# 4.1. VECTOR SPACES AND SUBSPACE.

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A **vector space** is a nonempty set $V$ of objects, called *vectors*, on which are defined two operations, called *addition* and *multiplication by scalars* (real numbers), subject to the ten axioms (or rules) listed below.[1] The axioms must hold for all vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $V$ and for all scalars $c$ and $d$.

1. The sum of $\mathbf{u}$ and $\mathbf{v}$, denoted by $\mathbf{u}+\mathbf{v}$, is in $V$.
2. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$.
3. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$.
4. There is a **zero** vector $\mathbf{0}$ in $V$ such that $\mathbf{u}+\mathbf{0}=\mathbf{u}$.
5. For each $\mathbf{u}$ in $V$, there is a vector $-\mathbf{u}$ in $V$ such that $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$.
6. The scalar multiple of $\mathbf{u}$ by $c$, denoted by $c\mathbf{u}$, is in $V$.
7. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$.
8. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$.
9. $c(d\mathbf{u})=(cd)\mathbf{u}$.
10. $1\mathbf{u}=\mathbf{u}$.

## * Subspace.

Vector space V 의 Subspace는
아래 3가지 성질을 만족하는 Vector space V의 부분집합 H이다.

① zero vector가 H set에 존재해야한다.

② H는 벡터덧셈에 닫혀있다

⇒ H안에있는 u,v 를 더한 u+v가 H안에 있어야한다.

③ H는 스칼라곱에 닫혀있다

⇒ H안에있는 임의의 벡터 u에 스칼라 c와 곱한 값 cu가 H안에 있어야한다.

## * Span.

If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are in a vector space $V$, then Span $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a subspace of $V$.

$V_1 \cdots V_p$가 $V$안에있다면, span$\{V_1 \cdots V_p\}$는 $V$의 Subspace.

1.3장에서 Span$\{V_1, \cdots, V_p\}$는 $C_1V_1 + C_2V_2 + \cdots + C_pV_p$ 형태로 나타낼수있는 모든벡터들의 집합이라고 정의함.

4.1. EXERCISES

**9.** Let $H$ be the set of all vectors of the form $\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix}$. Find a vector $\mathbf{v}$ in $\mathbb{R}^3$ such that $H = \text{Span}\{\mathbf{v}\}$. Why does this show that $H$ is a subspace of $\mathbb{R}^3$?

sol).

$$\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix} = s\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$\therefore V = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ 는 $\mathbb{R}^3$ 공간안에 있다!
(3x1).

$H = \text{Span}\left\{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}\right\}$ $\Rightarrow$ H는 $\mathbb{R}^3$공간의 Subspace.