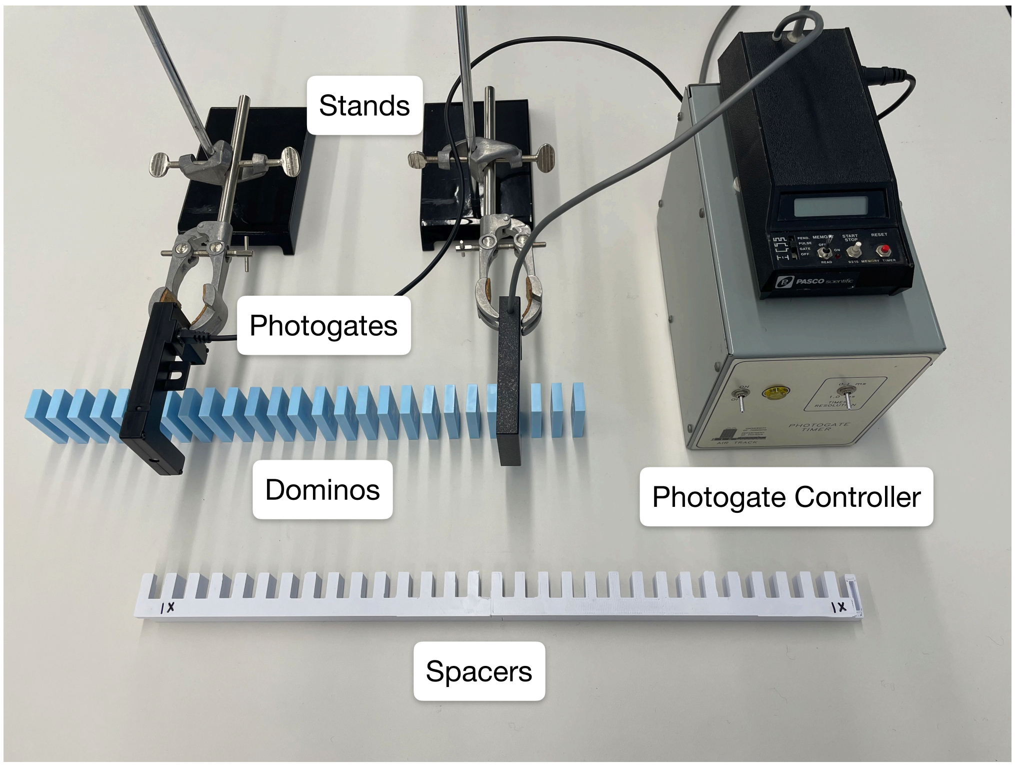
Physics
UNIVERSITY OF TORONTO

The Domino Chain Effect



Revisions

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Safety Reminder

- Food or drink are not allowed in any of our Advanced Lab Experiment rooms. There may be traces of radioactive materials and lead on the benches and these are the most dangerous when they are ingested or inhaled.
- There are some electronic components used in the lab. Make sure you are aware of loose/ hanging wires, exposed connections, and potential differences. If any equipment appears damaged or doesn't function properly, ask Laboratory Technicians for assistance. finger or a foot if dropped
- Wash your hands when you leave the lab.

NOTE: *This is not a complete list of all hazards; we cannot warn against every possible dangerous ignorance, e.g. opening plugged-in electrical equipment, juggling cryostats, etc. Experimenters must constantly use common sense to assess and avoid risks, e.g. if you spill liquid on the floor it will become slippery, sharp edges may cut you, etc. If you are unsure whether something is safe, ask the supervising professor, the lab technologist, or the lab coordinator. If an accident or incident happens, you must let us know. More safety information is available at* http://www.ehs.utoronto.ca/resources.htm.

Objective

To observe and quantify asymptotic velocity of propagation in a domino chain as a function of spacing between the domino prisms, as well as scaling relations of the domino effect.

Introduction

The domino effect can be described as "the destabilization of marginally stable systems" [1], which has been observed in both social and natural sciences. We are interested in the problem of classical mechanics involving the toppling of a set of slender rectangular prisms standing on their side with the least area. These rectangular prisms, or dominoes, are assumed to have uniform physical properties and are evenly spaced along a straight line. As one domino falls, energy and momentum are transferred onto the next, and thus the process continues down the entire chain. The physics of a falling domino chain does not have one agreed upon theory, but a number of theories that seek to describe this nonlinear system have been developed by physicists over the past century.

As with all problems in classical mechanics, researchers are interested in the equation of motion of each individual block, and to ultimately be able to predict the behaviours of the domino chain. Models have been formulated by different researchers, each with their own set of assumptions that are often contradictory to each other. There are two main approaches: one is to model each collision using equations and conservation laws from classical mechanics, another is to perform dimensional analysis for the asymptotic velocity of the domino propagation. There is no unified theory that predicts the behaviour of dominoes, so the ultimate goal of this lab is to compare experimental results with existing theories.

Ideally, one should be able to measure the precise motion of each domino as it falls and collides with the next, as a way to extract maximum information from the experiment; however, that can be quite difficult to measure, as the time interval between collisions is on the order of 0.01 s. To make this study feasible for an undergraduate lab, we will focus on measuring the average time interval between collisions to determine the asymptotic velocity of the domino chain; theory predicts that as the the wave propagates down the chain, the velocity reaches a constant, in which case one can simply measure the time interval over a range of collisions to find the asymptotic velocity.

Since the standard dominoes are not very massive it is difficult to find a surface on which the domino will not slip at all. Standard practice for the experiments is to use sandpaper as the base, but even that method does not guarantee that slipping is eliminated as the domino falls, especially when it strikes the next one. The possibility of slipping allows for the rotation about the center axis parallel to the height of the prism. Because of that the falling domino may make contact with the next one with a corner rather than full width of the top edge, causing less energy to be transferred in the process.

Another factor that contributes to variation in the velocity of the domino chain is related to the fact that with increasing rotation speed, the possibility for the prism to overcome friction and slip on the surface rather than tilt. As a result, the next domino would be hit lower, decreasing the torque exerted on it.

In this lab, students have the opportunity to explore asymptotic behaviours of the domino effect, where they will determine the location at which asymptotic velocity is reached and the value of the velocity for various spacings on different surfaces, including the lab table and sandpaper.

Theory

In this section, we will review some papers that approach the domino effect from different perspectives. The first method is to simply perform dimensional analysis on the asymptotic velocity and use experimental data to subsequently determine the dimensionless constants. The other approach models each domino block individually as a mechanical object and derives the equation of motion for the rotation of each block under assumptions such as no-slip at the contact point between the domino and the surface as well as the assumption that two adjacent domino blocks will remain in contact after their initial collision. Of all the models, Efthimiou and Johnson [2] made the most limiting assumptions, with the most unrealistic being consideration of a domino as a point mass atop a massless rod. Larham [3] has shown since that this model does not agree with experimental data, as it vastly overestimates the speed of the domino wave, especially when the dominos are set up closer together (separation ratio $\frac{d}{h} < 0.6$, where h is the height of a domino block and d is the separation between the prisms). Since no other models make this assumption, it is reasonable to acknowledge it for completion, but ignore it if comparisons between various models are made.

The most important assumption any model must make is the nature of the collisions between the dominoes. If you choose to focus on the computational aspect of this experiment - paper by student Angela Xiang is a great starting point to analyzing the existing models.

Dimensional analysis approach

Dimensional analysis allows for a more straightforward understanding of the underlying physics involved in the complex mechanical interaction between colliding dominoes. The simplest model on the domino effect is by McLachlan et al. [1]. It assumed that the domino thickness has a negligible effect on the propagation, so they are essentially treated as 2D objects. Through dimensional analysis, it was found that the non-dimensional wave speed

$$\frac{v}{\sqrt{gh}} = f\left(\frac{d}{h}\right) \tag{1}$$

for some function f to be determined from experiments, where h is the height of each domino, and d is the spacing between each pair of dominoes. Figure 1 shows the plot of result reported in this paper [1]. It demonstrates that the asymptotic velocity of the domain is dependent only on the ratio of spacing to height. The experiment involved about 100 dominoes, and timed with a stopwatch, where timing began when the hand pushed on the first domino and stopped when the last domino in the chain fell. Thus, this is not exactly the asymptotic velocity, as it includes the transient state behaviour at the start of the chain. However, if the transient process were short and the chain was relatively long, the time measured would still be dominated by the asymptotic behaviour, and the transient time would be considered as a systematic error. An important point to note is that this research implies that the asymptotic behaviour is independent of the initial conditions, i.e. the inconsistency from the initial push is neglected.

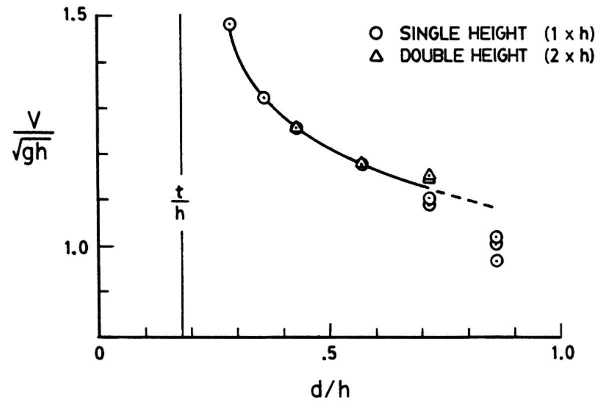

Figure 1: The experimental results for the dimensionless asymptotic velocity as a function of the ratio of domino spacing and its height [1].

A more modern work in 2020 by Sun [4] considers the dimensional analysis approach more rigourously. The motion of dominoes can be considered as a two-dimensional mechanics problem, where the domino wave propagates along the x axis, while the dominoes are placed vertically along the z axis, and therefore, a total of three dimensions (L_x, L_z, T) are required. Table 1 summarizes the variables and their dimensions.

Table 1: A summary of variables used when considering the domino propagation and their dimensions.

Variable	Symbol	Dimensions
speed of propagation	v	$L_z T^{-1}$
height	h	L_z
thickness	t	L_x
spacing	d	L_x
gravitational acceleration	g	$L_z T^{-2}$

Since there are five variables and three dimensions, two dimensionless variables will be generated as

$$\Pi_1 = v h^{a_1} d^{b_1} g^{c_1} \tag{2}$$

and

$$\Pi_2 = t h^{a_2} d^{b_2} g^{c_2}. \tag{3}$$

(a_i, b_i, c_i) can be determined by the fact that both Π_1 and Π_2 are dimensionless, which then follows that $(a_1, b_1, c_1) = (1/2, -1, 1/2)$ and $(a_2, b_2, c_2) = (0, -1, 0)$. By Buckingham Π theorem,

$$v = d\sqrt{\frac{g}{h}} f\left(\frac{t}{d}\right). \tag{4}$$

A further analysis shows that the width of the domino blocks have no effect on the propagation speed. Experimental results have shown that

$$v = C\sqrt{\frac{td}{h}g}, \tag{5}$$

where the proportionality constant C would be obtained from curve fitting.

Theoretical derivation

The model by van Leeuwen [5] makes the assumption that the edge of the domino does not slip against the surface while it is toppling over.

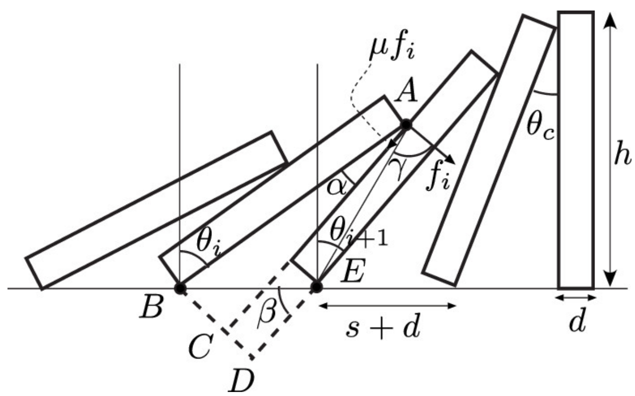

Figure 2: The diagram depicting the toppling of dominoes. Note that in this diagram the separation between the blocks is labeled as s and the domino's thickness is identified by d

Figure 2 visualizes the variables used in the derivation. Since we are mostly concerned about the rotation of each domino block, we first derive a relationship between the the angular position of two adjacent dominoes. The angle θ_i is defined as the angle between the vertical axis and the long side of the resting domino. We further assume that the collision is inelastic, where two dominoes always remain in contact after their initial collision. From the geometry,

$$h\sin(\theta_i - \theta_{i+1}) = (s+d)\cos\theta_{i+1} - d.. \tag{6}$$

Note that near the back of the chain, θ approaches the stacking angle, which is the angle of dominoes at the rest position after they have toppled over when the chain is infinitely long. The recursive equation (6) will help us eliminate all variables before the n-th collision. Defining

$$\theta_i(\theta_n) = \psi_{n-i}(\theta_n), \tag{7}$$

for some function ψ that is defined recursively, which expresses θ_i in terms of θ_n, and thus enables us to express

$$\omega_i = \frac{\mathrm{d}\theta_i}{\mathrm{d}\theta_n}\frac{\mathrm{d}\theta_n}{\mathrm{d}t}. \tag{8}$$

There are two important components of of this mechanics problem: finding the initial angular velocity of the frontmost domino immediately after a collision based on the dynamics of the chain, and deriving an equation describing the motion of the frontmost domino before it hits the next one. To derive the equation of motion, we consider

$$I\frac{\mathrm{d}\omega_n}{\mathrm{d}t} = T_n + f_{n-1}a_{n-1}, \tag{9}$$

$$I\frac{\mathrm{d}\omega_i}{\mathrm{d}t} = T_n + f_{i-1}a_{i-1} - f_i b_i, \tag{10}$$

where I is the moment of inertia of the dominoes, T_i is the torque due to gravity, f_i is the force exerted to the $i+1$-th domino and a_i is the length of moment arm, whereas b_i the length of moment arm for the force exerted on the i-th domino by the $i+1$-th domino. We model friction using the coefficient of friction as $f_f = \mu f$, and therefore this can be incorporated in equation (9) and (10). No assumptions regarding conservation laws are made, and the equation of motion are derived solely from Newton's laws. It is possible to eliminate all other variables other than ones with subscript n, where we obtain

$$\frac{1}{2}A_n(\theta)\frac{\mathrm{d}\omega_n^2}{\mathrm{d}\theta} + B_n(\theta)\omega_n^2 = C_n(\theta) \tag{11}$$

for some A_n, B_n, and C_n defined recursively. This is a first order ODE that can be solved numerically. The time between collisions is then

$$t_n = \int_0^{\theta_c} \frac{\mathrm{d}\theta}{\omega_n}, \tag{12}$$

where θ_c is the critical angle at which the domino touches the next one down the chain. The velocity of propagation can be defined as $v_n = (s+d)/t_n$. Asymptotically, the speed will approach a constant depending only on the geometry of the setup.

At each collision, we assume that the process is quick and the angle does not change while the collision is taking place. The initial angular velocity of the next domino is derived by considering angular impulse due to forces exerted by the current frontmost domino, and we obtain

$$A_n(0)\omega_n(0) = \left[A_n(0) - 1\right]\omega_{n-1}(\theta_c). \tag{13}$$

The precise derivation for the model can be found in the respective publications the publication.

The python code for this model can be downloaded from the appropriate module on Quercus page.

Experimental setup and Apparatus

Dominoes, Spacers, and Surfaces

You are provided with a uniform set of plastic dominoes that measure 24 mm × 7.5 mm × 48 mm, as shown in Figure 3). Wooden dominoes are also available, but they are thinner and will slide around in the spacers provided. They are also less uniform both in shape and in the weight distribution, but could provide an interesting study subject for comparisons.

For quicker setup, several spacing templates (or "spacers") are provided at 0.75×, 1×, 2×, and 3× thicknesses, where '2×' means that the space between the dominoes is twice the thickness of each domino. These 3D printed, comb-like tools will speed up your setup significantly, and help ensure a uniform spacing between each domino. They also hold the chain stable while you align the photogates. For each thickness, the spacers interlock so that longer chains can be built. If you need longer chains, more can be printed using the CAD files provided.

Additionally, two kinds of sandpaper (120 and 400 grit) are provided to allow for testing on high-friction surfaces, bringing the setup closer to the theoretical no-slip condition.

Photogate and Ring Stands

The photogate sensors ("gates") can be used to measure the time it takes for a number of dominoes to fall. The gates can be clamped into the ring stands such that they are suspended above the chain of dominoes, so the very top of a falling domino will break the beam and trigger the sensors. Use the "gate" setting on the photogate so that when the sensor is first broken on either gate, the timer begins, and when either sensor is triggered again, the timer stops.

One of the gates has a red light that turns on when the sensor is triggered, which you should use to align it. Simply place the gate between two dominoes, and then slowly move towards the next domino until the light turns on, then back away very slightly. This helps ensure that the sensor will be triggered right after impact, and that your data will record properly. You can use the red light and the stopwatch to help you do this; slowly move the gate towards the domino until the light turns, and then back away slightly so the sensor beam remains unbroken.

The other gate does not have a light, but you can repeat the same process by first resetting and starting the stopwatch, and taking the moment when the stopwatch stops as the exact configuration where the gate would be triggered.

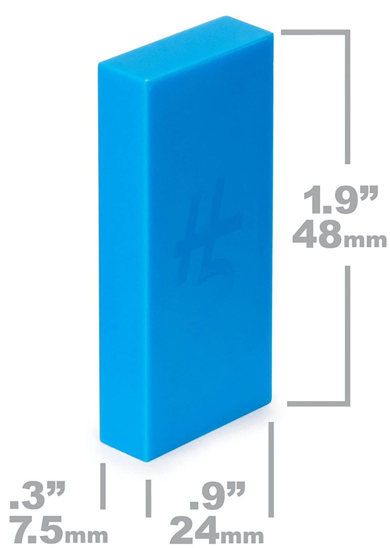

Figure 3: Dimensions of the plastic dominoes.

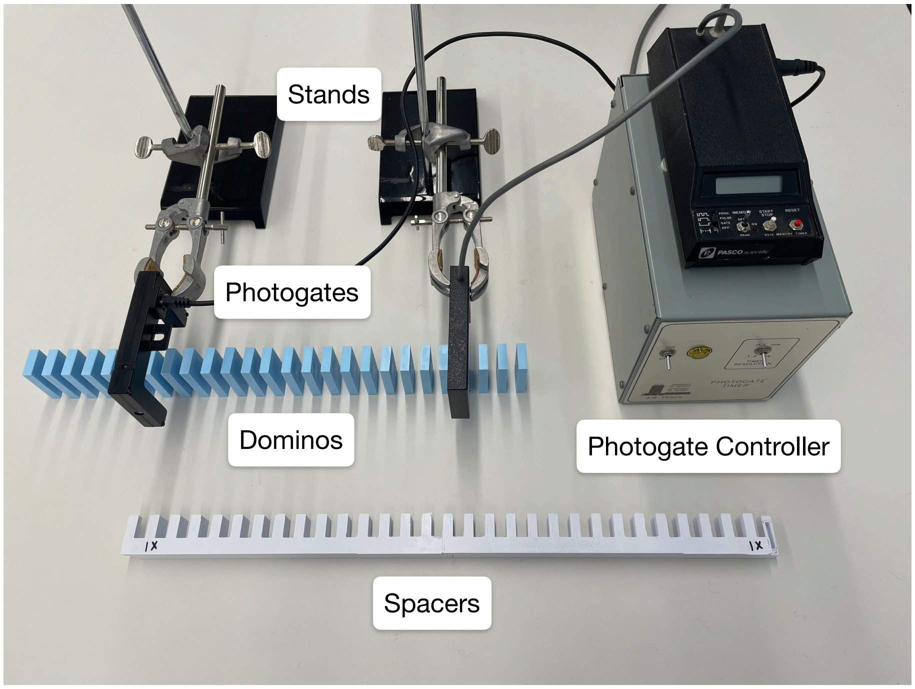

Figure 4: Set up and materials of the experiment. Note that in order to observe the asymptotic behaviour the chain of dominos should be significantly longer than shown in the figure above.

Basic Procedure

Using the spacers to facilitate domino setup, you will arrange a line of dominoes with one photogate adjacent to a range of dominoes in the chain. You may choose to set up the photogates before the first and after the last domino of the chain, or you may choose to study a specific region of the chain, such as the first N dominoes, the last N dominoes, or some range in between.

The photogates should be set up in such a way that when the domino closest to the first photogate is pushed, the first photogate will detect the solid body passing before it, starting a timer. When the domino before the second photogate falls, it will be detected by the second photogate, and stop the timer.

These time measurements can be made in succession with domino chains of increasing lengths to determine the asymptotic velocity of the system. This can be done for a number of domino spacings.

Below is a list of the steps required for completing a measurement:

1. Prepare a surface to set up the domino chain. If you're just starting out, you will likely want to start with dominoes directly on the lab table.
2. Place your photogates in the clamps provided, if not done for you. (This will likely take you some time at the beginning, but will be quicker in future lab sessions.) Using the knobs on the stand, make sure each photogate is securely attached. Both gates should be at the same height off the table, just lower than the top of the domino.

3. Choose a spacing for testing. We recommend starting with 1×.
4. Slot your dominoes into the spacers provided to set up your chain. Push the spacer gently in the direction of the teeth to make sure the dominoes are against the back edge of the slot.
5. Turn on the photogate timer and set the measurement increment to 0.1 ms.
6. Check that the photogates work as you expect (i.e., the stopwatch starts at the first beam break,and stops at the second beam break). Move the photogates to bracket the range you are studying (i.e., example, dominoes 1-20, or 25-30.) Move the beaker stands and align the gates so that as soon as the domino immediately beside it tips over, the sensor is triggered.
7. Gently pull the spacers back away from the chain. You will need to move slowly and in a perpendicular direction to keep the dominoes straight. Alternatively, you can rotate the spacer 90° away from the table.
8. Reset the stopwatch and press start again. If it starts, both sensors are clear and you are ready to proceed. If it does not, your spacer removal has shifted a domino and tripped one of the sensors. Readjust and repeat this step until you are confident you are ready to take a measurement.
9. Gently tip over the first domino to start the chain reaction!
10. Record your time and any observations in your lab notebook.
11. Repeat the same measurement more than once to obtain an average measurement with an uncertainty.

Tips and Troubleshooting

Though the data collection process for this experiment is quite straightforward compared to some other labs, it can be tricky to get started. Below are some tips that may help you.

Photogates

- If your recorded times are much, much lower than you expect, adjust the gates to be perpendicular to the chain. If they are askew, one domino might trigger the same gate twice, causing low and unhelpful readings. Make note of when this occurs; if it is happening consistently, you may have placed the gates too close to the table vertically.
- Note that one of the photogates is attached with a long cable, while the built-in cable is very short. You may need to move the photogate base during your experiment to account for this.

Spacers

- If a domino is getting stuck in a spacer, pull the spacer back halfway and adjust the dominoes that are caught before removing completely. If this happens repeatedly in the same space, use tweezers or some of the sandpaper provided to remove whatever the domino is catching on.
- If using sandpaper, you will not be able to slide the spacers like you can on the table. Tape the sandpaper into place, making sure it is flat, and place the spacer directly atop the sandpaper, then lift it slightly before removing.

Setup

- Use different coloured dominoes to mark the locations where you will place the gates later for easy measuring. For example, set up 10 purple dominoes, followed by 10 green, followed by 10 purple.
- You should try to set up the dominoes in the spacer in the position where you will be conducting the trial; however, you can move the chain around the table if needed. Simply use a ruler to secure the dominoes in their slots, and hold the ruler and the spacer together to slide around.

Investigation Questions

1. Estimate the average propagation speed at various points in the chain and try to show that the chain reaches an asymptotic velocity, and repeat for at least one other spacing. Do you see evidence of an asymptotic velocity? At what point in the chain does it begin, and how does it change when the spacing between the dominoes is varied? You should start here, before you attempt any other investigations.
2. Create a plot comparing the average dimensionless velocity as a function of the domino number, overlaying your results for each of the domino spacings. What trends do you notice?
3. Create a plot of dimensionless asymptotic velocity as a function of the dimensionless domino separation. Does your fit agree with the existing theories (i.e., equation (5) and (1))? Perform a fit to check. For equation (1), the exact form of f is not known a priori, so try fitting with different types of functions (polynomial, power functions, etc.).
4. Compare your results to the van Leeuwen theory [5] using the simulation code provided. An important variable to set in the code is selecting an appropriate coefficient of friction between the dominoes. Are your results in agreement with the theory? Which author do you most agree with?
5. Replicate previous trials using a sandpaper mat as the surface the dominoes lie on to better simulate the no-slip condition. How do your results change with the increased friction, and what do you think causes this change? Do these results better match existing theories than your previous results?
6. Another question to investigate is the dependence of the inclination on the dominoes. At what inclination can you place the domino chain such that it will not fall over without a push? How does inclining the dominoes affect the speed in terms of inclining it upwards or downwards with respect to the domino motion?
7. Feel free to explore ideas from other research papers such as [6] [7], and check if your results agree with conclusions from these papers.

References

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