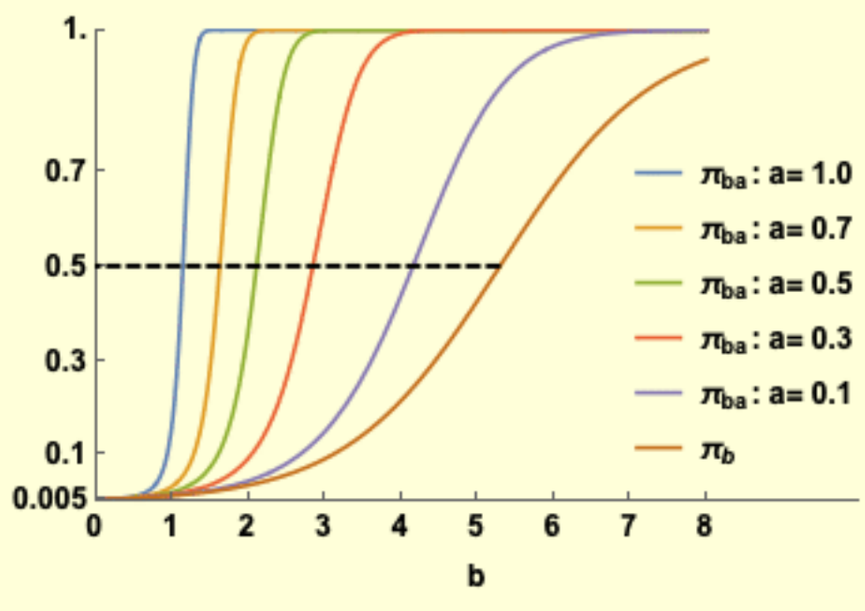
# Figures from KPAGL paper

Comparison of the posterior probabilities of a second action potential for various levels of apical input, together the posterior probability of an initiating action potential.

Fig. 2

```
post1[b_, a_]:= 1/(1+ Exp[5.2933 -0.5*b*(1 +Exp[0.5*b*a*(1+Exp[a])])])

Plot[{post1[b, 1.0], post1[b, 0.7], post1[b, 0.5], post1[b, 0.3], post1[b, 0.1], post1[b, 0]},
{b, 0, 8},  FrameTicks→{{0, 1, 2, 3, 4, 5, 6, 7, 8}, {0.005, 0.1, 0.3, 0.5, 0.7, 1.0}},
Frame→{{True, False}, {True, False}}, FrameLabel→{{"", None}, { "b", None}},
LabelStyle→{12, Bold, Black}, TicksStyle→Directive[Bold, 10], PlotRange→{{0,10},{0,1}},
Epilog→{Directive[{Thick,Dashed}],Line[{{-0.1, 0.5}, {5.3, 0.5}}]},
PlotLegends→Placed[{ "πba: a= 1.0","πba: a= 0.7", "πba: a= 0.5", "πba: a= 0.3",
"πba: a= 0.1" , "πb"}, {0.85,0.4}]]
```




# Binarised data based on Shai et al. (2015)

A plot of the binary data that is read from the file *spbin.csv*, which was created in Figures1.R. The thresholds which were found using penalised logistic regression are superimposed on the plot.

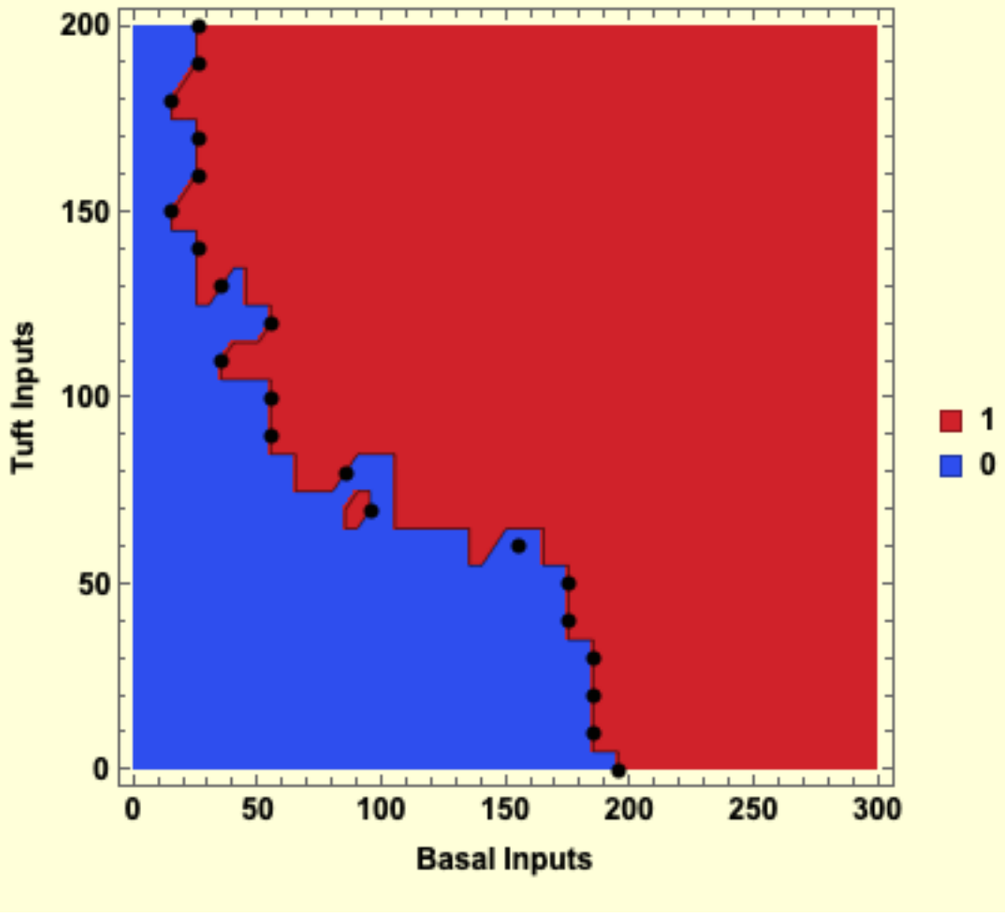
Fig. 3(a)

```
bindat = Import[ "/Users/jk/Desktop/KPAGL/spbin.csv"];
threshdat = Import[ "/Users/jk/Desktop/KPAGL/threshdat.csv"];

g1 = ListContourPlot[bindat, DataRange→{{ 0, 300}, { 0, 200}},
 ColorFunction→ ColorData["TemperatureMap"],Contours→1,
FrameLabel→{"Basal Inputs", "Tuft Inputs"}, LabelStyle→{12, Bold, Black},
PlotLegends→SwatchLegend[{ ColorData["TemperatureMap",1],
ColorData["TemperatureMap",0]}, {"1", "0"}]];

g2 = ListPlot[threshdat, FrameLabel→{"Basal Inputs", "Tuft Inputs"}, LabelStyle→{6, Black},
FrameLabel→{"Basal Inputs", "Tuft Inputs"}, PlotStyle→{Thick, Black}, AspectRatio→1];

Show[g1, g2]
```



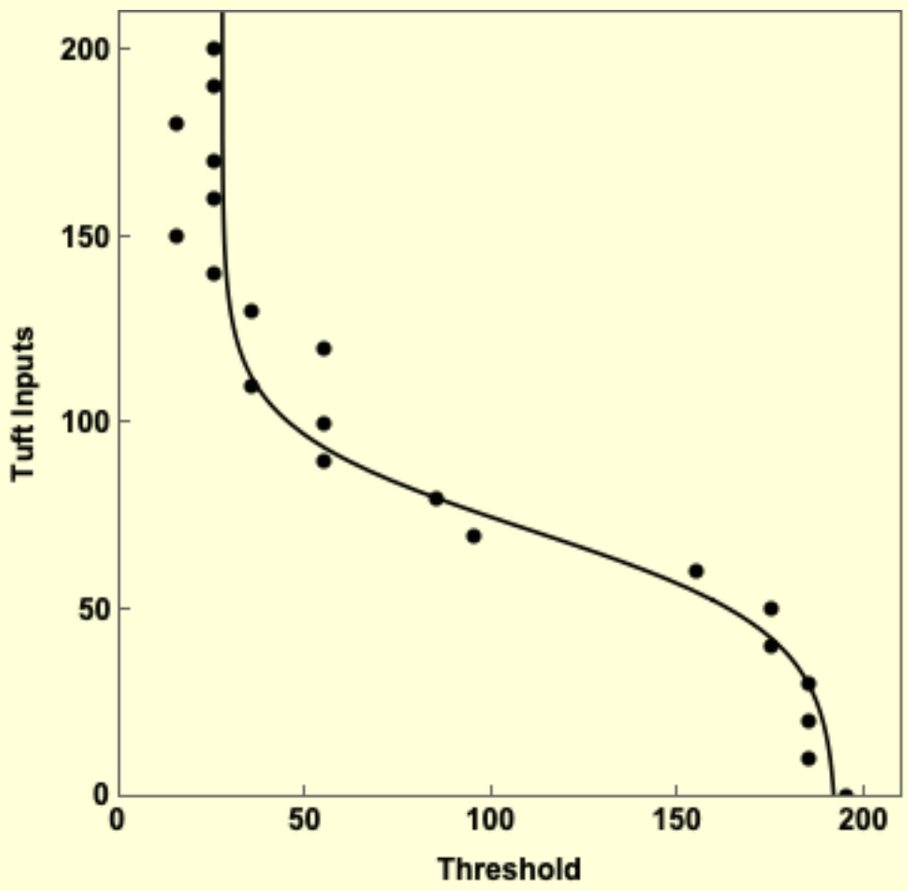

The threshold points are again plotted and the inverse of the *thresh* function is superimposed

Fig. 3(b)

```
b1 = 27.665; b2 = 164.851; b3= 71.301; b4 = -13.686;
thresh[y_]:= b1 + b2 /(1 + Exp[-(y-b3)/b4]);

g3 =Plot[InverseFunction[thresh][y], {y, 0, 300}, PlotRange→{{0, 210},{0, 210}},
FrameLabel→{ "Threshold", "Tuft Inputs"}, LabelStyle→{12, Bold, Black},
FrameTicks→{{{0, 50, 100, 150, 200}, None}, {{0, 50, 100, 150, 200}, None}},
Frame→True, PlotStyle→{Black}, AspectRatio→1];

Show[g3, g2]
```




## Threshold model for binarised data based on Shai et al. (2015)

Region, contour and surface plots of the posterior probability of a second action potential for given values of the basal and apical inputs are given below. We first define the posterior probability based on the threshold model.

The parameter estimates of b1, b2, b3, b4 are taken from the output of the fitted weighted nonlinear regression model in the R script, Figures1.R, and they are given in Eq (7) in the manuscript.

```
b1 = 27.665; b2 = 164.851; b3 = 71.301; b4 = -13.686;
thresh[y_] := b1 + b2 / (1 + Exp[-(y - b3) / b4]);
thpost[x_, y_] := 1 / (1 + Exp[-(x - thresh[y] - 5.2933)])
```

Fig. 4(a) -- removed in the revised version.

```
RegionPlot[thpost[x, y] >= 0.5, {x, 0, 300}, {y, 0, 200}, LabelStyle → {12, Bold, Black},
 BaseStyle → {Font -> "Bold", FontSize -> 12}, FrameLabel → {"Basal Inputs", "Tuft Inputs"},
 Epilog → {Arrow[{{50, 50}, {80, 83}}],
   Inset[Text[Style["Threshold Curve", Bold, 12, FontFamily -> "Arial"]], {50, 45}]}]
```

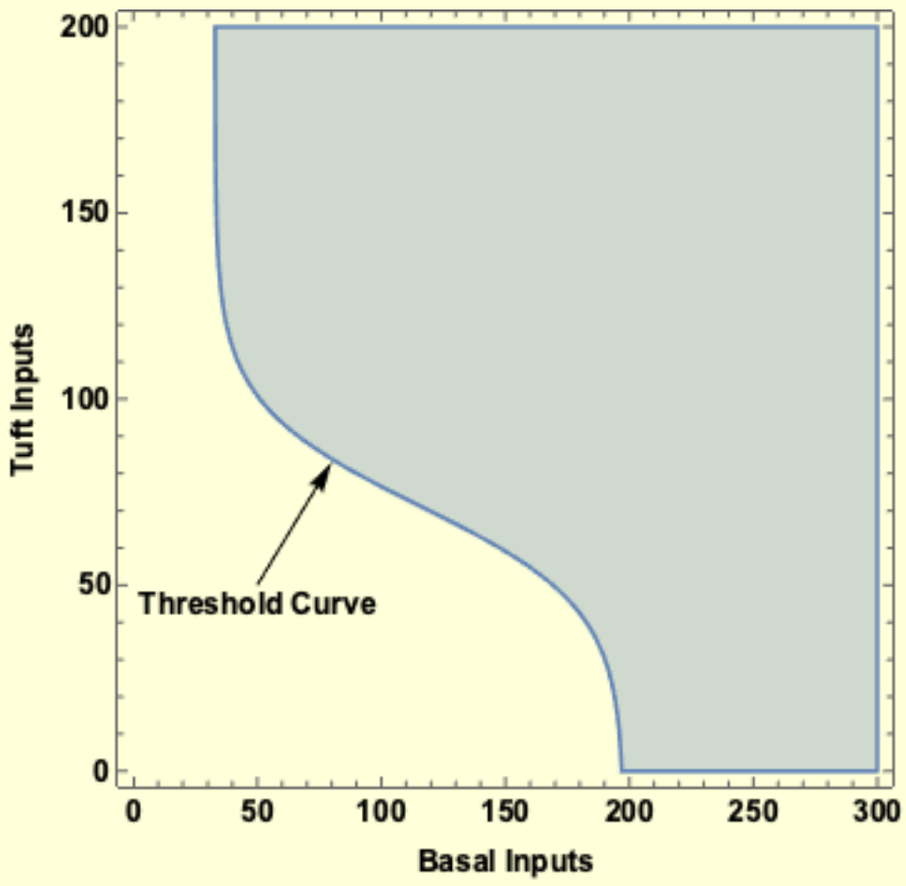



Fig. 4(c) -- Fig. 4(a) in the revised version.

```
ContourPlot[thpost[x, y], {x, 0, 300}, {y, 0, 200},
 ColorFunction → ColorData["TemperatureMap"], Contours → 20,  PlotPoints → 50,
 FrameLabel → {"Basal Inputs", "Tuft Inputs"}, LabelStyle → {12, Bold, Black},
 PlotLegends → BarLegend[{Automatic, {0, 1}}, {Automatic, 10}]]
```

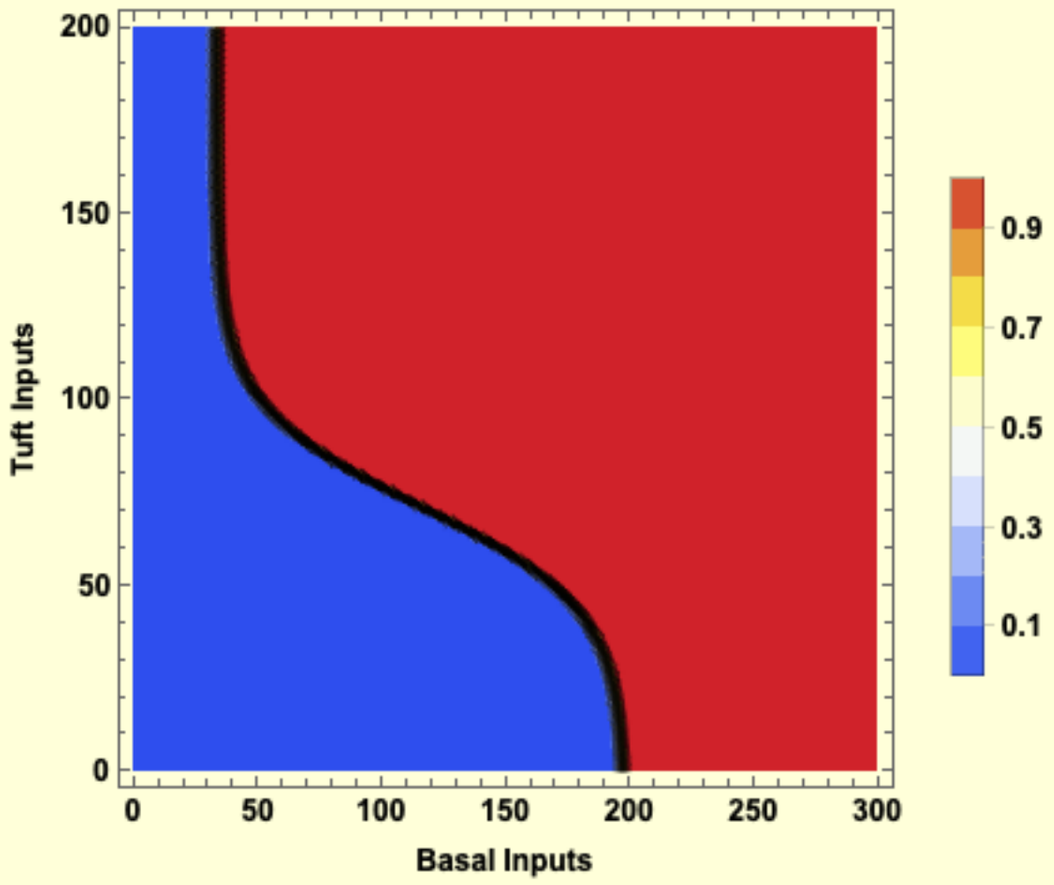
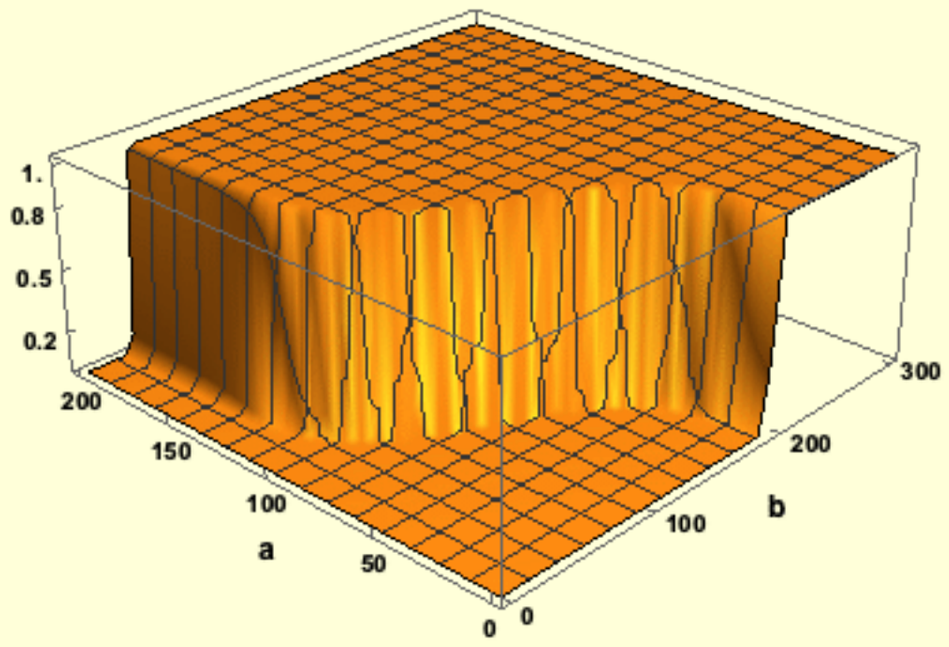



Fig. 4(e) -- Fig. 4(c) in the revised version.

```
Plot3D[thpost[x, y], {x, 0, 300}, {y, 0, 200},  AxesLabel → {"b", "a", " "}, LabelStyle → {12, Bold, Black},
 TicksStyle → Directive[Bold, 10],
 Ticks → {{0, 100, 200, 300}, {0, 50, 100, 150, 200}, {0.2, 0.5, 0.8, 1.0}}, PlotPoints → 50,
 PlotRange -> {0, 1.05}]
```




# General model for binarised data based on Shai et al. (2015)

Region, contour and surface plots of the posterior probability of a second action potential for given values of the basal and apical inputs are given below. We first define the posterior probability based on the threshold model.

The parameter estimates of a2, a3, a4 are taken from the output of the Bayesian logistic regression nonlinear model in the R script, Figures2.R, and given after Eq (11) in the manuscript.

```
a2 = 1.226*10^(-2); a3 = 3.707*10^(-5); a4 = 1.571*10^(-2); t = 10.0;
act[x_, y_]:= -5.2933 + a2*x*(1 + Exp[a3*x*y*(1 +Exp[a4*y])]);
act1[x_, y_]:= If[act[x,y] >t, t, act[x,y]];
act2[x_, y_]:= If[act1[x,y] < -t, -t, act1[x,y]];
gmpost[x_, y_]:= 1/(1 +Exp[-act2[x,y]])
```

Fig. 4(b) -- omitted in the revised version.

```
RegionPlot[gmpost[x, y] ≥ 0.5, {x, 0, 300}, {y, 0, 200},
 LabelStyle → {12, Bold, Black}, FrameLabel → {"Basal Inputs", "Tuft Inputs"},
 Epilog → {Arrow[{{50, 50}, {84, 87}}],
   Inset[Text[Style["Threshold Curve", Bold, 12, FontFamily -> "Arial"]], {50, 45}]} ]
```

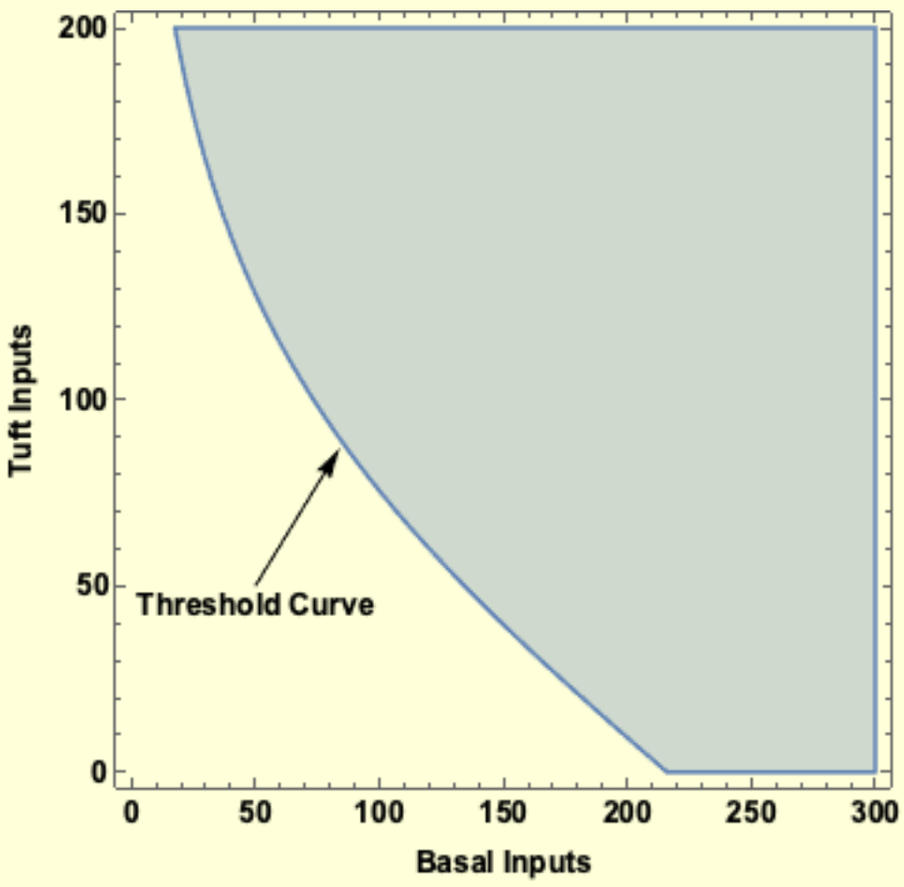



Fig. 4(d) -- Fig. 4(b) in the revised version.

```
ContourPlot[gmpost[x, y], {x, 0, 300}, {y, 0, 200},
 ColorFunction → ColorData["TemperatureMap"], Contours → 20,  PlotPoints → 50,
 FrameLabel → {"Basal Inputs", "Tuft Inputs"}, LabelStyle → {12, Bold, Black},
 PlotLegends → BarLegend[{Automatic, {0, 1}}, {Automatic, 10}]]
```

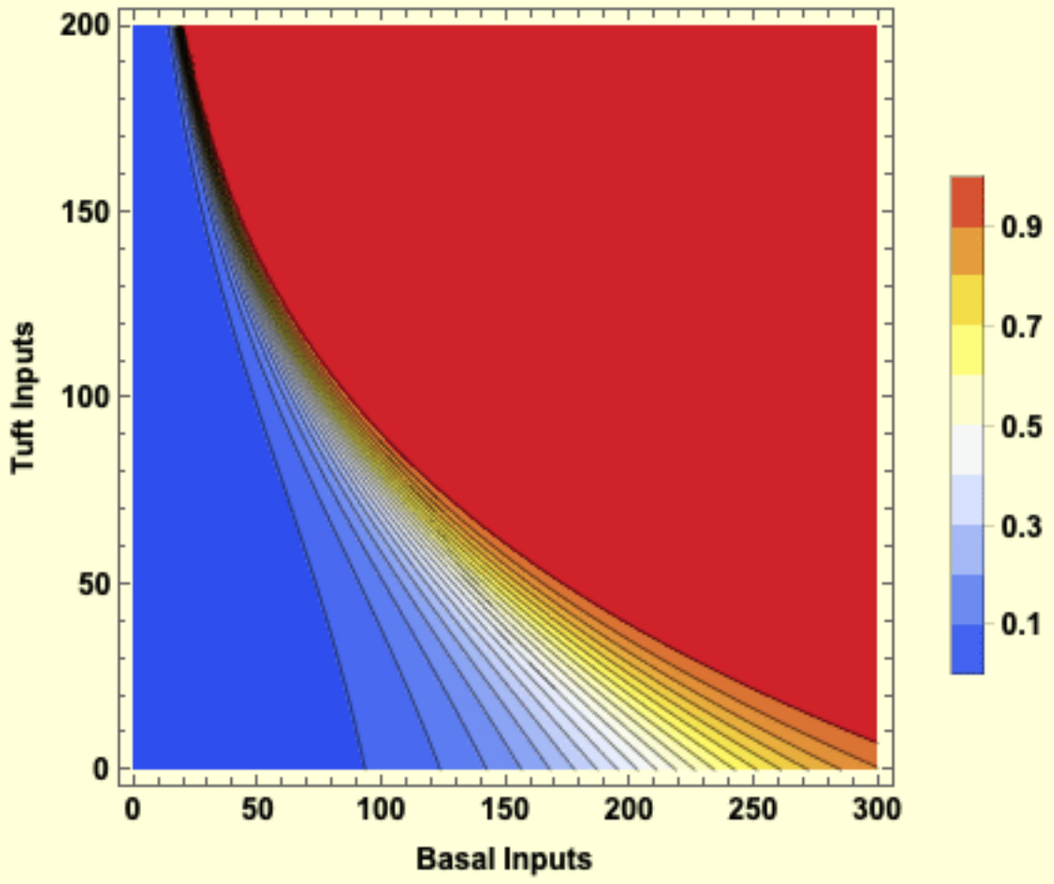
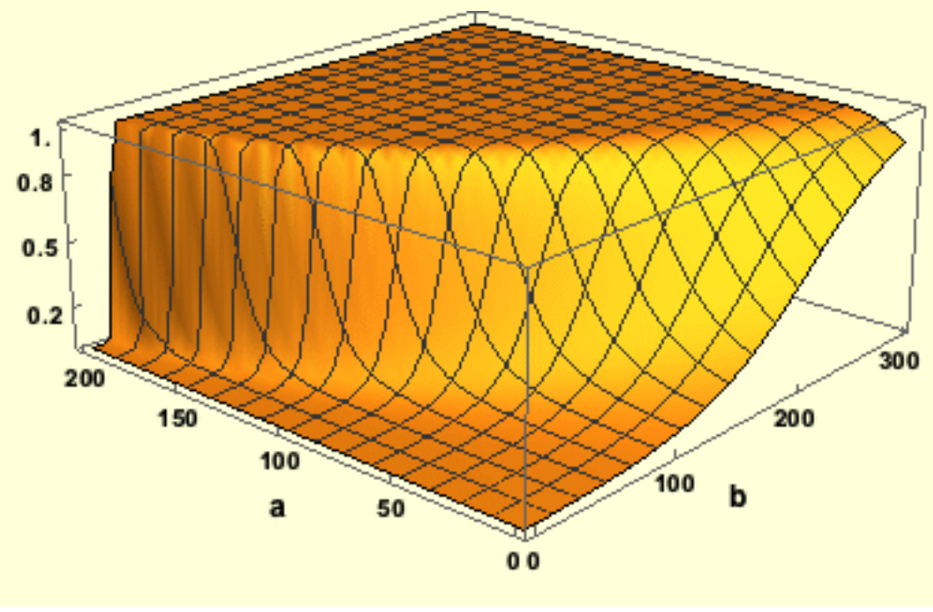



Fig. 4(f) -- Fig. 4(d) in the revised version.

```
Plot3D[gmpost[x, y], {x, 0, 300}, {y, 0, 200}, AxesLabel → {"b", "a", " "}, LabelStyle → {12, Bold, Black},
 TicksStyle → Directive[Bold, 10],
 Ticks → {{0, 100, 200, 300}, {0, 50, 100, 150, 200}, {0.2, 0.5, 0.8, 1.0}}, PlotPoints → 50,
 PlotRange -> {0, 1.05}]
```



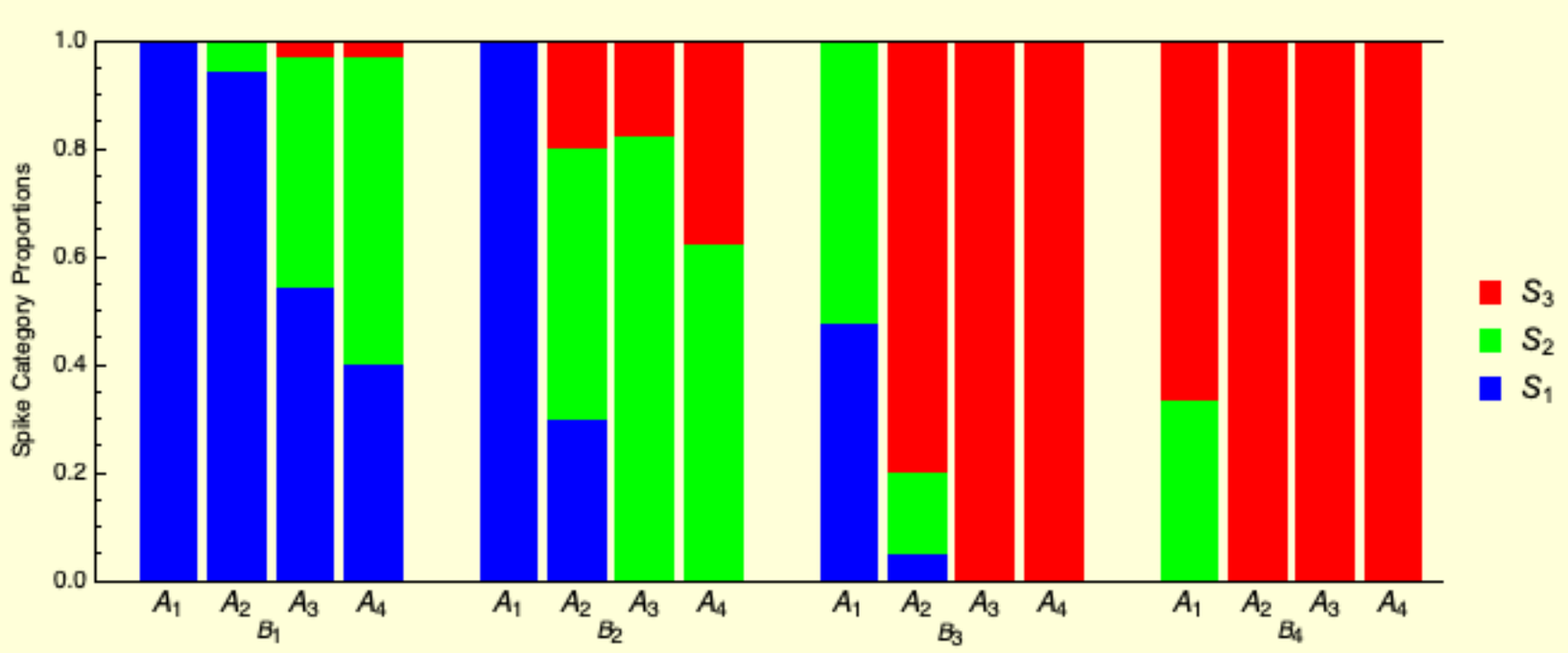

# Categorised data based on Shai et al. (2015)

Bar Chart of the categorised action potential data from Figures3.R.

Fig. 5

```
data=N[{{1,0,0},{33,2,0}/35,{19,15,1}/35,{14,20,1}/35,{0,0,0},{1,0,0},{12,20,8}/40,
{0, 33, 7}/40,{0, 25, 15}/40,{0,0,0},{23,25,0}/48,{2,6,32}/40,{0,0,1},{0,0,1},{0,0,0},
{0, 16, 32}/48,{0,0, 1},{0, 0, 1},{0, 0, 1}}];

BarChart[data,AspectRatio→0.4,BaseStyle→Directive[FontFamily→"Helvetica",10,GrayLevel[0.6]],
BarSpacing→0.2,ChartBaseStyle→Directive[EdgeForm[], Black],ChartLayout→"Stacked",
LabelStyle→{Black},ChartLabels→{Placed[Style[#,12]&/@{"A1", "A2", "A3", "A4", "
", "A1", "A2", "A3", "A4", " ","A1", "A2", "A3", "A4", " ","A1", "A2", "A3", "A4", " "},Below],
None},ChartStyle→{Blue, Green, Red},Frame→{{True,False},{True,True}},
FrameLabel→{None,Style["Spike Category Proportions",Black],None,None},
FrameStyle→Black,FrameTicksStyle→Black,GridLinesStyle→GrayLevel[0.7],
ImageSize→600,PlotRangePadding→{{0.3,0.3},{0,0}},Ticks→{None,Automatic},
ChartLegends→{"S1", "S2", "S3"}, FrameTicks->{{{2.5,"\nB1",0},{7.5,"\nB2",0},{12.5,"\nB3",0},
{17.5,"\nB4",0}},Automatic,None,None}]
```



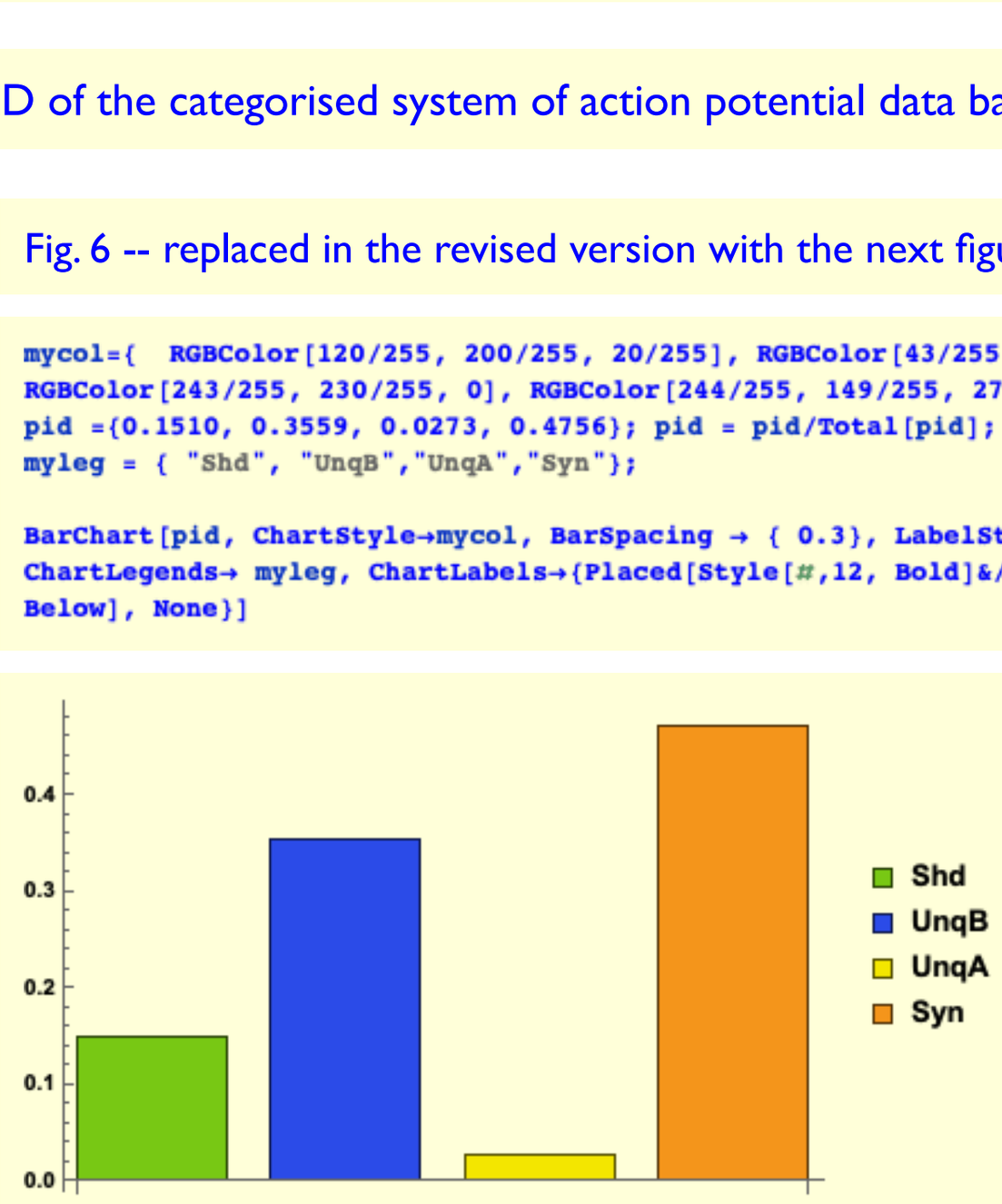

# Partial Information Decomposition

PID of the categorised system of action potential data based on Shai et al. (2015).

Fig. 6 -- replaced in the revised version with the next figure.

```
mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
pid ={0.1510, 0.3559, 0.0273, 0.4756}; pid = pid/Total[pid];
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[pid, ChartStyle→mycol, BarSpacing → { 0.3}, LabelStyle→{Black, Bold},
ChartLegends→ myleg, ChartLabels→{Placed[Style[#,12, Bold]&/@{"Shd", "UnqB", "UnqA", "Syn"},
Below], None}]
```

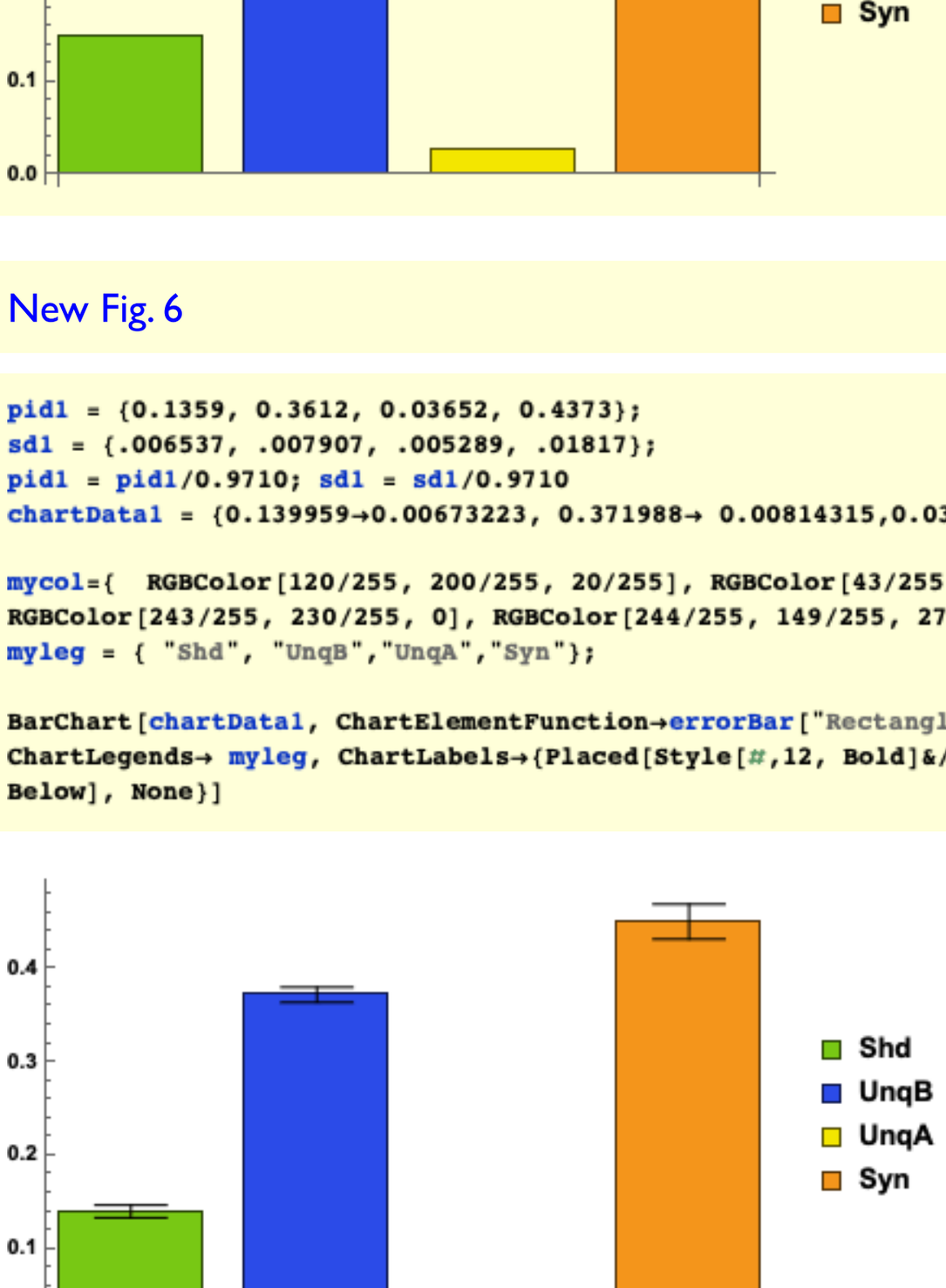



New Fig. 6

```
pid1 = {0.1359, 0.3612, 0.03652, 0.4373};
sd1 = {.006537, .007907, .005289, .01817};
pid1 = pid1/0.9710; sd1 = sd1/0.9710
chartData1 = {0.139959→0.00673223, 0.371988→ 0.00814315,0.0376107→0.00544696, 0.45036→ 0.0187127}

mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[chartData1, ChartElementFunction→errorBar["Rectangle"], ChartStyle→mycol, BarSpacing → { 0.3}, LabelStyle→{Black, Bold},
ChartLegends→ myleg, ChartLabels→{Placed[Style[#,12, Bold]&/@{"Shd", "UnqB", "UnqA", "Syn"},
Below], None}]
```

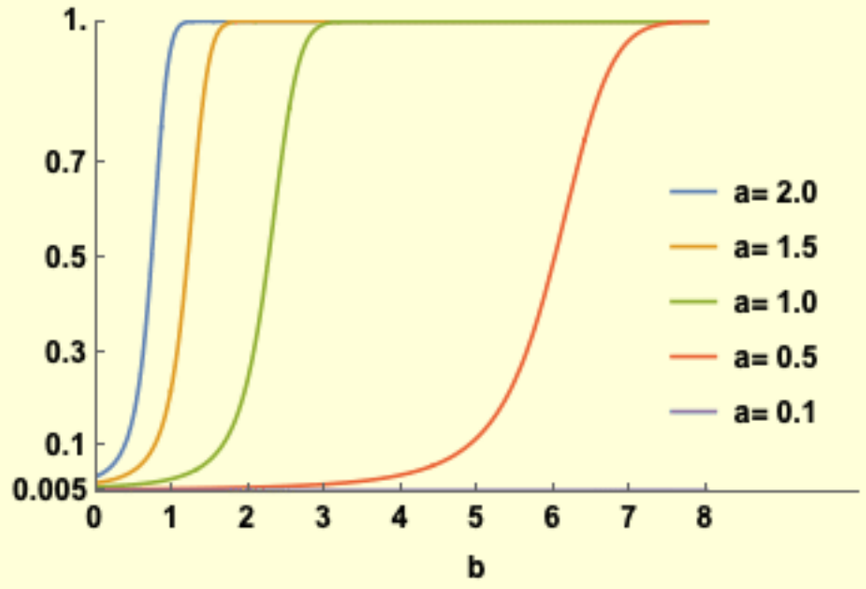


Comparison of posterior probabilities of an action potential for various levels of apical input when the apical input is driving, without an initiating backpropagated action potential having been generated.

Fig. 7(a)

```
post2[b_, a_]:= 1/(1+ Exp[5.2933 -0.5*a*(1 +Exp[b*a])])

Plot[{post2[b, 2.0], post2[b, 1.5], post2[b, 1.0], post2[b, 0.5], post2[b, 0.1]},
{b, 0, 8},  FrameTicks→{{0, 1, 2, 3, 4, 5, 6, 7, 8}, {0.005, 0.1, 0.3, 0.5, 0.7, 1.0}},
Frame→{{True, False}, {True, False}}, FrameLabel→{{"", None}, { "b", None}},
LabelStyle→{12, Bold, Black}, TicksStyle→Directive[Bold, 10], PlotRange→{{0,10},{0,1}},
PlotLegends→Placed[{ "a= 2.0","a= 1.5", "a= 1.0", "a= 0.5",
"a= 0.1" }, {0.85,0.4}]]
```

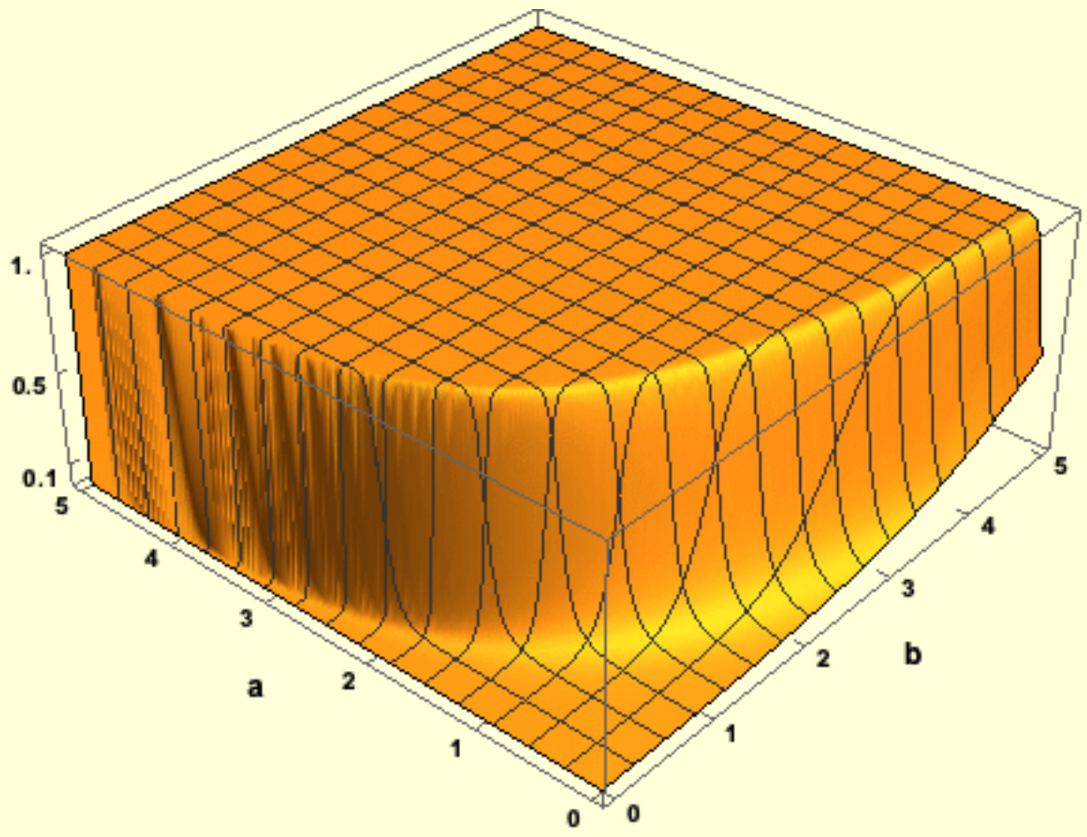



Surface of the posterior probability of a second action potential as a function of the levels of basal and apical inputs, using Eq (20). Here an initiating backpropagated action potential has been generated and BAC firing triggered, and the initial fdrive comes from the basal input.

Fig 7(b)

```
act3[b_, a_] := -5.2933 + 0.5*b*(1 +Exp[0.5*b*a*(1 +Exp[a])]); t = 10.0;
act4[b_, a_] := If[act3[b, a] > t, t, act3[b, a]];
surf1[b_, a_] := 1 / (1 + Exp[-act4[b, a]]);

Plot3D[surf1[b, a], {b, 0, 5}, {a, 0, 5}, AxesLabel→{"b", "a", " "}, LabelStyle→{12, Bold, Black},
TicksStyle→Directive[Bold, 10], Ticks→{{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 5}, {0.1, 0.5, 1.0}},
PlotPoints→50, PlotRange -> {0, 1.05}]
```

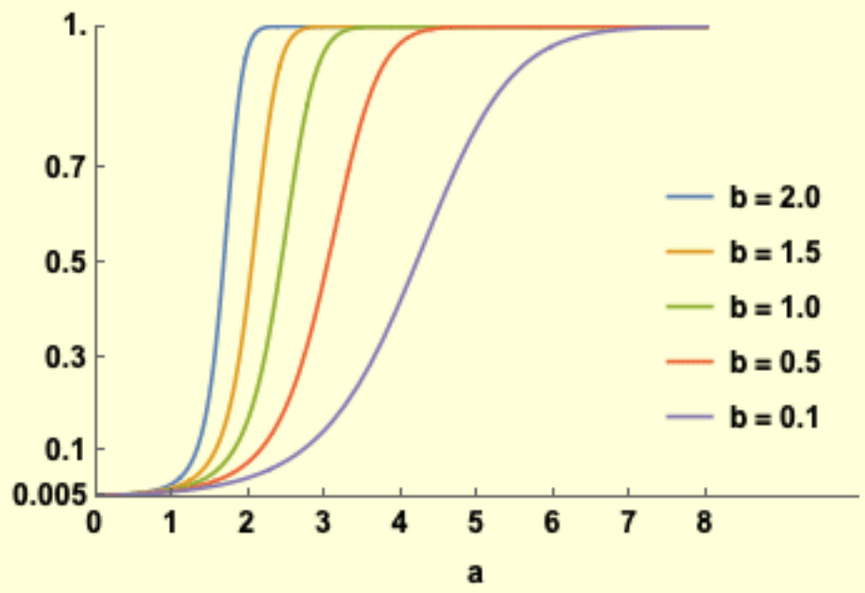



Comparison of posterior probabilities of an action potential for various levels of basal input when the apical input is driving, without an initiating backpropagated action potential having been generated.

Fig. 7(c)

```
post2[b_, a_]:= 1/(1+ Exp[5.2933 -0.5*a*(1 +Exp[b*a])])

Plot[{post2[1.0, a], post2[0.7, a], post2[0.5, a], post2[0.3, a], post2[0.1, a]},
{a, 0, 8},  FrameTicks→{{0, 1, 2, 3, 4, 5, 6, 7, 8}, {0.005, 0.1, 0.3, 0.5, 0.7, 1.0}},
Frame→{{True, False}, {True, False}}, FrameLabel→{{"", None}, { "a", None}},
LabelStyle→{12, Bold, Black}, TicksStyle→Directive[Bold, 10], PlotRange→{{0,10},{0,1}},
PlotLegends→Placed[{ "b = 2.0","b = 1.5", "b = 1.0", "b = 0.5",
"b = 0.1" }, {0.85,0.4}]]
```

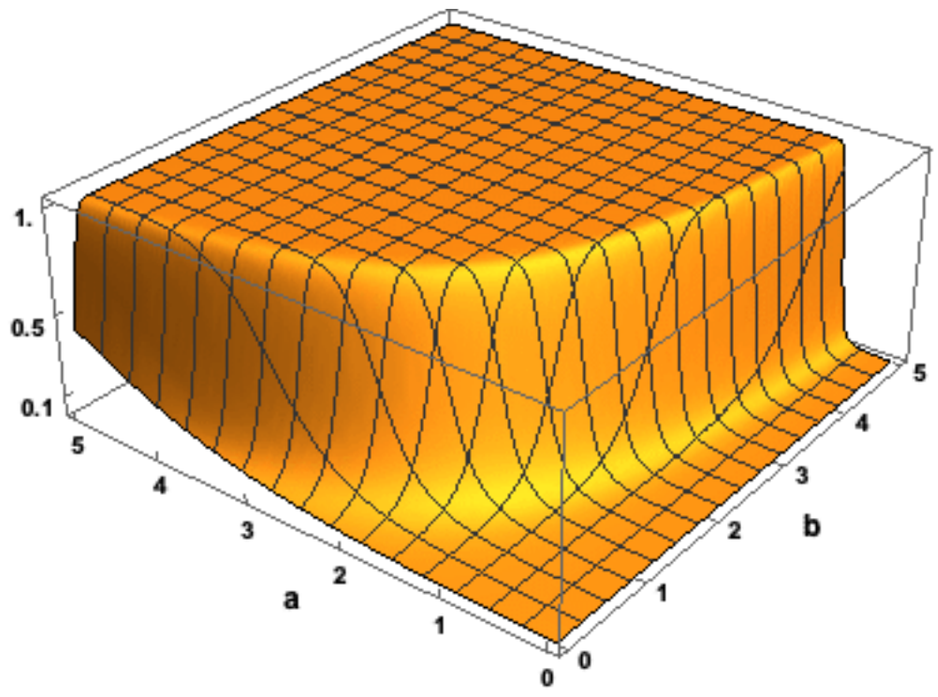



Surface of the posterior probability of an action potential as a function of the levels of basal and apical inputs, using Eq (32). Here the drive is from the apical input, without an initiating backpropagated action potential having been generated.

Fig7(d)

```
act5[b_, a_] := -5.2933 + 0.5*a*(1 + Exp[b*a]); t = 10.0;
act6[b_, a_] := If[act5[b, a] > t, t, act5[b, a]];
surf2[b_, a_] := 1 / (1 + Exp[-act6[b, a]]);

Plot3D[surf2[b, a], {b, 0, 5}, {a, 0, 5}, AxesLabel → {"b", "a", " "}, LabelStyle → {12, Bold, Black},
TicksStyle → Directive[Bold, 10], Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 5}, {0.1, 0.5, 1.0}},
PlotPoints → 50, PlotRange -> {0, 1.05}]
```

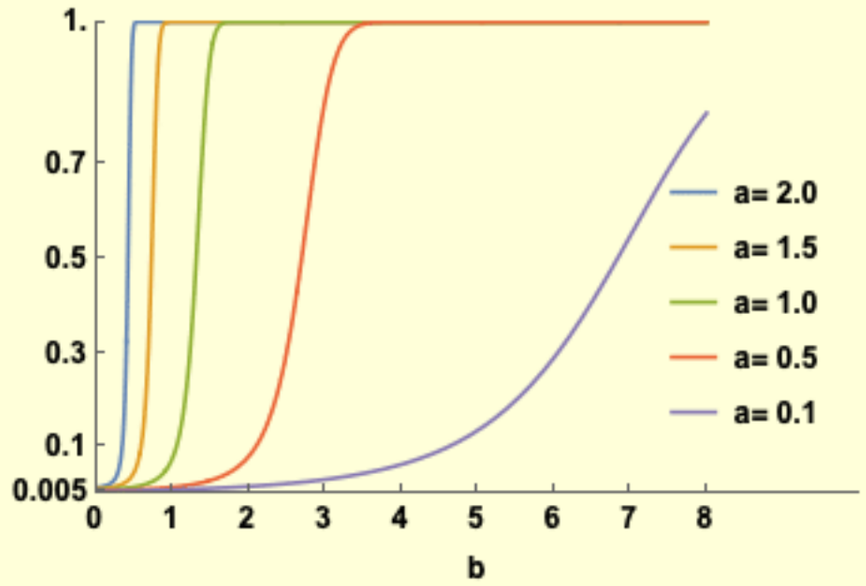



Comparison of posterior probabilities of an action potential for various levels of apical input, in the scenario where amplification results from a mixture of basal and apical inputs.

Fig. 8(a)

```
act8[b_, a_]:= -5.2933 + 0.25*b*(1 + Exp[0.5*b*a*(1 +Exp[a])]) +0.25*a*(1+Exp[b*a]);
t=10.0;
act9[b_,a_]:= If[ act8[b, a]>t, t, act8[b,a]];
post3[b_, a_]:= 1/(1+ Exp[-act9[b, a]]);

Plot[{post3[b, 2.0], post3[b, 1.5], post3[b, 1.0], post3[b, 0.5], post3[b, 0.1]},
{b, 0, 8},  FrameTicks→{{0, 1, 2, 3, 4, 5, 6, 7, 8}, {0.005, 0.1, 0.3, 0.5, 0.7, 1.0}},
Frame→{{True, False}, {True, False}}, FrameLabel→{{"", None}, { "b", None}},
LabelStyle→{12, Bold, Black}, TicksStyle→Directive[Bold, 10], PlotRange→{{0,10},{0,1}},
PlotLegends→Placed[{ "a= 2.0","a= 1.5", "a= 1.0", "a= 0.5",
"a= 0.1" }, {0.85,0.4}]]
```

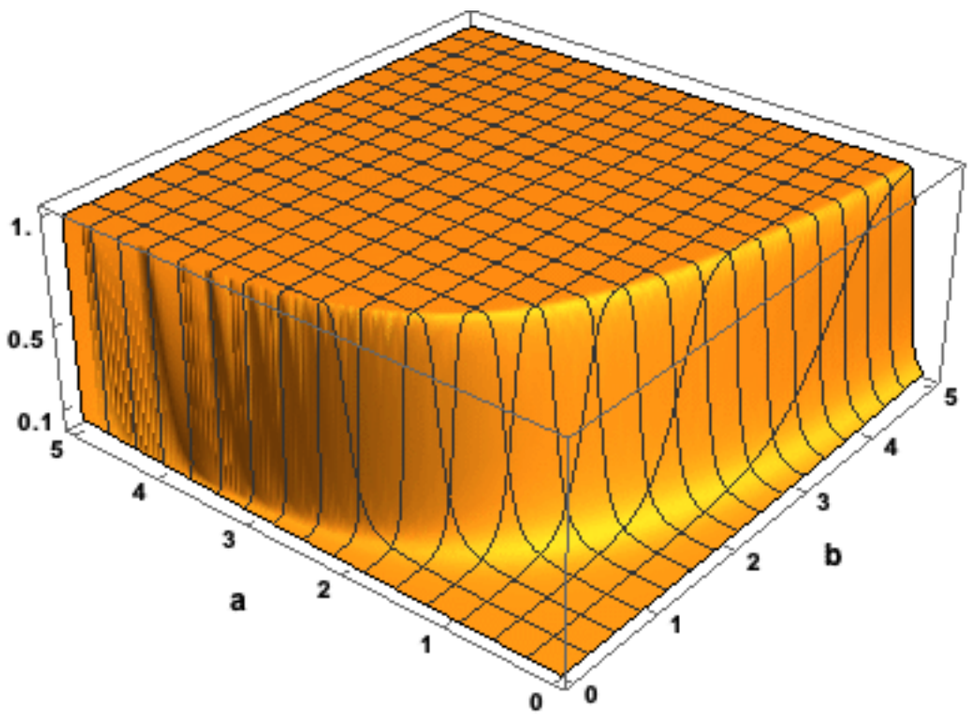



Surface of posterior probability of an action potential in the scenario where amplification results from a mixture of basal and apical inputs.

## Fig 8(b)

```
act10[b_, a_] := -5.2933 + 0.25 * b * (1 + Exp[0.5 * b * a * (1 + Exp[a])]) + 0.25 * a * (1 + Exp[b * a]);
act11[b_, a_] := If[act10[b, a] > t, t, act10[b, a]];
surf3[b_, a_] := 1 / (1 + Exp[-act11[b, a]]);

Plot3D[surf3[b, a], {b, 0, 5}, {a, 0, 5}, AxesLabel → {"b", "a", " "}, LabelStyle → {12, Bold, Black},
TicksStyle → Directive[Bold, 10], Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 5}, {0.1, 0.5, 1.0}},
PlotPoints → 50, PlotRange -> {0, 1.05}]
```

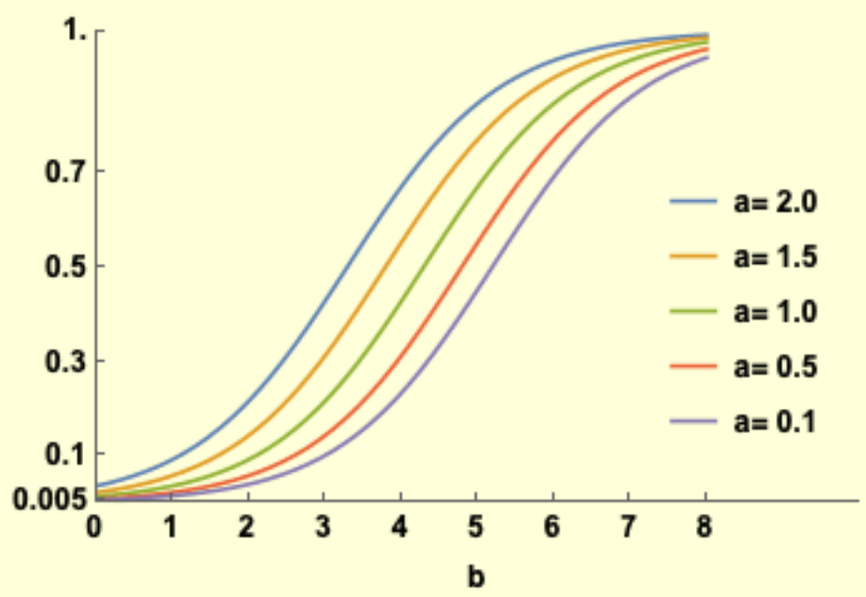



Comparison of posterior probabilities of an action potential for various levels of basal input, in the scenario where amplification results from an additive mixture of basal and apical inputs.

## Fig. 8(c)

```
post4[b_, a_]:= 1/(1+ Exp[5.2933 - a - b])

Plot[{post4[b, 2.0], post4[b, 1.5], post4[b, 1.0], post4[b, 0.5], post4[b, 0.1]},
{b, 0, 8},  FrameTicks→{{0, 1, 2, 3, 4, 5, 6, 7, 8}, {0.005, 0.1, 0.3, 0.5, 0.7, 1.0}},
Frame→{{True, False}, {True, False}}, FrameLabel→{{"", None}, { "b", None}},
LabelStyle→{12, Bold, Black}, TicksStyle→Directive[Bold, 10], PlotRange→{{0,10},{0,1}},
PlotLegends→Placed[{ "a= 2.0","a= 1.5", "a= 1.0", "a= 0.5",
"a= 0.1" }, {0.85,0.4}]]
```

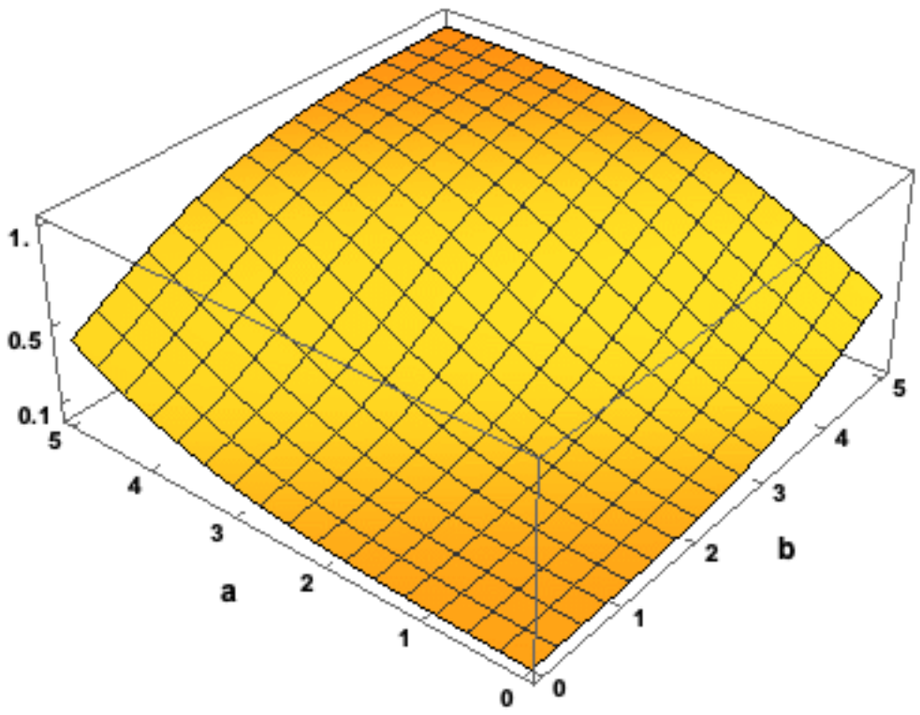


Surface of posterior probability of an action potential in the scenario where amplification results from ad additive mixture of basal and apical inputs.

Fig 8(d)

```
act12[b_, a_] := -5.2933 + a + b;
surf4[b_, a_] := 1 / (1 + Exp[-act12[b, a]]);

Plot3D[surf4[b, a], {b, 0, 5}, {a, 0, 5}, AxesLabel → {"b", "a", " "}, LabelStyle → {12, Bold, Black},
TicksStyle → Directive[Bold, 10], Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1, 2, 3, 4, 5}, {0.1, 0.5, 1.0}},
PlotPoints → 50, PlotRange -> {0, 1.05}]
```

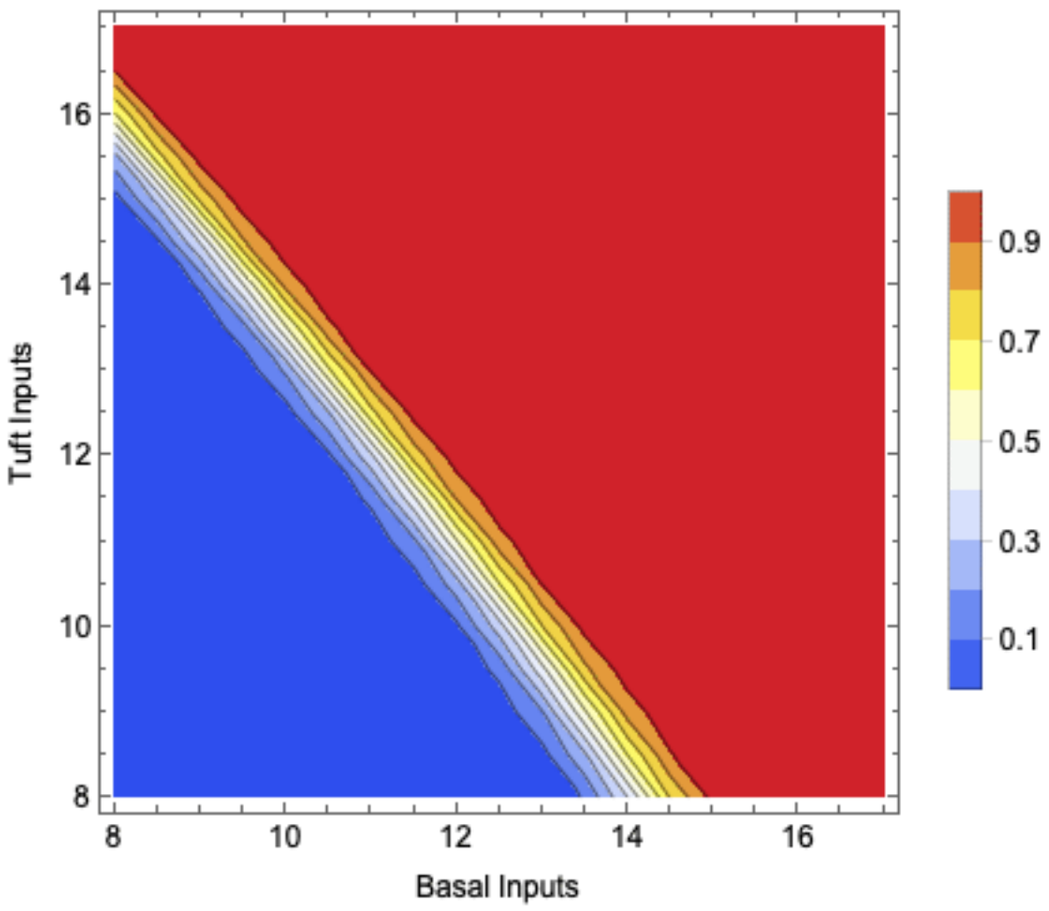



## Supplementary material: S2 File

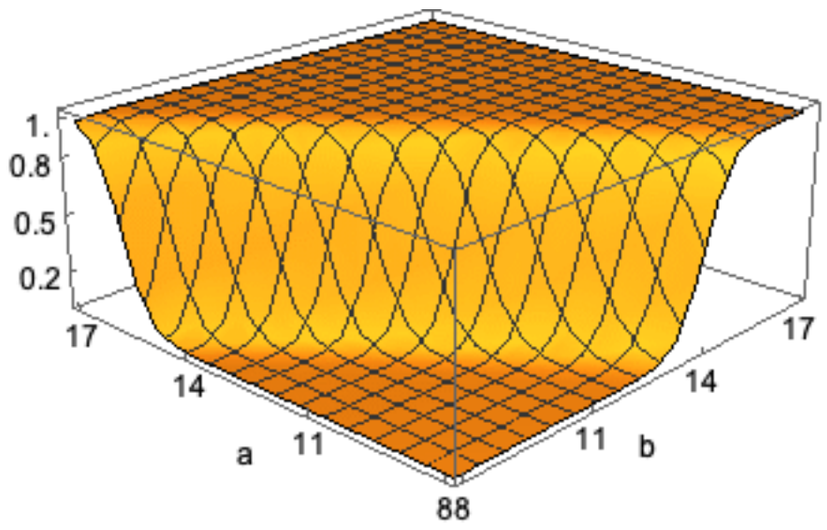
Contour and surface plots of the posterior predictive probability of a second action potential for given values of the basal and apical inputs are given below. The posterior predictive probabilities were computed in the R script Supp-S2-BN.R, and were saved to the file PPdat.csv. We first import the data.

```
Baydat = Import["/Users/jk/Desktop/KPAGL/PPdat.csv"];
```

```
ListContourPlot[Baydat, Contours → 10,
ColorFunction → ColorData["TemperatureMap"], FrameLabel → {"Basal Inputs", "Tuft Inputs"},
LabelStyle → {12, Black}, PlotLegends → BarLegend[{Automatic, {0, 1}}, {Automatic, 10}]]
```




```
ListPlot3D[Baydat, AxesLabel → {"b", "a", " "}, LabelStyle → {12, Black}, TicksStyle → Directive[12],
 Ticks → {{8, 11, 14, 17}, {8, 11, 14, 17}, {0.2, 0.5, 0.8, 1.0}}, PlotRange -> {0, 1.05}]
```




# Supplementary Figure, S4 Fig

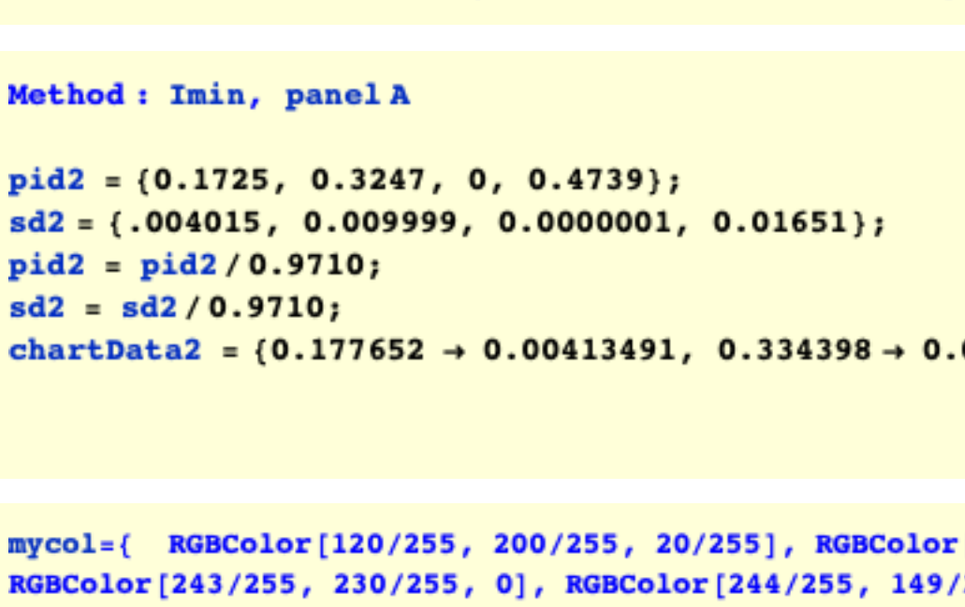
Partial information decompositions obtained using four other methods.

```
Method : Imin, panel A

pid2 = {0.1725, 0.3247, 0, 0.4739};
sd2 = {.004015, 0.009999, 0.0000001, 0.01651};
pid2 = pid2 / 0.9710;
sd2 = sd2 / 0.9710;
chartData2 = {0.177652 → 0.00413491, 0.334398 → 0.0102976, 0 → 0.000000, 0.4739 → 0.01651}
```

```
mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[chartData2, ChartElementFunction→errorBar["Rectangle"], ChartStyle→mycol,
BarSpacing → {0.3}, LabelStyle→{Black, Bold}
]
```

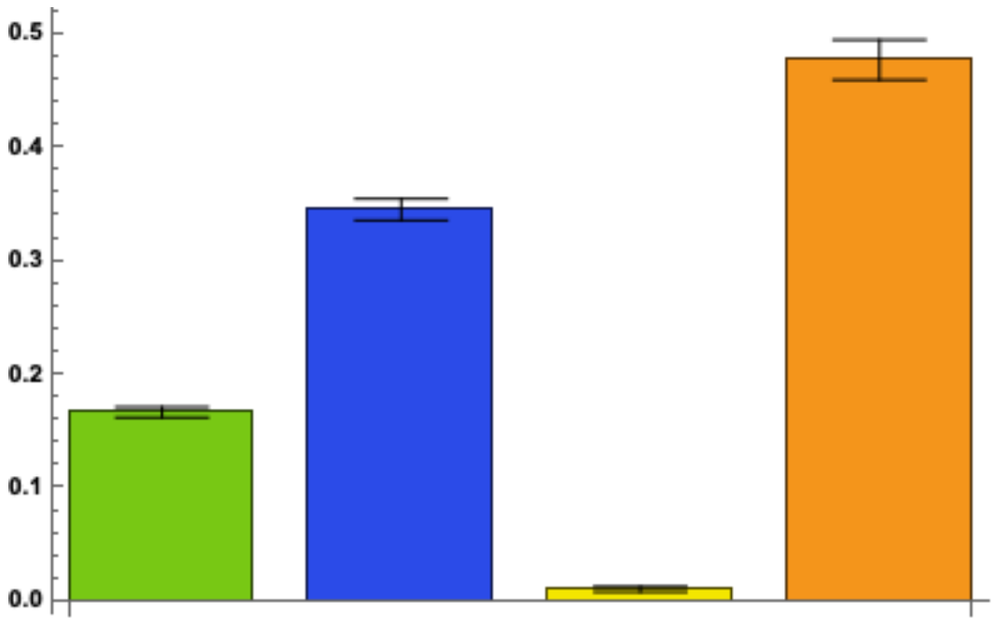



```
Method : Iproj, panel B

pid3 = {0.1619, 0.3353, 0.0106, 0.4633};
sd3 = {0.004482, 0.009212, 0.002282, 0.01715};
pid3 = pid3 / 0.9710;
sd3 = sd3 / 0.9710;
chartData3 = {0.166735 → 0.00461586, 0.345314 → 0.00948713, 0.0109166 → 0.00235015, 0.477137 → 0.0176622}
```

```
mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[chartData3, ChartElementFunction→errorBar["Rectangle"], ChartStyle→mycol,
BarSpacing → { 0.3}, LabelStyle→{Black, Bold}
]
```

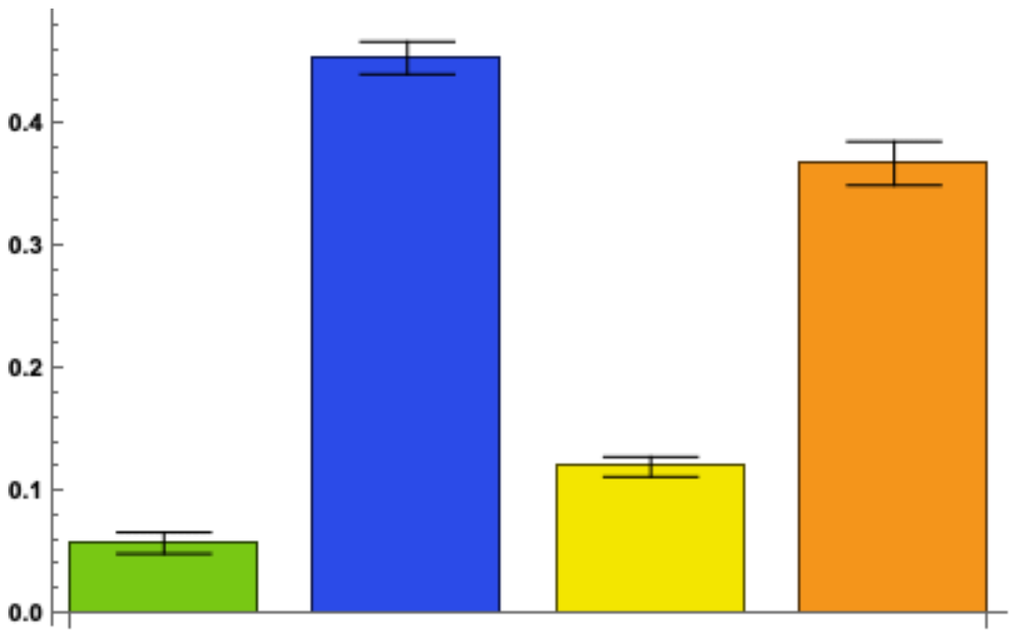



```
Method: Iccs, panel C

pid4 ={0.05600, 0.4411, 0.1165, 0.3574};
sd4={0.008466, 0.01293, 0.007929, 0.01736};
pid4=pid4/.9710;
sd4=sd4/.9710;
chartData4 ={0.0576725→ 0.00871885, 0.454274→ 0.0133162, 0.119979→ 0.00816581, 0.368074→ 0.0178785}
```

```
mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[chartData4, ChartElementFunction→errorBar["Rectangle"], ChartStyle→mycol,
BarSpacing → { 0.3}, LabelStyle→{Black, Bold}]
```




```
Method: Idep, panel D

pid5 ={0.07115, 0.4260, 0.1013, 0.3726};
sd5 ={0.002542, 0.009066, 0.002009, 0.01509};
pid5 =pid5/.9710;
sd5=sd5/.9710;
chartData5 ={0.073275→ 0.00261792, 0.438723→ 0.00933677, 0.104325→0.002069, 0.383728→0.0155407}
```

```
mycol={  RGBColor[120/255, 200/255, 20/255], RGBColor[43/255, 75/255, 232/255],
RGBColor[243/255, 230/255, 0], RGBColor[244/255, 149/255, 27/255] };
myleg = { "Shd", "UnqB","UnqA","Syn"};

BarChart[chartData5, ChartElementFunction→errorBar["Rectangle"], ChartStyle→mycol,
BarSpacing → { 0.3}, LabelStyle→{Black, Bold}]
```