AE4441-16: modeling and solving a MILP workshop

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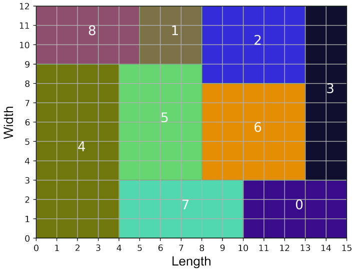

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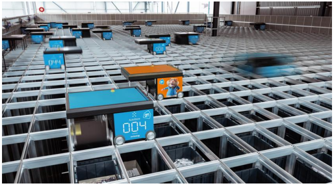

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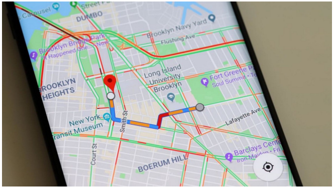

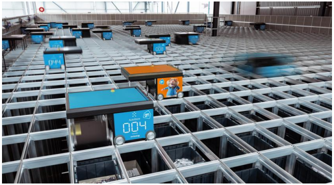

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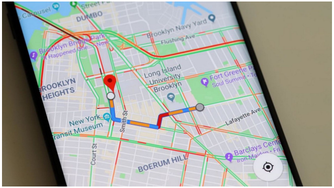

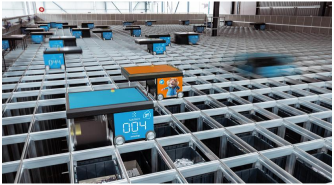

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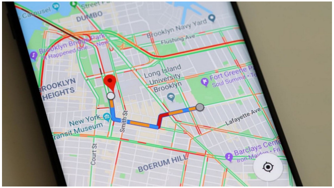

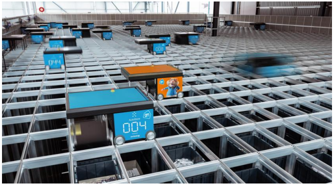

All these practical situations can be solved with **Operations Research (OR)** techniques

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1. translation of a practical problem into a **mathematical model**
2. solution of the mathematical model via **ad-hoc algorithms** $\rightarrow$ The goal is to find the best (**optimal**) solution according to some criteria
3. translation of the solution into **actions to improve the outcome of the practical problem we started from**

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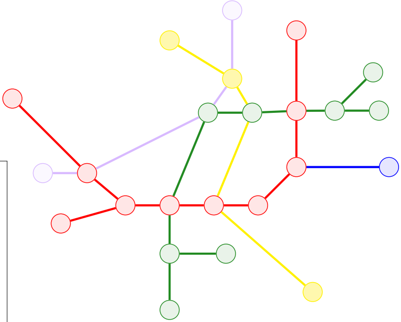

$\mathcal{G} = (\mathcal{N}, \mathcal{E}) \rightarrow \min \sum_{e \in \mathcal{E}} C_e y_e$

$\frac{\partial x}{\partial t} = f_1(x)$, $\frac{\partial^2 x}{\partial t^2} = f_2(x, \frac{\partial x}{\partial t})$

Real-life problem → **Model and solution** → **Assumptions, simplifications, unforeseen circumstances**

Revise model?

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The **cost of going from i to j**, whose distance is $D_{i,j}$ and traveling time is $T_{i,j}$, is $C_{i,j}$

Vehicle Routing Problem (VRP): decision variables

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t_c: visit time of customer c

Vehicle Routing Problem (VRP): objective function

$$\min \quad \underbrace{\sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}^-_{i,k}} C_{i,j} D_{i,j} x^k_{i,j}}_{\text{A}} + \underbrace{\sum_{k\in\mathcal{K}} C_k y_k}_{\text{B}} \tag{1}$$

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- **A**: cost due to routing of vehicle fleet

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- **A**: cost due to routing of vehicle fleet
- **B**: cost due to deployment of vehicle fleet

Vehicle Routing Problem (VRP): constraints

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}_{i,k}^{-}} x_{c,j}^{k} = 1 \qquad \forall c \in \mathcal{C} \qquad (2)$$

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Vehicle Routing Problem (VRP): constraints

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Every customer should be visited exactly once.

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Every customer should be visited exactly once. Note: we can impose this constraint either on outgoing edges from customer c (as above), or equivalently on ongoing edges. Because of the conservation of flow requirement, it is the same

Vehicle Routing Problem (VRP): constraints

$$\sum_{j \in \mathcal{N}^-_{0,k}} x^k_{0,j} = y_k \qquad \forall k \in \mathcal{K} \tag{3}$$

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If a vehicle leaves the depot, then it is flagged as used

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$$\sum_{j \in \mathcal{N}^{+}_{0,k}} x^{k}_{j,0} = y_k \qquad \forall k \in \mathcal{K} \tag{4}$$

Vehicle Routing Problem (VRP): constraints

$$\sum_{j \in \mathcal{N}_{0,k}^{-}} x_{0,j}^{k} = y_k \qquad \forall k \in \mathcal{K} \tag{3}$$

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$$\sum_{j \in \mathcal{N}_{0,k}^{+}} x_{j,0}^{k} = y_k \qquad \forall k \in \mathcal{K} \tag{4}$$

If a vehicle was flagged as used, then it must return to the depot

Vehicle Routing Problem (VRP): constraints

$$\sum_{j \in \mathcal{N}^{-}_{c,k}} x^{k}_{c,j} - \sum_{j \in \mathcal{N}^{+}_{c,k}} x^{k}_{j,c} = 0 \qquad \forall c \in \mathcal{C}, k \in \mathcal{K} \tag{5}$$

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Conservation of flow per customer node and vehicle

Vehicle Routing Problem (VRP): constraints

$$t_j \geq t_i + T_{i,j} - (1 - x_{i,j}^k)M \qquad \forall k \in \mathcal{K}, i \in \mathcal{C}, j \in \mathcal{N}_{i,k}^- \cap \mathcal{C} \tag{6}$$

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Time precedence constraints. If we go from i to j with vehicle k, then we cannot arrive at j sooner than when we arrived at i, plus the traveling time from i to j.

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$$t_j \geq +T_{0,j} - (1 - x_{0,j}^k)M \qquad \forall k \in \mathcal{K}, j \in \mathcal{N}_{0,k}^- \qquad (7)$$

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Time precedence constraints from the origin (we assume we can leave the depot at time 0. **This might not be the case if we have time restrictions on some vehicles**)

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$$T_c^{LB} \leq t_c \leq T_c^{UB} \qquad \forall c \in \mathcal{C} \tag{8}$$

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Every customer should be visited within the specified time-window

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Maximum range constraints. Note that we consider every possible edge we can transverse with vehicle k

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$$\sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{N}^-_{c,k}} C_c x^k_{c,j} \leq Q_k \qquad \forall k \in \mathcal{K} \qquad (10)$$

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Payload capacity constraints We consider only edges stemming from customers we can transverse with vehicle k.

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Payload capacity constraints We consider only edges stemming from customers we can transverse with vehicle k. The depot does not have a required demand per se. So, we need to ensure the necessary demand we load there is sufficient to meet the requirements of the customers we will visit, without exceeding the maximum payload

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- **VRP with backhaul**. First we only perform deliveries, then only pickups to **go back to the depot with goods (operationally more efficient)**
- **electric fleet** with battery discharge and potential to be recharged along the route

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