Reach avoid

No Author Given

No Institute Given

Abstract. The problem of synthesizing a controller for nonlinear systems that is guaranteed to be both provably safe and goal-reachable is important. Reinforcement learning (RL for short) is a popular approach to effectively train controllers from user-defined reward functions encoding desired system requirements. The main challenge with this approach is that we can hardly formally verify the safety or goal-reaching of such systems under learnt DNN controllers. To address this issue, we design a special hybrid polynomial-DNN controller which is easy to verify and without losing its expressiveness and flexibility, and propose an efficient method to synthesize the hybrid controller using RL, low-degree polynomial fitting and knowledge distillation. We evaluate the proposed hybrid controller synthesis method on a set of benchmark examples. We present very encouraging results of finding verified safe and goal-reaching controllers on 12 commonly used nonlinear examples (with 2-7 dimensional state space).

Keywords: Formal verification, Reinforcement learning, Barrier certificate, Lyapunov-like function

A Benchmark Example

A.1 Cartpole system

The equation of motion for cartpole system is

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{u+m_p sin\theta(l\dot{\theta}^2 - gcos(\theta))}{m_c + m_p sin^2(\theta)} \\ \frac{ucos(\theta)+m_p l\dot{\theta}^2 cos\theta sin\theta - (m_c+m_p)gsin\theta}{l(m_c+m_p sin^2\theta)} \end{bmatrix}$$

with the constants set as $m_c = 1, m_p = 1, m_p = 1, g = 1, l = 1$.Define the state space $\mathbf{x} = [x, \theta, \dot{x}, \dot{\theta}]$,where

$$\Psi = \{\mathbf{x} \in \mathbb{R}^4 \mid [-1.3, -1.3, -1.3, -1.3]^T \leq \mathbf{x} \leq [1.3, 1.3, 1.3, 1.3]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^4 \mid 0.0 \leq ||\mathbf{x}||_2 \leq 0.7\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^4 \mid 1.0 \leq ||\mathbf{x}||_2 \leq 1.3\}.$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^4 \mid 0.0 \leq ||\mathbf{x}||_2 \leq 0.1\}.$$

we get the

$$\begin{aligned}k_0(\mathbf{x}) =& -1.4954x_1 - 0.6050x_2 - 2.6590x_3 - 1.2333x_4 - 0.1398x_1^2 + 0.0463x_1x_2 \\ & + 1.0553x_1x_3 + 0.0182x_1x_4 + 0.2287x_2^2 + 1.8307x_2x_3 - 0.1025x_2x_4 \\ & + 2.6455x_3^2 + 1.1557x_3x_4 + 0.014x_4^2\end{aligned}$$

$$\begin{aligned}B(\mathbf{x}) =& -0.002x_1^4 + 0.0008x_1^3x_2 + 0.0007x_1^3x_3 - 0.0015x_1^3x_4 + 0.0014x_1^3 - 0.0049x_1^2x_2^2 \\ & - 0.0025x_1^2x_2x_3 + 0.0084x_1^2x_2x_4 + 0.0019x_1^2x_2 - 0.0004x_1^2x_3^2 - 0.0021x_1^2x_3x_4 \\ & - 0.0035x_1^2x_3 - 0.0125x_1^2x_4^2 + 0.0013x_1^2x_4 - 0.0021x_1^2 - 0.0052x_1x_2^3 + 0.011x_1x_2^2x_3 \\ & + 0.0239x_1x_2^2x_4 + 0.0017x_1x_2^2 + 0.0018x_1x_2x_3^2 - 0.008x_1x_2x_3x_4 + 0.008x_1x_2x_3 \\ & - 0.0116x_1x_2x_4^2 + 0.0031x_1x_2x_4 + 0.0017x_1x_2 - 0.0008x_1x_3^3 + 0.0003x_1x_3^2x_4 \\ & - 0.0013x_1x_3^2 + 0.0055x_1x_3x_4^2 - 0.0032x_1x_3x_4 - 0.0007x_1x_3 + 0.0053x_1x_4^3 \\ & + 0.0008x_1x_4^2 - 0.002x_1x_4 - 0.001x_1 - 0.0082x_2^4 + 0.0007x_2^3x_3 + 0.0037x_2^3x_4 \\ & + 0.0014x_2^3 - 0.0063x_2^2x_3^2 - 0.0169x_2^2x_3x_4 + 0.0071x_2^2x_3 - 0.0155x_2^2x_4^2 + 0.0039x_2^2x_4 \\ & + 0.0001x_2^2 - 0.0003x_2x_3^3 - 0.0066x_2x_3^2x_4 - 0.0015x_2x_3^2 - 0.0054x_2x_3x_4^2 \\ & - 0.0047x_2x_3x_4 + 0.0012x_2x_3 - 0.0034x_2x_4^3 + 0.0088x_2x_4^2 + 0.0003x_2x_4 - 0.0016x_2 \\ & - 0.0007x_3^4 + 0.0013x_3^3x_4 - 0.0014x_3^3 - 0.0051x_3^2x_4^2 + 0.0025x_3^2x_4 - 0.0044x_3^2 \\ & - 0.0065x_3x_4^3 - 0.0037x_3x_4^2 + 0.0055x_3x_4 - 0.0003x_3 - 0.0032x_4^4 - 2.1051e - 5x_4^3 \\ & - 0.0009x_4^2 - 0.0014x_4 + 0.0024\end{aligned}$$

$$\begin{aligned}V(\mathbf{x}) =& 0.0004x_1^4 + 0.0035x_1^3x_2 - 0.0035x_1^3x_3 + 0.002x_1^3x_4 - 0.0035x_1^3 + 0.0098x_1^2x_2^2 \\ & - 0.0198x_1^2x_2x_3 + 0.0176x_1^2x_2x_4 - 0.0133x_1^2x_2 + 0.0128x_1^2x_3^2 - 0.0047x_1^2x_3x_4 \\ & + 0.0187x_1^2x_3 + 0.0009x_1^2x_4^2 - 0.0107x_1^2x_4 + 0.0084x_1^2 + 0.0063x_1x_2^3 - 0.0205x_1x_2^2x_3 \\ & + 0.0094x_1x_2^2x_4 - 0.0127x_1x_2^2 + 0.0223x_1x_2x_3^2 - 0.0236x_1x_2x_3x_4 + 0.0281x_1x_2x_3 \\ & - 0.0009x_1x_2x_4^2 - 0.0104x_1x_2x_4 + 0.0034x_1x_2 - 0.004x_1x_3^3 + 0.0008x_1x_3^2x_4 \\ & - 0.0092x_1x_3^2 + 0.0266x_1x_3x_4^2 + 0.017x_1x_3x_4 - 0.0075x_1x_3 - 0.0136x_1x_4^3 \\ & + 0.0004x_1x_4^2 + 0.006x_1x_4 + 2.7139e - 6x_1 + 0.0017x_2^4 - 0.0084x_2^3x_3 + 0.0034x_2^3x_4 \\ & - 0.0028x_2^3 + 0.0173x_2^2x_3^2 - 0.0092x_2^2x_3x_4 + 0.0117x_2^2x_3 + 0.0038x_2^2x_4^2 - 0.0028x_2^2x_4 \\ & + 0.0018x_2^2 - 0.017x_2x_3^3 + 0.0106x_2x_3^2x_4 - 0.0245x_2x_3^2 - 0.0022x_2x_3x_4^2 + 0.017x_2x_3x_4 \\ & - 0.0053x_2x_3 + 0.0035x_2x_4^3 - 0.0045x_2x_4^2 + 0.002x_2x_4 + 0.0001x_2 + 0.0048x_3^4 \\ & + 0.0049x_3^3x_4 + 0.0124x_3^3 - 0.0167x_3^2x_4^2 - 0.0054x_3^2x_4 + 0.0081x_3^2 + 0.0222x_3x_4^3 \\ & + 0.0034x_3x_4^2 - 0.0066x_3x_4 - 9.687e - 5x_3 - 0.0051x_4^4 - 0.0045x_4^3 + 0.0016x_4^2 \\ & + 9.4336e - 5x_4 - 6.9873e - 5\end{aligned}$$

A.2 Dubin's Car

The equation of motion for Dubin's Car system is

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} \sin(x_2) \\ -u \end{bmatrix}$$

where u is the control input.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^2 \,|\{-6 \leq x_1 \leq 6, -7\pi/10 \leq x_2 \leq 7\pi/10\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^2 \,|\{-1 \leq x_1 \leq 1, -\pi/16 \leq x_2 \leq \pi/16\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^2 \,|\{-5 \leq x_1 \leq 5, -\pi/2 \leq x_2 \leq \pi/2\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 0.1\}.$$

$$k_0(\mathbf{x}) = 0.057 + 4.8268x_1 + 7.1136x_2 - 0.3278x_1^2 - 2.6499x_1x_2 - 3.1174x_2^2$$

$$B(\mathbf{x}) = -0.0993x_1^2 - 0.071x_1x_2 - 0.0118x_1 - 0.1016x_2^2 + 0.0603x_2 + 0.1174$$

$$V(\mathbf{x}) = 0.0254x_1^2 + 0.0096x_1x_2 + 0.0002x_1 + 0.0444x_2^2 - 0.0001x_2 - 0.0001$$

A.3 Pendulum system

The equation of motion for Pendulum system is

$$\ddot{\alpha} = -\frac{g}{l} sin(\alpha) - \frac{d}{ml^2}\dot{\alpha} + \frac{u}{ml^2}$$

with the constants set as g=10, l=1, m=1, d=0.1. Define the state space $\mathbf{x} = [\alpha, \dot{\alpha}]$, where

$$\Psi = \{\mathbf{x} \in \mathbb{R}^2 \,|\, [-\pi, -5]^T \leq \mathbf{x} \leq [\pi, 5]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 2\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 2.5 \leq ||\mathbf{x}||_2 \leq 3\}.$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 0.1\}.$$

$$\begin{aligned} k_0(\mathbf{x}) = &- 0.72054 - 2.66857x_1 - 10.6991x_2 - 0.13387x_1^2 + 1.31456x_1x_2 \\ &+ 1.04989x_2^2 \end{aligned}$$

$$\begin{aligned} B(\mathbf{x}) = &- 0.011761x_1^2 - 0.003369x_1x_2 - 0.001453x_1 - 0.012896x_2^2 \\ &- 0.003091x_2 + 0.063334 \end{aligned}$$

$$\begin{aligned} V(\mathbf{x}) = &6.1379x_1^2 + 10.7155x_1x_2 - 0.0051x_1 \\ &+ 13.9549x_2^2 + 0.0068x_2 - 0.1711 \end{aligned}$$

33 A.4 Vehicle path tracking system

The kinematic model of a wheeled vehicle tracking a reference path is given by

$$\begin{aligned}
\dot{s} &= \frac{v\cos(\theta_e)}{1 - d_e k(s)} \\
\dot{d}_e &= v\sin(\theta_e) \\
\dot{\theta}_e &= \frac{v\tan(u)}{L} - \frac{vk(s)\cos(\theta_e)}{1 - d_e k(s)}
\end{aligned}$$

with the constants set as $v = 6$ and $L = 1$, where

$$\Psi = \{\mathbf{x} \in \mathbb{R}^2 \,|\, [-0.8, 0.8]^T \leq \mathbf{x} \leq [0.8, 0.8]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 0.5\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0.6 \leq ||\mathbf{x}||_2 \leq 0.8\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^2 \,|\, 0 \leq ||\mathbf{x} - \mathbf{x}_g||_2 \leq 0.2\} with \mathbf{x}_g = [-0.2, 0]^T$$

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$$k_0(\mathbf{x}) = -0.286x_1 - 1.3352x_2 + 0.2347x_1^2 + 0.3372x_1x_2 - 0.1027x_2^2$$

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$$B(\mathbf{x}) = -0.6573x_1^2 + 0.0128x_1x_2 + 0.008x_1 - 0.812x_2^2 + 0.012x_2 + 0.2239$$

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$$V(\mathbf{x}) = -29.8168x_1^2 + 26.4377x_1x_2 - 11.9429x_1 - 29.8581x_2^2 + 5.305x_2 - 0.5327$$

37 A.5 UAV control

The motion of equation for a UAV flying in planar is given by

$$\begin{aligned}
\ddot{x} &= \frac{-(u_1 + u_2)\sin\theta}{m} \\
\ddot{y} &= \frac{(u_1 + u_2)\cos\theta - mg}{m} \\
\ddot{\theta} &= \frac{r(u_1 - u_2)}{I}
\end{aligned}$$

with the constants are set as $m = 0.1, g = 0.1, I = 0.1, r = 0.1$. Define the state space $\mathbf{x} = [x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}]^T$ and the control variable $u = [u_1, u_2]^T$.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^6 \,|\, [-1, -1, -1, -1]^T \leq \mathbf{x} \leq [1, 1, 1, 1]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^6 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 0.3\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^6 \,|\, 0.9 \leq ||\mathbf{x}||_2 \leq 1\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^6 \,|\, 0 \leq ||\mathbf{x}||_2 \leq 0.1\}$$

$$
\begin{aligned}
k_0^1(\mathbf{x}) = & -0.0002 + 0.8191x_2 + 0.8255x_3 - 1.986x_4 + 2.4404x_5 - 0.2194x_6 - 1.8463x_6 \\
& + 0.0116x_2^2 + 0.0966x_2x_3 - 0.038x_2x_4 + 0.0588x_2x_5 + 0.0289x_2x_6 - 0.0658x_2x_6 \\
& - 0.0411x_3^2 - 0.1863x_3x_4 - 0.0235x_3x_5 - 0.2933x_3x_6 - 0.0407x_3x_6 + 0.0334x_4^2 \\
& - 0.0939x_4x_5 - 0.0984x_4x_6 + 0.0691x_4x_6 - 0.0585x_5^2 - 0.1217x_5x_6 + 0.0918x_5x_6 \\
& - 0.0396x_6^2 - 0.0428x_6x_6 + 0.0267x_6^2
\end{aligned}
$$

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$$
\begin{aligned}
k_0^2(\mathbf{x}) = & - 0.0002 + 0.916x_2 - 1.0967x_3 - 1.8236x_4 - 0.0922x_5 - 5.1911x_6 + 0.3104x_6 \\
& - 0.0249x_2^2 - 0.027x_2x_3 + 0.0901x_2x_4 - 0.0011x_2x_5 + 0.0808x_2x_6 + 0.0881x_2x_6 \\
& + 0.0197x_3^2 + 0.022x_3x_4 + 0.0888x_3x_5 + 0.0953x_3x_6 - 0.0358x_3x_6 - 0.069x_4^2 \\
& - 0.0286x_4x_5 - 0.1482x_4x_6 - 0.152x_4x_6 + 0.1788x_5^2 - 0.001x_5x_6 - 0.0251x_5x_6 \\
& - 0.1162x_6^2 - 0.1691x_6x_6 - 0.0391x_6^2
\end{aligned}
$$

$$
\begin{aligned}
B(\mathbf{x}) = &-0.4345x_1^4 - 0.5793x_1^3x_2 + 0.3745x_1^3x_3 + 1.123x_1^3x_4 + 0.7019x_1^3x_5 + 0.3989x_1^3x_6 \\
&- 0.1529x_1^3 - 1.4809x_1^2x_2^2 + 1.8892x_1^2x_2x_3 + 2.1724x_1^2x_2x_4 + 1.4596x_1^2x_2x_5 \\
&+ 2.4046x_1^2x_2x_6 - 0.6032x_1^2x_2 - 0.9883x_1^2x_3^2 - 1.7759x_1^2x_3x_4 - 0.2175x_1^2x_3x_5 \\
&- 2.254x_1^2x_3x_6 - 0.0252x_1^2x_3 - 1.7972x_1^2x_4^2 - 2.371x_1^2x_4x_5 - 2.2248x_1^2x_4x_6 \\
&+ 0.0314x_1^2x_4 - 1.5084x_1^2x_5^2 + 0.2135x_1^2x_5x_6 + 0.4066x_1^2x_5 - 1.8248x_1^2x_6^2 \\
&- 0.5275x_1^2x_6 - 0.2344x_1^2 - 1.6119x_1x_2^3 + 2.7513x_1x_2^2x_3 + 2.083x_1x_2^2x_4 \\
&+ 0.8904x_1x_2^2x_5 + 3.6571x_1x_2^2x_6 - 0.3836x_1x_2^2 + 0.242x_1x_2x_3^2 - 2.3503x_1x_2x_3x_4 \\
&+ 0.0135x_1x_2x_3x_5 - 0.0466x_1x_2x_3x_6 + 1.5712x_1x_2x_3 - 1.7175x_1x_2x_4^2 \\
&- 3.5298x_1x_2x_4x_5 - 3.129x_1x_2x_4x_6 + 0.3032x_1x_2x_4 - 1.7376x_1x_2x_5^2 \\
&- 2.3867x_1x_2x_5x_6 + 1.3476x_1x_2x_5 - 0.4814x_1x_2x_6^2 + 1.4621x_1x_2x_6 - 0.049x_1x_2 \\
&- 1.1407x_1x_3^3 + 0.5019x_1x_3^2x_4 + 1.2769x_1x_3^2x_5 - 1.9035x_1x_3^2x_6 - 0.3521x_1x_3^2 \\
&+ 1.7903x_1x_3x_4^2 + 0.405x_1x_3x_4x_5 + 0.2268x_1x_3x_4x_6 - 0.4213x_1x_3x_4 - 1.0768x_1x_3x_5^2 \\
&+ 1.5791x_1x_3x_5x_6 - 0.4497x_1x_3x_5 - 2.775x_1x_3x_6^2 + 1.3557x_1x_3x_6 + 0.2687x_1x_3 \\
&+ 1.2751x_1x_4^3 + 2.9956x_1x_4^2x_5 + 2.4008x_1x_4^2x_6 + 0.0465x_1x_4^2 + 3.1552x_1x_4x_5^2 \\
&+ 3.1009x_1x_4x_5x_6 - 1.7613x_1x_4x_5 + 0.3725x_1x_4x_6^2 + 0.3473x_1x_4x_6 - 0.0727x_1x_4 \\
&+ 1.2645x_1x_5^3 + 0.4994x_1x_5^2x_6 - 1.3555x_1x_5^2 + 1.1616x_1x_5x_6^2 - 1.5689x_1x_5x_6 \\
&- 0.1821x_1x_5 - 0.5453x_1x_6^3 + 1.0723x_1x_6^2 + 1.1063x_1x_6 - 0.146x_1 - 0.8599x_2^4 \\
&+ 0.8801x_2^3x_3 + 1.2057x_2^3x_4 - 0.4969x_2^3x_5 + 1.0418x_2^3x_6 - 0.1108x_2^3 + 0.5104x_2^2x_3^2 \\
&+ 0.1684x_2^2x_3x_4 + 0.1378x_2^2x_3x_5 - 1.0782x_2^2x_3x_6 - 0.0226x_2^2x_3 - 2.0549x_2^2x_4^2 \\
&- 1.7267x_2^2x_4x_5 + 0.3111x_2^2x_4x_6 + 2.2068x_2^2x_4 - 1.3046x_2^2x_5^2 + 0.2919x_2^2x_5x_6 \\
&+ 1.494x_2^2x_5 - 0.4956x_2^2x_6^2 - 0.6366x_2^2x_6 - 0.8323x_2^2 - 0.0311x_2x_3^3 - 2.2141x_2x_3^2x_4 \\
&- 0.9843x_2x_3^2x_5 - 0.1587x_2x_3^2x_6 + 0.9089x_2x_3^2 + 1.2116x_2x_3x_4^2 + 3.959x_2x_3x_4x_5 \\
&+ 0.6739x_2x_3x_4x_6 - 3.2578x_2x_3x_4 + 0.9295x_2x_3x_5^2 - 1.8115x_2x_3x_5x_6 - 3.2641x_2x_3x_5 \\
&- 2.0519x_2x_3x_6^2 - 1.169x_2x_3x_6 + 1.6001x_2x_3 + 1.4537x_2x_4^3 + 1.7019x_2x_4^2x_5 \\
&+ 1.1033x_2x_4^2x_6 - 2.0299x_2x_4^2 + 1.544x_2x_4x_5^2 + 4.6231x_2x_4x_5x_6 - 1.2405x_2x_4x_5 \\
&+ 1.6348x_2x_4x_6^2 - 1.4774x_2x_4x_6 + 0.8622x_2x_4 + 0.3765x_2x_5^3 + 2.6338x_2x_5^2x_6 \\
&- 0.8094x_2x_5^2 + 2.4308x_2x_5x_6^2 - 3.4079x_2x_5x_6 + 0.1075x_2x_5 - 0.5658x_2x_6^3 - 2.453x_2x_6^2 \\
&+ 1.0101x_2x_6 - 0.3329x_2 - 0.8071x_3^4 + 0.2488x_3^3x_4 + 1.7399x_3^3x_5 + 0.8537x_3^3x_6 \\
&- 0.9774x_3^3 + 0.0263x_3^2x_4^2 - 1.5681x_3^2x_4x_5 - 0.6946x_3^2x_4x_6 + 0.3614x_3^2x_4 - 2.4618x_3^2x_5^2 \\
&- 2.3113x_3^2x_5x_6 + 1.7323x_3^2x_5 - 2.3809x_3^2x_6^2 + 2.3117x_3^2x_6 - 0.8675x_3^2 - 0.7775x_3x_4^3 \\
&- 1.3864x_3x_4^2x_5 - 2.043x_3x_4^2x_6 + 1.4189x_3x_4^2 + 1.1469x_3x_4x_5^2 - 0.8022x_3x_4x_5x_6 \\
&+ 1.6059x_3x_4x_5 - 0.3974x_3x_4x_6^2 + 1.2958x_3x_4x_6 - 1.1481x_3x_4 + 1.3424x_3x_5^3 \\
&+ 0.1173x_3x_5^2x_6 - 0.8208x_3x_5^2 + 0.1761x_3x_5x_6^2 - 1.1217x_3x_5x_6 - 0.3513x_3x_5 \\
&- 1.3584x_3x_6^3 + 0.8985x_3x_6^2 - 0.3517x_3x_6 + 0.0761x_3 - 0.5975x_4^4 - 0.8975x_4^3x_5 \\
&- 0.8533x_4^3x_6 + 0.5852x_4^3 - 1.5863x_4^2x_5^2 - 2.9748x_4^2x_5x_6 + 0.484x_4^2x_5 - 1.9129x_4^2x_6^2 \\
&+ 0.1045x_4^2x_6 - 0.5699x_4^2 - 1.1772x_4x_5^3 - 2.0738x_4x_5^2x_6 + 1.75x_4x_5^2 - 2.6867x_4x_5x_6^2 \\
&+ 3.262x_4x_5x_6 + 0.2539x_4x_5 - 1.1435x_4x_6^3 + 2.0961x_4x_6^2 - 0.16x_4x_6 + 0.0572x_4 \\
&- 0.4348x_5^4 - 0.2832x_5^3x_6 + 0.6945x_5^3 - 1.4542x_5^2x_6^2 + 1.3075x_5^2x_6 - 0.3605x_5^2 \\
&- 0.7009x_5x_6^3 + 1.9816x_5x_6^2 - 1.2957x_5x_6 - 0.1684x_5 - 0.8587x_6^4 + 0.6134x_6^3 - 1.6042x_6^2 \\
&+ 0.3673x_6 + 0.2234
\end{aligned}
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$$
\begin{aligned}
V(\mathbf{x}) = &- 1.7447x_1^4 + 2.9423x_1^3x_2 + 0.6604x_1^3x_3 - 0.014x_1^3x_4 + 4.9473x_1^3x_5 + 0.776x_1^3x_6 \\
&- 3.1295x_1^3 - 3.2721x_1^2x_2^2 + 0.9953x_1^2x_2x_3 + 4.2895x_1^2x_2x_4 - 5.4042x_1^2x_2x_5 \\
&+ 0.3052x_1^2x_2x_6 + 7.6336x_1^2x_2 + 3.652x_1^2x_3^2 + 1.7176x_1^2x_3x_4 + 0.3067x_1^2x_3x_5 \\
&- 1.6623x_1^2x_3x_6 + 2.9577x_1^2x_3 + 0.8254x_1^2x_4^2 + 1.3362x_1^2x_4x_5 + 1.4501x_1^2x_4x_6 \\
&- 0.3323x_1^2x_4 - 3.1555x_1^2x_5^2 - 2.8955x_1^2x_5x_6 + 5.1722x_1^2x_5 - 0.9249x_1^2x_6^2 \\
&+ 3.6219x_1^2x_6 - 2.2984x_1^2 + 0.6644x_1x_2^3 + 3.4051x_1x_2^2x_3 - 4.5719x_1x_2^2x_4 \\
&+ 0.4404x_1x_2^2x_5 - 2.1213x_1x_2^2x_6 - 2.526x_1x_2^2 - 4.3237x_1x_2x_3^2 + 0.7812x_1x_2x_3x_4 \\
&+ 1.538x_1x_2x_3x_5 + 1.2035x_1x_2x_3x_6 - 2.0407x_1x_2x_3 - 2.7631x_1x_2x_4^2 \\
&- 8.9635x_1x_2x_4x_5 - 0.7957x_1x_2x_4x_6 + 3.8295x_1x_2x_4 - 0.9756x_1x_2x_5^2 \\
&+ 1.644x_1x_2x_5x_6 - 2.9519x_1x_2x_5 + 0.6812x_1x_2x_6^2 - 3.4435x_1x_2x_6 + 2.3405x_1x_2 \\
&+ 0.4827x_1x_3^3 - 0.6215x_1x_3^2x_4 - 3.1464x_1x_3^2x_5 - 2.8288x_1x_3^2x_6 + 2.4783x_1x_3^2 \\
&+ 0.7829x_1x_3x_4^2 - 1.9372x_1x_3x_4x_5 + 0.1024x_1x_3x_4x_6 + 1.638x_1x_3x_4 + 1.1132x_1x_3x_5^2 \\
&- 1.9809x_1x_3x_5x_6 - 4.9951x_1x_3x_5 + 0.9601x_1x_3x_6^2 + 0.0576x_1x_3x_6 + 2.7646x_1x_3 \\
&+ 0.0917x_1x_4^3 - 1.3055x_1x_4^2x_5 - 0.9173x_1x_4^2x_6 + 2.6294x_1x_4^2 - 3.7031x_1x_4x_5^2 \\
&+ 1.0255x_1x_4x_5x_6 + 3.9779x_1x_4x_5 - 1.1606x_1x_4x_6^2 - 2.3282x_1x_4x_6 + 0.9027x_1x_4 \\
&- 0.9584x_1x_5^3 + 0.6698x_1x_5^2x_6 - 0.3323x_1x_5^2 + 1.2957x_1x_5x_6^2 - 1.0973x_1x_5x_6 \\
&+ 1.048x_1x_5 - 0.5067x_1x_6^3 - 1.5025x_1x_6^2 + 0.1816x_1x_6 + 0.0522x_1 - 0.6901x_2^4 \\
&- 1.1542x_2^3x_3 + 0.952x_2^3x_4 - 0.6367x_2^3x_5 + 1.3086x_2^3x_6 - 0.2746x_2^3 - 1.7491x_2^2x_3^2 \\
&- 2.8397x_2^2x_3x_4 - 0.4208x_2^2x_3x_5 - 2.2584x_2^2x_3x_6 - 1.0875x_2^2x_3 - 1.68x_2^2x_4^2 \\
&- 0.8576x_2^2x_4x_5 + 0.0547x_2^2x_4x_6 - 1.8605x_2^2x_4 - 1.2806x_2^2x_5^2 + 1.0226x_2^2x_5x_6 \\
&+ 1.1792x_2^2x_5 - 0.5951x_2^2x_6^2 - 0.1051x_2^2x_6 - 1.5088x_2^2 - 2.3729x_2x_3^3 - 5.8386x_2x_3^2x_4 \\
&- 2.4461x_2x_3^2x_5 + 1.9421x_2x_3^2x_6 - 4.2286x_2x_3^2 - 2.922x_2x_3x_4^2 + 3.5626x_2x_3x_4x_5 \\
&- 3.5031x_2x_3x_4x_6 - 7.7923x_2x_3x_4 + 2.9839x_2x_3x_5^2 + 1.3254x_2x_3x_5x_6 - 2.8165x_2x_3x_5 \\
&+ 0.2347x_2x_3x_6^2 - 2.7153x_2x_3x_6 - 1.8489x_2x_3 - 2.0567x_2x_4^3 - 3.9163x_2x_4^2x_5 \\
&- 0.2072x_2x_4^2x_6 - 1.2433x_2x_4^2 - 1.6428x_2x_4x_5^2 + 1.1804x_2x_4x_5x_6 - 2.2892x_2x_4x_5 \\
&+ 1.6673x_2x_4x_6^2 - 1.7086x_2x_4x_6 - 0.4904x_2x_4 - 0.9143x_2x_5^3 + 3.4008x_2x_5^2x_6 \\
&+ 1.389x_2x_5^2 - 0.2317x_2x_5x_6^2 - 4.5637x_2x_5x_6 - 0.5482x_2x_5 + 0.6269x_2x_6^3 + 1.4868x_2x_6^2 \\
&- 0.5146x_2x_6 + 0.1063x_2 - 1.1026x_3^4 - 3.2117x_3^3x_4 - 0.1513x_3^3x_5 - 0.5125x_3^3x_6 \\
&- 2.4391x_3^3 - 4.7054x_3^2x_4^2 - 3.9741x_3^2x_4x_5 + 1.5214x_3^2x_4x_6 - 3.7951x_3^2x_4 - 0.9299x_3^2x_5^2 \\
&+ 0.6776x_3^2x_5x_6 - 1.9056x_3^2x_5 + 0.9762x_3^2x_6^2 + 0.0867x_3^2x_6 - 1.351x_3^2 - 2.4163x_3x_4^3 \\
&- 0.038x_3x_4^2x_5 + 0.2165x_3x_4^2x_6 - 4.2091x_3x_4^2 + 4.1843x_3x_4x_5^2 + 1.4006x_3x_4x_5x_6 \\
&- 7.3685x_3x_4x_5 + 0.1382x_3x_4x_6^2 - 0.6713x_3x_4x_6 - 0.6878x_3x_4 + 1.0532x_3x_5^3 \\
&- 0.2561x_3x_5^2x_6 - 1.6565x_3x_5^2 - 0.5695x_3x_5x_6^2 + 3.8552x_3x_5x_6 - 1.948x_3x_5 - 0.4576x_3x_6^3 \\
&- 0.3309x_3x_6^2 - 1.1229x_3x_6 - 0.0435x_3 - 1.027x_4^4 - 1.7881x_4^3x_5 + 0.0638x_4^3x_6 - 0.554x_4^3 \\
&- 0.9129x_4^2x_5^2 + 1.439x_4^2x_5x_6 - 0.9181x_4^2x_5 + 0.6574x_4^2x_6^2 - 1.3518x_4^2x_6 - 0.5667x_4^2 \\
&- 0.8358x_4x_5^3 + 1.5696x_4x_5^2x_6 + 0.7499x_4x_5^2 + 0.2797x_4x_5x_6^2 - 0.1843x_4x_5x_6 \\
&- 1.574x_4x_5 + 0.1892x_4x_6^3 - 0.413x_4x_6^2 - 0.3876x_4x_6 - 0.2224x_4 - 0.8781x_5^4 \\
&+ 0.623x_5^3x_6 + 1.3757x_5^3 - 2.1807x_5^2x_6^2 + 1.2042x_5^2x_6 - 1.7249x_5^2 - 0.0817x_5x_6^3 \\
&+ 1.5404x_5x_6^2 - 0.0009x_5x_6 - 0.0028x_5 - 0.2112x_6^4 + 0.3998x_6^3 - 1.2385x_6^2 - 0.048x_6 \\
&+ 0.0235
\end{aligned}
$$

40 ### A.6 Academic 3D

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_3 + 8x_2 \\ -x_2 + x_3 \\ -x_3 - x_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

where u is the control input.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^3 \mid -2.2 \le x_1 \le 2.2, -2.2 \le x_2 \le 2.2, -2.2 \le x_3 \le 2.2\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^3 \mid -0.2 \le x_1 \le 0.2, -0.2 \le x_2 \le 0.2, -2.2 \le x_3 \le 2.2\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^3 \mid -2 \le x_1 \le 2, -2 \le x_2 \le 2, -2 \le x_3 \le 2\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^3 \mid 0 \le ||\mathbf{x}||_2 \le 0.1\}.$$

41

$$\begin{aligned} k_0(\mathbf{x}) =& 0.1247 - 3.3332x_1 - 5.726x_2 - 10.6688x_3 + 1.9106x_1^2 + 1.2121x_1x_2 + 2.1376x_1x_3 \\ & - 1.3317x_2^2 - 10.0699x_2x_3 - 12.9515x_3^2 \end{aligned}$$

42

$$\begin{aligned} B(\mathbf{x}) =& -0.032x_1^2 - 0.0168x_1x_2 - 0.001x_1x_3 - 0.0074x_1 - 0.0149x_2^2 + 0.0364x_2x_3 \\ & - 0.0001x_2 - 0.0641x_3^2 - 0.0075x_3 + 0.0246 \end{aligned}$$

43

$$\begin{aligned} V(\mathbf{x}) =& -0.0004x_1^2 + 0.0042x_1x_2 - 0.0034x_1x_3 - 0.0004x_1 + 0.0119x_2^2 + 0.0083x_2x_3 \\ & - 0.0001x_2 - 0.0042x_3^2 - 0.0001x_3 - 0.0001 \end{aligned}$$

44 ### A.7 Space rendezvous

The equation of motion for Space rendezvous system is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{v_x} \\ \dot{v_y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ n^2x + 2nv_y + \frac{\mu}{r^2} - \frac{\mu}{r^3}(r + x) + \frac{u_x}{m_c} \\ n^2y - 2nv_x - \frac{\mu}{r^3}y + \frac{v_x}{m_c} \end{bmatrix}$$

where $\mathbf{x} = [x, y, v_x, v_y]$ is the control input,and the constants set as $\mu = 3.986 \times 10^{14} \times 60^2[m^3/min^2]$,$r = 42164 \times 10^3[m]$,$m_c = 500[kg]$ and $n = \sqrt{\frac{\mu}{r^3}}$.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^4 \mid [-200, -200, -200, -200]^T \le \mathbf{x} \le [200, 200, 200, 200]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^4 \mid [-50, -50, 0, 0]^T \le \mathbf{x} \le [25, 25, 0, 0]^T\}$$

$$X_u = \{\mathbf{x} \in \mathbb{R}^4 \mid [-100, -100, -200, -200]^T \le \mathbf{x} \le [100, 100, 200, 200]^T\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^4 \mid [-10, -10, -10, -10]^T \le \mathbf{x} \le [10, 10, 10, 10]^T\}$$

45

$$\begin{aligned} k_0^1(\mathbf{x}) =& -28.8306x_1 + 0.1045x_2 - 1450.0508x_3 + 0.1559x_4 - 0.0x_1^2 + 0.0x_1x_2 \\ & - 0.0002x_1x_3 - 9e - 05x_1x_4 + 0.0x_2^2 + 0.001x_2x_3 - 0.0006x_2x_4 - 0.0097x_3^2 \\ & + 0.0367x_3x_4 - 0.0343x_4^2 \end{aligned}$$

$$
\begin{aligned}
k_0^2(\mathbf{x}) = & -0.0873x_1 - 33.2556x_2 + 0.0086x_3 - 1451.4999x_4 - 0.0x_1^2 + 0.0x_1x_2 \\
& - 0.0002x_1x_3 - 7e - 05x_1x_4 - 0.0x_2^2 + 0.001x_2x_3 - 0.0009x_2x_4 - 0.0104x_3^2 \\
& + 0.0404x_3x_4 - 0.0388x_4^2
\end{aligned}
$$

$$
\begin{aligned}
B(\mathbf{x}) = & -0.8538x_1^4 + 0.9518x_1^3x_2 - 0.8331x_1^3x_3 - 0.1601x_1^3x_4 + 2.2889x_1^3 + 0.7599x_1^2x_2^2 \\
& + 2.3631x_1^2x_2x_3 + 4.1786x_1^2x_2x_4 - 2.2201x_1^2x_2 - 1.6886x_1^2x_3^2 - 2.9481x_1^2x_3x_4 \\
& + 5.169x_1^2x_3 - 5.3382x_1^2x_4^2 + 3.8832x_1^2x_4 - 2.7864x_1^2 + 0.0451x_1x_2^3 - 0.0174x_1x_2^2x_3 \\
& + 2.5711x_1x_2^2x_4 - 1.3929x_1x_2^2 + 1.6645x_1x_2x_3^2 - 7.0398x_1x_2x_3x_4 - 2.5403x_1x_2x_3 \\
& + 3.0704x_1x_2x_4^2 - 10.3496x_1x_2x_4 + 0.8463x_1x_2 - 0.5476x_1x_3^3 + 0.5042x_1x_3^2x_4 \\
& - 5.7843x_1x_3^2 - 3.052x_1x_3x_4^2 - 3.3958x_1x_3x_4 + 0.6559x_1x_3 - 3.2031x_1x_4^3 \\
& + 10.8095x_1x_4^2 - 0.374x_1x_4 + 2.2921x_1 - 0.6271x_2^4 - 0.1216x_2^3x_3 - 2.1319x_2^3x_4 \\
& - 3.1195x_2^3 - 6.5096x_2^2x_3^2 + 0.7821x_2^2x_3x_4 + 3.1089x_2^2x_3 - 4.0768x_2^2x_4^2 - 8.8325x_2^2x_4 \\
& - 6.5364x_2^2 + 10.5536x_2x_3^3 + 16.6991x_2x_3^2x_4 - 12.8039x_2x_3^2 + 15.9803x_2x_3x_4^2 \\
& - 4.8689x_2x_3x_4 + 5.2072x_2x_3 + 0.0512x_2x_4^3 - 13.9612x_2x_4^2 + 3.0023x_2x_4 - 1.9047x_2 \\
& - 8.945x_3^4 - 17.7636x_3^3x_4 + 7.248x_3^3 - 20.6102x_3^2x_4^2 + 11.9733x_3^2x_4 - 1.6713x_3^2 \\
& - 8.2966x_3x_4^3 + 15.7038x_3x_4^2 - 8.8341x_3x_4 - 2.0216x_3 - 2.8143x_4^4 + 4.2948x_4^3 \\
& - 13.1546x_4^2 - 4.7168x_4 + 2527.2016
\end{aligned}
$$

$$
\begin{aligned}
V(\mathbf{x}) = & -0.2596x_1^2 - 0.2608x_1x_2 - 0.5691x_1x_3 - 0.2672x_1x_4 - 0.1136x_1 - 0.303x_2^2 \\
& - 0.4228x_2x_3 + 0.4163x_2x_4 - 0.1374x_2 - 0.1177x_3^2 + 0.2929x_3x_4 + 0.109x_3 \\
& - 0.2964x_4^2 + 0.7002x_4 + 7.6771
\end{aligned}
$$

A.8 Oscillator

Van der Pol's oscillator is a 2-dimensional non-linear benchmark system, which can be expressed as

$$
\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = \gamma(1 - x_1^2)x_2 - x_1 + u \end{cases} \tag{1}
$$

with the constants set as sampling period δ=0.1, and damping coefficient $\gamma = 1$.

$$
\Psi = \{\mathbf{x} \in \mathbb{R} \,|\, [-2, -2]^T \leq \mathbf{x} \leq [2, 2]^T\}
$$

$$
X_0 = \{\mathbf{x} \in \mathbb{R}^2 \,|\, [-0.51, 0.51]^T \leq \mathbf{x} \leq [-0.49, 0.49]^T\}
$$

$$
X_u = \{\mathbf{x} \in \mathbb{R}^2 \,|\, [-0.3, 0.2]^T \leq \mathbf{x} \leq [-0.25, 0.35]^T\}
$$

$$
X_g = \{\mathbf{x} \in \mathbb{R}^2 \,|\, [-0.05, 0.05]^T \leq \mathbf{x} \leq [-0.05, 0.05]^T\}
$$

$$
k_0(\mathbf{x}) = -0.0133 + 1.2057x_1 + 1.2299x_2 + 13.5323x_1^2 + 81.1611x_2x_2 + 57.9627x_2^2
$$

$$
B(\mathbf{x}) = 0.1837x_1^2 + 0.1128x_1x_2 + 0.083x_1 + 0.1181x_2^2 - 0.0197x_2 + 0.0115
$$

$$
V(\mathbf{x}) = 0.0352x_1^2 - 0.027x_1x_2 - 0.0001x_1 + 0.0448x_2^2 - 0.0001x_2 - 0.0001
$$

55 A.9 Bicycle Steering

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{ml}{J}(g \cdot \sin(x_1)) + \frac{v^2}{b}\cos(x_1)\tan(x_3) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{amlv}{Jb} \cdot \frac{\cos(x_1)}{\cos^2(x_3)} \\ 1 \end{bmatrix} u$$

where u is the control input. By introducing a new neural network controller $\tilde{u}$ such that $u = \tilde{u}\cos^2(x_3) - 20\cos(x_3)\sin(x_3)$, the original system is transformed into the following one:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_2 \\ 30\sin(x_1) + 15\tilde{u}\cos(x_1) \\ -20\cos(x_3)\sin(x_3) + \tilde{u}cos^2(x_3) \end{bmatrix}$$

with the constants set as $m = 20, l = 1, b = 1, J = \frac{mb^2}{3}, v = 10, g = 10, a = 0.5$.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^3 \mid -2.2 \le x_1 \le 2.2, -2.2 \le x_2 \le 2.2, -2.2 \le x_3 \le 2.2\}$$
$$X_0 = \{\mathbf{x} \in \mathbb{R}^3 \mid -0.2 \le x_1 \le 0.2, -0.2 \le x_2 \le 0.2, -0.2 \le x_3 \le 0.2\}$$
$$X_u = \Psi - \{\mathbf{x} \in \mathbb{R}^3 \mid -2 \le x_1 \le 2, -2 \le x_2 \le 2, -2 \le x_3 \le 2\}$$
$$X_g = \{\mathbf{x} \in \mathbb{R}^3 \mid -0.1 \le x_1 \le 0.1, -0.1 \le x_2 \le 0.1, -0.1 \le x_3 \le 0.1\}$$

56
$$\begin{aligned} k_0(\mathbf{x}) =& 0.0066 - 7.1534x_1 - 4.7381x_2 - 15.2619x_3 + 1.4062x_1^2 - 6.9605x_1x_2 + 0.4192x_1x_3 \\ & - 4.1475x_2^2 + 6.7059x_2x_3 - 6.1452x_3^2 \end{aligned}$$

57
$$\begin{aligned} B(\mathbf{x}) =& -0.008x_1^2 - 0.1012x_1x_2 - 0.0364x_1x_3 - 0.1916x_1 + 0.0026x_2^2 - 0.0369x_2x_3 \\ & - 0.0438x_2 + 0.0393x_3^2 - 0.0162x_3 + 0.053 \end{aligned}$$

58
$$\begin{aligned} V(\mathbf{x}) =& 0.0035x_1^4 + 0.0059x_1^3x_2 + 0.0187x_1^3x_3 - 0.0036x_1^3 + 0.0063x_1^2x_2^2 + 0.0099x_1^2x_2x_3 \\ & - 0.0156x_1^2x_2 + 0.0354x_1^2x_3^2 - 0.0081x_1^2x_3 + 0.0042x_1^2 - 0.0125x_1x_2^3 + 0.0116x_1x_2^2x_3 \\ & + 0.0149x_1x_2^2 + 0.0484x_1x_2x_3^2 - 0.0039x_1x_2x_3 + 0.0026x_1x_2 + 0.0305x_1x_3^3 \\ & - 0.034x_1x_3^2 + 0.0075x_1x_3 - 0.0001x_1 - 0.0115x_2^4 + 0.0526x_2^3x_3 + 0.0143x_2^3 \\ & - 0.0288x_2^2x_3^2 - 0.0776x_2^2x_3 + 0.0204x_2^2 + 0.0297x_2x_3^3 + 0.0138x_2x_3^2 + 0.0081x_2x_3 \\ & - 0.0002x_2 + 0.0144x_3^4 - 0.0057x_3^3 + 0.023x_3^2 + 0.0002x_3 - 0.0002 \end{aligned}$$

59 A.10 LALO20

The Laub-Loomis model is for studying a class of enzymatic activities. The dynamics can be defined by the following ODE with 7 variables.

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \\ \dot{x_7} \end{bmatrix} = \begin{bmatrix} 1.4x_3 - 0.9x_1 \\ 2.5x_5 - 1.5x_2 \\ 0.6x_7 - 0.8x_2x_3 \\ 2 - 1.3x_3x_4 \\ 0.7x_1 - x_4x_5 \\ 0.3x_1 - 3.1x_6 \\ 1.8x_6 - 1.5x_2x_7 \end{bmatrix}$$

The system is asymptotically stable and the equilibrium is the origin.

$$\Psi = \{\mathbf{x} \in \mathbb{R}^7 \mid [-3.8, -3.95, -3.5, -2.6, -4., -4.9, -4.55]^T \leq \mathbf{x} \leq [6.2, 6.05, 6.5, 7.4, 6., 5.1, 5.45]^T\}$$

$$X_0 = \{\mathbf{x} \in \mathbb{R}^7 \mid [1.15, 1., 1.45, 2.35, 0.95, 0.05, 0.4]^T \leq \mathbf{x} \leq [1.25, 1.1, 1.55, 2.45, 1.05, 0.15, 0.5]^T\}$$

$$X_u = \Psi - \{\mathbf{x} \in \mathbb{R}^7 \mid [-3.3, -3.45, -3., -2.1, -3.5, -4.4, -4.05]^T \leq \mathbf{x} \leq [1.25, 1.1, 1.55, 2.45, 1.05, 0.15, 0.5]^T\}$$

$$X_g = \{\mathbf{x} \in \mathbb{R}^7 \mid [0.77, 0.27, 0.46, 2.65, 0.12, -0.02, 0.17]^T \leq \mathbf{x} \leq [0.97, 0.47, 0.66, 2.85, 0.32, 0.18, 0.37]^T\}$$

$$\begin{aligned} k_0(\mathbf{x}) = &- 0.6859 + 0.6218x_1 + 0.7227x_2 - 0.0839x_3 + 0.3256x_4 - 1.1222x_5 + 0.0745x_6 \\ &- 0.4122x_7 - 0.0286x_1^2 - 0.4843x_1x_2 - 0.2439x_1x_3 - 0.089x_1x_4 + 0.4708x_1x_5 \\ &- 0.3794x_1x_6 + 0.6159x_1x_7 + 0.0793x_2^2 + 0.318x_2x_3 - 0.1582x_2x_4 - 0.3194x_2x_5 \\ &+ 0.585x_2x_6 - 0.2161x_2x_7 + 0.0962x_3^2 + 0.0211x_3x_4 - 0.1527x_3x_5 + 0.1073x_3x_6 \\ &- 0.3344x_3x_7 - 0.0332x_4^2 + 0.2425x_4x_5 - 0.6464x_4x_6 - 0.0177x_4x_7 + 0.2825x_5^2 \\ &+ 0.9822x_5x_6 - 0.1117x_5x_7 - 0.6376x_6^2 + 0.5657x_6x_7 + 0.4215x_7^2 \end{aligned}$$

$$\begin{aligned} B(\mathbf{x}) = &- 0.0377x_1^2 - 0.0024x_1x_2 + 0.0442x_1x_3 + 0.0489x_1x_4 - 0.0044x_1x_5 + 0.0418x_1x_6 \\ &- 0.045x_1x_7 - 0.0361x_1 - 0.052x_2^2 + 0.0058x_2x_3 + 0.0278x_2x_4 + 0.0242x_2x_5 \\ &- 0.0519x_2x_6 - 0.0179x_2x_7 - 0.0656x_2 - 0.0203x_3^2 - 0.0408x_3x_4 - 0.0134x_3x_5 \\ &- 0.0078x_3x_6 + 0.0274x_3x_7 + 0.0315x_3 - 0.0296x_4^2 - 0.0094x_4x_5 + 0.0199x_4x_6 \\ &+ 0.0383x_4x_7 + 0.0611x_4 - 0.0306x_5^2 + 0.0386x_5x_6 + 0.0019x_5x_7 + 0.0114x_5 \\ &- 0.0878x_6^2 + 0.0737x_6x_7 - 0.0902x_6 - 0.0505x_7^2 + 0.0175x_7 + 0.2026 \end{aligned}$$

$$\begin{aligned} V(\mathbf{x}) = &- 0.0002x_1^2 - 0.0201x_1x_2 - 0.0106x_1x_3 + 0.0009x_1x_4 - 0.0081x_1x_5 - 0.0056x_1x_6 \\ &- 0.0043x_1x_7 + 0.0152x_1 - 0.0158x_2^2 - 0.0247x_2x_3 + 0.0185x_2x_4 - 0.0173x_2x_5 \\ &- 0.0006x_2x_6 + 0.0298x_2x_7 - 0.0112x_2 - 0.0134x_3^2 + 0.0036x_3x_4 - 0.0118x_3x_5 \\ &- 0.0101x_3x_6 + 0.0036x_3x_7 + 0.0257x_3 - 0.0002x_4^2 - 0.0009x_4x_5 - 0.0026x_4x_6 \\ &- 0.0046x_4x_7 - 0.0061x_4 - 0.0022x_5^2 - 0.0128x_5x_6 + 0.0012x_5x_7 + 0.0247x_5 \\ &+ 0.0007x_6^2 - 0.0008x_6x_7 + 0.0206x_6 + 0.0018x_7^2 + 0.0026x_7 - 0.0086 \end{aligned}$$