Stokes drift through corals

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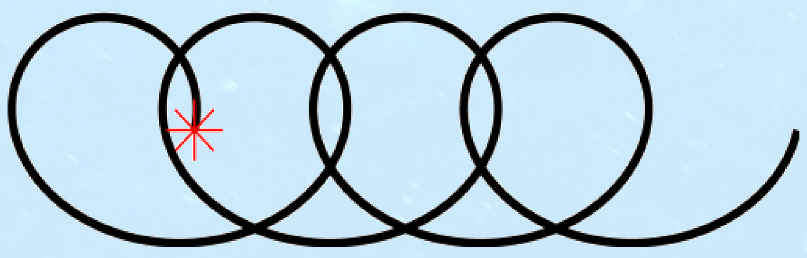
Stokes drift

- Waves in the ocean result in a drift effect of water under the surface, in a phenomenon known as *Stokes drift*, named after Sir George Gabriel Stokes FRS (1819-1903, pictured).
  If we follow the path of an individual 'parcel' of water below the surface undergoing wave motion, we find that it spirals along, drifting with the direction of the wave propagation. In the adjacent diagram, a fluid parcel starts at the position of the red asterisk, and travels with the wave to the right.



- The drift is an entirely horizontal effect, and is well-documented, in, for example, Stokes[1] (1848) and Phillips[2] (1977). In a shallow sea, of depth *1*m, and with waves travelling at ~*1.5*ms^{-1}, with ω=2s^{-1}, a typical velocity is ~*0.05*ms^{-1}.

Applications

Koehl *et al.*[3] and other papers emphasise the importance of flow of water through coral reefs to bring nutrients and oxygen to inhabitants – understanding the flow profile is vital for understanding these ecosystems. We plan to compare these results with those of fieldwork.

Explanation

- We describe the fluid flow in two distinct regimes – the porous layer, dominated by viscous effects and described by Darcy's Law, and the upper layer, which is essentially inviscid.
- Matching the layers at their interface, we derive the wavenumber k as a function of frequency ω, which is a complex number – the waves have both an oscillatory part and a decaying effect due to damping.

Frequency of waves | Viscosity of water | Depth of upper layer

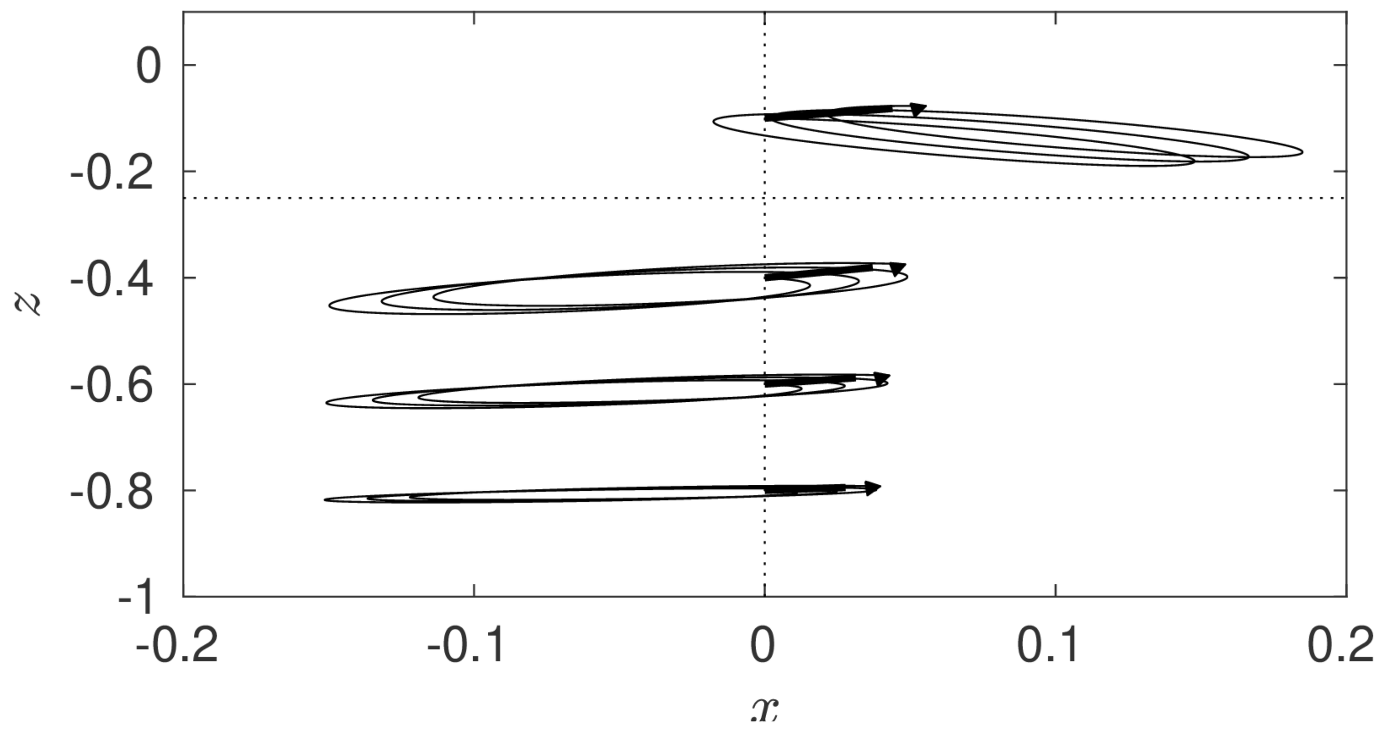
$$\kappa\omega \tanh k\left(D-d\right) - \mathrm{i}\nu \tanh\left[\operatorname{arctanh}\left(\frac{\omega^2}{gk}\right) - kd\right] = 0$$

Permeability of porous layer | Depth of porous layer | Gravity | **Wavenumber**

- The vertical drift can be understood by considering a small particle undergoing wave motion. The particle moves **forwards and down**, followed by **backwards**, and then **back up again**. But as the magnitude of vertical velocity reduces with distance due to damping, there is net motion perpendicular to the wave propagation.

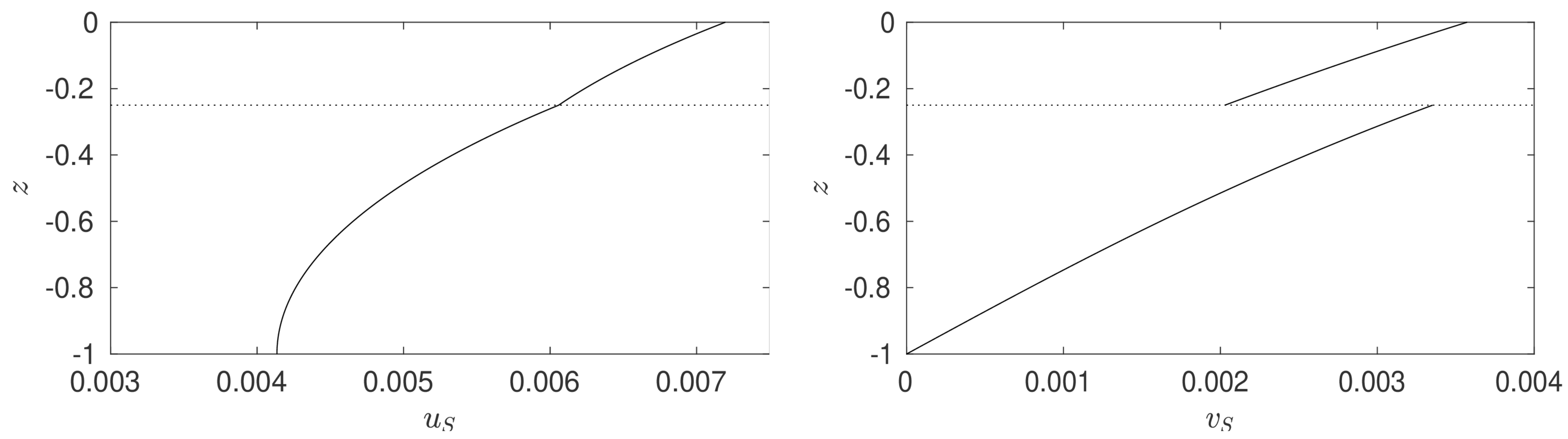
Effect of a porous layer

- Placing a porous layer, like a coral reef, below the wave surface, as shown in the diagram below, damps the waves and means that the amplitude decreases as horizontal distance increases in the direction of wave propagation, denoted *x*.
- As the horizontal drift speed is dependent on the amplitude of the waves, the horizontal drift speed reduces with distance in the direction of wave propagation.
- **Most notably, a vertical drift effect is introduced as a result of the damping.**
- This small drift effect can be seen by tracing out particle paths – in the below case, *D=1*, *d=0.25*.

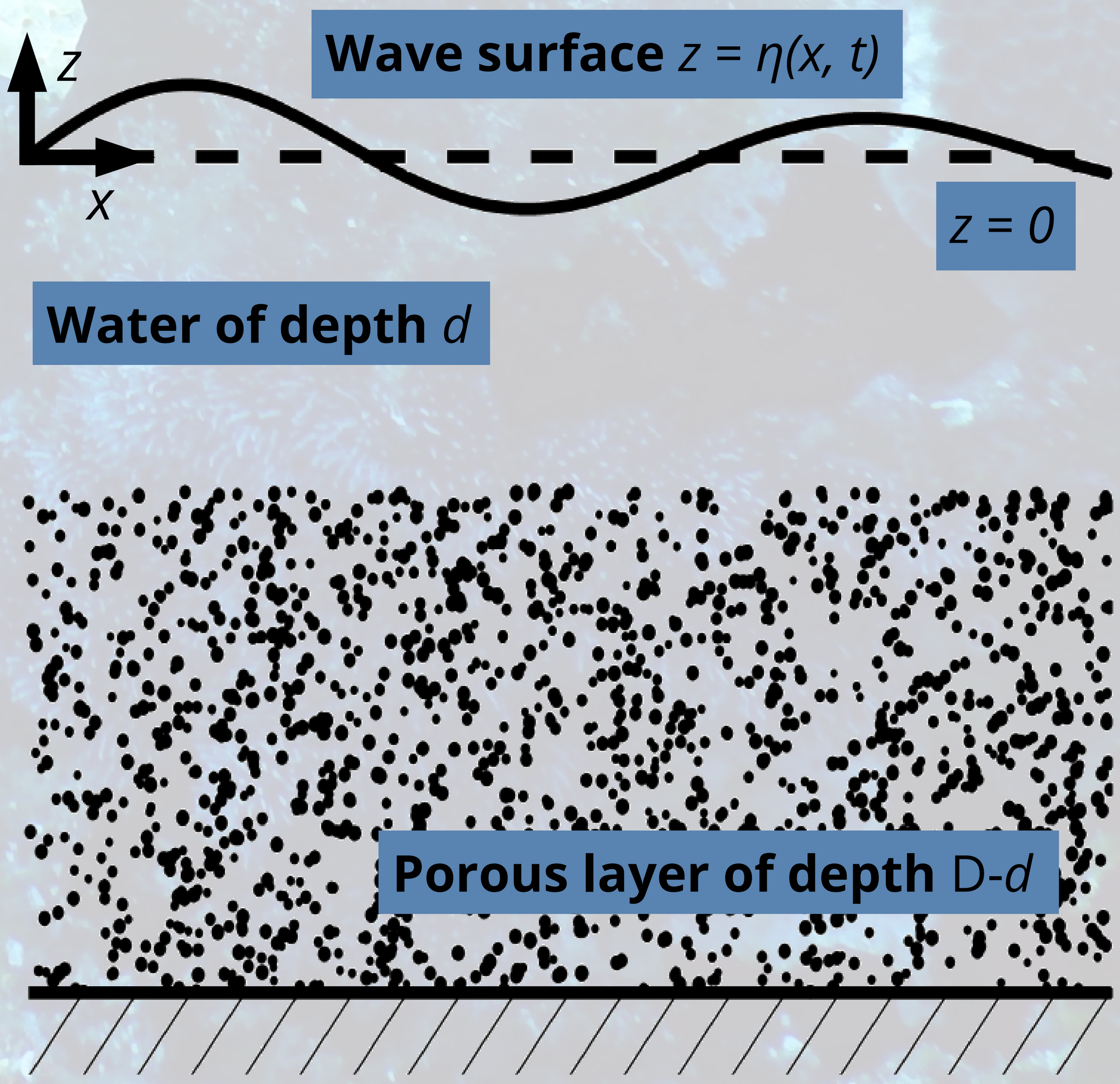


Velocity profiles

Fluid is naturally seen to drift faster in the upper layer, where there is no viscous resistance to its motion. The velocity profiles reduce to those for no reef in the limits as the reef becomes more porous, or thinner.



Stokes drift velocities in the horizontal (left) **and vertical** (right) **directions at** *x = 0.00* **when** *D = 1.00*, *d = 0.25* **and waves have a frequency** ω*=2*



(1) **Stokes, G.G.** 1847 On the theory of oscillatory waves. *Trans. Camb. Philos. Soc.* **8**, 441-455

(2) **Phillips, O.M.** 1977 *The Dynamics of the Upper Ocean*. Cambridge University Press

(3) **Koehl, M.A.R., Powell, T.M. & Dobbins, E.L.** 1997 Effects of algal turf on mass transport and flow microhabitat of ascidians in a coral reef lagoon. *Proc. 8th Int. Coral Reef Symp.* **2**, 1087-1092

based on **Webber, J.J. & Huppert, H.E.** Stokes drift through corals (submitted to Journal of Fluid Mechanics)

Image of coral reef by Tom Fisk on Pexels – free for commercial use

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