Numbers
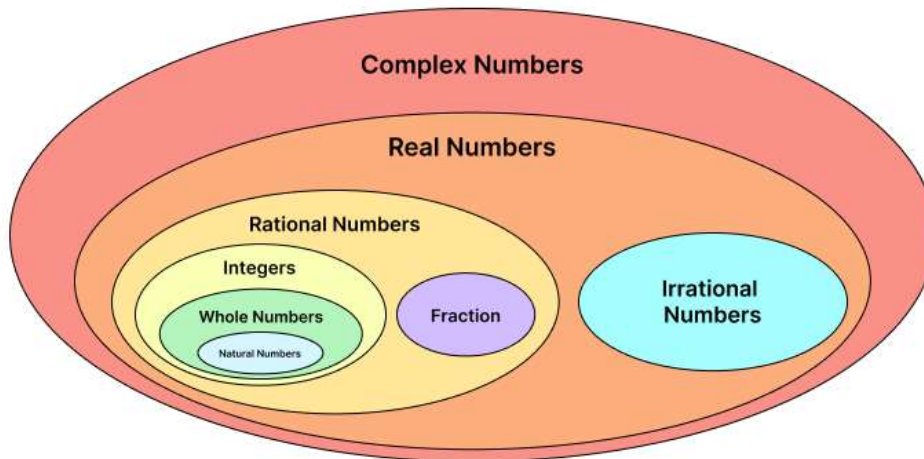
Classification of Numbers

Complex number = a + ib Value of i = √-1 Rational numbers form = p/q, q is not equal to 0 Integer range:- -infinity to + infinity Whole number range :- 0 to + infinity Real numbers range:- 1 to + infinity

Divisibility Rules 2 :- The last number should be divisible by 2 4 :- The last 2 numbers should be divisible by 4 8 :- The last 3 numbers should be divisible by 8
3 :- The sum of the numbers should be divisible by 3 9 :- The sum of the numbers should be divisible by 9
5 :- The last number should be 0 or 5 10 :- The last number should be 0
6 :- Any number which is divisible by 2 & 3 is divisible by 6 as well 12 :- Any number which is divisible by 3 & 4 is divisible by 12 as well
11:- The difference between the sum of odd-placed numbers and even-placed numbers should be equal to 0 or 11 multiple. 7:- The last number should be doubled and subtracted from the remaining numbers. If the resultant is divisible by 7. Then the whole number is divisible by 7

Power cycles 2:- {2,4,8,6} 3:- {3,9,7,1} 4:- {4,6} 5:- {5} 6:- {6} 7:- {7,9,3,1} 8:- {8,4,2,6} 9:- {9,1}

Remainder cycles
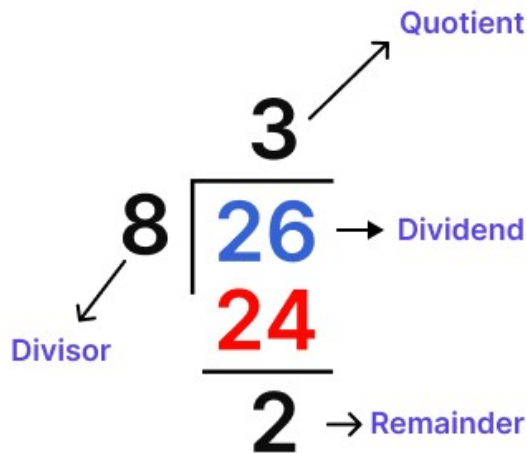
For example: The remainder cycle of $6^n / 7 = \{6,1\}$

LCM & HCF

Factors Let prime factorization $N = a^p \times b^q \times c^r$ where, a,b,c are prime numbers. Number of factors for $N = (p + 1) \times (q +1) \times (r +1)$ Sum of factors $= (a^{p+1}-1)/(a-1) \times (b^{q+1}-1)/(b-1) \times (c^{r+1}-1)/(c-1)$ Product of factors of $N = N^{(\text{Total No. of Factors})/2}$

- P is said to be a factor of Q if P divides Q without leaving the remainder.
- Factors of a number are always less than or equal to that number
- HCF of two numbers is the highest factor which is common to both numbers.

Multiples

- P is said to be multiple of Q if Q divides P without leaving remainder.
- Multiples of a number are always greater than or equal to that number.
- LCM of two numbers is the smallest multiple which is common to both the numbers

Properties Product of two Numbers = LCM x HCF If the ratio of numbers is a:b, and the HCF of numbers is H then, LCM of the numbers = H x a x b = HCF x Product of ratios
LCM of fractions =(LCM of Numerators)/(HCF of denominators) HCF of fractions =(HCF of Numerators)/(LCM of denominators)

Percentages

Basic Calculations a percentage of b = (a/b)*100

Decimal to Fraction Conversion

1/2=50%	1/3=33.33%	1/4=25%	1/5=20%	1/6=16.66%	1/8=12.5%	1/9=11.11%
	2/3=66.66%	2/4=50%	2/5=40%	2/6=33.33%	2/8=25%	2/9=22.22%
		3/4=75%	3/5=60%	3/6=50%	3/8=37.5%	3/9=33.33%
			4/5=80%	4/6=66.66%	4/8=50%	4/9=44.44%
				5/6=83.33%	5/8=62.5%	5/9=55.55%
					6/8=75%	6/9=66.66%
					7/8=87.5%	7/9=77.77%
						8/9=88.88%

1/7 = 14.2857% 2/7 = 28.5714% 3/7 = 42.8571% 4/7 = 57.1428% 5/7 = 71.4285% 6/7 = 85.7142%

Percentage Increase/Decrease

$$\text{Percentage Increase} = \left[\frac{(\text{New number} - \text{Original number})}{\text{Original number}} \right] * 100$$

$$\text{Percentage Decrease} = \left[\frac{(\text{Original number} - \text{New number})}{\text{Original number}} \right] * 100$$

Successive Increase/Decrease Net % Change or Overall Percentage Change = x + y + (xy/100) Here, x and y are the percentage increase or decrease. Use (+)ve sign for increase and (-)ve sign for decrease.

Profit and Loss

Terminology and Relations Selling price = Cost Price +profit Selling price = Cost price - Loss
Profit Percent(P%) =[(SP - CP)/CP]*100 Loss Percent(L%) = [(CP - SP)/CP)]*100
CP - price at which seller buys SP - price at which seller sells
Profit - amount gain by selling above CP Loss - amount loss by selling below CP

Discount Discount = Marked Price(MP) - Selling Price(SP) Discount Percent(D%) = (Discount/Marked Price) x 100 = [(MP - SP)/MP] x 100 Effective Discount on successive discount% = [- X - Y + (XY))/100]
Where, X = First discount Y = Second discount -ve sign represents reduction in price.

- Marked Price - Price on the product at which it is supposed to be sell.

- Discount - Amount by which marked price is reduced

- Discount percentage - Percentage by which the price of an item is reduced

Discount types:

- Up to - May vary from 0 to given amount

- Flat - Exact amount is given

- Successive - Discounts are given in successive order

Simple Interest & Compound Interest

Definitions Compound Interest(CI) is the interest that is calculated both on the principal and the previously earned interest. Simple Interest (SI) is the interest that is calculated only on the principal. For the same principle, Rate, and Time period CI > SI

Formulas Amount(A) = Principle(P) + Simple/Compund Interest(SI or CI)
SI = (P x R x T)/100 Where, P → Principle R → Rate of Interest T → Time (in years)
To calculate the Compound Interest(CI) we have to calculate the total amount.

$$A = P\left(1 + \frac{r}{100}\right)^{nt}$$

CI = A-P Where, A → Amount P → Principle t → Time period r → Rate of Interest n → number of times interest applied per time period

Ratios and Proportions

Ratios Ratio is a comparison of two or more quantities of the same unit. It is unitless quantity means it has no unit.
ak:bk = a:b a/k:b/k = a:b if a:b = c:d then b:a = d:c a:b = c:d ⇒ ad = bc

Proportions If we have the ratio a:b and it's equal to the ratio c:d, a:b=c:d, then we can say that a:b is proportional to c:d If two ratios a:b and c:d are equal then they are represented as, a:b :: c:d Where, a and d → extreme term b and c → mean term
For two ratios in proportion. a/b = c/d, can be written as a/c = b/d. i.e a x d = b x c

Mixtures and Allegations

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{CP of Dearer - Mean Price}}{\text{Mean Price - CP of Dearer}}$$

Cost Price of Cheaper [C] Cost Price of Dearer [D]

Cost Price of Mixture [M]
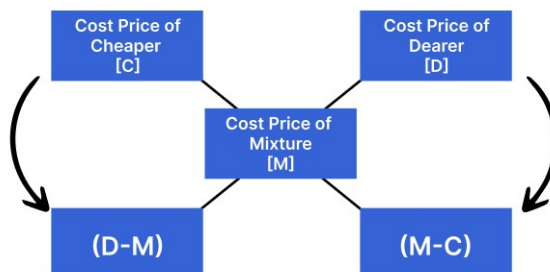
(D-M)   (M-C)

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{(D-M)}{(M-C)}$$

When we are mixing or combining two or more different substances then the resultant solution is known as mixture

Allegation helps us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price

Partnerships

Profit = investment × time Tip: Always try to find ratio of profits of all the partners and distribute total profit among them in the same ratio.

Ages

Age is the amount of time that has passed since someone or something was born or created. It's usually measured in years.

- If someone is 'a' years old now, in 'n' years they will be (a + n) years old.

- If we go back 'n' years, they were (a - n) years old.

- If we compare two people's ages as a ratio, like p:q, then one person's age is 'pa' and the other's is 'qa', where 'a' will be a constant.

- If someone is 'a' years old now, after 'n' times that age, they will be a*n years old.

- If someone is 'a' years old now, then one part out of 'n' parts of their age is a/n.

Averages

Average = Sum of Observation(Sn) / Number of Observation(N) = Sn/N Addition or subtraction of constant k to each item in the list will result in increase or decrease of average by k. Multiplication or division of all number by a constant k which result the average to be multiplied or divided by k respectively.
If the given list of numbers are in AP then average will be: (middle element) if number of elements are odd, (average of middle two element) if number of elements are even.

Time Speed and Distance

Basic Terminology Distance is the total length of the path traveled by an object. Time refers to the duration over which an object has been moving to cover a certain distance. Speed is a measure of how fast something is moving. It is calculated as the distance covered per unit of time Speed(S) = Distance(D)/Time(T)

Units

- Distance is measured in units like meters, kilometers, miles, etc.

- Time is a fundamental quantity measured in seconds, minutes, hours, etc.

- The standard units of speed are meters per second (m/s) and kilometers per hour km/hr.

Unit conversion 1 Km = 1000 meters 1 Hr = 3600 seconds
x km/hr = x*(5/18) m/s x m/s = x*(18/5) km/hr

Average speed Average Speed = (Total distance traveled)/(Total time taken)

- When a body travels at a speeds of x and y for equal distances, the average speed is (2xy)/(x+y)

- When the body travels at speeds of x and y for equal time durations, the average speed is (x+y)/2.

Relative Speed

- Relative speed is the speed of one object or body in relation to another object.

- If two objects are travelling opposite to each other then the relative speed is sum of individual speeds.

- If two objects are travelling in the same direction then the relative speed is difference between individual speeds.

Effective speed

- If two objects are traveling opposite to each other then the effective speed is the difference between individual speeds.

- If two objects are traveling in the same direction then the effective speed is the sum of individual speeds.

Boats and Streams Upstream: Against the direction of the stream Downstream: Same direction of the stream Upstream speed = Speed of the boat - Speed of steam Downstream speed = Speed of the boat + Speed of steam

Trains

- The distance traveled by two trains traveling in the same direction or in different directions is equal to the sum of their respective lengths.

- When a train crosses a stationary object, the distance covered by the train is equal to the length of the train.

- The total distance covered by a train when it passes a platform or a bridge is equal to the sum of the train's length and the length of the bridge or platform.

Races

- A beats B implies that A has successfully overcome or performed better than B in the given task.

- In a race, A and B both reach the finish line simultaneously, which is known as DEADHEAT

- HEADSTART is an advantage or lead granted to one individual or team over another

- Linear track is a straight path with one starting point and one ending point used for races.

- Circular track is a rounded path designed for racing where participants continuously follow the circular route.

Time and Work

Pipes and Cisterns If pipe(A) can fill a tank in x hours and pipe(B) can fill it in y hours. If they are open together to fill the tank, then time taken by them to fill the tank completely is $xy/(x+y)$

If pipe(A) can fill a tank in x hours, pipe(B) can fill in y hours and pipe(C) can empty it in z hours. If they are open together to fill the tank, hours it will take to fill the tank will be:

Number of hours taken by A = x Number of hours taken by B = y Number of hours taken by C = -z (- indicate negative work)

Total capacity of the tank = LCM (x,y,z)

Efficiency of A = LCM/(hours taken by A) Efficiency of B = LCM/(hours taken by B) Efficiency of C = LCM/(hours taken by C) Time taken by pipe A,B and C to fill the tank = LCM / (Efficiency of A + Efficiency of B+ Efficiency of C) If pipe(A) is filled a tank in x hours, then the part of tank filled in one hour= $1/x$

Man - Days Concept (M1 x D1 x H1 x E1) / W1 = (M2 x D2 x H2 x E2) / W2 Where, M = Number of workers D = Number of days H = Number of working hours in a day E = Efficiency of workers W = Units of work If a worker can complete a work in x days, he will complete $1/x$ of work in a single day. On the other hand, if a worker completes $1/x$ work in a single day, it will take x days to finish the work.

Permutations & Combinations

Fundamental Counting Principle 0! = 1 n! = 1 x 2 x 3 ...... x n

And = Multiplications Or = Addition

Linear Permutations nPr = n! / (n-r)!

Circular Permutations Here, P(n) is Circular permutation of n items. Clockwise and Anti-Clockwise Order is Different P(n) = (n-1)!

Clockwise and Anti-Clockwise Order is Identical P(n) = (n-1)!/2

Question on combinations nCr = n! / [r!*(n-r)!)] nCr = nC(n-r) nC0 = nCn = 1

Probabilty

Basic Definitions Probability = Number of favourable outcomes /Total number of outcomes With Replacement : With replacement means that each time an item is selected from a set, it is put back before the next selection, allowing it to be chosen again.

Without Replacement : Without replacement ensures that each selected item from a set is not put back, hence cannot be selected again. It means each item can be chosen once.

Note: If the question does not specify whether to consider with or without replacement, then assume without replacement

Questions on coins Total number of outcomes = 2^n n=number of coins. A coin has 2 faces : Head (H) and Tail (T)
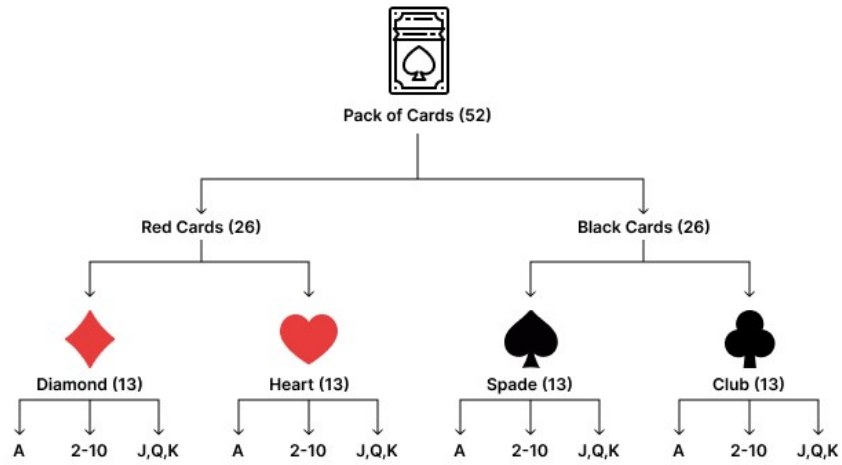
Questions on Dices A Dice has 6 faces. Total number of outcomes = 6^n n = number of dice

Die1/Die2	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Triangle representing no of favorable outcomes for the given sums respectively:
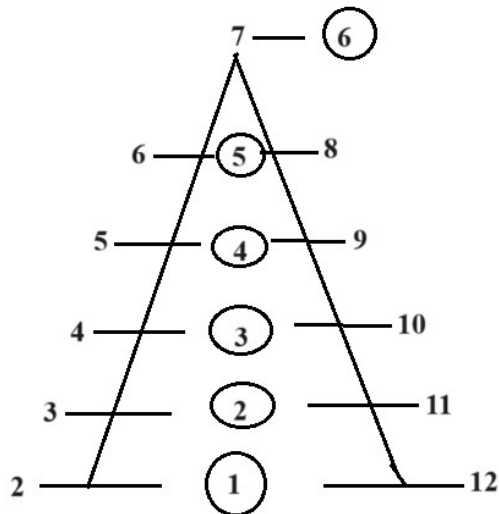
Questions on cards

Each set of 13 cards has an Ace(A) , the numbers 2 to 10 and then Jack(J), Queen(Q), and King(K). J, Q and K cards are called face cards.