

Last Period 1

Problem 1:

P	q	$P \text{ NAND } q$	$P \text{ NOR } q$	$P \geq q$
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

Problem 2:

P	q	$P \rightarrow q$	$\text{NOT } P \text{ OR } q$	E
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

P	q	r	$\overbrace{r \text{ OR NOT } p}^a$	$\overbrace{\overline{q} \rightarrow a}^b$	$P \rightarrow b$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

P	Q	$P \text{ OR } Q$	$P \text{ AND } Q$	$(P \text{ OR } Q) \rightarrow (P \text{ AND } Q)$
0	0	0	0	1
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

Unim

interior
Exapt

Problem 3:

exclus. v.

Problem 4:

$$(P \overset{\circ}{\text{NAND}} \overset{\circ}{Q}) \overset{\circ}{\text{NAND}} R \stackrel{?}{=} 0 \quad \text{NAND} \not\equiv \text{ass.}$$

$$\overset{\circ}{P} \overset{\circ}{\text{NAND}} (\overset{\circ}{Q} \overset{\circ}{\text{NAND}} R) \stackrel{?}{=} 1$$

$$(\overset{\circ}{P} \rightarrow \overset{\circ}{Q}) \rightarrow \overset{\circ}{R} \stackrel{?}{=} 0 \quad \rightarrow \not\equiv \text{ass.}$$

$$\overset{\circ}{P} \rightarrow (\overset{\circ}{Q} \rightarrow \overset{\circ}{R}) \stackrel{?}{=} 1$$

$$(\overset{\circ}{P} \overset{\circ}{\text{NOR}} \overset{\circ}{Q}) \overset{\circ}{\text{NOR}} \overset{\circ}{R} \stackrel{?}{=} 0$$

$$\overset{\circ}{P} \overset{\circ}{\text{NOR}} (\overset{\circ}{Q} \overset{\circ}{\text{NOR}} \overset{\circ}{R}) \stackrel{?}{=} 1$$

NOR $\not\equiv$ ass.

1111
0000

Problem 5

TRUE

False

NOT p

NOT q

Problem 6

Raise AND $\rightarrow P \text{ NOT}(q \rightarrow p)q \text{ XOR OR NOR} \equiv \bar{q} q \rightarrow p \bar{p} p \rightarrow q \text{ NAND TRUE}$

8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
4	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
2	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Problem 7

$$\begin{array}{c|cc} p & q & \otimes \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array} \quad p \otimes q = q \otimes p$$
$$(p \otimes q) \otimes r = 0$$
$$p \otimes (q \otimes r) = 0$$

commutative And associative

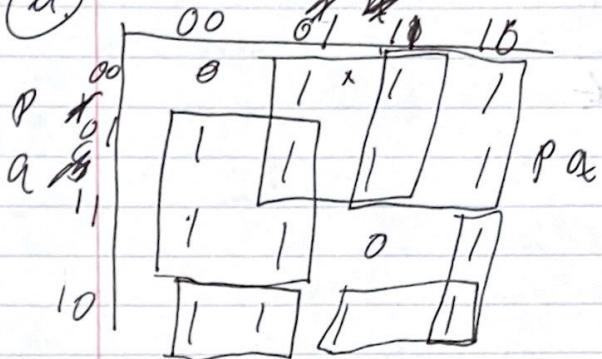
Problem 8

$$a = \bar{p}\bar{q}r + \bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + p\bar{q}\bar{r} + pq\bar{r}$$

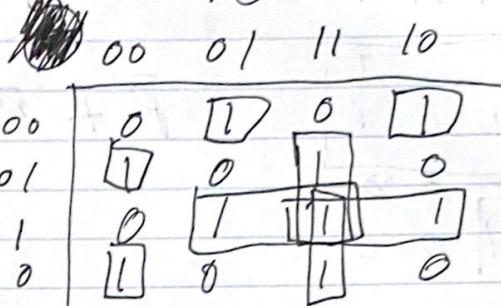
$$b = \bar{p}q\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r}$$

Program 11/12

(a)



rs



$$\overline{R} \overline{S} \overline{P} \overline{A} + \overline{R} \overline{S} \overline{P} \overline{A} + \overline{R} \overline{S} P \overline{A} + \overline{R} S \overline{P} \overline{A} + \dots$$

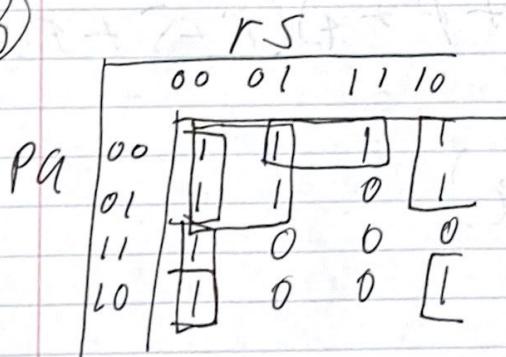
~~... + SPq + RSa + RSA + RS P + RPa~~

~~PAR + PRS + RPA +~~

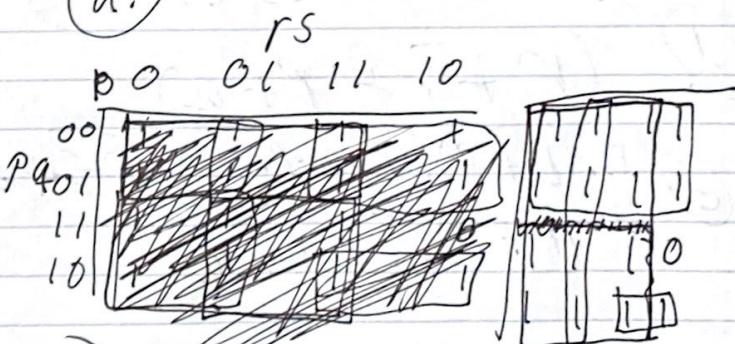
~~CAP + SPA + RP~~

Prime Implicants

(b)



(d.)



$$\overline{S} \overline{P} \overline{A} + \overline{R} \overline{S} + S \overline{P} \overline{A} + \dots$$

~~+ PS + RP~~

Prime Implicants

$$\overline{P} + \overline{P} \overline{R} + \overline{R} S + \overline{P} \overline{A} R$$

Q. $r s$

	00	01	11	10
00	1		1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

$$\bar{P} + \bar{r}\bar{a}$$

Problem ~~13/14~~:

13/14 (a) $(\bar{r} + \bar{s} + \bar{p} + \bar{q})(rs + p + q)$

13/14 (b) ~~$(r + s + q)(r + s + p)(p + q + s)(p + q + r)$~~

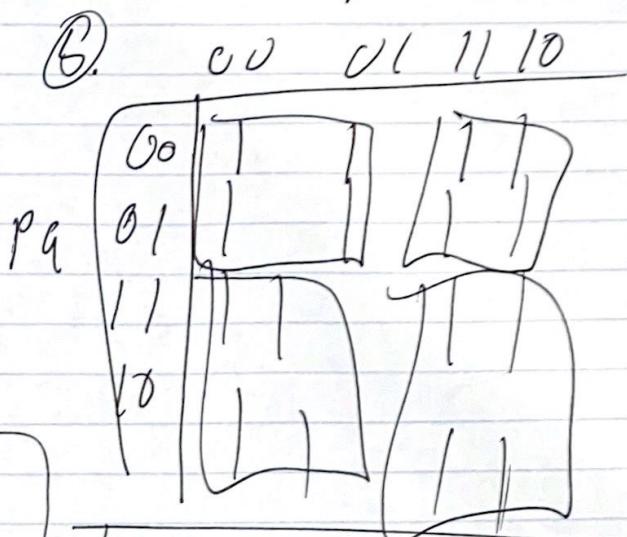
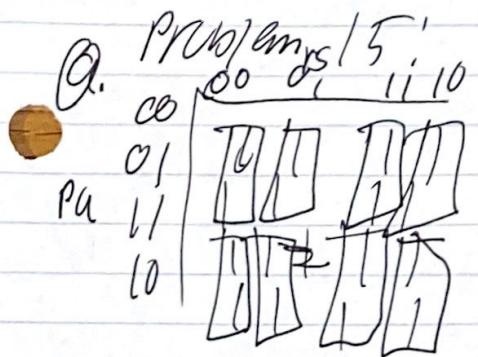
~~$(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p}) (\bar{r} + \bar{s} + \bar{q}) (\bar{r} + \bar{s} + \bar{p} + \bar{q})$~~

13/14 (c) $(\bar{p}a + \bar{r}s)(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})$
 $\cdot (\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})$

13/14 (d) $(r + \bar{s} + p + q)$

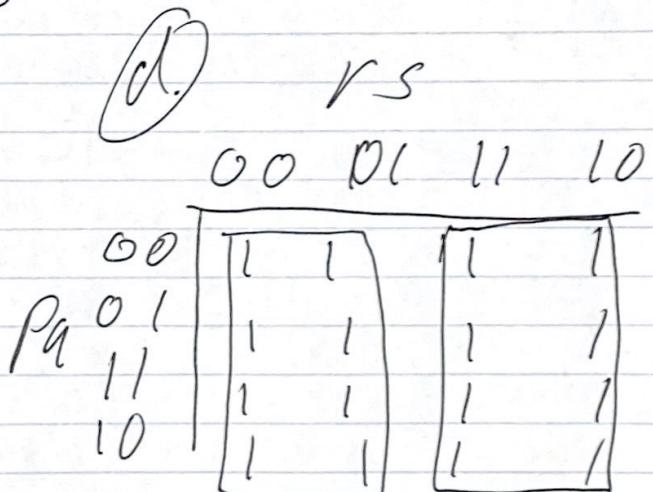
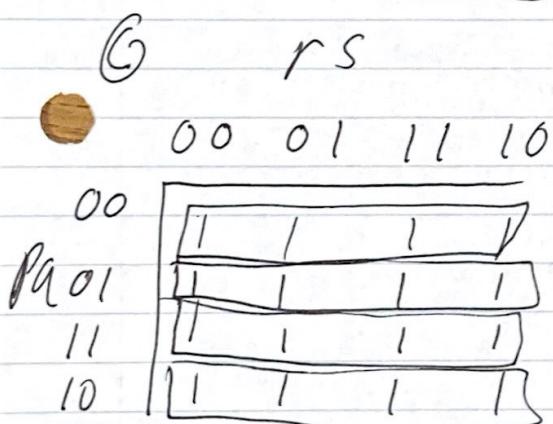
13. $(r + p)(\bar{r} + s + p + q)(\bar{r} + \bar{s} + \bar{p} + \bar{q})$

14. $(r + p)(\bar{r} + p + q)$



$$\begin{aligned}
 & \bar{r} \bar{s} \bar{p} + \bar{r} \bar{s} p + \dots \\
 & \dots + \bar{r} s \bar{p} + \bar{r} s p + \dots \\
 & \dots + r s \bar{p} + r s p + \dots \\
 & \dots + r \bar{s} \bar{p} + r \bar{s} p + \dots
 \end{aligned}$$

$$\bar{r} \bar{p} + r p + \bar{r} p + r \bar{p}$$



$$\bar{p} \bar{q} + p \bar{q} + \bar{p} q + p q$$

$$\bar{r} + r$$

Problem 16:

S.

a.

(A)	pqr	$\overline{p \wedge q}$	$\overline{par} \rightarrow \overline{porq}$	$\overline{(B)} / (p \rightarrow q) (a \rightarrow r)) \rightarrow (p \rightarrow r,$
	0 0 0	0	1	
	0 0 1	0	1	
	0 1 0	1	1	
	0 1 1	1	1	
	1 0 0	1	1	
	1 0 1	1	1	
	1 1 0	1	1	
	1 1 1	1	1	

Tautology C Tautology

	$p q$	$(p \rightarrow q) \rightarrow p$	$(p \equiv (q \vee r)) \rightarrow q \rightarrow p \wedge r$
	0 0	0	
	0 1	0	
	1 0	1	
	1	1	

not tautology D not tautology

Program 17:

Batch expressing one each to tree

$P \wedge q$	$P \equiv P$	$P \vee Q \wedge \emptyset$	$P \vee Q \wedge \emptyset \equiv P$	$(P \equiv P) \equiv (P \vee Q \wedge \emptyset \equiv P)$
0 0	* 1	0	1	1
0 1	* 1	0	1	1
1 0	* 1	1	1	1
1 1	* 1	1	1	1

Problem 18:

P	$P \equiv P$
0	1
1	1

p	q	$(P \equiv q)$	$(q \equiv P)$	$a \equiv b$
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	1	1	1

$p \text{ and } r$	$(P \equiv q)$	$(q \equiv r)$	$(a \equiv r)$	$\neg(P \rightarrow r)$	$(P \equiv r)$
0 0 0	1	1	1	1	1
0 0 1	1	0	0	1	0
0 1 0	0	0	0	1	1
0 1 1	0	0	1	1	1
1 0 0	0	0	1	1	0
1 0 1	0	0	0	1	0
1 1 0	1	0	0	1	1
1 1 1	1	1	1	1	1

$\neg(P \equiv q) \text{ and } (a \equiv r) \rightarrow (P \equiv r)$

p	q	$\neg p$	$\neg q$	$(P \equiv q)$	$(\bar{P} \equiv \bar{q})$	$(P \equiv q) \equiv (\bar{P} \equiv \bar{q})$
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

5.

P	q	$P \wedge q$	$\neg P$	$P \vee q$
0	1	0	1	1
0	0	0	1	0
1	1	1	0	1
1	0	0	0	1

6.

P	q	$P \wedge 1$	$(q \wedge 1)$	$\neg (P \wedge q)$	$(P \wedge q) \wedge 1$	1	E
0	0	0	0	1	0	0	1
0	1	0	0	1	0	0	1
1	0	1	0	1	0	0	1
1	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
0	0	0	0	1	0	0	1
0	1	0	0	1	0	0	1
1	1	1	1	1	1	1	1

7.

$P \neg q$	$P \vee \neg q$	\equiv	$\neg q \vee P$	E
0 0	0	1	0	1
0 1	1	1	1	1
1 0	1	1	1	1

$P \text{ and } Q$	$(P \text{ OR } Q)$	$(Q \text{ OR } R)$	\equiv	$((P \text{ OR } Q) \text{ OR } R)$
0 0 0	0 0	0 0	0	0 0 0
0 0 1	0 1	1 0	1	0 1 0
0 1 0	1 0	1 1	1	1 1 0
0 1 1	1 1	1 1	1	1 1 1
1 0 0	1 0	0 1	0	0 0 1
1 0 1	1 1	1 1	1	1 1 1
1 1 0	1 1	1 1	1	1 0 1
1 1 1	1 1	1 1	1	1 1 1

q.

$P \text{ and } Q$	$\overline{Q \text{ and } R}$	$P \text{ and } \overline{Q}$	$\overline{P \text{ and } Q}$	$\overline{P \text{ and } R}$	$\overline{Q \text{ or } R}$	$\overline{R \text{ or } C}$	$C \equiv C$
0 0 0	0	0	0	0	0	0	1
0 0 1	1	0	0	0	0	0	1
0 1 0	0	0	0	0	0	0	1
0 1 1	1	0	0	0	0	0	1
1 0 0	0	0	0	0	0	0	1
1 0 1	1	1	0	1	1	1	1
1 1 0	1	1	1	0	1	1	1
1 1 1	1	1	1	1	1	1	1

⑩

P	$\overline{P \text{ AND } Q}$	$a = P$
0	0	1
1	1	1

⑪

P	$\overline{P \text{ OR } Q}$	$a \neq P$
0	0	1
1	1	1

⑫

P	$\overline{P \text{ AND } Q}$	$a \equiv P$
0	0	1
1	0	1

⑬

P	$\neg P$	$\neg \neg P$	$a \equiv P$
0	1	0	1
1	0	1	1

14.

<u>PQR</u>	<u>ar</u>	<u>$P+Q R$</u>	<u>$P \oplus Q R$</u>	<u>$P \otimes R Q$</u>	<u>$b and d$</u>	<u>$d \equiv a$</u>
0 0 0	0	0	0	0	0	1
0 0 1	0	0	0	0	0	1
0 1 0	0	1 0	1	0	0	1
0 1 1	1	1	1	1	1	1
1 0 0	0	1	1	1	1	1
1 0 1	0	1	1	1	1	1
1 1 0	0	1	1	1	1	1
1 1 1	1	1	1	1	1	1

<u>P</u>	<u>$1 OR P$</u>	<u>$1 OR P = 1$</u>
0	1	1
1	1	1

<u>P</u>	<u>$P and P$</u>	<u>$P P = P$</u>
0	0	0
1	1	1

<u>P</u>	<u>$P OR P$</u>	<u>$P + P = P$</u>
0	0	0
1	1	1

<u>P</u>	<u>q</u>	<u>$P and q$</u>	<u>$Pq + P$</u>	<u>$E P = q$</u>
0 0	0	0	0	0
0 1	0	0	0	0
1 0	0	0	1	0
1 1	1	1	1	1

<u>P</u>	<u>q</u>	<u>$P and q$</u>	<u>$a and p$</u>	<u>$b = p$</u>
0 0	0	0	0	0
0 1	1	0	0	0
1 0	0	0	1	1
1 1	1	1	1	1

19. (a) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \neg p & \neg p \text{ OR } q & p \wedge q & p \wedge q & E \\ \hline 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline \end{array}$

(b) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \bar{p} & \bar{p} \wedge q & \bar{p} \vee q & p \text{ OR } q & E \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hline \end{array}$

20. (a) $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline p & q & \bar{p} \text{ and } q & \neg q \wedge r & \neg p & \neg q & \neg p \vee \neg q & a \equiv b \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(b) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \neg p & \neg q & \text{NOT}(p \wedge q) & \neg p \vee \neg q & E \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(c) $\begin{array}{|c|c|c|c|c|c|} \hline p & \neg p & \neg p \text{ and } p & \text{not } p \vee \text{not } p & E \\ \hline 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(d)	$P \mid \neg P$	$\overline{P \wedge P}$	$\overline{\text{not}(P) \mid P \text{ or } \neg P}$	E
0	0	1	0	
1	0	0	0	

(21.)

$p \mid q$	$\overline{p \rightarrow p}$	$\overline{p \rightarrow q}$	a and b	$\overline{p \equiv q}$	E
0 0	1	1	0	0	
0 1	0	1	0	0	
1 0	1	0	0	0	
1 1	1	1	1	1	

$p \mid q$	$\overline{p \equiv q}$	$\overline{p \rightarrow q}$	$\overline{q \rightarrow b}$
0 0	1	1	1
0 1	0	1	1
1 0	0	0	1
1 1	1	1	1

(23.)	$p \mid q \mid r$	$\overline{p \rightarrow a}$	$\overline{q \rightarrow b}$	$\overline{a \text{ and } b}$	$\overline{p \rightarrow r}$	$\overline{c \rightarrow d}$	$c \rightarrow d$
0 0 0	1	1	1	1	1	1	1
0 0 1	1	1	1	1	1	1	1
0 1 0	1	0	0	0	1	1	1
0 1 1	1	1	1	1	1	1	1
1 0 0	0	1	0	0	0	0	0
1 0 1	0	1	0	0	1	1	1
1 1 0	1	0	0	0	0	1	1
1 1 1	1	1	1	1	1	1	1

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(A)

P	q	$P \rightarrow q$	$\neg P$	$\neg P \text{ OR } q$	E
0	0	1	1	1	
0	1	1	1	1	
1	0	0	0	0	
1	1	1	0	1	

(B)

P	q	$\frac{P \text{ and } P}{\neg q}$	$\neg q \rightarrow q$	$\neg P \text{ OR } P$	$\neg q \text{ OR } q$	E
0	0	0	1	0	0	
0	1	0	1	0	0	
1	0	1	0	1	1	
1	1	1	0	1	1	

problem 1g : $x+y = p$ $yz = q$ $r = (x)$

1. $x+y \equiv x+y$

2. $(x+y \equiv yz) \equiv (yz \equiv x+y)$

3. $((x+y \equiv yz) \text{ AND } (yz \not\equiv x)) \rightarrow (x+y) \equiv (x)$

4. $(x+y \equiv yz) \equiv (\text{Not}(x+y) \equiv \text{Not}(yz))$

5. $(x+y)(y) \equiv ((*) (x+y))$

6. $(x+y)((yz)(x)) \equiv ((x+y)(yz))(x)$

7. $((x+y) + (yz)) \equiv ((yz) + (x+y))$

8. $((x+y) + (yz + x)) \equiv (((*) + yz) + x)$

9. $(x+y)(yz) + (x) \equiv (x+y)(yz) + ((x+y)(yz) + (x+y)(x))$

10. $(x+y) \text{ AND } 1 \equiv x+y$

11. $(x+y) \text{ OR } 0 \equiv x+y$

12. $((x+y) \text{ AND } 0) \equiv 0$

13. $(\text{Not} \text{Not}(x+y)) \equiv x+y$

14. $((x+y) + (yz))(x) \equiv (((x+y) + (xy))((x+y) + (x)))$

15. $(1 \text{ OR } (x+y)) \equiv 1$

16. $(x+y)(\bar{x}+y) \equiv x+y$

$$17. (X+Y) + (X+Y) \equiv X+Y$$

18.

$$a. (X+Y) + (X+Y) \equiv X+Y$$

$$b. (X+Y)((X+Y) + YZ) \equiv X+Y$$

~~18. a. $(X+Y) + (X+Y) \equiv X+Y$
b. $(X+Y)((X+Y) + YZ) \equiv X+Y$~~

19.

$$a. (X+Y)(\text{NOT}(X+Y) + YZ) \equiv (X+Y)(YZ)$$

$$b. (X+Y) + \text{NOT}(X+Y)(YZ) \equiv (X+Y) + (YZ)$$

$$20. a. \text{NOT}((X+Y)(YZ)) \equiv \text{NOT}(X+Y) + \text{NOT}(YZ)$$

$$b. \text{NOT}(X+Y) + YZ \equiv \text{NOT}((X+Y)YZ)$$

$$c. (\text{NOT}(X+Y)(X+Y)) \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$$

$$d. (\text{NOT}(X+Y) + (X+Y)) \equiv (\text{NOT}(X+Y) \text{ OR } \text{NOT}(X+Y))$$

$$21. ((X+Y) \rightarrow YZ) \text{ AND } (YZ \rightarrow (X+Y)) \equiv (X+Y) \equiv YZ$$

$$22. ((X+Y) \equiv YZ) \rightarrow ((X+Y) \rightarrow YZ)$$

$$23. ((X+Y) \rightarrow YZ) \text{ AND } (YZ \rightarrow (X+Y)) \rightarrow (X+Y \rightarrow X)$$

24.

$$a. (X+Y \rightarrow YZ) \equiv (\text{NOT}(X+Y) + YZ)$$

$$b. ((X+Y)(X+Y)) \rightarrow YZ \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$$

Problem 20:

(a) The d.istributive law

PQRS

$P(q+r)$ distributive law = $(Pq)(Pr)$
 $P(q+s)$ distributive law = $(Pq)(Ps)$

PQRS via distributive law

$$\overline{(P+R)(P+S)(Q+R)(Q+S)}$$

(b) $\overbrace{Pq + Pqr} \rightarrow P(Q+r)$
cause, in reason

1. $P(q+\bar{q}r) \neq$ d.istributive
2. $P(q+r)$ Elimination of certain negation

Problem 21:

~~P + PQ~~ a. $P(P+q)$

~~P + Pq~~ b. $P(P+q)$

~~P + Pq~~ c. $P(P+q)$

~~P + Pq~~ d. ~~d.istributive
Andance of AND
distributive~~

~~OR P + PQ~~ e. $d.istr. b.inary$ Comm. law of AND

~~OR P + PQ~~ f. $d.istr. b.inary$ Comm. law of OR

Problem 21:

a. $P + pq \equiv P$

① $P + Pa$

② $P = P$

③ $Pp + Pa$

④ ~~Pa~~

⑤ P

Independence of OR

reflexive

distributive

②

Indulgence or AND

b. $P(P+q) \equiv P$

① $PP + Pq$

② $P + Pa$

③ $(P+P)(P+q)$

④ $P(P+q) \equiv P$

⑤ P

distributive

Indulgence of and
distributive

Indulgence of OR

law of OR

Problem 22.

a. $\text{NOT } Pa + \bar{P}\bar{r}$

$\text{NOT}(\overline{Pa} \cap \bar{r})$

~~$\overline{Pa} \overline{P}\bar{r}$~~

b. $\text{NOT}(\text{NOT } P + q \cap \text{NOT}(r + \bar{s}))$

$\text{NOT}(P + q \cap \text{NOT}(r + s)) \Rightarrow$

$\text{NOT}(P + q \cap \bar{r}s)$

~~$\boxed{Pa(r\bar{s})} \quad \bar{P} \bar{q} (r\bar{s})$~~

Problem 23:

①

$$\cancel{w\bar{x} + w\bar{x}\gamma + \bar{z}\bar{x}w}$$

$$①. \cancel{w\bar{x}} + w\bar{x}\gamma + \bar{z}\bar{x}w \quad \text{Base}$$

$$② \quad w\bar{x} + \bar{z}\bar{x}w + \cancel{w\bar{x}\gamma} \quad \text{Subsumption}$$

③

$$\cancel{\bar{x}w} + \bar{z}\bar{x}w$$

Commut.

④

$$\cancel{w\bar{x}}$$

Subsumption

②

$$① (w+x)(w+\gamma+\bar{z})(\bar{w}+x+\bar{\gamma})(\bar{x}) \quad \text{Base}$$

$$② (\bar{x})(w+\gamma+\bar{z})(\bar{w}+\bar{x}+\bar{\gamma}) \quad \text{Subsumption}$$

$$③ (w\bar{x}) + (\bar{x}\gamma) + (\bar{x}\bar{z})$$

Replacement