

cs3821 Assignment 3

Jennifer Xu (z5207930)

Question 3

3.1 Table

| x_1 | x_2 | x_3 |
|-------|-------|-------|
| 4 | 0 | 0 |
| 2 | 1 | 0 |
| 1 | 0 | 3 |
| 0 | 2 | 0 |

3.2 Brute Force Algorithm - Pseudo Code

```
nSolutions = 0
def formS(array, target):
    Global nSolutions
    # Base Case: Array is empty
    if (len(array) == 0):
        if (target != 0): return False
        nSolutions += 1
        return True

    for i in range(target + 1):
        # Recursively call formS
        # each time taking out the last item in the array
        formS(array[:-1], target - i * array[-1])
```

Total number of solutions is found with the variable `nSolutions`.

3.3 Time Complexity

M is the smallest value in C .

N is the length of C .

S is the target number.

Each recursive call has a single subproblem. Each subproblem has a max of S/M recursive calls. The recursion terminates after being called N times. Therefore it's S/M problems wide and N problems wide. Hence the upper asymptotic bound is $N^{(S/M)}$. Note: Since our target sum is able to keep decreasing, the expected time complexity is much smaller.

Therefore, the big-O complexity is $O(n^n)$.

3.4 Dynamic Programming Algorithm

Our subproblem, $P(n)$, would be to find the number of solutions for our target sum n using our coefficients C .

Hence, the number of solutions $P(n)$ would be found by adding the amount we already know of to the amount of to $P(n - C[i])$.

For example, given $S = 4$, $C = [1, 2, 3]$,

$$\begin{aligned} P(3) &= P(3 - C[0]) + P(3 - C[1]) + P(3 - C[2]) \\ &= P(2) + P(1) + P(0) \end{aligned}$$

...And so on, repetitively using our subcases and building up the number of ways we can find $P(n)$.

To do this, we utilise a table P , size S , to record the number of solutions for $P(n)$. First we iterate through C , and then in a nested iteration, we iterate through $P(c[i])$ to $P(S)$.

Therefore by the time we finish our nested while loop, $P[S]$ will hold the number of solutions to the number of combinations of the linear equation represented by C to achieve the target S .