SHG phase matching

Phase matching :

$$\Delta k = 0$$

$$\Rightarrow \frac{n_1 \omega_1}{c} + \frac{n_2 \omega_2}{c} = \frac{n_3 \omega_3}{c}$$

for SHG,

$$\omega_1 = \omega_2 \quad \Rightarrow \quad n(\omega) = n(2\omega)$$

$\therefore$ We need to consider phase matching such that incident & generated pulses have different polarizations & thus experience different refractive indices

$$n_o(\omega)\,\omega + n_o(\omega)\,\omega = n_e(2\omega) \cdot 2\omega$$

i.e. $n_o(\omega) = n_e(2\omega)$ [$n_e(\omega) < n_o(\omega)$ negative uniaxial crystal]

Now,

$$\left(\frac{1}{n_e(\theta)}\right)^2 = \frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}$$

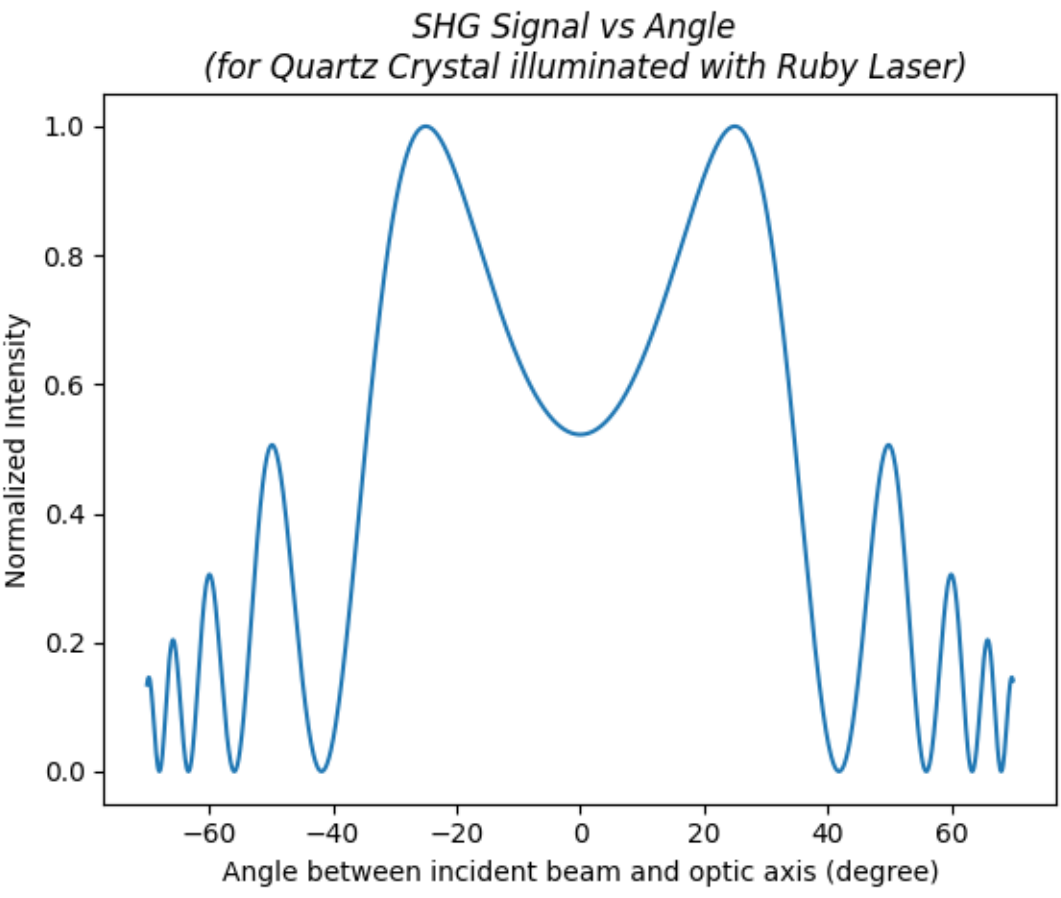
$$\therefore \frac{1}{n_e(\theta)} = \sqrt{\frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}}$$

$$\therefore \Delta k = n_e(2\omega) \cdot \frac{2\omega}{c} - 2n_o(\omega) \cdot \frac{\omega}{c}$$

Since, the crystal is tilted

$$L_{eff} = \frac{L}{\cos(\theta)} \quad \text{(path length changes)}$$

$$\therefore \frac{I}{I_{max}} = \left[\frac{\sin\left(\Delta k \cdot L_{eff}/2\right)}{\Delta k \cdot L_{eff}/2}\right]^2$$


SHG Signal vs Angle
(for Quartz Crystal illuminated with Ruby Laser)
Normalized Intensity
1.0
0.8
0.6
0.4
0.2
0.0
−60
−40
−20
0
20
40
60
Angle between incident beam and optic axis (degree)