Solutions Manual for:
An Introduction to Tensors and Group Theory for Physicists

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Contents

Chapter 1

Linear Algebra and Tensors

Summary of Exercises

There are no problems listed for chapter 1.

Chapter 2

Vector Spaces

2.1. Levels of structure

For each of the below, V means all vector spaces, H means only those with non-degenerate Hermitian forms, and P means only those with inner products.

(a) P - unit vector $\hat{x}$ requires the norm

(b) V - all vector spaces can have a basis (in fact, it is possible to prove that all vector spaces have a basis)

(c) V

(d) P - norm requires positive-definite form, therefore inner products

(e) V

(f) H - orthogonality

(g) V - any vector space has a least the inclusion linear operator $\iota : V \hookrightarrow V$ where $\iota(v) = v$ for all v in V.

(h) P - angle requires norm, norm requires inner product

(i) V - though a hermitian form might offer a natural choice for the definition of a dual, all vector spaces have a dual space

(j) P - metric dual requires a metric

2.2. $L^2\left([-a, a]\right)$ closed under addition

Given $f, g \in L^2\left([-a, a]\right)$, we need to show that the addition of f and g also is in $L^2\left([-a, a]\right)$. Specifically, we must show $h(x) = (f + g)(x) \in L^2\left([-a, a]\right)$.

Begin by using the Triangle Inequality

$$\int_{-a}^{a} |h|^2 \, dx = \int_{-a}^{a} |f+g|^2 \, dx \leq \int_{-a}^{a} (|f| + |g|)^2 \, dx$$

Using the given inequality $0 \leq \int_{-a}^{a} (|f| + \lambda |g|)^2 \, dx$ for all $\lambda \in \mathbb{R}$. If we expand the inequality and choose $\lambda = 1$, we obtain the fairly general result 2.1:

$$0 \leq \int_{-a}^{a} (|f| + \lambda |g|)^2 \, dx = \int_{-a}^{a} |f|^2 \, dx + \lambda^2 \int_{-a}^{a} |g|^2 \, dx + 2\lambda \int_{-a}^{a} |f| \, |g| \, dx$$

$$2 \int_{-a}^{a} |f| \, |g| \, dx \leq \int_{-a}^{a} |f|^2 \, dx + \int_{-a}^{a} |g|^2 \, dx < \infty \tag{2.1}$$

Now that we've shown the cross-term is finite, we can establish our original objective:

$$\int_{-a}^{a} |h|^2 \, dx = \int_{-a}^{a} |f|^2 \, dx + \int_{-a}^{a} |g|^2 \, dx + 2 \int_{-a}^{a} |f| \, |g| \, dx < \infty$$

2.3. [WIP] Spherical harmonics basis

2.4. [WIP] Angular momentum generates rotations

2.5. [WIP] Double dual space

2.6. [WIP] Transpose of linear operator

2.7. [WIP] Hermitian adjoint

2.8. [WIP] Metric dual of the metric

2.9. [WIP] Bases of $P(\mathbb{R})$

Bibliography

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