0 Group Theory

0.1 Basic Axioms and Examples

Proposition 0.1. If G is a group under the operation $\cdot$, then

1. The identity of G is unique
2. for each $a \in G$, a^{-1} is uninuely determined
3. $(a^{-1})^{-1} = a$ for all $a \in G$
4. $(a \cdot b)^{-1} = (b^{-1}) \cdot (a^{-1})$
5. for any $a_q, a_2, \ldots, a_n \in G$ the value of $a_1 a_2 \cdots a_n$ is independent of how the expresion is bracketed

Proposition 0.2. Let G be a group and let $a, b \in G$. The equations $ax = b$ and $ya = b$ have unique solutions for $x, y \in G$. In particular, the left and right cancelation laws hold in G, i.e.,

1. if $au = av$, then $u = v$, and
2. if $ub = vb$, then $u = v$.