Assignment 2: Word Vectors

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1. Basics

   a. Prove softmax is invariant to constant offset in the input.

$$((soft\max(x+c))_i = \frac{\exp(x_i+c)}{\sum_{j=1}^{\dim(x)}\exp(x_j+c)} = \frac{\exp(x_i)*\exp(c)}{\sum_{j=1}^{\dim(x)}(\exp(x_j)*\exp(c))}$$

$$= \frac{\exp(x_i)*\exp(c)}{\exp(c)*\sum_{j=1}^{\dim(x)}\exp(x_j)} = \frac{\exp(x_i)}{\sum_{j=1}^{\dim(x)}\exp(x_j)} = soft\max(x)_j$$

   b. See softmax.py

   c.

Sigmoid :	$\sigma(x) = \frac{1}{1+e^{-x}}$
Derivative of sigmoid:	$\frac{d}{dx}(1+e^{-x})^{-1} = (-1)*(1+e^{-x})^{-2}*\frac{d}{dx}(1+e^{-x}) = \frac{-1}{(1+e^{-x})^2}\frac{d}{dx}(e^{-x})$ $= \frac{-1}{(1+e^{-x})^2}*(-1)*e^{-x} = \frac{e^{-x}}{(1+e^{-x})^2}$
$\sigma(x)(1-\sigma(x))$	$\frac{1}{1+e^{-x}}(1-\frac{1}{1+e^{-x}}) = \frac{e^{-x}}{(1+e^{-x})^2}$

2. Word2vec

a	$J=\sum_{i=1}^{w} y_i \log(\frac{\exp(u_i^T v_c)}{\sum_{w=1}^{W}\exp(u_w^T v_c)})=\sum_{i=1}^{w} y_i[u_i^T v_c-\log(\sum_{w=1}^{W}\exp(u_w^T v_c))$
	since y is one-hot vector, there is only one non-zero item in the summation. If say oth is the current word, $y_o=1$, and $y_{i\neq o}=0$. The loss could be written as :
	$J=(u_o^T v_c-\log(\sum_{w=1}^{W}\exp(u_o^T v_c))$
	$\frac{\partial J}{\partial v_c}=-[u_o-\frac{\partial(\log(\sum_{w=1}^{W}\exp(u_w^T v_c))}{\partial v_c}]=-[u_o-\frac{\sum_{w=1}^{W}\exp(u_w^T v_c)u_w}{\sum_{w=1}^{W}\exp(u_w^T v_c)}]$ $=\sum_{w=1}^{W}\frac{\exp(u_w^T v_c)u_w}{\sum_{w=1}^{W}\exp(u_w^T v_c)}-u_o=\sum_{w=1}^{W}\hat{y}_w u_w-u_o=U\hat{y}-Uy=U(\hat{y}-y)$
b	$\frac{\partial J}{\partial u_w}=-[v_c-\frac{\partial(\log(\sum_{w=1}^{W}\exp(u_w^T v_c))}{\partial u_w}]=-[v_c-\frac{\sum_{w=1}^{W}\exp(u_w^T v_c)v_c}{\sum_{w=1}^{W}\exp(u_w^T v_c)}]=$ $\sum_{w=1}^{W}\frac{\exp(u_w^T v_c)v_c}{\sum_{w=1}^{W}\exp(u_w^T v_c)}-v_c=\sum_{w=1}^{W}\hat{y}_w v_c-v_c=(\hat{y}-1)v_c=(\hat{y}-y)v_c$
c	Since o ∉ {1, ..., K } , the second part is 0 $\frac{\partial J}{\partial u_o}=-\frac{\sigma(u_o^T v_c)(1-\sigma(u_o^T v_c))v_c}{\sigma(u_o^T v_c)}$ $=-(1-\sigma(u_o^T v_c))v_c$
d	$\frac{\partial J_{skip-gram}(word_{c-m,...,c+m})}{\partial U}=\sum_{-m\leq j\leq m, j\neq 0}\frac{\partial F(w_i,\hat{v})}{\partial U}$
	$\frac{\partial J_{skip-gram}(word_{c-m,...,c+m})}{\partial v_c}=\sum_{-m\leq j\leq m, j\neq 0}\frac{\partial F(w_i,\hat{v})}{\partial v_c}$
	$\frac{\partial J_{skip-gram}(word_{c-m,...,c+m})}{\partial v_j}=0$, for all $j\neq c$

g. In the run.py file, I tested the KNN function by computing the 5 nearest neighbor of the first word(**'great'**) among the visualizeWords, which are: [”great”, "cool", "brilliant", "wonderful", "well", "amazing", "worth", "sweet", "enjoyable", "boring", "bad", “dumb","annoying", "female", "male", "queen", "king", "man", "woman", "rain", "snow", "hail", "coffee", “tea”], and the results are ['well', 'annoying', 'amazing', 'bad', ‘queen’]

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(py37) MacBook-Pro-3:a2 joey$ python run.py
sanity check: cost at convergence should be around or below 10
training took 0 seconds
the nearest words to great are ['well', 'annoying', 'amazing', 'bad', 'queen']
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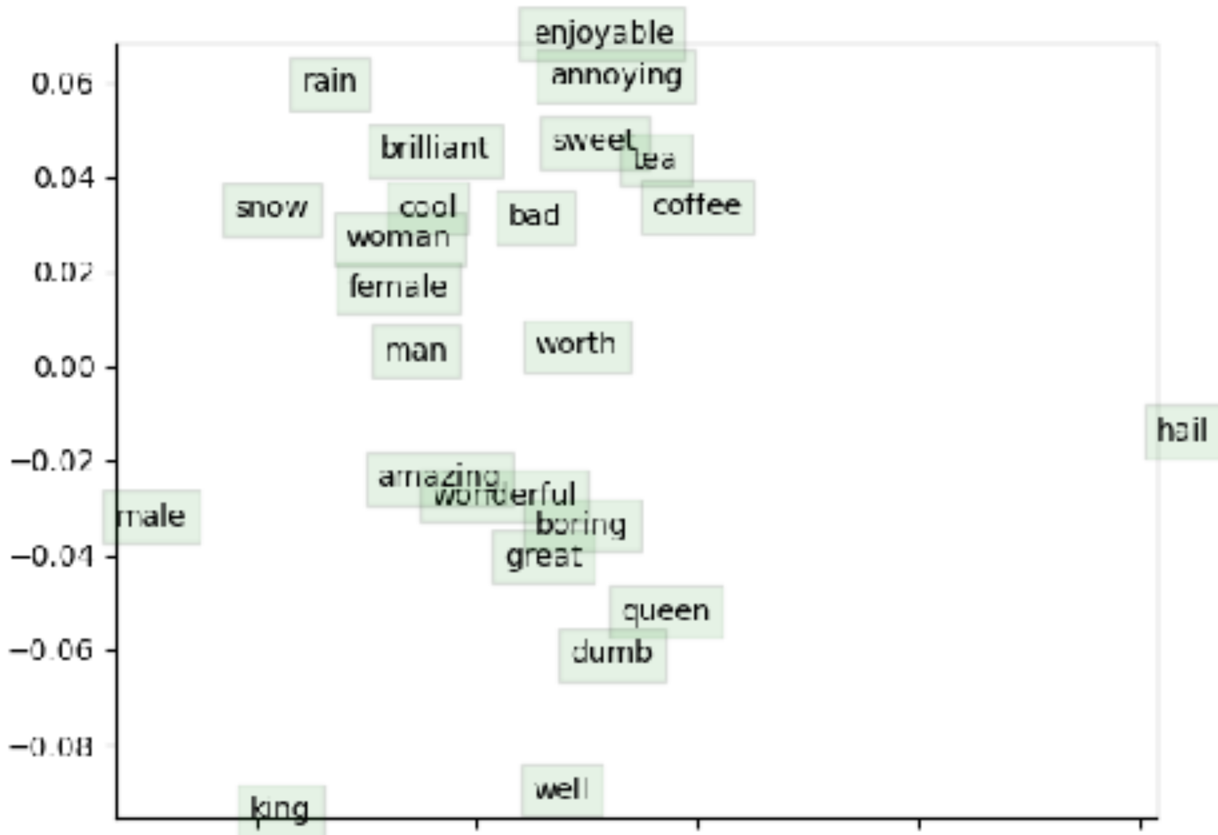
enjoyable
rain
annoying
brilliant
sweet
tea
snow
cool
bad
coffee
woman
female
man
worth
hail
amazing
wonderful
male
boring
great
queen
dumb
well
king
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.1
0.0
0.1
0.2
0.3