UPPSALA
UNIVERSITET

Institutionen för informationsteknologi

Besöksadress:
hus 10, Lägerhyddsvägen 1

Postadress:
Box 337
751 05 Uppsala

Telefon:
018–471 0000 (växel)

Telefax:
018-471 20 00

Hemsida:
http:/www.it.uu.se/

Department of Information Technology

Visiting address:
hus 10, Lägerhyddsvägen 1

Postal address:
Box 337
SE-751 05 Uppsala
SWEDEN

Telephone:
+46 18–471 0000 (switch)

Telefax:
+46 18-471 20 00

Web page:
http:/www.it.uu.se/

Assignment - Monte Carlo Simulation to price European Options

Assignment

- Implement Euler's method in MATLAB to price a European call option when the underlying stock follows the dynamics

$$dS_t = rS_t dt + \sigma S_t^\gamma dW_t, \tag{1}$$

The diffusion coefficient, σS_t^γ, models the hypothesis that the volatility depends on the stock price. If $\gamma < 1$, the volatility and the price are inversely related, which has been suggested by empirical evidence. Make use of your results from the Seminar. You find help in MATLAB on the use of normally distributed random numbers by typing `help randn`. As an example you can use the following parameters: $S_0 = 14$, $K = 15$, $r = 0.1$, $\sigma = 0.25$, $T = 0.5$, $\gamma = 0.8$.

- The error in the computed solution has two parts:

  - *The sample error* which is due to the fact that you can't use an ∞ number of sample paths.

  - *The discretization error/bias* which is due to the fact that you can't use an ∞ number of time-steps.

Use your code to make experiments and measure the error in the computed solution. Try to say something about how the error depends on the number of sample paths and on the time-step. You can use the code `bsexact.m` to compute the exact value of the option for $\gamma = 1$. This code uses the fact that the exact value of the option $V(s,t)$ for this value of γ is given by

$$
\begin{array}{rcl}
V(s,t) & = & sN(d_1) - Ke^{-r(T-t)}N(d_2), \\
d_1 & = & \frac{ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \\
d_2 & = & d_1 - \sigma\sqrt{T-t}.
\end{array}
$$

Here $N(x)$ denotes the standard normal cumulative distribution function and is given by $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$.

It might also be helpful in these experiments to note that for $\gamma = 1$ you can integrate (1) exactly in time.

UPPSALA
UNIVERSITET

- Now try to reduce the error in your computed solution. Read about antithetic variates on p.243 in Hirsa (section 6.9.2) and use this method to try to reduce the sample error. Then go on reading section 6.6 and try to reduce the bias by using another discretization scheme.

- Vary γ between 0.5 and 1. What happens to the value of the option?

- You don't have to write a full report, but you should hand in your program code and the following plots:

    - The sample error as a function of the number of sample paths.

    - The discretization error as a function of the time step.

    - A plot showing what happens when using antithetic variates.

    - A plot showing what happens when using another discretization scheme.

    - A plot of V as a function of γ.

    All plots should be *commented* with respect to its behavior. How do your obtained results relate to the theory?

Your results should be handed in through the student portal no later than 10/9.

During 10-12/9 there will be a questionnaire open in the student portal that you all have to answer individually.

Good luck!