

1 P : The Users enter a valid pass word

q : Access is granted

r : The user has paid the Subscription fee

a. $P \wedge q$

b. $P \wedge q \rightarrow r$

c. $\neg q \rightarrow \neg q$

d. $\neg P \vee r \rightarrow q$

2 a. one-to-one but not onto

$$f(x) = x^2$$

b. onto but not one-to-one

$$f(x) = \frac{x}{2}$$

c. onto and one-to-one (differ from Identity funct)

$$f(x) = \begin{cases} x-1, & x \text{ odd} \\ x+1, & x \text{ even} \end{cases}$$

d. $f(x) = \frac{x}{0}$



3 Prove $3n < n!$ $n > 6$

Test for $n=7$

$$3(7) < 7!$$

$$21 < 5040 \quad \checkmark$$

Assume $n=k$ $k > 6$

$$3k < k!$$

$$3k < (k+(k+1)) \text{ and } k > 6$$

$$3k < k(6-1)$$

$$3k < 5k$$

\therefore By mathematical induction $3n < n!$ with $n > 6$ is true //

4 $n=7$ women, 9 men

$$a. A = {}^{16}C_5 - {}^9C_5 = ({}^{16}C_5) - ({}^9C_5)$$

$$= \frac{16!}{5!11!} - \frac{9!}{5!4!} = \frac{16!}{5!11!} - \frac{9!}{5!4!}$$

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$$= 4368 - 126 = 4242$$

$$= 4242$$

$$b. B = {}^{16}C_5 - ({}^9C_5 + {}^7C_5)$$

$$= \frac{16!}{5!11!} - \left(\frac{9!}{5!4!} + \frac{7!}{5!2!} \right)$$

$$= 4368 - (126 + 21)$$

$$= 4221$$



$$5 \quad A = 2^5 = 32$$

$$n = 1$$

\therefore The probability of 4 heads appear after the first flip came up tail is $\frac{1}{32}$ //

6

a.

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix} \quad -2A = \begin{bmatrix} -2 & -6 & -16 \\ -4 & -8 & -22 \\ -2 & -4 & -10 \end{bmatrix}$$

b.

$$B - 2A = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & 3 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -6 & -16 \\ -4 & -8 & -22 \\ -2 & -4 & -10 \end{bmatrix} = \begin{bmatrix} 9 & 1 & 15 \\ 3 & 4 & 25 \\ 4 & 4 & 11 \end{bmatrix}$$

c.

$$C \cdot A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 4 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+1 & 3+8+2 & 8+22+5 \\ -2+2+2 & -6+4+4 & -16+11+10 \\ 4+6+2 & 12+12+4 & 32+33+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 13 & 35 \\ 2 & 2 & 5 \\ 12 & 28 & 75 \end{bmatrix}$$

7 $A = \left[\begin{array}{ccc|ccc} 1 & 3 & 8 & 1 & 0 & 0 \\ 2 & 4 & 11 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{array} \right]$

$R_1 = R_1$ $\left[\begin{array}{ccc|ccc} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & -2 & -5 & -2 & 1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{array} \right]$

$R_2 = R_2 - 2R_1$

$R_3 = R_3 - R_1$

$R_1 = R_1$ $\left[\begin{array}{ccc|ccc} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & 1 & 5/2 & 1 & -1/2 & 0 \\ 0 & 0 & -1/2 & 0 & -1/2 & 1 \end{array} \right]$

$R_2 = R_2 / -2$

$R_3 = R_3 + R_2 / -2$

$R_1 = R_1$ $\left[\begin{array}{ccc|ccc} 1 & 3 & 8 & 1 & 0 & 0 \\ 0 & 1 & 5/2 & 1 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{array} \right]$

$R_2 = R_2$

$R_3 = 2R_3$

$R_1 = R_1 - 8R_3$ $\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & -8 & 16 \\ 0 & 1 & 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{array} \right]$

$R_2 = R_2 - R_3 \cdot \frac{5}{2}$

$R_3 = R_3$

$R_1 = R_1 - 3R_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 1 & -3 & 5 \\ 0 & 0 & 1 & 0 & 1 & -2 \end{array} \right]$

$R_2 = R_2$

$R_3 = R_3$



$$A^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -3 & 5 \\ 0 & 1 & -2 \end{bmatrix} //$$

$$8 \quad \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$$

$$\text{Det } A = \begin{vmatrix} 4 & 11 \\ 2 & 5 \end{vmatrix} + (-2) \begin{vmatrix} 3 & 8 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 8 \\ 4 & 11 \end{vmatrix}$$

$$(20 - 22) + (-2)(15 - 16) + (33 - 32)$$

$$-2 + 2 + 1$$

$$\text{Det } A = 1 //$$

$$9 \quad x = \begin{bmatrix} 4 & -3 & 1 & -7 \\ 5 & -2 & 5 & -3 \\ -6 & 2 & -3 & 10 \end{bmatrix}$$

$$\begin{aligned} R_1 &= R_1 & \begin{bmatrix} 4 & -3 & 1 & -7 \\ 5 & -2 & 5 & -3 \\ -6 & 2 & -3 & 10 \end{bmatrix} \\ R_2 &= R_2 \\ R_3 &= R_3 + R_2 \end{aligned}$$

$$\begin{array}{l} \square R_1 = R_1 + 3R_3 \\ \square R_2 = R_2 + 5R_3 \\ \square R_3 = 4R_3 + R_1 \end{array} \quad \begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & -2 & 15 & 32 \\ 0 & -3 & 9 & 21 \end{bmatrix}$$

$$\begin{array}{l} \square R_1 = R_1 \\ \square R_2 = R_2 \\ \square R_3 = R_3 - R_2 \cdot \frac{3}{2} \end{array} \quad \begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & -2 & 15 & 32 \\ 0 & 0 & -12.5 & -27 \end{bmatrix}$$

$$\begin{array}{l} \square R_1 = R_1 \\ \square R_2 = R_2 / -2 \\ \square R_3 = R_3 / -12.5 \end{array} \quad \begin{bmatrix} 1 & -3 & 7 & 14 \\ 0 & 1 & -7.5 & -16 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{l} \square R_1 = R_1 - 7R_3 \\ \square R_2 = R_2 + R_3 \cdot 15/2 \\ \square R_3 = R_3 \end{array} \quad \begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{l} \square R_1 = R_1 + 3R_2 \\ \square R_2 = R_2 \\ \square R_3 = R_3 \end{array} \quad \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$



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$$A = \begin{bmatrix} 4 & -3 & 1 \\ 5 & -2 & 5 \\ -6 & 2 & -3 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix}$$

$$\det A = (24 + 90 + 10) - (12 + 40 + 45)$$

$$= 124 - 97$$

$$= 27$$

$$X = \begin{bmatrix} -7 & -3 & 1 \\ -3 & -2 & 5 \\ 10 & 2 & -3 \end{bmatrix}$$

$$\det X = (-42 - 150 + 6) - (-20 - 70 - 27)$$

$$= -196 + 117$$

$$= -81$$

$$Y = \begin{bmatrix} 4 & -7 & 1 \\ 5 & -3 & 5 \\ -6 & 10 & -3 \end{bmatrix}$$

$$\det Y = (36 + 210 + 50) - (10 + 200 + 10)$$

$$= 296 - 323$$

$$= -27$$

$$Z = \begin{bmatrix} 4 & -3 & -7 \\ 5 & -2 & -3 \\ -6 & 2 & 10 \end{bmatrix}$$

$$\det Z = (-80 - 54 - 70) - (-84 - 24 - 150)$$

$$= -204 + 258$$

$$= 54$$

$$X = \frac{\det X}{\det A} = \frac{-81}{27} = -3$$

$$Y = \frac{\det Y}{\det A} = \frac{-27}{27} = -1$$

$$Z = \frac{\det Z}{\det A} = \frac{54}{27} = 2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$