Linear Algebra, HW3, 강주연

## 2.1. MATRIX OPERATIONS

diagonal entries.(대각원소)

$$\begin{array}{cc} & \begin{array}{ccccc} & & \text{Column } j & & \end{array} \\ \text{Row } i & \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} = A \\ & \begin{array}{ccccc} \uparrow & & \uparrow & & \uparrow \\ \mathbf{a}_1 & & \mathbf{a}_j & & \mathbf{a}_n \end{array} \end{array}$$

**FIGURE 1** Matrix notation.

**Theorem 1**

Let $A$, $B$, and $C$ be matrices of the same size, and let $r$ and $s$ be scalars.

a. $A + B = B + A$
b. $(A + B) + C = A + (B + C)$
c. $A + 0 = A$
d. $r(A + B) = rA + rB$
e. $(r + s)A = rA + sA$
f. $r(sA) = (rs)A$

• Matrix Multiplication.

B라는 linear transform　　　A라는 linear transform.

A(Bx) ←

=

(AB)x. ←

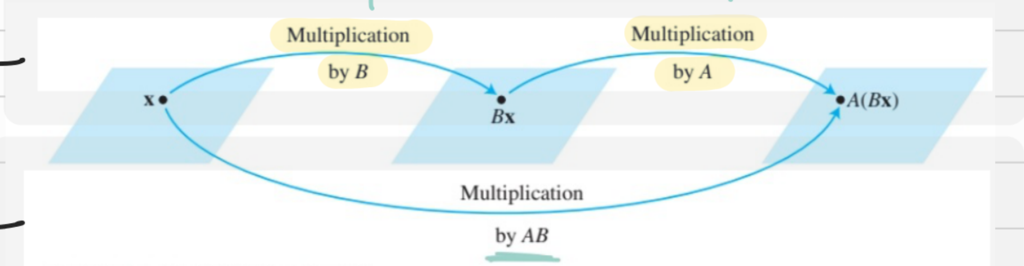


**FIGURE 3** Multiplication by $AB$.

⇓

$A(Bx) = [Ab_1 \ \ Ab_2 \ \cdots \ Ab_p]x = (AB)x.$

DEFINITION

If $A$ is an $m \times n$ matrix, and if $B$ is an $n \times p$ matrix with columns $\mathbf{b}_1, \ldots, \mathbf{b}_p$, then the product $AB$ is the $m \times p$ matrix whose columns are $A\mathbf{b}_1, \ldots, A\mathbf{b}_p$. That is,

$$AB = A[\mathbf{b}_1 \ \ \mathbf{b}_2 \ \cdots \ \mathbf{b}_p] = [A\mathbf{b}_1 \ \ A\mathbf{b}_2 \ \cdots \ A\mathbf{b}_p]$$

이때까지는 $b_1$ 하나의 vector 에대해 곱했는데
→ 얘는 b가 $b_1 \cdots b_p$ 여러개.

EXAMPLE 3.

$$\begin{array}{ccc} A & & \mathbf{b}_1 \end{array} \qquad \begin{array}{ccc} A & & \mathbf{b}_2 \end{array} \qquad \begin{array}{ccc} A & & \mathbf{b}_3 \end{array}$$

$$A\mathbf{b}_1 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad A\mathbf{b}_2 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad A\mathbf{b}_3 = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ -1 \end{bmatrix} \qquad = \begin{bmatrix} 0 \\ 13 \end{bmatrix} \qquad = \begin{bmatrix} 21 \\ -9 \end{bmatrix}$$

Then

$$AB = A[\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3] = \begin{bmatrix} 11 & 0 & 21 \\ -1 & 13 & -9 \end{bmatrix}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ A\mathbf{b}_1 & A\mathbf{b}_2 & A\mathbf{b}_3 \end{array}$$

AB의 각 Column은 A의 Column에 B의 Column을 선형결합 한것.

= weight.

AB의 행 = A의 행, AB의 열 = B의 열.

## · Properties of Matrix Multiplication.

Theorem 2. Let $A$ be an $m \times n$ matrix, and let $B$ and $C$ have sizes for which the indicated sums and products are defined.

a. $A(BC) = (AB)C$ (associative law of multiplication)

b. $A(B + C) = AB + AC$ (left distributive law)

c. $(B + C)A = BA + CA$ (right distributive law)

d. $r(AB) = (rA)B = A(rB)$ for any scalar $r$

e. $I_m A = A = AI_n$ (identity for matrix multiplication)

**WARNINGS:**

1. In general, $AB \neq BA$.
2. The cancellation laws do *not* hold for matrix multiplication. That is, if $AB = AC$, then it is *not* true in general that $B = C$. (See Exercise 10.)
3. If a product $AB$ is the zero matrix, you *cannot* conclude in general that either $A = 0$ or $B = 0$. (See Exercise 12.)

2. $A = \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -6 & 0 \\ -3 & 1 \end{bmatrix}$ 일때 $AB = BC = \begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ 일때 $AB = 0$.

- Powers of a Matrix.

$$A^k = \underbrace{A \cdots A}_{k}, \quad A^0 = I \text{ (identity matrix).}$$

- The transpose of a Matrix. 전치행렬

The transpose → Column과 row를 바꾼것.

Theorem 3.

Let $A$ and $B$ denote matrices whose sizes are appropriate for the following sums and products.

a. $(A^T)^T = A$
b. $(A + B)^T = A^T + B^T$
c. For any scalar $r$, $(rA)^T = rA^T$
d. $(AB)^T = B^T A^T$

## 2.1. EXERCISES.

**20.** Suppose the second column of $B$ is all zeros. What can you say about the second column of $AB$?

sol).

편의를 위해 B가 3X3 Matrix라 했을때

$$\begin{array}{c} B = \begin{bmatrix} b_{00} & 0 & b_{02} \\ b_{01} & 0 & b_{12} \\ b_{02} & 0 & b_{22} \end{bmatrix} \\ \quad\; \vec{b_1} \quad \vec{b_2} \quad \vec{b_3} \end{array}$$

$$AB = A[\vec{b_1}\ \vec{b_2}\ \vec{b_3}] = \begin{bmatrix} Ab_{00} & A\cdot 0 & Ab_{02} \\ Ab_{01} & A\cdot 0 & Ab_{12} \\ Ab_{02} & A\cdot 0 & Ab_{22} \end{bmatrix}$$

$$= \begin{bmatrix} Ab_{00} & 0 & Ab_{02} \\ Ab_{01} & 0 & Ab_{12} \\ Ab_{02} & 0 & Ab_{22} \end{bmatrix}$$

# 2.2. THE INVERSE OF A MATRIX. 역행렬.

$$A^{-1}A = I \quad \text{and} \quad AA^{-1} = I.$$

- $A$와 $A^{-1}$의 크기는 $n \times n$으로 같다.
- $A^{-1}$는 unique.

- matrix
  - → not invertible → Singular matrix (역행렬은 만들수 X)
  - → invertible → nonsingular matrix.

**Theorem 4.**

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $ad - bc \neq 0$, then $A$ is invertible and

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, then $A$ is not invertible.

- $A$의 determinant ⇒ $\det A = ad - bc$

**Theorem 5.**

A가 역행렬이 존재하는 $n \times n$ 행렬이라면, $\mathbb{R}^n$ 공간의 $b$에 대해 식 $Ax = b$는 unique solution $x = A^{-1}b$를 가짐.

$Ax = A(A^{-1}b) = (AA^{-1})b = Ib = b$

**Theorem 6.** (역행렬의 성질)

a. If $A$ is an invertible matrix, then $A^{-1}$ is invertible and

$$(A^{-1})^{-1} = A$$

b. If $A$ and $B$ are $n \times n$ invertible matrices, then so is $AB$, and the inverse of $AB$ is the product of the inverses of $A$ and $B$ in the reverse order. That is,

$$(AB)^{-1} = B^{-1}A^{-1}$$

c. If $A$ is an invertible matrix, then so is $A^T$, and the inverse of $A^T$ is the transpose of $A^{-1}$. That is,

$$(A^T)^{-1} = (A^{-1})^T$$

## • Elementary Matrices. (기본행렬)

: Elementary matrix는 identity matrix에 row operation을 적용해서 얻어진다.

EXAMPLE 1.

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$$

⇒ replacement가 필요.
즉, $4 \times$ row1 + row3 연산 필요

이때,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

있을때 A에 $4\times$ row1 + row3 연산을 동일하게 적용한것과

$\parallel$

$E_1A$ 결과와 동일!

$$E_1A = \begin{bmatrix} a & b & c \\ d & e & f \\ g-4a & h-4b & i-4c \end{bmatrix}$$

• $m \times n$ matrix A에 적용한 row operation을 $m \times m$ Identity matrix에 적용하면 Elementary Matrix E가 만들어진다.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

에 적용한 row operation($4\times$row1 + row3)을 적용하면 $E_1$이랑 동일!

• Elementary Matrix E가 invertible이면 E의 inverse는 E를 I로 변환하는 elementary Matrix 이다.

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}, \quad E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ +4 & 0 & 1 \end{bmatrix}$$

## Theorem 7.

n×n matrix A를 $I_n$ 으로 변환하는 row operation을 $I_n$에 적용하면 $A^{-1}$가 나온다.

$$\underbrace{E_p \cdots E_1}_{\text{row operation}} A = I_n$$

$$\Rightarrow E_p \cdots E_1 = A^{-1}, \quad E_p \cdots E_1 I_n = A^{-1}$$

$I_n$에 같은 row operation을 적용,,.

- **An Algorithm for Finding $A^{-1}$**

: Augmented matrix $[A \;\; I] \xrightarrow{\text{Row operation}} [I \;\; A^{-1}]$

**EXAMPLE 7** Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

**SOLUTION**

$$[A \;\; I] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & 0 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 & -4 & 1 \end{bmatrix}$$

row operation 반복

$$\sim \begin{bmatrix} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3/2 & -2 & 1/2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -9/2 & 7 & -3/2 \\ 0 & 1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & 3/2 & -2 & 1/2 \end{bmatrix} = [I \;\; A^{-1}]$$

I (Echelon form)

Theorem 7 shows, since $A \sim I$, that $A$ is invertible, and

$$A^{-1} = \begin{bmatrix} -9/2 & 7 & -3/2 \\ -2 & 4 & -1 \\ 3/2 & -2 & 1/2 \end{bmatrix}$$

## 2.2. EXERCISES.

**21.** Explain why the columns of an $n \times n$ matrix $A$ are linearly independent when $A$ is invertible.

sol).

Theorem 5

: A가 invertible n×n matrix이고, b가 $\mathbb{R}^n$ 공간벡터이면 $Ax=b$ 식에대한 unique solution $x=A^{-1}b$가 존재.

$\Rightarrow$ linearly independent.

$$Ax=b \Rightarrow AxA^{-1}=bA^{-1}$$

$\Rightarrow x=A^{-1}b$. 로 정리하는수있고,

정리된 x는 $Ax=A(A^{-1}b)=(AA^{-1})b=Ib=b.$

+

**EXAMPLE 7** Find the inverse of the matrix $A=\begin{bmatrix}0&1&2\\1&0&3\\4&-3&8\end{bmatrix}$, if it exists.

**SOLUTION**

$$\begin{array}{cc} & \begin{array}{cc} A & \quad\quad\quad I \end{array} \\ [A\ \ I]= & \left[\begin{array}{ccc|ccc}0&1&2&1&0&0\\1&0&3&0&1&0\\4&-3&8&0&0&1\end{array}\right] \end{array} \sim \begin{bmatrix}1&0&3&0&1&0\\0&1&2&1&0&0\\4&-3&8&0&0&1\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&3&0&1&0\\0&1&2&1&0&0\\0&-3&-4&0&-4&1\end{bmatrix} \sim \begin{bmatrix}1&0&3&0&1&0\\0&1&2&1&0&0\\0&0&2&3&-4&1\end{bmatrix}$$

row operation 반복

$$\sim \begin{bmatrix}1&0&3&0&1&0\\0&1&2&1&0&0\\0&0&1&3/2&-2&1/2\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&0&-9/2&7&-3/2\\0&1&0&-2&4&-1\\0&0&1&3/2&-2&1/2\end{bmatrix} = [I\ A^{-1}]$$

I (Echelon form)

Theorem 7 shows, since $A \sim I$, that $A$ is invertible, and

$$A^{-1}=\begin{bmatrix}-9/2&7&-3/2\\-2&4&-1\\3/2&-2&1/2\end{bmatrix}$$

A가 row operation을 통해 reduced echelon form으로 만들었을때 pivot 수 = Column수, 즉 Free variable이 없다.

$\Rightarrow$ linear independent.

## 2.3. CHARACTERIZATIONS OF INVERTIBLE MATRICS.

### Theorom 8. The Invertible Matrix Theorem.

A는 n×n matrix일때, ( either all true or all False).

a. A는 역행렬을 가지고있다.
b. A에 row operation을해서 Reduced Echelon Form을만들면 $I_n$이다.
c. A는 n개의 pivot position을 가진다.

d. $Ax=0$ 은 trivial solution ($x=0$)을 가진다.
e. A는 linear Independent.
f. $x \mapsto Ax$ 은 one-to-one 한다.

g. $Ax=b$ 방정식에서 $R^n$ 공간의 b는 solution으로가짐.
h. $R^n$공간에 A는 span함.
i. $x \mapsto Ax$의 map $R^n$은 $R^n$ onto $R^n$

j. $CA=I$ 를만족하는 n×n matrix가 있다
k. $AD=I$ 를만족하는 n×n matrix가 있다.
l. $A^T$ 도 역행렬을 가지고있다.

trivial Solution → linear Independent
→ n pivot position ( Free variable 없음).

A는 Solution이 있고, $R^n$공간의 b는 Solution으로 가지니까
A는 $R^n$ 공간에 Span.

A가 Invertible한지 확인하려면 pivot position 체크!

## · Invertible Linear Transformation.

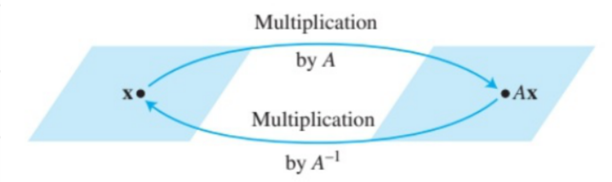


**FIGURE 2** $A^{-1}$ transforms $A\mathbf{x}$ back to $\mathbf{x}$.

· linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ 이 invertible 하다

$\Downarrow$

$$S(T(\mathbf{x})) = \mathbf{x} \quad \text{for all } \mathbf{x} \text{ in } \mathbb{R}^n$$
$$T(S(\mathbf{x})) = \mathbf{x} \quad \text{for all } \mathbf{x} \text{ in } \mathbb{R}^n$$

x의 image에 S를 취하면 x로 돌아온다.

S함수가 존재할때 T는 invertible 하다고 함.

S는 T의 inverse $= T^{-1}$

· linear transform은 항상 standard matrix (A)가 존재.

$T: x \mapsto Ax$ 에서 T가 invertible 이면, A도 invertible.

$(T: \mathbb{R}^n \to \mathbb{R}^n)$

이때 linear transform S는 $S(x) = A^{-1}x$.

$S(T(x)) = S(Ax) = A^{-1}(Ax) = x$.

2.3. EXERCISES.

**23.** If an $n \times n$ matrix $K$ cannot be row reduced to $I_n$, what can you say about the columns of $K$? Why?

sol) $n \times n$ matrix $k \xrightarrow{\text{row reduced}} I_n$ 만들수없음.

⇒ pivot position 이 n개 미만

⇒ Free variable 이 존재한다.

⇒ linear dependent ⇒ nontrivial solution

⇒ not invertible.

# 2.4. PARTITIONED MATRICES.

· Partitioned matrix (= block matrix).

큰 matrix들은 작은 matrix들의 조합으로 표현하면.

$$A = \left[\begin{array}{ccc|cc|c} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \\ \hline -8 & -6 & 3 & 1 & 7 & -4 \end{array}\right]$$

· Multiplication of Partitioned Matrics.

AB가 블록단위로 곱해지려면 A파티션의 column과 B파티션의 row가 일치해야함.

$$A = \begin{array}{c} \begin{array}{ccc|cc} & 3 & & \quad 2 & \end{array} \\ \left[\begin{array}{ccc|cc} 2 & -3 & 1 & 0 & -4 \\ 1 & 5 & -2 & 3 & -1 \\ \hline 0 & -4 & -2 & 7 & -1 \end{array}\right] \end{array} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{array}{c} 3 \\ \\ \\ 2 \\ \\ \end{array} \left[\begin{array}{cc} 6 & 4 \\ -2 & 1 \\ -3 & 7 \\ \hline -1 & 3 \\ 5 & 2 \end{array}\right] = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{bmatrix} = \left[\begin{array}{cc} -5 & 4 \\ -6 & 2 \\ \hline 2 & 1 \end{array}\right]$$

⇓

**Column–Row Expansion of $AB$**

If $A$ is $m \times n$ and $B$ is $n \times p$, then

$$AB = \begin{bmatrix} \underbrace{\mathrm{col}_1(A)}_{m \times 1} & \mathrm{col}_2(A) & \cdots & \mathrm{col}_n(A) \end{bmatrix} \begin{bmatrix} \overset{1 \times p}{\mathrm{row}_1(B)} \\ \mathrm{row}_2(B) \\ \vdots \\ \mathrm{row}_n(B) \end{bmatrix} \qquad (1)$$

$$= \underbrace{\mathrm{col}_1(A)}_{m \times 1}\, \underbrace{\mathrm{row}_1(B)}_{1 \times p} + \cdots + \mathrm{col}_n(A)\, \mathrm{row}_n(B)$$

⇒ m×p matrix 만들어짐.

- Inverses of Partitioned Matrix.

EXAMPLE 5.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ 0 & I_p \end{bmatrix}$$

$B = A^{-1}$

$$\Rightarrow \begin{aligned} A_{11}B_{11} + A_{12}B_{21} &= I_p \\ A_{11}B_{12} + A_{12}B_{22} &= 0 \\ A_{22}B_{21} &= 0 \\ A_{22}B_{22} &= I_q \end{aligned}$$

전개하면 식이나오는데
이식을가지고 계산하면
연산을 줄이고 역행렬을 구할수있음.

Thus

$$\Rightarrow B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = A^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}$$

## 2.4. EXERCISES.

Q In Exercises 11 and 12, mark each statement True or False. Justify each answer.

**11.** a. If $A = [A_1 \quad A_2]$ and $B = [B_1 \quad B_2]$, with $A_1$ and $A_2$ the same sizes as $B_1$ and $B_2$, respectively, then $A + B = [A_1 + B_1 \quad A_2 + B_2]$.

sol) $\boxed{\text{True.}}$ //

$$A+B = [A_1 \ A_2] + [B_1 \ B_2] = [A_1+B_1 \ \ A_2+B_2].$$

$A = [\underbrace{1 \ 1}_{A_1} \ \underbrace{3 \ 2}_{A_2}]$, $B = [\underbrace{8 \ 8}_{B_1} \ \underbrace{4 \ 2}_{B_2}]$ 라고 할때

$A+B$는 $[1 \ 1 \ 3 \ 2] + [8 \ 8 \ 4 \ 2] = \boxed{[9 \ 9 \ 7 \ 4]}$.

$$[A_1+B_1] = [1 \ 1] + [8 \ 8] = [9 \ 9]$$

$$[A_2+B_2] = [3 \ 2] + [4 \ 2] = [7 \ 4]$$

$$\therefore \ [A_1+B_1 \ \ A_2+B_2] = \boxed{[9 \ 9 \ 7 \ 4]}$$

Q b. If $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, then the partitions of $A$ and $B$ are conformable for block multiplication.

Sol) $\boxed{\text{False}}$ (파티션내부 col/row는 알수없음).

$$AB = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{bmatrix}.$$

$$\begin{cases} A_{11} \text{ col} = B_1 \text{ row} \\ A_{21} \text{ col} = B_1 \text{ row} \end{cases}$$

$$\begin{cases} A_{12} \text{ col} = B_2 \text{ row} \\ A_{22} \text{ col} = B_2 \text{ row} \end{cases}$$

즉, AB가 Blocked 단위로 곱해지려면 A파티션의 Col = B파티션 row.