1 Preliminaries

1.1 Basics

Proposition 1. Let $f\colon A \to B$.

1. The map f is injective if and only if f has a left inverse.
2. The map f is surjective if and only if f has a right inverse.
3. The map f is a bijection if and only if there exist $g\colon B \to A$ such that $f \circ g$ is the identity map on B and $g \circ f$ is the identity map on A.
4. If A and B are finite sets with the same number of elements the $f\colon A \to B$ is bijective if and only if f is injective if and only if f is surjective.

Proposition 2. Let A be a nonempty set.

1. If $\sim$ defines an equivalence relation on A then the set of equivalence classes of $\sim$ form a partition of A.
2. If $\{A_i \mid i \in I\}$ is a partition of A then there is an equivalence relation on A whose equivalence classes are precisely the sets $A_i, i \in I$