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On the continuous function defined on a sphere.

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The following theorem is proved for every dimension n. Let $f(X)$ be a real-valued continuous function defined on an n-dimensional sphere S^n in an $(n+1)$-dimensional Euclidean space E^{n+1} with center O. Then there exist $n+1$ points $X_1,\cdots,X_{n+1}$ on S^n such that (i) the vectors $OX_1,\cdots,OX_{n+1}$ are perpendicular to one another and (ii) $f(X_1)=\cdots=f(X_{n+1})$. This theorem was originally conjectured by Rademacher and the case $n=2$ was proved by the reviewer [Ann. of Math. (2) **43,** 739–741 (1942); MR0007267]. From this follows, as a corollary, that for any bounded convex body in E^{n+1} there exists a circumscribing cube around it. The authors prove this theorem by reducing it to the following lemma which can be proved by induction on n. Let ${S_0}^n, {S_1}^n$ be two concentric n-dimensional spheres in E^{n+1} with the same center O. Let L be a closed subset of E^{n+1} contained between ${S_0}^n$ and ${S_1}^n$ which intersects any continuous curve joining ${S_0}^n$ and ${S_1}^n$. Then L contains $n+1$ points $X_1,\cdots,X_{n+1}$ such that the vectors $OX_1,\cdots,OX_{n+1}$ are perpendicular to one another and have the same length.