# 0 Group Theory

## 0.1 Basic Axioms and Examples

**Definition.**

1. A *binary operation* $\star$ on a set $G$ is a function $\star\colon G \times G \to G$. For any $a, b \in G$ we shall write $a \star b$ for $\star(a, b)$.

2. A binary operation $\star$ on a set $G$ is associative if for all $a, b, c \in G$ we have $a \star (b \star c) = (a \star b) \star c$.

3. If $\star$ is a binary operation on a set $G$ we say elements $a$ and $b$ of $G$ commute if $a \star b = b \star a$. We say $\star$ (or $G$) is *commutative* if for all $a, b \in G$, $a \star b = b \star a$.

**Proposition 1.** If G is a group under the operation $\cdot$, then

1. The identity of G is unique

2. for each $a \in G$, $a^{-1}$ is uninuely determined

3. $(a^{-1})^{-1} = a$ for all $a \in G$

4. $(a \cdot b)^{-1} = (b^{-1}) \cdot (a^{-1})$

5. for any $a_q, a_2, \ldots, a_n \in G$ the value of $a_1 a_2 \cdots a_n$ is independent of how the expresion is bracketed

**Proposition 2.** Let G be a group and let $a, b \in G$. The equations $ax = b$ and $ya = b$ have unique solutions for $x, y \in G$. In particular, the left and right cancelation laws hold in G, i.e.,

1. if $au = av$, then $u = v$, and

2. if $ub = vb$, then $u = v$.