**Robust Observer-Based Adaptive Fuzzy Sliding Mode Controller**

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**Abstract**

In this paper, a new observer-based adaptive fuzzy integral sliding mode controller is proposed based on the Lyapunov stability theorem. The plant under study is subjected to a square-integrable disturbance and is assumed to have mismatch uncertainties both in state- and input-matrices. In addition, a norm-bounded time varying term is introduced to address the possible existence of un-modelled/nonlinear dynamics. Based on the classical sliding mode controller, the equivalent control effort is obtained to satisfy the sufficient requirement of sliding mode controller and then the control law is modified to guarantee the reachability of the system trajectory to the sliding manifold. The sliding surface is compensated based on the observed states in the form of linear matrix inequality. In order to relax the norm-bounded constrains on the control law and solve the chattering problem of sliding mode controller, a fuzzy logic inference mechanism is combined with the controller. An adaptive law is then introduced to tune the parameters of the fuzzy system on-line. Finally, by aiming at evaluating the validity of the controller and the robust performance of the closed-loop system, the proposed regulator is implemented on a real-time mechanical vibrating system.

**Keywords:** Lyapunov stability; Robust control; Disturbance rejection; Sliding mode, Fuzzy system

1. **Introduction**

The uncertainty as a perturbation of the real system model can significantly affect the dynamics of the closed-loop system and system response. As a result, it is crucial to address these uncertainties within the modelling procedure in order to increase the applicability and reliability of the model. On the other hand, the control methods mostly rely on simplified models in order to be implemented on real time configurations. These two complex modelling boundaries forced the control strategies to develop based on norm-bounded constraints of un-modelled dynamics, modelling uncertainties, and external disturbances and therefore robust controller can be considered as a general control methodology that fulfills this requirement. The sliding mode controller (SMC) is regarded as a robust technique due to the rejection of external disturbance and insensitivity with respect to bounded perturbation [1]. In addition, SMC is proven to have fast reaction with low control order [2]. An important issue of the SM technique is the sensitivity of the closed-loop system to the perturbation in the reaching initial period, during which the dynamic trajectory of the system moves toward the sliding surface [3]. An alternative approach known as integral SMC (ISMC) is contributed with aiming at solving this issue throughout elimination of the reaching phase [4]. However, even ISMC method cannot generally solve the matching problem and therefore it is used in combination with other robust techniques [5]. Recently, some researchers donated their attention toward the effects of these mismatch uncertainties in robust performance of ISMC. Cao and Xu limited their control development to the case of mismatch uncertainty only in state matrix [6]. Qu and Wang presented a SMC on a delayed system with mismatch perturbation in state-matrix in a linear matrix inequality (LMI) frame-work by use of Lyapunov method [7]. Choi proposed a new ISMC in an LMI form for the systems with mismatch uncertainty in both state and input matrices. The controller is proven to be able to guaranty the asymptotic stability as well as satisfying the -stability constraint [8]. Plestan et al., presented a new methodology for adaptive SMC without the requirement of the prior knowledge of the numerical values for the bounds of the uncertainties [9]. Their algorithms are limited to single input single output (SISO) systems with a limited order adaptive law. Ha et al., combined the fuzzy logic with SMC in order to obtain a robust disturbance rejection controller on mismatch systems. Fuzzy system is introduced to address the chattering problem of the SMC [10]. Ho et al., presented an adaptive fuzzy SMC (AFSMC) for a class of continuous time unknown nonlinear systems. Their controller guaranties the robust stability of the closed-loop system and is not subjected to chattering by the introduction of the fuzzy system [11]. Oveisi and Gudarzi used an ideal controller based on SMC and introduced a fuzzy system to mimic this controller. The robust stability is guaranteed by designing the controller based on compensation of the difference between the fuzzy controller and the ideal controller by use of Lyapunov theory [12]. In this paper a new observer-based AFISMC is introduced in order to solve the problem for a class of systems with mismatch uncertainties in state and input matrices. The proposed controller is based on Lyapunov stability theorem and can reject the output disturbance in the presence of un-modelled/nonlinear bounded dynamics. In order to solve the mismatch and the chattering problem of SMC, the fuzzy system is introduced which is combined with an adaptive rule.

Most of the mentioned researches have limited their case study to some numerical examples. However, Wai et al. adopted an AFISMC to control the position of an electrical servo drive [13]. Gholami and Markazi introduced a new AFSM observer for a class of nonlinear MIMO systems and implemented their observer on a modular and reconfigurable robot (MRR) system [14]. Mechanical vibration control is an important application of compensator design [15]. Hasheminejad et al. implemented an AFSMC scheme to actively suppress the two-dimensional vortex-induced vibrations (VIV) of an elastically mounted circular cylinder which is free to move in in-line and cross-flow directions [16]. Gudarzi et al. designed a robust LMI-based controller to reduce the vibration magnitude of a plate by using piezoelectric actuator/sensors [17]. In this paper, the proposed controller is implemented on a piezolaminated beam which is vibrating due to the existence of undesired external disturbance. The dynamics of the system is obtained by use of subspace system identification method. In the rest of the manuscript, represents the identity matrix with appropriate dimension, shows the norm of the corresponding matrix, and stands for the set of real numbers.

1. **Problem formulation**

Consider the open loop plant of the system in uncertain state-space form of Eq. (1)

|  |  |
| --- | --- |
|  | (1) |

where , , and are the state, input, and output vectors, respectively. In addition, and represent the vector of un-modelled/nonlinear dynamics and square-integrable disturbance, respectively. Moreover, , , , and are the state, control input, disturbance input, and output matrices, correspondingly. and are the perturbation terms that are considered as modelling source of uncertainty. It is assumed that the uncertainty matrices and un-modelled dynamics are norm-bounded as , , and with , , and being positive measurable constants. The uncertainty of disturbance input matrix can be considered as a new source of disturbance and therefore it does not appear in the model of the plant explicitly. By assuming the system to be stabilizable with nominal input matrix of full rank, the following system is introduced as the dynamics of the observer

|  |  |
| --- | --- |
|  | (2) |

in which, and are the observed state- and output-vectors, respectively. Also, symbolizes the observer gain. An alternative representation for observer dynamics is also used later based on the full-order Luenberger observer.

* 1. **Sliding mode control**
     1. **Sliding surface and equivalent control input design**

The sliding surface is defined as

|  |  |
| --- | --- |
|  | (3) |

where, . It is assumed that the initial condition is in order to satisfy . In addition, can be calculated by solving the following dynamic equation

|  |  |
| --- | --- |
|  | (4) |

with being the designed observed-state feedback gain which moves the trajectory toward the sliding surface and keeps it on the manifold of Eq. (3). Derivation of the switching function in Eq. (3) with respect to time and using Eqs. (4), (2) gives

|  |  |
| --- | --- |
|  | (5) |

Without losing the generality of the problem, let us introduce the uncertain terms as

|  |  |
| --- | --- |
|  | (6) |

where , , , and Based on the definition of the uncertain terms, there exist positive constants such that , , and . It should be noted that is required to be nonsingular for calculating the equivalent control law, which is guaranteed by assuming . By substituting Eq. (6) in Eq. (5) the dynamics of the sliding function can be written as

|  |  |
| --- | --- |
|  | (7) |

The essential constraint of sliding mode is satisfied if and and therefore, the equivalent control effort of sliding mode can be expressed as

|  |  |
| --- | --- |
|  | (8) |

As it can be seen in Eq. (8), the sliding mode control effort is only the function of the observed states. By substituting Eq. (8) in Eq. (1), the dynamics of the system can be described as

|  |  |
| --- | --- |
|  | (9) |

where . By defining and using Eqs. (9), (2), the dynamics of the observation error can be calculated as

|  |  |
| --- | --- |
|  | (10) |

* + 1. **Stability analysis of the observer-based SMC**

**Theorem 1.** *(Sufficient condition)**The uncertain system (1) with observer system (2) is quadratically stable and satisfies the -norm bounded function of if there exist symmetric positive definite matrices and , positive constants , and such that the following LMI is satisfied*

|  |  |
| --- | --- |
|  | (11) |

*where*

|  |  |
| --- | --- |
|  | (12) |

*The controller and the observer gain can be calculated as and , respectively.*

*Proof.* Consider the following Lyapunov function

|  |  |
| --- | --- |
|  | (13) |

by using Lemma 1 in Appendix A, the following inequality should be satisfied

|  |  |
| --- | --- |
|  | (14) |

substituting Eqs. (9) and (10) into Eq. (14), one can obtain

|  |  |
| --- | --- |
|  | (15) |

Lemma 2 in Appendix A can be then employed for the following terms

|  |  |
| --- | --- |
|  | (16) |

using the inequalities in Eq. (16) and introducing Eq. (15) can be reformatted in matrix form as

|  |  |
| --- | --- |
|  | (17) |

where

|  |  |
| --- | --- |
|  | (18a) |
|  | (18b) |

after pre- and post-multiplying Eq. (18a) by and defining , , and , and then successive use of Schur Complement (A.3) in Appendix A, LMI (11) can be achieved and therefore, the proof is complete.

■

**Remark 1.** In Eq. (12), appears explicitly even after the introduction of , therefore the following iterative algorithm should be used in order to find the sub optimal feasibility solution of LMI (11).

**Algorithm 1:**

1. Consider an initial guess for and , namely and solve the LMI (11), in which all the terms in which explicitly is present, should be replace by .
2. Define and use from previous step in Eq. (12) to satisfy the feasibility problem (11) together with satisfying the linear matrix equality (LME) .
3. Replace with being the step size of -norm bounded constraint of stability and go to step 1.
4. If for desired the solution for in step 2 is found, then exit: Based on the values of and in the last iteration associated with the observer gain can be calculated as .

**Remark 2.** The results in theorem 1 and algorithm 1 are obtained based on the observer dynamics (2) which uses the maximum information that is available about the plant. In the case when the perturbation terms in Eq. (2) are ignored, the following corollary can be used instead of theorem 1 and algorithm 1 to obtain the controller and observer gains.

**Corollary 1.** *The system (1) with state and input perturbations together with the state observer is quadratically stable and satisfies the -norm bounded function of if there exist , positive constants , , , and such that the LMI/LME (19) is satisfied for , and the obtained satisfies the LMI/LME (20)*

|  |  |
| --- | --- |
|  | (19) |

*Then which should be substituted in (19)*

|  |  |
| --- | --- |
|  | (20) |

*The controller and the observer gain can be calculated as using (19) and , respectively.*

*Proof*. Instead of decomposing the perturbation matrices in form of Eq. (6), are used in which , and are known matrices with appropriate dimensions and are the time varying unknown terms. The equivalent control law can be calculated by following the same treatments as in Eqs. (7) and (8) to be equal to . Then, replacing the equivalent control law in the state equation as well as the error dynamics and using the same Lyapunov function as in Eq. (13) and following the same steps as in theorem 1 an LMI condition can be obtained. In this LMI equation, the terms with input and state perturbation matrices as in equivalent equation (16) in theorem 1 for which Lemma 2 is employed, Lemma 4 in appendix A is used instead. Next applying the Schur Complement (A.3) in Appendix A successively and finally introducing the LME condition , the LMI/LME Eqs. (19, 20) are obtained and therefore, the proof is complete.

■

**Remark 3.** The LMI conditions (19) and (20) are affined on the subject to the defined matrices by the introduction of the LME conversion (), which transforms a non-convex problem to a convex one. The non-singularity of is guaranteed by the assumption that the input matrix is a full column rank.

After obtaining the sufficient condition for quadratic stability of the system on sliding manifold of Eq. (3), the reachability condition should be guaranteed by means of ISMC law.

* + 1. **Reachability of ISMC**

**Theorem 2.** *(Reachability) Consider the uncertain system (1) with the switching function (3), for which the controller gain is obtained by satisfying LMI (11) (sufficient condition is guaranteed). Then a stable sliding mode exists for the following SMC control effort*

|  |  |
| --- | --- |
|  | (21) |

*in which, for a positive scalar , is introduced as*

|  |  |
| --- | --- |
|  | (22) |

*Proof.* Consider the new Lyapunov function as Using Eq. (7), one can obtain

|  |  |
| --- | --- |
|  | (23) |

then replacing from Eq. (21) into Eq. (23), can be expressed as

|  |  |
| --- | --- |
|  | (24) |

Thus, the proof is complete.

■

**Remark 4.** Control law (21) has the major issue of difficulty in confirming the upper bound of the controller gain and the observed states. Therefore, a fuzzy system is introduced by replacing the switching term of Eq. (21) with a fuzzy logic interface.

* + 1. **Fuzzy SMC design**

The switching function (3) can be decomposed as . Allowing for to be the input linguistic variable and the output linguistic variable, the fuzzy sets are defined as P (positive), Z (zero), and N (negative) for and PE (positive effort), ZE (zero effort), and NE (negative effort) for . The fuzzy linguistic rules are introduced as

* Rule 1: If is P, the is PE.
* Rule 2: If is Z, the is ZE.
* Rule 3: If is N, the is NE.

In this paper the input membership functions and output membership functions are assumed to be triangle-type and singleton-type, respectively [5]. The output of the defuzzification module can be expressed as

|  |  |
| --- | --- |
|  | (25) |

where are the firing strength of rule . The center of the output membership functions is presumed to be , , and [18]. Utilizing special input membership functions () together with Eq. (25), the fuzzy controller can be reduced to Eq. (26)

|  |  |
| --- | --- |
|  | (26) |

Now by replacing the second term of integral sliding mode control effort in Eq. (21) with the new fuzzy sliding mode control law as and substituting this in Eq. (23), one can obtain

|  |  |
| --- | --- |
|  | (27) |

where and . Then defining , the Lyapunov function (27) can be reformatted as

|  |  |
| --- | --- |
|  | (28) |

Using Eq. (26), the following inequality can be stated as for Eq. (28)

|  |  |
| --- | --- |
|  | (29) |

Based on (29), for , the following inequality should be satisfied

|  |  |
| --- | --- |
|  | (30) |

It is proven by Wang [18] that there exists an optimal solution for as , which cannot be determined explicitly because of the unknown uncertainty bounds. Hence, the following adaptive law is introduced to address this drawback.

**Theorem 3.** *For the uncertain system (1) with control input , where is obtained from solving the feasibility problem of (11) and from Eq. (24), can be replaced with an adaptive parameter described as follows*

|  |  |
| --- | --- |
|  | (31) |

*where are positive scalars, then the stable adaptive fuzzy sliding mode controller exists that guaranties the -performance.*

*Proof.* By defining the estimation error of as , the following Lyapunov function is used to perform the stability analysis

|  |  |
| --- | --- |
|  | (32) |

Thus using Eqs. (28) and (31), we obtain

|  |  |
| --- | --- |
|  | (33) |

By using the introduced fuzzy control law of Eq. (26), fuzzy inequality law (30), and the definition for , Eq. (33) can be rewritten as

|  |  |
| --- | --- |
|  | (34) |

This completes the proof.

■

1. **Experimental and mathematical system**

In this section the performance of the observer-based controller is evaluated by experimental implementation of the control system on a vibrating clamped-free beam (440×40×3 mm). The elastic beam is assumed to be isotropic with Young’s modulus 70 GPa and density 2.7 g/cm³. Two piezoelectric patches (DuraActTM P-876.A15) are attached on one side of the beam acting as the actuation elements. In order to address the optimality of the actuation power in vibration control, the actuator placement is carried out by employing the method that is presented by Nestorović and Trajkov [22]. Accordingly, as shown in Fig. 1, the first piezo-patch is in 20 mm distance from the clamped end and the second patch stands in 80 mm from the first one. The measurement channel is realized by the velocity measurement that is obtained by using a scanning digital laser Doppler vibrometer (VH-1000-D). Since the first vibration mode shape of the structure plays an important role in overall response of the system, the scanning digital laser is placed at the free end of the beam with the distance of 238 mm from the beam at its rest mode.

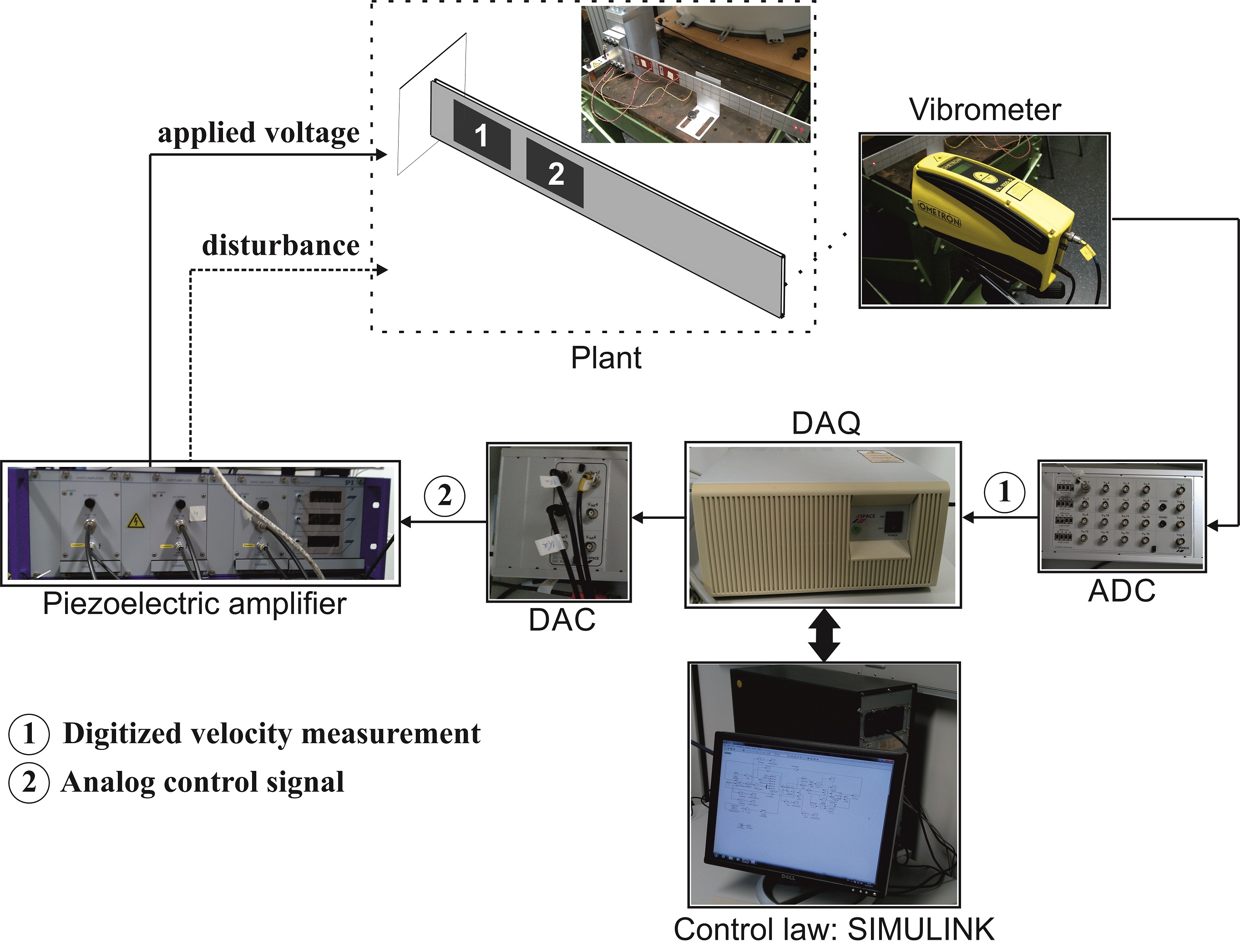


Fig. 1. Sketch of the experimental setup

External disturbance is assumed to act through an independent channel by use of a third piezo-patch that is placed on the other side of the beam. The real-time analyses are carried on a dSPACE digital data acquisition (DAQ) with DS1005 PPC board. The front end of the DAQ system consists of an analog to digital converter (dSPACE ADC DS2004) and a digital to analog converter (dSPACE DAC DS2102). An amplifier (PI E-500) which is placed between the DAC DS2102 board and the piezo-actuators, acts over the DAQ output signal in the working range of () to provide the sufficient voltage to drive the actuation elements to 750 V) with a constant gain (100). Both of the controller and observer systems are modelled on SIMULINK platform and then the created model is compiled and uploaded to the DAQ system in real-time (see Figs. 1 and 2).

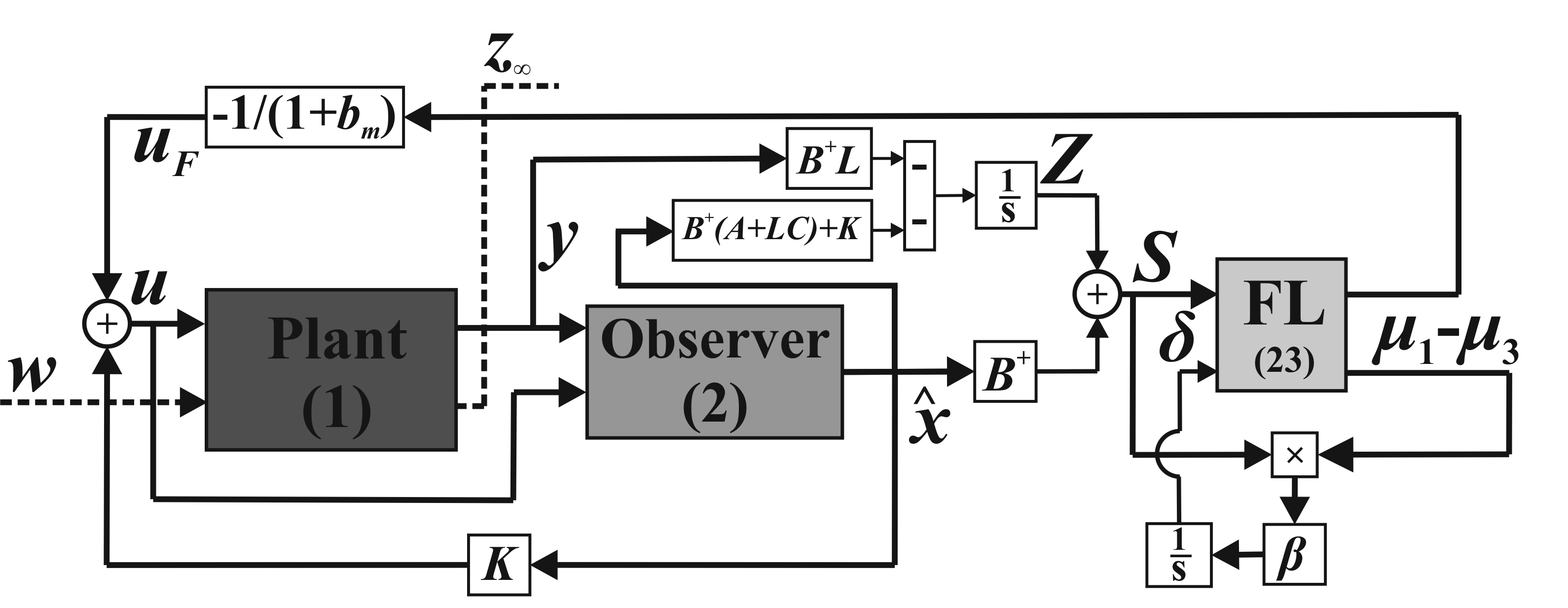


Fig. 2. Schematic configuration of the control system

In Fig. 2, represents the designer-defined -constraint on the transfer function (A.1), which in this paper is considered to be the transfer function from the external disturbance to the measured output (). In order to extract the mathematical model of the system, the subspace identification method is used. Interested reader can refer to [19-22]. It should be mentioned that identification is carried out by neglecting the torsional modes of the beam for the reason that they are not relevant for the bending vibration [23].

1. **Experimental implementation**

In this section, the experimental results of the implementation of the designed control system in real-time is evaluated on the smart structure shown in Fig. 1. For design purposes, a nominal reduced order system is identified based on the consideration of three mode-shapes of the piezolaminated beam. The remaining mode shapes are considered as the source of uncertainty in form of a norm bounded time dependent terms. The identified nominal system as expected is a sixth-order state space model with the system matrices shown in Appendix B. For investigation of the robust performance of the uncertain closed-loop system with the designed controller (11), algorithm 1 can be used. By the experimental implementation of the control law on the system, the prospect of the effective vibration control performance is evaluated on the full order system (see Fig. 2). In order to design a full state feedback controller and implement the obtained control law on the experimental setup, an observer is designed simultaneously based on (2) and by solving convex optimization (11) to estimate the states of the system based on the measurement from laser Doppler vibrometer. The optimal solution for LMI/LME in (11, 21) is obtained by using Scilab [24] with the numerical values given in Appendix B. For this purpose after constructing the control system in SIMULINK and compiling it to dSPACE RTI platform, the system is excited through the disturbance channel with a sweep sine signal. The frequency of the excitation signal is changed from zero to 170 Hz (1068.1 rad/s) in two cases of the open loop and closed-loop system. The open loop and the closed-loop systems are implemented on the real-time DAQ of the dSPACE with sampling frequency of 10 kHz. Investigations are carried out in time domain by means of the experimental setup shown in Fig. 1.

**Remark 5.** In Eqs. (1) and (2), mathematically, can be converted to or included in the form of unmodelled dynamics () and the main purpose of this decomposition is the available uncertainty quantification methods in practice. In other words, uncertainty in structural vibrations can essentially be obtained based on the application of DRC. If the disturbance is active in high frequencies, then the effect of unmodelled/nonlinear dynamics in the nominal plant model is more important. In this frequency range, after effective modelling of dynamics of low frequency nature, the bound of the unknown additive uncertainty function, , in plant model can be obtained following the experimental method in Refs. [25-27]. Experimental results in this area are based on generating several nominally identical samples of structural systems, preparing identical and repetitive experimental procedure in exciting the structure, recording the response, and extracting the frequency response function (FRF) of the system which hands the uncertainty bounds based on the structure of uncertainty. For instance, this method is employed by Kompella and Bernhard [28] to measure FRF at driver microphones for 57 pickup trucks. Similarly, Fahy [29] utilized measurements of FRF of 41 nominally identical beer cans to calculate this uncertainty bound. Similar results are reported by Refs. [30, 31]. In contrast if the frequency of the disturbance is not higher than the highest natural frequency of the vibrating system, stochastic finite element method (SFEM) is an appropriate method for calculating the known matrices in mismatch uncertainty of and [32-34].

The response of the system for controlled and uncontrolled case is shown in Fig. 3a in time domain based on the measurement signal generated from Doppler vibro-meter, which is further fed to the dSPACE ADC board (DS2004). In addition, the frequency response function (FRF) of the open loop system is compared with the closed-loop one in frequency range of in Fig. 3b.

|  |  |
| --- | --- |
| Fig. 3a. Experimental comparison of measured outputs in time domain | Fig. 3b. Experimental comparison of FRF of the systems |

Fig. 3a shows the measured voltage that is collected by the dSPACE ADC board based on the measurement of the laser vibrometer with the sensitivity factor of . It can be seen that the controller design, based on the reduced order identified model, suppressed the vibration magnitude within the considered frequency range. In addition, the corresponding control efforts that are generated for piezo-actuator patches by the dSPACE DAC board are shown in Fig. 4.



Fig. 4. Control effort of the piezo-patch actuators

Fig. 4 shows that, the control efforts applied to each of the piezo-actuators are limited to maximum 50 V without any sudden jump. The experimental results show that the robust control system performs well in attenuating the vibration amplitude in the presence of structured uncertainties in system matrices. In addition, the observation error of the output is depicted in Fig. 5.



Fig. 5. Observation error of the observer system

In order to investigate the robust performance of the designed system in high frequency, where the unmodelled dynamics can affect the response of the system, the real structure is excited with a chirp signal with the frequency between and . The measurement output and the applied control effort are presented in Figs. 6 and 7, respectively. It should be mentioned that, in order to reject the measurement noise a low-pass fourth order Butterworth filter with passband frequency equal to is implemented on the measurement channel.

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Fig. 6. Experimental comparison of measured outputs for unmodelled dynamics

It can be seen, in Figs. 6 and 7, that the observer-based controller can handle the unmodelled dynamics with a limited control effort. In addition, the peak value in Figs. 6 and 7 is representing the higher order natural frequency of the system which is equal to . In order to study the spillover effect, the structure is excited by mechanical initial displacement. The comparison of the open loop and closed-loop systems is depicted in Fig. 8a. In addition, the corresponding applied control signals are shown in Fig. 8b for the two piezo-actuators.



Fig. 7. Control effort of the piezo-patch actuators for unmodelled dynamics

|  |  |
| --- | --- |
| Fig. 8a. Comparison of measured outputs | Fig. 8b. Control effort of the piezo-patch actuators |

The fast Fourier transformation (FFT) analysis is performed on the applied control efforts to identify the dominant frequencies of the controller. Fig. 9 shows that, the applied control efforts in frequency domain between zero and have their significant effect around the first and second eigen-frequencies, however, as one can see, the third mode shape of the structure with natural frequency around (see Fig. 3b), has no significant effect in the controller input. Moreover, the higher order dynamic that is studied in Figs. 6 and 7 is not excited by the control input which shows the rejection of the spillover effect by the proposed control system.

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Fig. 9. FFT of control effort for initial displacement excitation

Finally, the evolution of the sliding surface, () in Eq. (3), for two cases of forced vibration under chirp excitation and free vibration with initial displacement are depicted in Figs. 10a and 10b. Figures 10a and 10b indicate a stable switching function from the initial time for closed-loop system based on the proposed observer-based AFISMC.

|  |  |
| --- | --- |
| Fig. 10a. Switching functions: free vibration | Fig. 10b. Switching functions: forced vibration |

**Conclusion**

In this paper, an observer-based robust controller is introduced based on LMI approach. The plant under study is considered to have uncertainties in state and input matrices with an additional norm-bounded source of nonlinearity or un-modelled dynamics. The robust AFISM disturbance rejection controller is obtained by using classical Lyapunov stability theorem. The performance of the control method is studied by implementing the control system on a mechanical vibrating system. The results show that the controller has a robust performance in the presence of uncertainty, disturbance and un-modelled dynamics.

**Appendix A**

Some preliminary Lemmas are introduced that are used in the main results.

**Lemma 1.** (-*performance* [25]) *The LTI system of* *satisfies* *the* *norm constraint* *with Lyapunov function* *if for* ,

|  |  |
| --- | --- |
|  | (A.1) |

**Lemma 2.** [8] *For two arbitrary vectors with appropriate dimension such as the following inequality is valid for*

|  |  |
| --- | --- |
|  | (A.2) |

**Lemma 3.** [35] *For a given matrix* *with symmetric* *and symmetric negative definite* *the following two statements are equivalent*

|  |  |
| --- | --- |
|  | (A.3) |

**Lemma 4.** [36] For real matrices and symmetric matrix , the following first statement can be guaranteed if and only if the second one holds for a positive scalar and ,

|  |  |
| --- | --- |
|  | (A.4) |

**Appendix B.**

The following state matrices are used as the reduced order identified system in state-space form

|  |  |
| --- | --- |
| , | (B.1) |

In addition, it is assumed that the effect of higher order mode-shapes on the vibration is less than one percent of the states () and , , and and the following controller/observer gains are obtained

|  |  |
| --- | --- |
|  | (B.2) |

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