Assignment 2:
Part A
ECON 409 & ECON 442B

Submit a single Jupyter Notebook converted to PDF format that contains all answers to the assigned questions.

All code cells must be fully executed prior to submission, and all outputs must be visible in the PDF. Ensure that code, tables, and figures are not cut off at the edges of notebook cells after conversion.

All figures and plots should be professionally formatted and clearly labeled with appropriate titles, axis labels, and legends where applicable.

Written answers, explanations, and equations must be provided in Markdown cells, not as comments within code cells. Clearly label each response (e.g., Question 1, Question 2, etc.).

Theoretical Questions

1. Consider the state-space model presented in the lecture:

$$y_t = x_t + v_t$$

$$x_t = ax_{t-1} + w_t$$

$$|a| < 1$$

With $\mathrm{v_t} \sim \mathrm{N}(0, \sigma_{\mathrm{v}}^2)$, $\mathrm{w_t} \sim \mathrm{N}(0, \sigma_{\mathrm{w}}^2)$, and v_t, w_t is independent white noise.

   a. Write down the recursive formula for the filtered estimate $\widehat{x_t}$ using a constant gain $k \in [0,1]$.

   b. Explain intuitively what happens to the filtered series $\widehat{x_t}$ and the resulting multi-step forecasts $\widehat{y_{t+n}} = a^n \widehat{x_t}$ when k is very close to 0 versus when k is very close to 1.

   c. State the formula for the optimal Kalman gain k^{Kalman} and briefly explain when it lies between 0 and 1.

2. The Sharpe Ratio:
   a. Define the Sharpe ratio and carefully explain what it measures in economic terms. Why is the risk-free rate subtracted in the numerator?

b. Explain the pitfall of the Sharpe ratio related to and discuss why it can make the Sharpe ratio misleading when comparing strategies or assets.
c. Why is it common practice to convert annual risk-free rates to the same frequency as the portfolio returns (e.g., divide by 12 for monthly) before computing monthly excess returns and the Sharpe ratio?

3. Consider the CAPM Model:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

Where R_i(t) and R_M(t) are excess returns (over the risk-free rate) of the strategy/asset i and the market proxy, respectively, and $E[e_i(t)] = 0$.

a. Provide a precise economic interpretation of α_i and β_i in the context of evaluating an active investment strategy.
b. Explain why a negative beta combined with positive alpha is particularly attractive to investors, especially during periods of market stress.
c. Why might a strategy with a very high positive alpha but a beta close to 1.0 still underperform a passive market index during a prolonged bull market, even if the alpha is statistically significant?

4. Consider the definitions of skewness and kurtosis presented in the lecture:

$$\text{skew} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\frac{R_t - \bar{R}}{\hat{\sigma}}\right)^3, \qquad \text{kurtosis} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\frac{R_t - \bar{R}}{\hat{\sigma}}\right)^4 - 3$$

a. Provide the economic interpretation of a large negative skewness in monthly excess returns of a portfolio or trading strategy. What does it imply about the distribution of outcomes an investor should expect?
b. Explain why, under the assumption of normally distributed returns (skew = 0, kurtosis = 0), the mean and standard deviation are sufficient statistics for describing risk and expected performance, but this breaks down when skewness and/or excess kurtosis are materially nonzero.

5. A 30-year US Treasury bond has the following characteristics:

    - Face value (par value): $1,000
    - Annual coupon rate: 4% (paid semiannually)
    - Current market price: $950 (clean price, quoted per $1,000 face value)

    (a) Write the bond pricing equation that must be solved for the semiannual yield to maturity i(the constant semiannual discount rate that equates the present value of all promised cash flows to the current market price of $950). Specify the number of periods $N$and the size of each coupon payment.

    (b) If the bond were trading at par ($1,000) instead of $950, what would the semiannual YTM be? What does this tell you about the relationship between coupon rate, market price, and YTM?

6. Futures contracts are standardized versions of forward contracts that address the two main limitations of forwards: illiquidity and counterparty risk. Explain clearly what each of these two limitations means in the context of forward contracts and why they pose practical problems for market participants (e.g., hedgers or speculators).

7. In the standard Taylor rule:

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - \bar{y_t})$$

    a. Explain clearly what the coefficients 0.5 (on inflation) and 0.5 (on output gap) imply about the Federal Reserve's relative concern for inflation versus output stabilization.
    b. If the actual federal funds rate is persistently below the rate prescribed by the Taylor rule for a long period, how is monetary policy conventionally described? What are the main arguments that central banks give to justify such a situation?

8. Find all the text changes in the text of FOMC statements
    - September 18, 2024
    - December 18, 2024
    - January 29, 2025

    https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm

    Explain how those changes would affect expectations of feds funds rate over the next 12 months.

Coding and Bloomberg Questions

You will need to schedule time with a Bloomberg terminal to answer the following questions and to construct the necessary dataset.

1. Log into the Bloomberg Terminal and search for PCE CYOY Index. Plot the data from 01/01/1959-11/30/2025. Plot the Actual Economic Release Values on the Series Release Date (see the Bloomberg Terminal Hints guide for how to do this).
    a. When was the last data point released?
    b. What was the reference month (the month for which inflation was being measured)?
    c. Turn off the series release date setting. When is it indexing the last release now?
    d. Describe how changes in the dates you use can affect the forecasts you make.
2. The *hw2_data.csv* contains columns for SHY (a 1–3-year bond ETF), PCE inflation (%), the CFNAI index, and the fed funds rate. The Chicago Fed National Activity index (CFNAI) is described as "A zero value for the index indicates that the national economy is expanding at its historical trend rate of growth; negative values indicate below-average growth; and positive values indicate above-average growth."

    For later questions we will be using this as our "output gap" variable.

    a. Plot the CFNAI variable.
    b. Think about how we used trend to construct the output gap using industrial production in class. Why might it make sense to use the CFNAI as a proxy for the output gap?

3. Predicting the Fed Funds Rate:

    a. Calculate the inflation gap, and assume we are using a target rate of inflation of 2%.
    b. Fit the following Taylor rule model over a rolling 3-year window and use it to make a prediction for the next out of sample fed funds rate:

$$\widehat{fedfunds}_t = \alpha + \beta_1 CFNAI_t + \beta_2(\pi_t - 2) + \nu_t$$

    c. Plot the predicted fed funds rate against the observed rate.

4. We will build a simple trading strategy that goes long on the SHY when the predicted fed funds rate is higher than the observed rate and short when the predicted fed funds rate is lower than the observed rate. Note that when you get a signal that's either 1 or -1, you hold it only for one time period.

    a. Generate a signal column that corresponds to the strategy described above.
    b. Plot the strategy cumulative returns.
    c. Calculate the Annualized Return.
    d. Calculate the skewness of the returns. What does this tell you about your strategy?