

Introduction to Financial Econometrics

Empirical Analysis – Forecasting 30 Day returns of the SP500 using the ARIMA model

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Overview

I will be investigating the SP500 Returns over time and predict a 30-day forecast using the best-fit ARIMA model. This is done with the help of econometric analysis using Python to process the data, as well as using Yahoo Finance as the source of historical data. To bypass any bias within fitting the ARIMA model I ensured that I only fit the ARIMA model to a training data and then evaluate the models' forecasted values based on the test data (Actual values).

approach of dividing the data into training and test sets to avoid model fitting bias is a sound methodology in time series forecasting

ARIMA model

A ARIMA model is a popular statistical/econometric model used for analyzing and forecasting time series data. It is composed of three components:

- AR, also known as autoregressive, which is denoted by the parameter p , predicts the future value of its data based on its past value by regressing its own prior lagged values.
- I, presenting the term Integrated, denoted as d , provides an overview of how much data is differentiated, removing trends and seasonality to achieve a stationary time series.
- MA, also known as moving average, that is denoted as q , computes the relationship between an observed value and the residual error on lagged observations.

Through the processing of data and relevant statistical analysis the best-fitted parameters ARIMA(p,d,q), are found to forecast 30 days of the log returns of the SP500.

ARIMA model is a good choice , is appropriate for the financial data you are analyzing, which tends to be non-stationary and seasonal.

Processing of Historical Data

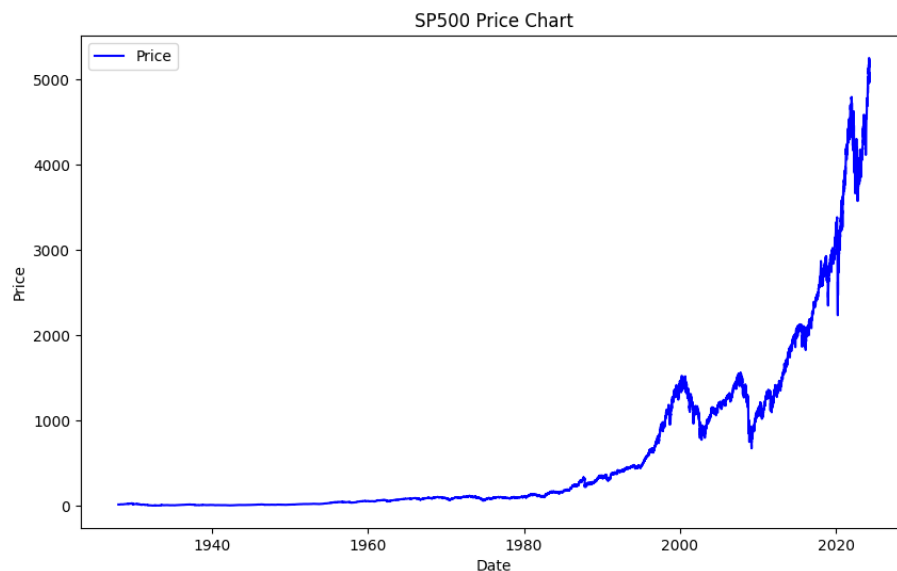


Fig 1: SP500 Historical Price Chart

At first, I took a brief overview of the historical price action of the SP500(Fig 1) and concluded that removing the data between 1930-1980 would provide a more appropriate modeling fit, to encapsulate better actual current market dynamics.

The choice to exclude data from 1930-1980 to better fit the model reflects an informed decision to focus on recent market dynamics. However, excluding a significant historical period might lead to the loss of valuable long-term trend data which could be important during unprecedented market shifts that resemble past conditions.

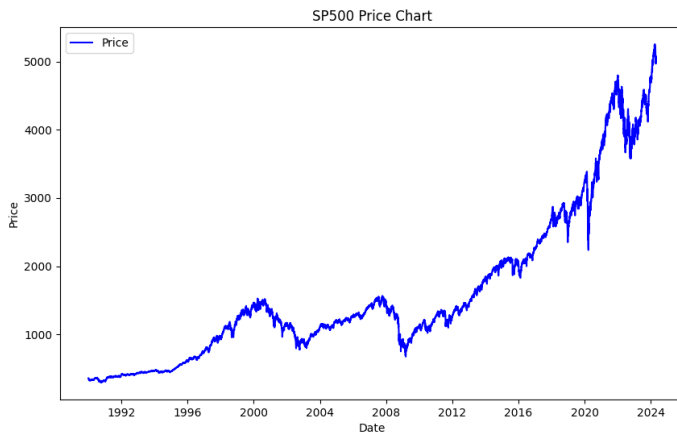


Fig 1.1: SP500 Historical Price Chart (1980-Present)

```
mean    1615.463638
std      1132.886016
min       295.459991
25%       900.940002
50%      1272.880005
75%      2068.560059
max      5254.350098
Name: Adj Close, dtype: float64
```

Fig 1.2: Basic Descriptive Statistics

Compute Logarithmic Returns

Since the aim of this empirical analysis is to forecast the returns of the SP500 I computed not only normal returns, but the logarithmic returns, since this is a common method to ensure stationarity and tends to provide a better fit to model ARIMA. Logarithmic returns mimic closer a stationary time series data. That's why usually returns especially also in this case logarithmic returns are forecasted instead of prices, since they provide more consistent statistical properties to model.

process to achieve stationarity through differencing ($d=1$) appears to be effective based on the ADF test results. However, it's crucial to ensure that the differencing level does not overly simplify the data, potentially masking some underlying patterns.

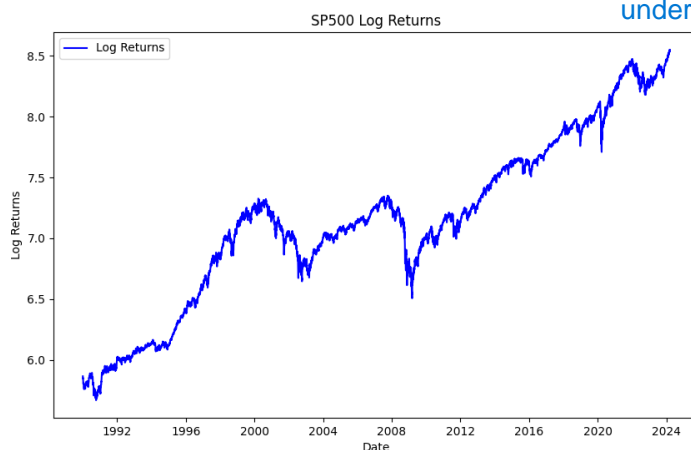


Fig 1.3: SP500 Log Returns

```
mean    7.142929
std      0.695555
min       5.669035
25%       6.793703
50%       7.142062
75%       7.628265
max       8.549995
Name: Log Returns, dtype: float64
```

Fig 1.4: Basic Descriptive Statistics

Based on the graph (Fig 1.3) the data does not exhibit a constant variance, nor constant variance or other constant statistical properties, therefore implying not stationary. Further investigation on seasonal decomposition as well as stationarity testing will help to find a good fit for the model.

Seasonal Decomposition of SP500

To investigate further the dynamics of the data I seasonally decomposed the data to Trend, Seasonality and Residuals. This is done to identify any seasonality within the data, as well as see if time-series data is stationary or not to further improve the forecasting accuracy to identify how to approach data-processing to fit the ARIMA model.

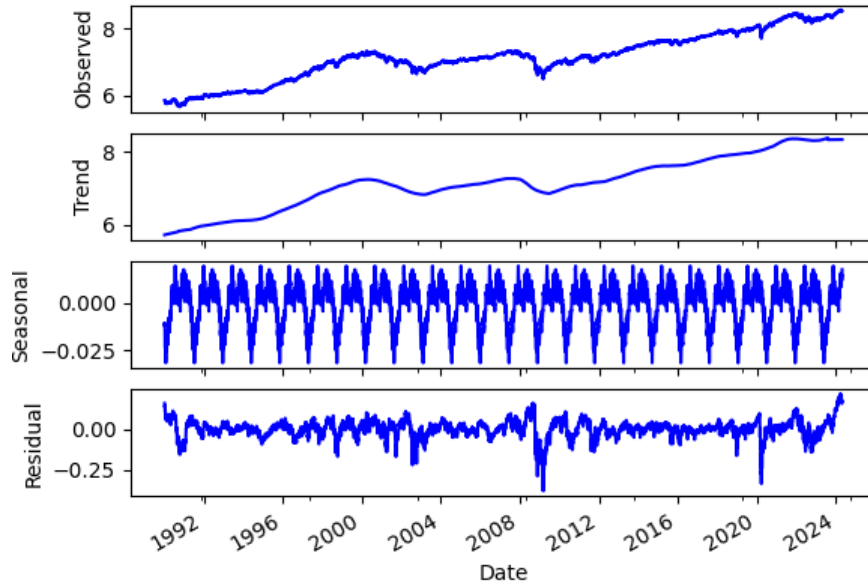


Fig 1.5: Seasonal Decomposition

Trend: Overall trend is positive and increasing with minor deviations

Seasonality: Seasonality pattern is present within the data with a full cycle approximately lasting every 2 years

Residuals: Shows some extreme values/outliers

Overall: Underlying patterns have revealed clear evidence of seasonality and residuals deviating out both to the upside and downside implying possible anomalies or outliers within the data. As well as non-stationary assumptions can be made since statistical properties, such as mean and variance change over time, based on visual interpretation.

Augmented Dickey Fuller Test (Stationarity Test)

To ensure stationarity of the data the ADF test also known as the Augmented Dickey Fuller Test is used which tests for the following hypothesis:

H0: Data = Non-stationary/Unit Root

H1: Data \neq non-stationary/unit root

The p-value is used in this case to interpret, whether to reject/accept the null hypothesis. The p-value reports the smallest significance level that leads to a rejection. In this case the critical region of 0.05 is used, implying that if the ADF test of the test-stationary data results in a p-value lower than 0.05, the null hypothesis is rejected, meaning that the data is stationary. The Integrated component (d) is then found based on when the null hypothesis is rejected.

I first computed the ADF test on the SP500 logarithmic returns without any order of differencing:

```
1. ADF : -0.6095522210071702
2. P-Value : 0.868808974930416
3. Num Of Lags : 34
4. Num Of Observations Used For ADF Regression and Critical Values Calculation
: 8607
5. Critical Values :
    1% : -3.4311099920233863
    5% : -2.8618758652800826
    10% : -2.566948776154556
```

Fig 1.6: ADF Test on SP500 Log Returns ($d=0$)

$p\text{-value} = 0.8688 > 0.05$

Since the p-value is above (Fig 1.6) the rejecting value of 0.05 I cannot reject the null hypothesis meaning that the data is non-stationary, and I need change the order of differencing to get a stationary time series data. After computing the logarithmic returns of the SP500 with first order of differencing ($d=1$) I got the following results:

```
1. ADF : -17.031821567461638
2. P-Value : 8.314721454703682e-30
3. Num Of Lags : 33
4. Num Of Observations Used For ADF Regression and Critical Values Calculation
: 8607
5. Critical Values :
    1% : -3.4311099920233863
    5% : -2.8618758652800826
    10% : -2.566948776154556
```

Fig 1.7: ADF Test on SP500 Log Returns ($d=1$)

$p\text{-value} = 8.3147e-30 < 0.05$

Since the p-value (Fig 1.7) is below the rejection we can reject the null hypothesis and therefore conclude that the data is stationary to fit an ARIMA model with first order of differencing ($d=1$). This would imply an ARIMA model with $d = 1$, still having to find the parameters for p and q .

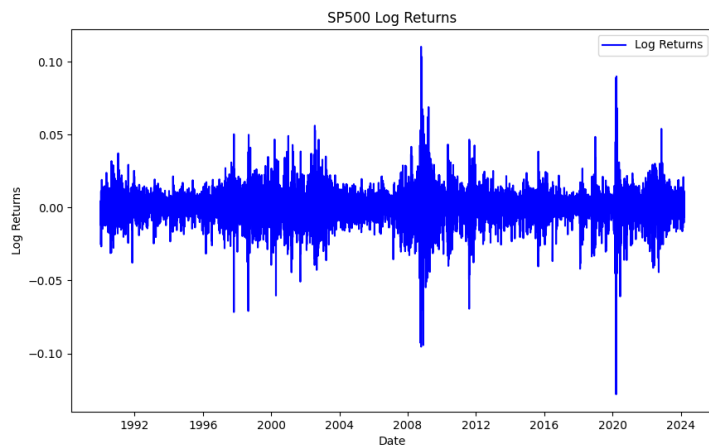


Fig 1.8: SP500 Log Returns after First Order of Differencing ($d=1$)

```
mean      0.000312
std       0.011513
min       -0.128083
25%       -0.004526
50%       0.000573
75%       0.005734
max       0.110376
Name: Log Returns, dtype: float64
```

Fig 1.9: Basic Descriptive Statistics

Fig 1.8 further helps in confirming a stationary time series, since log returns seem to act like white noise with relative constant statistical properties with less outliers than the log returns without any order of differencing.

ACF and PACF Plots

To investigate further the time-series data the ACF Plot showing autocorrelation of the series with its lags as well as PACF plot that shows partial correlation of the series with its lags are used to find any autocorrelation within the time-series data as well as any more seasonality or trend in data and interpret possibly how the MA component, looking at the ACF plot, and AR component, looking at the PACF plot, can be interpreted.

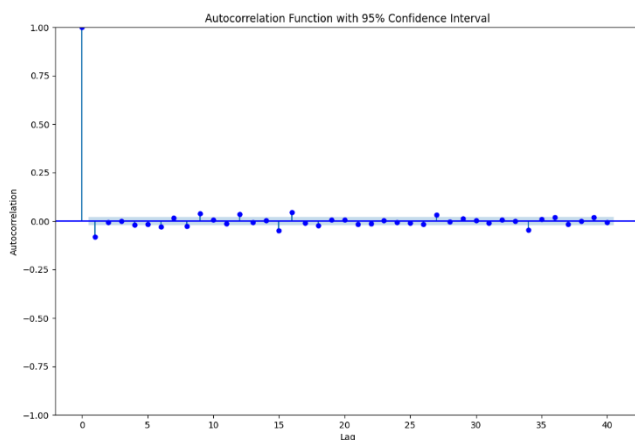


Fig 2: ACF Plot

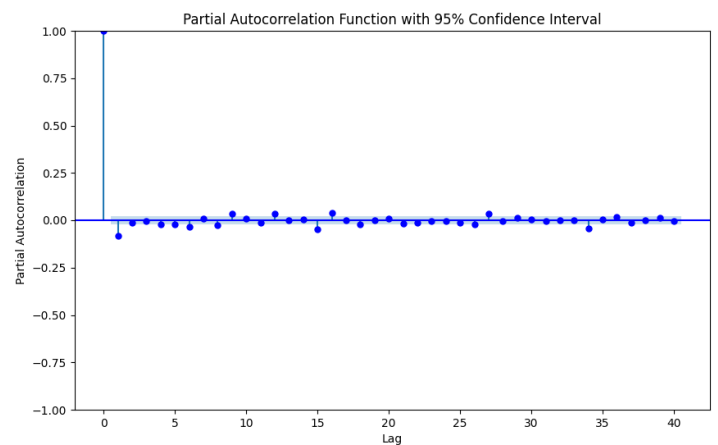


Fig 2.1: PACF Plot

From both the ACF and PACF plot I was able to conclude that there are minor deviations above the confidence interval of 95% after lag 0 implying a stationary time series with minimal seasonality.

Since it is usually a more challenging approach to identify exact AR and MA orders based on visual interpretation of the ACF and PACF since there is no clear pattern within the correlation coefficients I will be using Python that provides automation methods for model selection and parameter estimation, ensuring greater accuracy in estimating an ARIMA model for forecasting.

Parameter Estimation

Now since the data is processed to fit an ARIMA model I used a grid parameter estimation method to find the best parameters ranging for each parameter from 0-3. I fitted the training data to models with different parameters and determined their measure of goodness of fit based on the AIC and BIC. The AIC, also known as Akaike Information Criterion, which is a statistical measure that evaluates the quality of a given set of data and its goodness of fit. BIC, also known as Bayesian Information Criterion, does the same evaluations, like the AIC, but penalizes more complex models reducing any overfitting. The lower the AIC and BIC is, the better the fit of the model.

```
Parameters: (1, 1, 1) AIC: -52498.428083179 BIC : -52477.24534760355
Parameters: (1, 1, 2) AIC: -52499.094028239306 BIC : -52470.85038080537
Parameters: (1, 2, 1) AIC: -52483.38192815294 BIC : -52462.19954094885
Parameters: (2, 1, 1) AIC: -52496.3275963396 BIC : -52468.08394890567
Parameters: (2, 2, 1) AIC: -52483.09138238907 BIC : -52454.84819945029
Parameters: (1, 2, 2) AIC: -52426.422936838106 BIC : -52398.17975389932
Parameters: (2, 1, 2) AIC: -52498.65905302033 BIC : -52463.35449372791
Parameters: (2, 2, 2) AIC: -52472.52087009975 BIC : -52437.216891426266
Parameters: (1, 1, 3) AIC: -52494.602132528984 BIC : -52459.29757323656
Parameters: (1, 3, 1) AIC: -48667.795221265595 BIC : -48646.61318247333
Parameters: (3, 1, 1) AIC: -52494.303190916726 BIC : -52458.9986316243
Parameters: (1, 2, 3) AIC: -52483.37837053227 BIC : -52448.07439185879
Parameters: (2, 1, 3) AIC: -52492.22115972352 BIC : -52449.855688572614
Parameters: (2, 2, 3) AIC: -52469.24933104872 BIC : -52426.88455664054
Parameters: (1, 3, 1) AIC: -48667.795221265595 BIC : -48646.61318247333
Parameters: (1, 3, 2) AIC: -52391.783775342 BIC : -52363.541056952316
Parameters: (2, 3, 1) AIC: -49827.01046193038 BIC : -49798.767743540695
Parameters: (3, 1, 2) AIC: -52492.21667936981 BIC : -52449.851208218905
Parameters: (3, 2, 1) AIC: -52459.98698850088 BIC : -52424.6830098274
Parameters: (3, 2, 2) AIC: -52447.072515771004 BIC : -52404.70774136283
Parameters: (1, 3, 3) AIC: -51932.03652483568 BIC : -51896.73312684857
Parameters: (3, 3, 1) AIC: -50323.995829398 BIC : -50288.69243141089
Parameters: (3, 1, 3) AIC: -52490.21561120897 BIC : -52440.78922819958
Parameters: (2, 3, 3) AIC: -52434.86440795286 BIC : -52392.500330368326
Parameters: (3, 2, 3) AIC: -52456.858718622505 BIC : -52407.433148479635
Parameters: (3, 3, 2) AIC: -49778.00047842631 BIC : -49735.63640084177
Parameters: (3, 3, 3) AIC: -49019.83194447708 BIC : -48970.40718729513
```

Fig 2.2: Grid Parameter Estimation test for ARIMA (0-3, 0-3, 0-3)

Out of the following computation, I was able to conclude that the best parameter estimated based on the AIC and BIC and the range of 0-3 on each parameter is: ARIMA (1,1,1) AIC: -52498 BIC: -52477

The use of grid parameter estimation and subsequent selection of ARIMA(1,1,1) and ARIMA(0,1,5) based on AIC and BIC scores demonstrates a rigorous approach to model selection. However, the justification for the final model choice (ARIMA 0,1,5) over the initial (1,1,1) could be strengthened by discussing the impact of these parameters on model dynamics in more detail.

SARIMAX Results						
=====						
Dep. Variable:	Log Returns	No. Observations:	8613			
Model:	ARIMA(1, 1, 1)	Log Likelihood	26252.214			
Date:	Wed, 24 Apr 2024	AIC	-52498.428			
Time:	17:06:58	BIC	-52477.245			
Sample:	0	HQIC	-52491.204			
	- 8613					
Covariance Type:	opg					
=====						
	coef	std err	z	P> z	[0.025	0.975]

ar.L1	0.0843	0.052	1.606	0.108	-0.019	0.187
ma.L1	-0.1654	0.052	-3.162	0.002	-0.268	-0.063
sigma2	0.0001	8.37e-07	157.346	0.000	0.000	0.000

Fig 2.3: Model Summary ARIMA (1,1,1)

Log Likelihood: Shows how well the parameters explain observed data. A high value usually indicates a good fit for the model, which seems to be the case with these parameters.

AIC and BIC: From the parameter estimation this model had the lowest AIC and BIC indicating from the parameters used the best trade-off between goodness of fit and simplicity of model.

HQIC: Also, like AIC and BIC evaluates the balance trade-off between model fit and complexity. The lower the value, the better the fit.

Covariance type: opg (Outer Product of Gradients) type is used to estimate the variance-covariance matrix of the parameter estimates in a way that is robust to heteroskedasticity. This means that, like computing robust standard errors for linear regression, parameter estimates are obtained that are robust to violating any assumptions needed to model ARIMA like homoskedasticity.

Finding the Best-fit Parameters

By computing the auto_arma function using a library in python I found the best fitting ARIMA model for the training data, which resulted in the ARIMA model with parameters (0, 1, 5). To ensure that this is a better fit than the first proposed parameters I provided a statistical summary of the model on the data and compared it to the previous proposed model.

SARIMAX RESULTS

Dep. Variable:Log ReturnsNo. Observations:8612

Model:ARIMA(0, 1, 5)Log Likelihood26253.140

Date:Tue, 23 Apr 2024AIC-52494.280

Time:23:30:34BIC-52451.915

Sample:0HQIC-52479.833

- 8612

Covariance Type:opg

	coef	std err	z	P> z	[0.025	0.975]
ma.L1	-0.0810	0.006	-14.667	0.000	-0.092	-0.070
ma.L2	-0.0087	0.004	-1.962	0.050	-0.017	-7.76e-06
ma.L3	-0.0032	0.006	-0.567	0.571	-0.014	0.008
ma.L4	-0.0202	0.005	-3.890	0.000	-0.030	-0.010
ma.L5	-0.0181	0.005	-3.465	0.001	-0.028	-0.008
sigma2	0.0001	8.56e-07	153.692	0.000	0.000	0.000

Fig 2.4 Model Summary ARIMA (0,1,5)

Log Likelihood: With these parameters the log-likelihood value is even higher than the first proposed model fit concluding a better fit.

AIC and BIC: AIC and BIC values are also lower meaning that these parameters are a better fit.

HQIC: HQIC Value also lower in comparison to the first proposed model.

Overall: Proposes a better fit, if not the most-suited parameters to use for ARIMA modeling this training data to forecast.

Residual Analysis on ARIMA (0,1,5)

Before forecasting using the ARIMA model to further strengthen and acknowledge a good-fit model residuals must be analyzed to ensure that the underlying patterns and dynamics of the data are still encapsulated and if there are any possible model improvements to be made. Analyzing residuals ensures validity and reliability and accuracy of the ARIMA model by detecting any misspecifications.

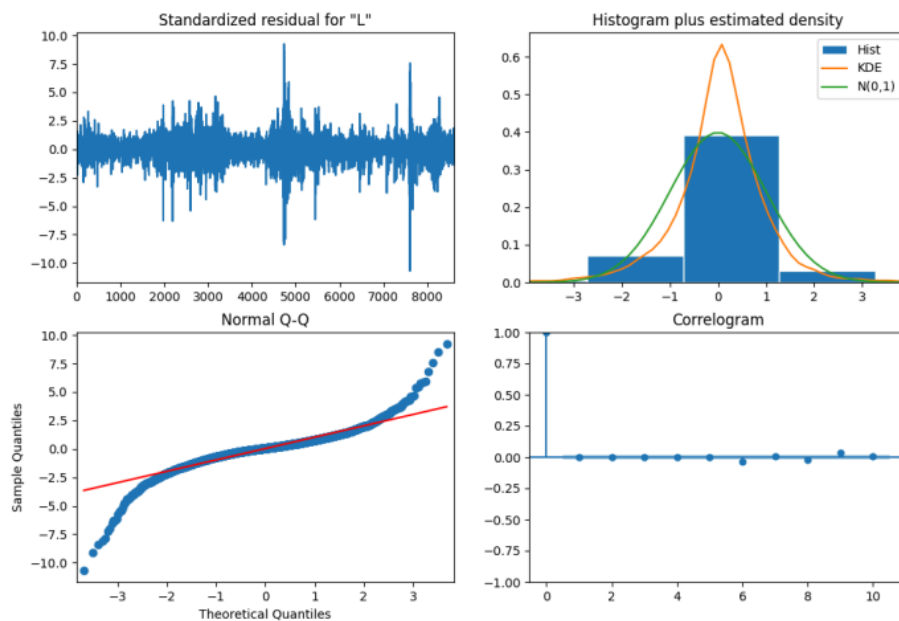


Fig 2.5: Residual Diagnostics

Standardized Residuals: Act very similar to white noise showing randomness and no serial correlation.

Histogram: Looks very similar to a normal distribution

Normal Q-Q: Follows the straight line, however, deviates on both ends confirming heavier tails within the distribution (probably Kurtosis value greater than 3)

Correlogram: Minor deviations above confidence intervals confirming stationarity.

Overall: Residuals follow relatively a normal distribution and act like white noise, however, there are still some extreme values within the standardized residuals as well as fatter tails on the normal Q-Q plot. Further tests are needed to see if there are any improvements to be made, such as investigating any autocorrelation within residuals through statistical tests since these would accrue bias towards the ARIMA model.

The analysis of residuals is thorough, indicating no significant autocorrelation and confirming the model's appropriateness. The presence of heavier tails in the Q-Q plot and the outcomes of the Ljung-Box test suggest that the model captures the data's randomness effectively, though further investigation into potential improvements could be beneficial, especially in handling extreme values or anomalies.

Ljung-Box Test (Autocorrelation Test)

The Ljung-Box test with 20 lags to test is used to further acknowledge a good-fit based on residual analysis. The Ljung-Box test essentially tests if data is independently distributed or serially correlated. This is done by testing the following hypothesis:

H0: Residuals = Independently Distributed

H1: Residuals \neq Independently Distributed

	lb_stat	lb_pvalue
1	0.021497	0.883433
2	0.049856	0.975380
3	0.052597	0.996842
4	0.090288	0.999011
5	0.106315	0.999811
6	0.109439	0.999974
7	0.240672	0.999953
8	0.297678	0.999982
9	0.363220	0.999992
10	0.380559	0.999998
11	0.380803	1.000000
12	0.399104	1.000000
13	0.576521	1.000000
14	0.576891	1.000000
15	0.619841	1.000000
16	0.627704	1.000000
17	0.632330	1.000000
18	0.646003	1.000000
19	0.656297	1.000000
20	0.738970	1.000000

Fig 2.6: Ljung-Box Test summary (20 lags, lb_stat = Ljung-Box test value, lb_pvalue = p-value)

Using the critical region of 0.05 we can conclude that the residuals are independently distributed. Most of the lags' p-value is closer to 1 and above 0.05, suggesting that residuals act accordingly to white noise, since we fail to reject the null hypothesis.

Model Evaluation

Statistical Evaluation

To evaluate the model, I computed the Mean Absolute Error (MAE) and the Root Mean Squared Error (RMSE) to see any significance within errors of the model. The Mean Absolute Error evaluates the average absolute differences between the forecasted values and the actual values in this case the forecasted log returns to the actual log returns. The Root Mean Squared Error is also looked at, which is the square root of the squared differences between the forecasted values and actual values decreasing the significance of any large errors/outliers. A base model is also used to compare MAE and RMSE values. In this case, the first proposed ARIMA (1,1,1) model is used and compared to the new ARIMA (0,1,5) model.

MAE: 0.013038768442033976
RMSE: 0.016170888538900093

Fig 2.7: Base Model: ARIMA (1,1,1)

MAE: 0.012440100102111865
RMSE: 0.015804848351589417

Fig 2.8: ARIMA (0,1,5)

Comparing the Base model with the ARIMA (0,1,5) model, we can conclude that ARIMA (0,1,5) provides a more accurate and valid forecasts with a lower MAE and RMSE.

The comparative analysis of the MAE and RMSE for different models is insightful. The visual representation in your forecasts provides a clear comparison between actual returns and predicted values, underscoring the model's accuracy. However, evaluating the model's performance against other types of models (like ARIMA-GARCH) or including out-of-sample forecasts could provide a more robust validation of your model's

Visual Evaluation

I forecasted 30 days of logarithmic returns with 95% confidence intervals to provide a confidence range of values in which the population parameter lies, using the ARIMA (0,1,5) model and compared them to the actual log returns:

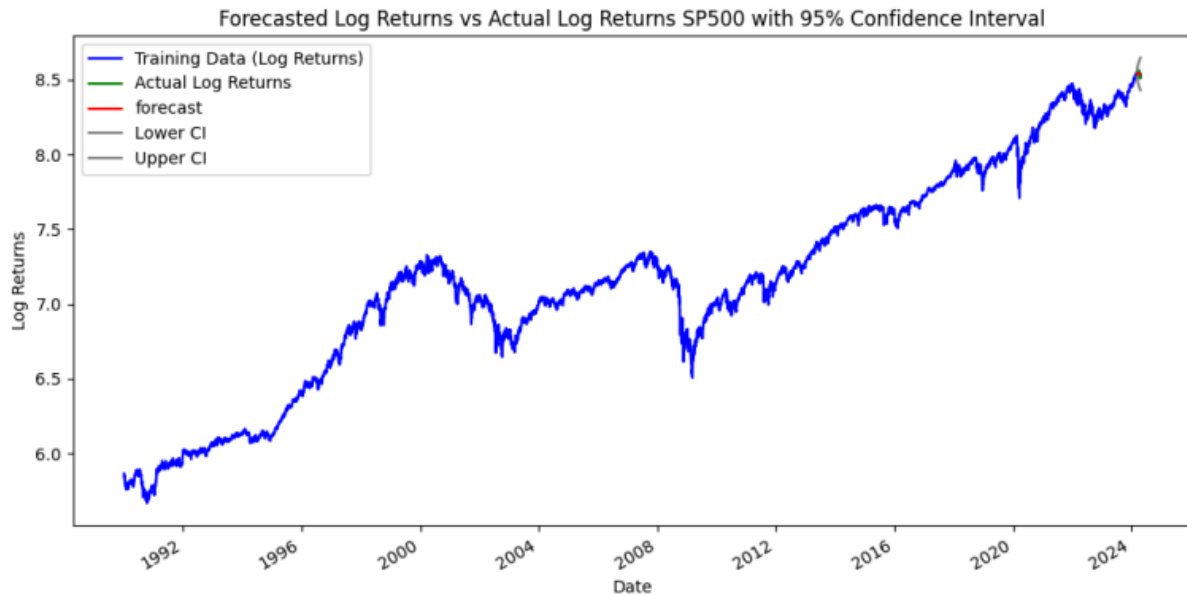


Fig 2.9: Forecast

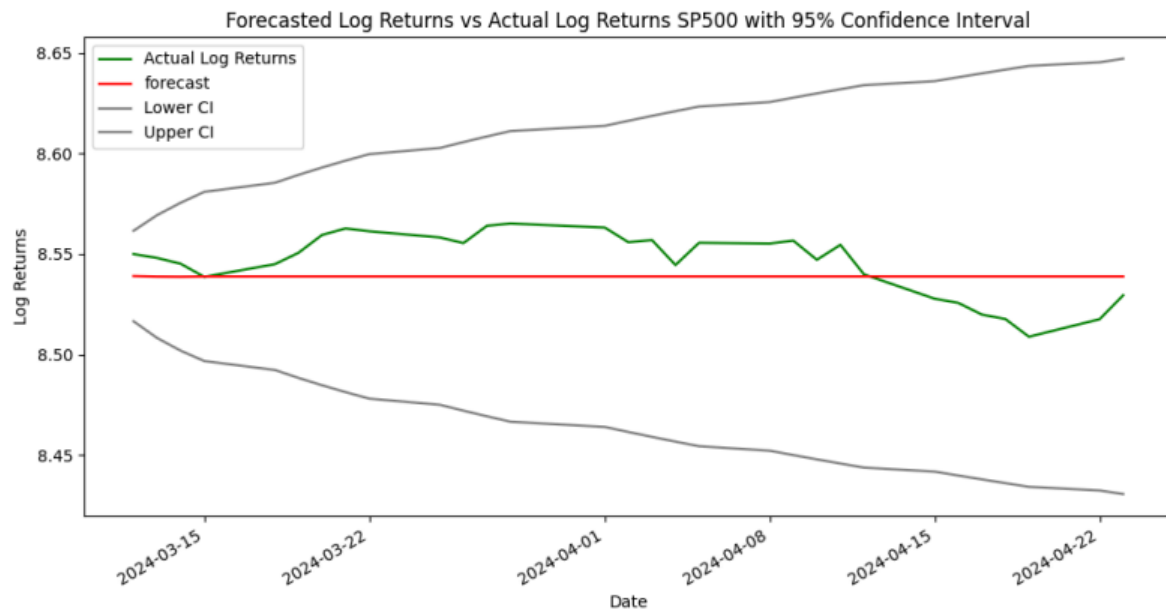


Fig 3: Zoomed in Forecast

The forecasted log returns (Fig 2.9 and Fig 3) followed a relatively similar direction to the Actual Log Returns, while the actual log returns had minor deviations above or below the forecasted line they followed the line very accurately and never deviated above or below the confidence intervals meaning that the model did indeed fit very well within a 30-day forecast.

Conclusion

The most-suited fit ARIMA model for the returns of the SP500 was ARIMA (0,1,5). With extensive analysis and investigation, I was able to conclude the best-fit model and provide relatively accurate results. Nonetheless there are significant challenges posed especially in modeling and forecasting financial markets, due to their nonlinearity or fat tails that can come from extreme events like any black swan events making it very difficult to incorporate that within the model and provide a perfect forecast, possibly more proposed models can be investigated at or even a hybrid model that could incorporate more of the volatility within financial markets, such as a ARIMA-GARCH hybrid model.

empirical analysis provides a solid foundation in financial econometrics with a cle

Appendix

<https://github.com/JVNI/ARIMA/blob/main/EmpiricalAnalysis/main.py>

```
import pandas as pd
import numpy as np
import yfinance as yf
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import adfuller
from statsmodels.tsa.seasonal import seasonal_decompose
from statsmodels.tsa.arima.model import ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.stats.diagnostic import acorr_ljungbox
from sklearn.metrics import mean_absolute_error, mean_squared_error
import statsmodels.api as sm
from statsmodels.tsa.holtwinters import ExponentialSmoothing
import warnings
import arch

#Download and Process Historical Data to Log Returns
sp500 = yf.download('^GSPC', start="1990-01-01", end="2024-04-25")
sp500['Returns'] = (sp500['Adj Close'] - sp500['Adj Close'].shift(1)) /
sp500['Adj Close']
print(sp500['Returns'])
sp500['Log Returns'] = np.log(sp500['Adj Close'] - np.log(sp500['Adj
Close'].shift(1)))
sp500['Log Returns'].dropna(inplace=True)
sp500.dropna(inplace=True)

train_data, test_data = sp500['Log Returns'][:8613], sp500['Log
Returns'][8613:]
train_data.dropna(inplace=True)
test_data.dropna(inplace=True)

# Plotting Price Function
def plot_price(ticker):
    plt.figure(figsize=(10,6))
    plt.plot(ticker.index, ticker['Adj Close'], label='Price', color='blue')
    plt.ylabel('Price')
    plt.xlabel('Date')
    plt.title('SP500 Price Chart')
    plt.legend()
    plt.show()
plot_price(sp500)

#Plotting Logarithmic Returns
```

```

def plot_logs(ticker):
    plt.figure(figsize=(10,6))
    plt.plot(ticker.index, ticker['Log Returns'], label='Log Returns',
color='blue')
    plt.ylabel('Log Returns')
    plt.xlabel('Date')
    plt.title('SP500 Log Returns')
    plt.legend()
    plt.show()

#Seasonal Decomposition of Data
def decompose_data(ticker):
    decompose_data = seasonal_decompose(ticker['Log Returns'].dropna(), model
= 'additive', period=365, extrapolate_trend='freq')
    daily_frequency = ticker.asfreq(freq='D')
    fig, axes = plt.subplots(4, 1, sharex=True)

    decompose_data.observed.plot(ax=axes[0], legend=False, color="blue")
    axes[0].set_ylabel('Observed')
    decompose_data.trend.plot(ax=axes[1], legend=False, color="blue")
    axes[1].set_ylabel('Trend')
    decompose_data.seasonal.plot(ax=axes[2], legend=False, color="blue")
    axes[2].set_ylabel('Seasonal')
    decompose_data.resid.plot(ax=axes[3], legend=False, color="blue")
    axes[3].set_ylabel('Residual')
    plt.show()

# ADF Test and Test Results
def stationarity_test(ticker):
    ticker['Log Returns'] = ticker['Log Returns'].diff().diff()
    dfctest = adfuller(ticker['Log Returns'].dropna(), autolag = 'AIC')
    print("1. ADF : ", dfctest[0])
    print("2. P-Value : ", dfctest[1])
    print("3. Num Of Lags : ", dfctest[2])
    print("4. Num Of Observations Used For ADF Regression and Critical Values
Calculation :", dfctest[3])
    print("5. Critical Values :")
    for key, val in dfctest[4].items():
        print("\t",key, ": ", val)

#Plotting ACF
def acf_plot(ticker):
    fig, ax = plt.subplots(figsize=(12,8))
    sm.graphics.tsa.plot_acf(ticker['Log Returns'].diff().dropna(), lags=40,
alpha=0.05, ax=ax, color='blue')
    ax.set_title('Autocorrelation Function with 95% Confidence Interval')
    ax.set_xlabel('Lag')
    ax.set_ylabel('Autocorrelation')

```

```

plt.show()

#Plotting PACF
def pacf_plot(ticker):
    fig, ax = plt.subplots(figsize=(10, 6))
    sm.graphics.tsa.plot_pacf(ticker['Log Returns'].diff().dropna(), lags=40,
alpha=0.05, ax=ax, color='blue')
    ax.set_title('Partial Autocorrelation Function with 95% Confidence
Interval')
    ax.set_xlabel('Lag')
    ax.set_ylabel('Partial Autocorrelation')
    plt.show()

#ARIMA Modeling and Plotting forecast
def arima_model(ticker, train, test, params):
    model = ARIMA(train, order=params)
    model_fit = model.fit()
    ticker['forecast log returns'] = model_fit.predict(start=8613, end=8643)
    forecast = model_fit.get_forecast(steps=30)
    ci = forecast.conf_int(alpha=0.05)
    plt.figure(figsize=(10, 6))
    plt.plot(ticker.index, ticker['Log Returns'], label='Log Returns')
    plt.plot(test.index, test, label='Actual Log Returns')
    plt.plot(ci.iloc[:, 0], label='Lower CI', color='gray')
    plt.plot(ci.iloc[:, 1], label='Upper CI', color='gray')
    plt.plot(ticker['forecast log returns'], label='Forecasted Log Returns')
    plt.fill_between(ci.index, ci.iloc[:, 0], ci.iloc[:, 1], color='gray',
alpha=0.25)
    plt.title('ARIMA Forecasted Log Returns vs Actual Log Returns with 95%
Confidence Interval')
    plt.legend()
    plt.xlabel('Date')
    plt.ylabel('Log Returns')
    plt.show()

#
def model_arima(ticker, train, test, params):
    model = ARIMA(train, order=params)
    model_fit = model.fit()
    forecast_series = model_fit.forecast(30, alpha=0.05)
    forecast = model_fit.get_forecast(30)
    ci = forecast.conf_int(alpha=0.05)

    plt.figure(figsize=(12,6))
    train.plot(color='blue', label='Training Data (Log Returns)')
    test.plot(color='green', label='Actual Log Returns')
    plt.plot(test.index, forecast_series, label='forecast', color='red')
    plt.plot(test.index, ci.iloc[:, 0], label='Lower CI', color='gray')

```



```

plt.plot(test.index, ci.iloc[:, 1], label='Upper CI', color='gray')
plt.legend()
plt.xlabel('Date')
plt.ylabel('Log Returns')
plt.title('Forecasted Log Returns vs Actual Log Returns SP500 with 95%
Confidence Interval')
plt.show()

# Close-up of Just Forecasts and Actual Log Returns
def model_close_arma(ticker, train, test, params):
    model = ARIMA(train, order=params)
    model_fit = model.fit()
    forecast = model_fit.get_forecast(30)
    ci = forecast.conf_int(alpha=0.05)
    forecast_series = model_fit.forecast(30, alpha=0.05)
    plt.figure(figsize=(12,6))
    test.plot(color='green', label='Actual Log Returns')
    plt.plot(test.index, forecast_series, label='forecast', color='red')
    plt.plot(test.index, ci.iloc[:, 0], label='Lower CI', color='gray')
    plt.plot(test.index, ci.iloc[:, 1], label='Upper CI', color='gray')
    plt.legend()
    plt.xlabel('Date')
    plt.ylabel('Log Returns')
    plt.title('Forecasted Log Returns vs Actual Log Returns SP500 with 95%
Confidence Interval')
    plt.show()

# Residual Analysis with Ljung-Box Test and Test Results
def arima_diagnostics(ticker, train, test, params):
    arima_model = ARIMA(train, order=params)
    arima_model_fit = arima_model.fit()
    arima_model_fit.plot_diagnostics(figsize=(12, 8))
    fig, axes = plt.subplots(4, 1, sharex=True)

    plt.show()
    residuals = arima_model_fit.resid
    Btest = acorr_ljungbox(residuals, lags=20, return_df=True)

    print(Btest)

# MAE and RMSE Test
def mae_rmse(train, test, params):
    model = ARIMA(train, order=params)
    model_fit = model.fit()
    forecast = model_fit.get_forecast(30)
    rmse = mean_squared_error(test, forecast.predicted_mean, squared=False)

```

```
mae = mean_absolute_error(test, forecast.predicted_mean)
print(f'MAE: {mae}\nRMSE: {rmse}')
```