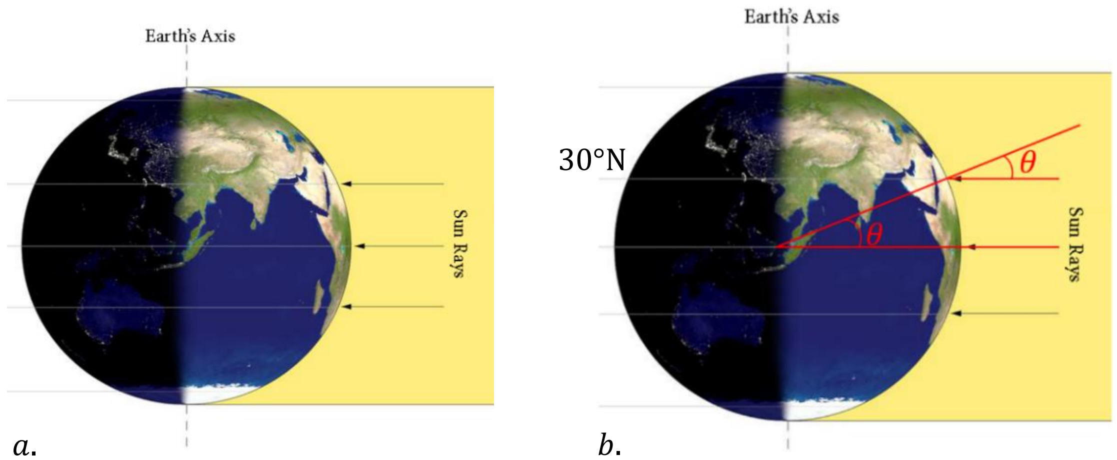
The optimum tilt angle being $33.17°$ can be explained by referring to the time of *year* the experiment was conducted on. The experiment was conducted on September 26, 27, and 28, which is very close to a special day in the year, September 22. On, or around, this day, the "Earth will reach a position known as *equinox*." (Earth-Sun Relationships) There are two equinoxes throughout the year, one on March 21 and the other on September 22. Note that the Earth's axis is tilted from its vertical at an angle of approximately $23.5°$, as shown in Figure 3.

Figure 3: Earth's Orbit and the Relation to Seasons



In the Figure above, the four positions of the Earth represent solstices and equinoxes. The summer solstice, around June 21, "occurs at the moment the Earth's tilt *toward* the sun is at a maximum," (The Seasons…) while the winter solstice, around December 21, occurs when the Earth's tilt *from* the sun is maximum. The vernal equinox and the autumnal equinox occur on around March 21 and September 22, respectfully. The equinox is a special event in the year because at this date, the "Earth's axis is tilted neither toward nor away from the sun," (The Seasons…) shown in Figure $4a$.

Figure 4: Idzkiewicz, Illumination of Earth by Sun on the Day of Equinox



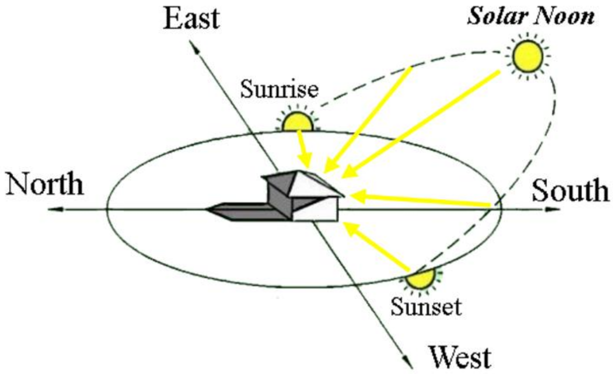
$a.$ $b.$

At the equinox, the sun's rays are perpendicular to the surface of Earth at the equator. However, at different latitudes, the angle between the sun rays and the normal to the point where the sun ray meets the surface of the Earth changes. For example, in Figure $4b$, the latitude equal to $30°\mathrm{N}$ is shown. When the sun ray hits the Earth at this latitude, an angle θ is formed with the normal at that point. Then, we can draw another line aimed to the center of the Earth and parallel to the sun ray. The same angle θ is formed, which is just the latitude angle. Therefore, at the equinox, the angle the sun's ray makes at a certain point on Earth's surface is equal to the latitude of that point!

Now, my experiment was conducted four to six days after autumnal equinox, so it is reasonable to assume that negligible changes occurred between these times. This means that the angle the sun was making with the surface of the Earth is equal to the latitude of the solar panel's position. According to *Get Latitude and Longitude*, the exact latitude coordinate of where the experiment was conducted is $33.86°\mathrm{N}$. Therefore, the sun rays made a $33.86°$ angle with the ground. The optimum tilt angle is the angle in which the sun ray makes a $0°$ angle with the normal of the solar panel. In order to do this, the tilt angle of the solar panel must also be $33.86°$! Then, the experimental optimum angle, $33.17°$, corresponds neatly with the theory.

Now for the time of day. The optimum time was found to be around $12{:}\,08\mathrm{PM}$. As explained in the Background Information, the intensity received on a surface depends on the angle the ray of light strikes with. Between $12{:}\,00\mathrm{PM}$ and $1{:}\,00\mathrm{PM}$, an event known as solar noon occurs, which is "when the sun has reached its zenith, or highest point in the sky, for that day." (Earth-Sun Relationships) A diagram showing the position of the sun at solar noon is shown below:

Figure 5: Rehman, "Solar Noon"



In Figure 5, the path the sun draws out during the day is called the sun's path and is represented by the dotted line; also, some sun rays are shown during different positions of the sun. Let's consider the sun rays at the position of sunrise and sunset. At these positions, the sun rays form very small angles with respect to the ground. Therefore, as stated in the Background Information section of this paper, the intensity decreases and less power is produced. However, as the sun's altitude, which "is the vertical angle the sun makes with the ground plane," (Solar Position) increases from sunrise to solar noon, the angle the sun rays make with the ground increase, thereby increasing the intensity of the sun rays. Since solar noon is the time during the day where the sun's altitude is maximum, then the angle the sun rays make with the ground is also maximum. At this point, "the irradiance from the sun is at its very highest and you can generate the most power." (Solar Angle Calculator) After solar noon, the sun's altitude decreases, the angle between the sun ray and the ground decreases, and the power output decreases as well. This is clearly shown in Graph 2; the increase in power output from 9: 00AM to around 12: 08PM, and then the decrease from 12: 0PM to 3: 00PM. The shape of the graph obtained corresponds beautifully with the theory.

In the case of the solar panel, the optimum power output occurs when the intensity of the insolation is maximum. This occurs, as mentioned earlier, at solar noon when the angle the sun rays make with the ground is maximum. During the days in which the experiment was conducted, the exact time solar noon occurred was around $12{:}\,28\mathrm{PM}$, according to the online database *Time and Date*. The experimental time in which solar noon occurred was found to be approximately $12{:}\,08\mathrm{PM}$, slightly earlier than the actual time.

CONCLUSION

The data that was obtained as a result of multiple trials and experimentation yielded two important points. The first is that the optimum angle for a solar panel to produce the maximum power output is approximately $33.17°$. The second is that the time during the day in which the insolation is greatest, and therefore the power output is optimum, occurs at around $12{:}\,08\mathrm{PM}$. Half of my hypothesis turned out to be true. The hypothesis that the optimum angle would be $45°$ is not true because the sun's elevation is not $45°$. If the experiment was conducted in Venice, Italy, which has a latitude of $45.44°$ according to *LatLong*, or anywhere on that latitude, only then would the optimum angle be approximately $45°$. The other half of my hypothesis, that the optimum time would be around $12{:}\,00\mathrm{PM}$, was an educated guess that was proven correct.

The percent error in the tilt angle and time of day can now be calculated. As mentioned earlier, the theoretical value for the tilt angle is $33.86°$ and for the time of day is $12{:}\,28\mathrm{PM}$. It is not possible to find the percent error of the time of day, however, the reason for the 20 minute deviation will be explained after the percent error in the tilt angle is found:

$$\%Error = \frac{33.86 - 33.17}{33.86} \times 100 = 2.04\%$$

The very small percent error indicates an accurate and successful experiment with low amounts of random error. However, it is still important to understand where these errors came from. For instance, the tilt angle depended on the angle in which the sun rays made with the ground. The assumption was made that the sun rays made an angle with the ground equal to the angle it made with Earth's atmosphere. In doing so, the "refraction of light caused by the atmosphere of Earth" (Gupta and Rahal) is completely neglected. Taking this effect into consideration may have made the data even more accurate. In the case of the time of day, the experimental time was found to be approximately 20 minutes earlier than the theoretical. This is due to the systematic errors in the experiment.

These findings are important because it helps solar panel companies and people who have installed solar panels in optimizing the tilt angle of solar panels to bring out the most in energy output. Furthermore, this information can be used in designing solar tracking systems, which are solar panels that "rotate from east to west throughout the day to follow the sun and optimize panel efficiency." (Solar Angles and Tracking Systems) For example, the optimum angle and time of day that was obtained would be the first step in designing a solar tracking system, as it would be coded in a way that at $12{:}\,08\text{PM}$, the solar panel will be tilted at an angle of $33.17°$. In doing so, it is possible to increase power output by as much as 30% compared to if the solar panels were fixed facing south. (Solar Angles and Tracking Systems)

STRENGTHS

a) The experiment was conducted on September 26, 27, and 28. The temperatures during these three days were between $23℃$ and $27℃$ (Time and Date) which is very close to the standard temperature of $25℃$.

b) Also, the experiment was carefully planned to be conducted on dates very close to the autumnal equinox. This allowed reasonable assumptions to be made and increased the overall accuracy of the data.

WEAKNESSES

a) A small part of my shadow would sometimes be on the solar panel. The shadow would cause less solar radiation to be incident on the panel, thus the power output is reduced as well as the accuracy of the data obtained.

b) Being outside for such a long time, the solar panel must have certainly accumulated dust particles on it, “which can reduce the system’s efficiency by up to 50%.” (Sulaiman) Therefore, the power outputs that I recorded are lower than the real ones, which adds to the systematic errors present in the experiment.

c) It was not *exactly* ever hour that I was collecting data; sometimes it would be an hour and a couple of minutes. This slightly affects the power output and increases the systematic errors.

IMPROVEMENTS

a) The experiment was conducted in September, which is quite a cloudy month. If the experiment was conducted earlier, the end of summer perhaps, then very few little clouds, if not none, will interfere with the solar radiation incident on the panel. Then the raw data will be more accurate and will lead to a better analysis overall.

b) Before beginning to record data at the start of every hour, it is best to wipe the dust that accumulated on the solar panel with a wet tissue to ensure no changes in voltage and current output.

c) A data logger can be used to drastically increase the accuracy of the experiment. The "Voltage and Current Data Logger" (Chamaa) from Voltaic Systems can give continuous voltage and current readings throughout the entire day. This would remove the assumption that was made in the experiment – that the power output calculated at the start of one hour remains the same until the start of the next hour – and would allow for a far more accurate analysis.

EXTENSIONS

It would be interesting to see the effect of the altitude and solar panel elevation on the power output. I conducted the experiment at an altitude of around 450 meters, so some factors such as the "emission of different gases from the masses [and] the usage of fossil fuels" (Panjwani and Narejo) did not limit the amount of solar radiation incident on the panel. Conduction the experiment at different altitudes would allow the study for factors such as humidity, gases in the air, and if there is a significant output effect in bringing the solar panel closer to the emitting source; the sun. All of these factors must be experimented with as soon as possible so that the use of fossil fuels for energy becomes obsolete in the modern world.

SOURCES

Works

Are Solar Panels Safe? The Solar Co, www.thesolarco.com/are-solar-panels-safe/.

“Cannaregio, Venice, Italy.” *LatLong*, www.latlong.net/place/cannaregio-venice-italy-531.html.

Chamaa, Karim. *Voltage and Current Data Logger*. Voltaic Systems, 4 Apr. 2017, www.voltaicsystems.com/blog/voltage-current-data-logger/.

Dhar, Michael. *How Do Solar Panels Work?* Live Science, 6 Dec. 2017, www.livescience.com/41995-how-do-solar-panels-work.html.

“Earth-Sun Relationships and Solar Energy.” *Fundamentals of Physical Geography*, by James F. Petersen et al., 1st ed., Cengage Learning, 2011, pp. 74–81, www.cengage.com/resource_uploads/downloads/0495555061_137179.pdf.

Get Latitude and Longitude. LatLong, www.latlong.net/.

Gupta, Rajan, and Rahul Jinda. *Atmospheric Refraction*. Fun Science, www.funscience.in/study-zone/Physics/RefractionOfLight/AtmosphericRefraction.php.

Mortimer, David. *Lambert's Cosine Law*. The Solar Bucket, 30 Jan. 2014, thesolarbucket.blogspot.qa/2014/01/lamberts-cosine-law.html.

Panjwani, Manoj Kumar, and Ghous Bukshsh Narejo. “Effect of Altitude on the Efficiency of Solar Panel.” *International Journal of Engineering Research and General Science*, vol. 2, no. 4, 2014, pp. 461–464., ijergs.org/files/documents/EFFECT-57.pdf.

Solar Angles and Tracking Systems. Teach Engineering, 2009, www.teachengineering.org/lessons/view/cub_pveff_lesson01.

"Solar Angle Calculator." *Solar Electricity Handbook 2017*, Greenstream Publishing, solarelectricityhandbook.com/solar-angle-calculator.html.

"Solar Position." *Sustainability Workshop*, AutoDesk, sustainabilityworkshop.autodesk.com/buildings/solar-position.

Sulaiman, Shaharin A., et al. "Effects of Dust on the Performance of PV Panels." *International Journal of Mechanical and Mechatronics Engineering*, vol. 5, no. 10, 2011, pp. 1–6., waset.org/publications/10305/effects-of-dust-on-the-performance-of-pv-panels.

"Sunrise, Sunset, and Daylength, September 2017." *Time and Date*, www.timeanddate.com/sun/lebanon/beirut?month=9&year=2017.

"Weather in September 2017 in Beirut, Lebanon." *Time and Date*, www.timeanddate.com/weather/lebanon/beirut/historic?month=9&year=2017.

Images

"Earth's Orbit and the Relation to Seasons." *The Seasons, the Equinox, and the Solstices*, National Weather Service, www.weather.gov/cle/seasons.

Idzkiewicz, Przemyslaw. "Illumination of Earth by Sun on the Day of Equinox (Vernal and Autumnal)." *Earth-Lighting-Equinox*, Wikipedia, 19 Apr. 2005, en.wikipedia.org/wiki/File:Earth-lighting-equinox_EN.png.

Petersen, James F., et al. *Sun Rays in Summer and Winter*. Cengage Learning, 1 Aug. 2012, www.cengage.com/resource_uploads/downloads/0495555061_137179.pdf.

Rehman, Naveed. “Solar Noon.” *Solar Energy Engineering*, LinkedIn Learning, 30 May 2017, www.slideshare.net/naveedurrehman85/solar-energy-engineering.