Jeevanjee Solutions

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Abstract

The following is a selection of solutions for various problems and exercises found in Introduction to Tensors and Group Theory for Physicists by Nadir Jeevanjee. These solutions were written by J. W. Kennington and updated last on 04-07-2019

Contents

1 Chapter 2

1.1 Problem 1

For each of the below, V means all vector spaces, H means only those with non-degenerate Hermitian forms, and P means only those with inner products:

- P - unit vector $\hat{x}$ requires the norm
- V - all vector spaces can have a basis (in fact, it is possible to prove that all vector spaces have a basis)
- V
- P - norm requires positive-definite form, therefore inner products
- V
- H - orthogonality
- V - any vector space has a least the inclusion linear operator $\iota : VV$ where $\iota(v) = v$ for all v in V.
- P - angle requires norm, norm requires inner product
- V - though a hermitian form might offer a natural choice for the definition of a dual, all vector spaces have a dual space
- P - metric dual requires a metric

1.2 Problem 2

Given $f, g \quad \in$, we need to show that the addition of f and g also is in . Specifically, we must show $h(x) = (f+g)(x) \quad \in$.

Begin by using the Triangle Inequality

$$\int_{-a}^{a} h^2 dx = \int_{-a}^{a} f + g^2 dx \leq \int_{-a}^{a} (f+g)^2 dx$$

Using the given inequality $0 \leq \int_{-a}^{a} (f + \lambda g)^2 \, dx$ for all $\lambda \in \mathbb{R}$.

If we expand the inequality and choose $\lambda = 1$, we obtain the fairly general result:

$$0 \leq \int_{-a}^{a} (f + \lambda g)^2 \, dx = \int_{-a}^{a} f^2 dx + \lambda^2 \int_{-a}^{a} g^2 dx + 2\lambda \int_{-a}^{a} fg dx$$

$$2 \int_{-a}^{a} fg dx \leq \int_{-a}^{a} f^2 dx + \int_{-a}^{a} g^2 dx < \infty$$

Now that we've shown the cross-term is finite, we can establish our original objective:

$$\int_{-a}^{a} h^2 dx = \int_{-a}^{a} f^2 dx + \int_{-a}^{a} g^2 dx + 2 \int_{-a}^{a} fg dx < \infty$$