1 P : The Users enter a valid password

q : Access is granted

r : The user has paid the Subscription fee

a. $P \land -q$

b. $P \land q \rightarrow r$

c. $-q \rightarrow -q$

d. $-P \lor r \rightarrow q$

2 a. one-to-one but not onto

$f(x) = x^2$

b. onto but not one-to-one

$f(x) = \frac{x}{2}$

c. onto and one-to-one (differ from Identity funct)

$$f(x) = \begin{cases} x-1, & x \text{ odd} \\ x+1, & x \text{ even} \end{cases}$$

d. $f(x) = \frac{x}{0}$

2 6 24 720 7 480

3 Prove $3n < n!$ $n > 6$

Test for $n = 7$

$3(7) < 7!$

$21 < 5040$ ✓

Assume $n = k$ $k > 6$

$3k < k!$

$3k < k(k-1)$ and $k > 6$

$3k < k(6-1)$

$3k < 5k$

$\therefore$ By mathematical Induction $3n < n!$ with $n > 6$ is true //

4 n: 7 women 9 men

a. $A = C(7,5) \cdot C(9,5) - C(9,5) \cdot C(7,3) \cdot C(9,2) + C(7,2) \cdot C($

$\frac{7!}{5!\,2!} \cdot \frac{9!}{5!\,4!} - \frac{9!}{4!\,5!} \cdot C(9,1)$

$21 \cdot 84 + 35 \cdot 36$

$21 \cdot 126 - 126$

$= 2,520$ //

5 $A = 2^5 = 32$

$n = 1$

$\therefore$ The probability of 4 heads appear after the first flip came up tail is $\frac{1}{32}$ //

6 a.

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix} \quad -2A = \begin{bmatrix} -2 & -6 & -16 \\ -4 & -8 & -22 \\ -2 & -4 & -10 \end{bmatrix}$$

b.

$$B - 2A = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & 3 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 6 & -16 \\ -4 & -8 & -22 \\ -2 & -4 & -10 \end{bmatrix} = \begin{bmatrix} 9 & 1 & 15 \\ 3 & 4 & 25 \\ 4 & 4 & 11 \end{bmatrix}$$

c.

$$C \cdot A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+1 & 3+8+2 & 8+22+5 \\ -2+2+2 & -6+4+4 & -16+11+10 \\ 4+6+2 & 12+12+4 & 32+33+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 13 & 35 \\ 2 & 2 & 5 \\ 12 & 28 & 75 \end{bmatrix}$$

7

$$A = \left[\begin{array}{ccc|ccc} 1 & 3 & 8 & 1 & 0 & 0 \\ 2 & 4 & 11 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_1 = R_1$
$R_2 = R_2 - 2R_1$
$R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & -2 & -5 & -2 & 1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{bmatrix}$$

$R_1 = R_1$
$R_2 = R_2 / -2$
$R_3 = R_3 + R_2 / -2$

$$\begin{bmatrix} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & 1 & 5/2 & 1 & -1/2 & 0 \\ 0 & 0 & -1/2 & 0 & -1/2 & 1 \end{bmatrix}$$

$R_1 = R_1$
$R_2 = R_2$
$R_3 = 2R_3$

$$\begin{bmatrix} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & 1 & 5/2 & 1 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

$R_1 = R_1 - 8R_3$
$R_2 = R_2 - R_3 \cdot \frac{5}{2}$
$R_3 = R_3$

$$\begin{bmatrix} 1 & 3 & 0 & 1 & -8 & 16 \\ 0 & 1 & 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

$R_1 = R_1 - 3R_2$
$R_2 = R_2$
$R_3 = R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 5 \\ 0 & 1 & -2 \end{bmatrix} //$$

8

$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$$

$$\text{Det } A = \begin{bmatrix} 4 & 11 \\ 2 & 5 \end{bmatrix} + -2\begin{bmatrix} 3 & 8 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 4 & 11 \end{bmatrix}$$

$$(20 - 22) + (-2(15 - 16)) + (33 - 32)$$

$$-2 \quad + \quad 2 \quad + \quad 1$$

$$\text{Det } A = 1 //$$

9

$$x = \left[\begin{array}{ccc|c} 4 & -3 & 1 & -7 \\ 5 & -2 & 5 & -3 \\ -6 & 2 & -3 & 10 \end{array}\right]$$

$R_1 = R_1$

$R_2 = R_2$

$R_3 = R_3 + R_2$

$$\begin{bmatrix} 4 & -3 & 1 & -7 \\ 5 & -2 & 5 & -3 \\ -1 & 0 & 2 & 7 \end{bmatrix}$$

$R_1 = R_1 + 3R_3$
$R_2 = R_2 + 5R_3$
$R_3 = 4R_3 + R_1$

$$\begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & -2 & 15 & 32 \\ 0 & -3 & 9 & 21 \end{bmatrix}$$

$R_1 = R_1$
$R_2 = R_2$
$R_3 = R_3 - R_2 \cdot \frac{3}{2}$

$$\begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & -2 & 15 & 32 \\ 0 & 0 & -13.5 & -27 \end{bmatrix}$$

$R_1 = R_1$
$R_2 = R_2 / -2$
$R_3 = R_3 / -13.5$

$$\begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & 1 & -7.5 & -16 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 = R_1 - 7R_3$
$R_2 = R_2 + R_3 \cdot 15/2$
$R_3 = R_3$

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 = R_1 + 3R_2$
$R_2 = R_2$
$R_3 = R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$

//

10

$$A = \begin{bmatrix} 4 & -3 & 1 \\ 5 & -2 & 5 \\ -6 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix}$$

$$\det A = (24 + 90 + 10) - (12 + 40 + 45)$$
$$= 124 - 97$$
$$= 27$$

$$X = \begin{bmatrix} -7 & -3 & 1 \\ -3 & -2 & 5 \\ 10 & 2 & -3 \end{bmatrix}$$

$$\det x = (-42 - 150 - 6) - (-20 - 70 - 27)$$
$$= -198 + 117$$
$$= -81$$

$$Y = \begin{bmatrix} 4 & -7 & 1 \\ 5 & -3 & 5 \\ -6 & 10 & -3 \end{bmatrix}$$

$$\det y = (36 + 210 + 50) - (18 + 200 + 10$$
$$= 296 - 323)$$
$$= -27$$

$$Z = \begin{bmatrix} 4 & -3 & -7 \\ 5 & -2 & -3 \\ -6 & 2 & 10 \end{bmatrix}$$

$$\det z = (-80 - 54 - 70) - (-84 - 24 - 150)$$
$$= -204 + 258$$
$$= 54$$

$$x = \frac{\det x}{\det A} = \frac{-81}{27} = -3$$
$$y = \frac{\det y}{\det A} = \frac{-27}{27} = -1$$
$$z = \frac{\det z}{\det A} = \frac{54}{27} = 2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$