The deadline for this submission is **17:59:59 Monday 9 March** (Sydney time). No submission will be accepted past the deadline.

Question 1

[**5 marks**] Suppose you are given a list of n unique points on a number line $S = [x_1, x_2, x_3, \ldots, x_n]$. You must answer n queries in the form *"How many points lie on the interval between x and y?"*, where $x, y \in S$ are distinct and are also points on the number line.

Design a $O(n \log n)$ algorithm that answers all of these queries and describe it in English.

Question 2

Given a sequence of n integers S, and an integer p, produce a $O(n \log n)$ algorithm to determine if there exist two values $x, y \in S$ where $\frac{x}{y} = p$. [**5 marks**] Produce another algorithm that solves this in *expected* $O(n)$ time. [**5 marks**]

Question 3

[**14 marks**] Suppose you have n metal balls of various weights, and n containers for these balls. Each container is designed to house balls of a specific weight. Your task is to assign the balls into the containers one-to-one so that in each case the ball's weight matches the container's expected weight. Although *some balls may have the same weight*, you are promised that a pairing exists: each container can be paired with a ball of the appropriate weight. Indeed, if several balls have the same weight, then the same number of containers will be targeting that weight.

Unfortunately, you do not have any direct way of comparing the weights of two balls against each other, or any comparison between two containers. The containers are advanced, they can identify if they are housing the correct ball by its weight, and indicate the result by lighting up. The container lights up red if the ball is lighter than expected, green if the ball is the correct weight, and blue if the ball is heavier than expected. After the container reacted to a ball's weight, you are free to remove the ball and put it elsewhere if you desire. Testing a ball in a container and observing the color of the light counts as an atomic operation.

3.1 Design a simple algorithm solving that matches each ball to an appropriate container and describe it in English.

3.2 We now look for an algorithm that would usually be more efficient. Design an algorithm with *expected* $O(n \log n)$ time complexity that matches each ball to its correct container. Describe your algorithm in pseudo-code.

Hint: Think of a modified quicksort algorithm.

3.3 Prove formally that your algorithm is correct.

Question 4

[**8 marks**] Read the review material on the class website on asymptotic notation and basic properties of logarithms, and then determine if $f(n) = \Omega(g(n))$, $f(n) = O(g(n))$ or $f(n) = \Theta(g(n))$.

You can assume n is an integer.

4.1 $f(n) = \Gamma(n)$ and $g(n) = n^n$.

4.2 $f(n) = \log_3(n^2 n^{n \log_3 n})$ and $g(n) = n(\log_3 n)^2$.

4.3 $f(n) = (\log_5 n)^{\frac{5}{3}}$ and $g(n) = \log_5 \sqrt[3]{n}$.

4.4 $f(n) = n^2(3n + \cos(\frac{\pi n}{2}))$ and $g(n) = n^3$.

Question 5

[**8 marks**] Determine the asymptotic growth rate of the solutions to the following recurrences. Solve these with or without using Master's Theorem.

5.1 $T(n) = 5T(\frac{n}{4}) + 2n + n \sin \frac{n}{2}$

5.2 $T(n) = 2T(\frac{n}{5}) + n^2\sqrt{n}$

5.3 $T(n) = 3T(\frac{n}{2}) + 3^{\log_2 \frac{n+3}{4}}$

5.4 $T(n) = T(n-1) - \log \frac{n-1}{n}$