Anmerkungen zu "Principles of Quantum Mechanics, Shankar" [1]

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Vektoren

Notation

Darstellung des Spalten("bra")-Vektors $< V \mid$ in der Basis $< i \mid$

$$< V \mid = \sum_i v_i < i \mid$$

Darstellung des Zeilen("ket")-Vektors $\mid W >$ in der Basis $\mid j >$

$$\mid W > = \sum_j w_j \mid j >$$

Inneres Produkt

Axiome[1]

Symmetrie

$$< V \mid W > = < W \mid V >^* \tag{1}$$

Semidefinitheit

$$\begin{aligned} < V \mid V > &> 0 \quad \textbf{if} \quad \mid V > \neq 0 \\ < V \mid V > &= 0 \quad \textbf{if} \quad \mid V > = 0 \end{aligned} \tag{2}$$

[1] * meint die komplexe Konjugation.

Linearität

$$< \ V \ | \ (a \mid \ W \ > + b \mid \ Z \ >) \ = \ a < \ V \ | \ W \ > + b < \ V \ | \ Z \ > \tag{3}$$

Definitionen

Allgemeine Basen

$$< \ V \ | \ W \ > = \sum_i \sum_j v_i^* w_j < \ i \ | \ j \ >$$

Orthogonale Basen

$$< \ V \ | \ W \ > = \sum_i \sum_j v_i^* w_j$$

Norm

$$| \ V \ |^2 \ = < \ V \ | \ V \ >$$

Orthogonale Basen

Schwarzsche Ungleichung

$$< \ V \ | \ W \ > \leq \ | \ V \ | \ | \ W \ |$$

Dreiecksungleichung

$$| \ V + W \ | \leq \ | \ V \ | + \ | \ W \ |$$

Adjungierte

Lineare Operatoren

Kommutator

$$[\Omega, \Lambda] \ = \ \Omega\Lambda - \Lambda\Omega$$

Regeln

$$[\Omega, \Theta\Lambda] \ = \ \Theta \, [\Omega, \Lambda] + [\Omega, \Theta] \, \Lambda \tag{4}$$

$$[\Theta\Omega, \Lambda] \ = \ \Theta \, [\Omega, \Lambda] + [\Theta, \Lambda] \, \Omega \tag{5}$$

Beweise

Regel 4

$$\begin{aligned}
\Omega\Theta\Lambda - \Theta\Lambda\Omega &= \Theta\left(\Omega\Lambda - \Lambda\Omega\right) + \left(\Omega\Theta - \Theta\Omega\right)\Lambda \\
&= \left(\Theta\Omega\Lambda - \Theta\Lambda\Omega\right) + \left(\Omega\Theta\Lambda - \Theta\Omega\Lambda\right) \\
&= \Omega\Theta\Lambda - \Theta\Lambda\Omega + \Theta\Omega\Lambda - \Theta\Omega\Lambda \\
&= \Omega\Theta\Lambda - \Theta\Lambda\Omega
\end{aligned}$$

Regel 5 dito

Literatur

[1] Principles of Quantum Mechanics; Shankar, R.; Springer Science+Business Media; 1980