Lab Period 1

Problem 1:

p	q	p NAND q	p NOR q	$p \equiv q$
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

Problem 2:

a).

p	q	$p \to q$	NOT p OR q	$\equiv$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

b).

p	q	r	$\overbrace{\text{r OR NOT p}}^{a}$	$\overbrace{q \to a}^{b}$	$p \to b$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

c.)

p	q	p OR q	p AND q	(p OR q) -> (p And q)
0	0	0	0	1
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

Union
intercut
Except

Problem 3:

exclusion

Problem 4:

$(\overset{0}{p} \text{ NAND } \overset{0}{q}) \text{ NAND } \overset{1}{r} = 0$

$\neq$ NAND $\neq$ associtam

$\overset{0}{p} \text{ NAND } (\overset{0}{q} \text{ NAND } \overset{1}{r}) = 1$

$(\overset{0}{p} \to \overset{0}{q}) \to \overset{0}{r} = 0$

$\neq$ $\to \neq$ associtive

$\overset{0}{p} \to (\overset{0}{q} \to \overset{0}{r}) = 1$

$(\overset{0}{p} \text{ NOR } \overset{0}{q}) \text{ NOR } \overset{1}{R} = 0$

NOR $\neq$ associtive

$\overset{0}{p} \text{ NOR } (\overset{0}{q} \text{ NOR } \overset{1}{R}) = 1$

1111
0000

Problem 5

True
Fals
NOT P
NOT q

Problem 6

	False	AND	->	P	NOT(q->P)	q	XOR	OR	NOR	$\equiv$	$\bar{q}$	q->P	$\bar{P}$	P->q	NAND	True
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
4	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
2	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Problem 7

P	q	$\otimes$
0	0	0
0	1	1
1	0	1
1	1	0

$P \otimes q \equiv q \otimes P$

$(\overset{1}{P} \otimes \overset{1}{q}) \otimes \overset{0}{r} = 0$

$\overset{1}{P} \otimes (\overset{1}{q} \otimes \overset{0}{r}) = 0$

Commmitive And Assciitive

Problem 8

$a = \bar{p}qr + p\bar{q}\bar{r} + pqr + pq\bar{r} + pqr$

$b = \bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r}$

Problem 9:

$a = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})$

$b = (p+\bar{q}+\bar{r})(\bar{p}+q+\bar{r})(\bar{p}+q+\bar{r})(p+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r})$

$c = (x+y+c)(x+\bar{y}+\bar{c})(\bar{x}+y+\bar{c})(\bar{x}+\bar{y}+c)$

Problem 10:

(a.)

pq \ rs	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

(b.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	1

(c.)

pq \ rs	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

(d.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

(e.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

Problem 11/12

(a.)

pq \ rs	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

$p\bar{q}r + pr\bar{s} + rp\bar{q} + \ldots$

$\bar{q}r + s\bar{p} + r\bar{p}$

Prime Implicants

(c.)

pq \ rs	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

$r\bar{s}\bar{p}\bar{q} + \bar{r}s\bar{p}\bar{q} + \bar{r}\bar{s}\bar{p}q + \bar{r}\bar{s}p\bar{q} + \ldots$

$\ldots + spq + rsq + rsp + rpq$

no prime implicants

(b)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	0	0	1

$\bar{s}\bar{p}\bar{q} + p\bar{r}\bar{s} + s\bar{p}\bar{q} + \ldots$

$\ldots + \bar{p}\bar{s} + \bar{r}\bar{p}$

Prime implicants

(d.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

$\cancel{\bar{p}\bar{r} + \bar{p}q + s\bar{p} + \bar{s}q + \ldots}$

$\bar{p} + p\bar{r} + prs + p\bar{q}r$

(e)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

$\bar{p} + \bar{r}\bar{q}$

Problem 13/14:

13/14 (a) $(\bar{r}+\bar{s}+\bar{p}+\bar{q})(r+s+p+q)$

(b) 14. $(r+s+q)(r+s+p)(p+q+s)(p+q+r)$

13. $(\bar{r}+s+p+q)(r+s+\bar{p}+q)(r+s+p+q)(r+s+\bar{p}+\bar{q})(r+\bar{s}+p+\bar{q})$

13/14 (c) $(\bar{p}+\bar{q}+\bar{r}+\bar{s})(\bar{r}+\bar{s}+\bar{p}+q)(\bar{r}+s+\bar{p}+q)(\bar{r}+s+p+\bar{q})\ldots$

$\ldots(r+s+\bar{p}+\bar{q})(r+\bar{s}+\bar{p}+q)(r+\bar{s}+p+\bar{q})$

13/14 (d) $(r+\bar{s}+p+q)$

(e) 13. $(r+p)(\bar{r}+s+p+q)(\bar{r}+\bar{s}+p+q)$

14. $(r+p)(\bar{r}+p+q)$

a. Problems 15:

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\bar{r}\bar{s}\bar{p} + \bar{r}\bar{s}p + ..$
$.. + \bar{r}sp + \bar{r}s\bar{p} + ..$
$.. + rsp + rs\bar{p} + ..$
$.. + r\bar{s}p + r\bar{s}\bar{p}$

b.

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\bar{r}\bar{p} + rp + \bar{r}p + r\bar{p}$

c.

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\bar{p}\bar{q} + p\bar{q} + \bar{p}q + pq$

d.

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$\bar{r} + r$

Problem 16:

a.

A.

p q r	p ~~AND~~ OR q	pqr -> p OR q
0 0 0	0	1
0 0 1	0	1
0 1 0	1	1
0 1 1	1	1
1 0 0	1	1
1 0 1	1	1
1 1 0	1	1
1 1 1	1	1

Tautology

b.

B. $((p \to q)\ (q \to r)) \to (p \to r)$

$((p \to q)\ (q \to r)) \to (p \to r)$
1
1
1
1
1
1
1
1

Tautology

C.

p	q	$(p \to q) \to p$
0	0	0
0	1	0
1	0	1
1	1	1

not tautology

D.

$(p = (q \vee r)) \to q \to p \wedge r$

$(p = (q \vee r)) \to q \to p \wedge r$
1
1
1
1
1
1
0
1

Not tautology

Problem 17:

Both expressions are equal to true

P	Q	$P \equiv P$	P OR Q	P OR Q $\equiv$ P	$(P \equiv P) \equiv (\text{P OR Q} \equiv P)$
0	0	~~0~~ 1	0	1	1
0	1	~~0~~ 1	0	1	1
1	0	~~1~~ 1	1	1	1
1	1	~~1~~ 1	1	1	1

Problem 18:

1.

p	$p \equiv p$
0	0
1	1

2.

p	q	$\overbrace{(p \equiv q)}^{a}$	$\overbrace{(q \equiv p)}^{b}$	$a \equiv b$
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	1	1	1

3.

p	q	r	$(p \equiv q)$	(AND)	$(q \equiv r)$	$(\rightarrow)$	$(p \equiv r)$
0	0	0	1	1	1	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	0	1	1
0	1	1	0	0	1	1	0
1	0	0	0	0	1	1	0
1	0	1	0	0	0	1	1
1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	1

$= (p \equiv q)$ And $(q \equiv r) \rightarrow (p \equiv r)$

4.

p	q	$\neg p$	$\neg q$	$(p \equiv q)$	$(\bar{p} \equiv \bar{q})$	$(p \equiv q) \equiv (\bar{p} \equiv \bar{q})$
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

5.

p	q	p ∧ q	q ∧ p	pq ≡ qr
0	1	0	0	1
0	0	0	0	1
1	1	1	1	1
1	0	0	0	1
				E

6.

r	p	q		p ∧	(q ∧ r)	≡	(p ∧ q)	∧	r	E
0	0	0	0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0	0	0	1
0	1	0	1	0	0	1	0	0	0	1
0	1	1	1	0	0	1	1	0	0	1
1	0	0	0	0	0	1	0	0	1	1
1	0	1	0	0	1	1	0	0	1	1
1	1	0	1	0	0	1	0	0	1	1
1	1	1	1	1	1	1	1	1	1	1

7.

p	q	p OR q	≡	q OR p	E
0	0	0	1	0	1
0	1	1	1	1	1
1	0	1	1	1	1
1	1	1	1	1	1

8.

p	q	r	(p	OR	(q or r))	≡	((p OR q)	or	r
0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	1	1	0	1	1
0	1	0	0	1	1	1	1	1	0
0	1	1	0	1	1	1	1	1	1
1	0	0	1	1	0	1	1	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1

tautology

9.

p q r	$\overbrace{q \text{ or } r}^{a}$	$\overbrace{p \text{ and } a}^{d}$	$\overbrace{p \text{ and } q}^{b}$	$\overbrace{p \text{ and } r}^{c}$	$\overbrace{b \text{ OR } c}^{e}$	$d \equiv e$
0 0 0	0	0	0	0	0	1
0 0 1	1	0	0	0	0	1
0 1 0	1	0	0	0	0	1
0 1 1	1	0	0	0	0	1
1 0 0	0	0	0	0	0	1
1 0 1	1	1	0	1	1	1
1 1 0	1	1	1	0	1	1
1 1 1	1	1	1	1	1	1

10.

p	$\overbrace{p \text{ AND } 1}^{a}$	$a \equiv p$
0	0	1
1	1	1

11.

p	$\overbrace{p \text{ OR } 0}^{a}$	$a \equiv p$
0	0	1
1	1	1

12.

p	$\overbrace{p \text{ AND } 0}^{a}$	$a \equiv 0$
0	0	1
1	0	1

13.

p	$\neg p$	$\overbrace{\neg\neg p}^{a}$	$a \equiv p$
0	1	0	1
1	0	1	1

14.

p q r	qr	$\overbrace{p+qr}^{a}$	$\overbrace{p+ra}^{b}$	$\overbrace{p \text{ OR } r}^{c}$	$\overbrace{b \text{ and } c}^{d}$	$d \equiv a$
0 0 0	0	0	0	0	0	1
0 0 1	0	0	0	1	0	1
0 1 0	0	0	1	0	0	1
0 1 1	1	1	1	1	1	1
1 0 0	0	1	1	1	1	1
1 0 1	0	1	1	1	1	1
1 1 0	0	1	1	1	1	1
1 1 1	1	1	1	1	1	1

15.

p	1 OR p	1 OR p ≡ 1
0	1	1
1	1	1

16.

p	p and p	pp ≡ p
0	0	1
1	1	1

17.

p	p OR p	p+p ≡ p
0	0	1
1	1	1

18.

a.

p q	p and q	$\overbrace{pq+p}^{a}$	E p ≡ a
0 0	0	0	1
0 1	0	0	1
1 0	0	1	1
1 1	1	1	1

b.

p q	$\overbrace{p \text{ or } q}^{a}$	$\overbrace{a \text{ and } p}^{b}$	b ≡ p
0 0	0	0	1
0 1	1	0	1
1 0	1	1	1
1 1	1	1	1

19. (a)

p	q	¬p	$\overbrace{\neg p \text{ OR } q}^{a}$	$p \wedge q$	$p \wedge q$	E
0	0	1	1	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	0	1
1	1	0	1	1	1	1

(b)

p	q	$\bar{p}$	$\bar{p} \wedge q$	$\bar{p} \wedge q \vee p$	p OR q	E
0	0	1	0	0	0	1
0	1	1	1	0	1	1
1	0	0	0	0	1	1
1	1	0	0	0	1	1

20. (a)

p	q	$\overbrace{p \text{ and } q}^{a}$	¬q	¬p	¬q	$\overbrace{\neg p \text{ OR } \neg q}^{b}$	$a \equiv b$
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

(b)

p	q	¬p	¬q	NOT(p OR q)	¬p and ¬q	E
0	0	0	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	0	0	1

(c)

p	¬p	not p and p...	not p OR not p	E
0	1	0	1	1
1	0	0	0	1

d.

p	¬p	p ∧ p (a)	not(a)	¬p or ¬p	E
0	1	0	1	1	1
1	0	1	0	0	1

21.

p	q	q -> p (a)	p -> q (b)	a and b	p ≡ q	E
0	0	1	1	1	1	1
0	1	0	1	0	0	1
1	0	1	0	0	0	1
1	1	1	1	1	0	1

22.

p	q	p ≡ q (a)	p -> q (b)	a -> b (E)
0	0	1	1	1
0	1	0	1	1
1	0	0	0	1
1	1	1	1	1

23.

p	q	r	p -> q (a)	q -> r (b)	a and b (c)	p -> r (d)	E (c -> d)
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

2a

(a)

p	q	p -> q	¬p	¬p OR q	E
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	7

(b)

p	q	$\underline{a}$ p and p	a -> q	$\underline{b}$ ¬p OR ¬p	b OR q	E
0	0	0	1	0	0	1
0	1	0	1	0	1	1
1	0	1	0	0	0	1
1	1	1	1	0	1	1

problem 1a: $x+y = p \quad yz = q \quad r = (x)$

1. $x+y \equiv x+y$
2. $(x+y \equiv yz) \equiv (yz \equiv x+y)$
3. $((x+y \equiv yz)\ AND\ (yz \equiv (x))) \rightarrow (x+y) \equiv (x)$
4. $(x+y \equiv yz)\ (Not(x+y) \equiv Not(yz))$
5. $(x+y)(\cancel{yz}) \equiv (\cancel{xz})(x+y)$
6. $(x+y)((yz)(x)) \equiv ((x+y)(yz))(x)$
7. $((x+y) + (yz)) \equiv ((yz) + (x+y))$
8. $((x+y) + (yz + (x)) \equiv (((x+y) + yz) + (x)$
9. $(x+y)((yz) + (x)) \equiv (x+y)(yz) + ((x+y)(yz) + (x+y)(x))$
10. $(x+y)\ AND\ 1 \equiv x+y$
11. $(x+y)\ OR\ 0 \equiv x+y$
12. $((x+y)\ AND\ 0) \equiv 0$
13. $(Not\ Not\ (x+y)) \equiv x+y$
14. $(x+y) + (yz)(x) \equiv (((x+y) + (xy))((x+y) + (x)))$
15. $(1\ OR\ (x+y)) \equiv 1$
16. $(x+y)(x+y) \equiv x+y$

17. $(X+Y) + (X+Y) \equiv X+Y$

18.

a. $((X+Y) + (X+Y) \equiv X+Y$

b. $(X+Y)((X+Y) + YZ) \equiv X+Y$

~~18 ((X+Y) + (X+Y)(YZ) ≡~~

19.

a. $(X+Y)(\text{NOT}(X+Y) + YZ) \equiv (X+Y)(YZ)$

b. $(X+Y) + \text{NOT}(X+Y)(YZ) \equiv (X+Y) + (YZ)$

20. a. $\text{NOT}((X+Y)(ZY)) \equiv \text{NOT}(X+Y) + \text{NOT}(YZ)$

b. $\text{NOT}((X+Y) + YZ) \equiv \text{NOT}((X+Y)YZ)$

c. $(\text{NOT}(X+Y)(X+Y)) \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$

d. $(\text{NOT}(X+Y) + (X+Y)) \equiv (\text{NOT}(X+Y) \text{ OR } (X+Y))$

21. $(((X+Y) \to YZ) \text{ AND } (YZ \to (X+Y))) \equiv ((X+Y) \equiv YZ)$

22. $((X+Y) \equiv YZ) \to ((X+Y) \to YZ)$

23. $(((X+Y) \to YZ) \text{ AND } (YZ \to (X))) \to (X+Y \to (X))$

24.

a. $(X+Y \to YZ) \equiv (\text{NOT}(X+Y) + XZ)$

b. $((X+Y)(X+Y)) \to YZ) \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$

Problem 20:

(a.) The distributive law

$pq + rs$

$pq + r$ distributive law $= \cancel{(p+q)}(p+r)$

$pq + s$ distributive law $= \cancel{(p+q)+(p+s)}$

$\therefore pq + rs$ via $\boxed{\text{distributive law}}$

$$= \boxed{(p+r)(p+s)(q+r)(q+s)}$$

(b.) $pq + p\bar{q}r \to p(q+r)$

conclusion	reason
1. $p(q+\bar{q}r)$	distributive
2. $p(q+r)$	Elimination of certain negation

~~Problem 21:~~

~~a. $p(\bar{p}+q)$~~

~~1. $p\bar{p}+pq$ distributive~~

~~2. $\bar{p}+pq$ Avoidance of AND~~

~~3. pq~~

~~1. $p\bar{p}+pq$ distributive commutative law AND (1)~~

~~2. commutative law of OR (2)~~

Problem 21:

a. $p + pq \equiv p$

① $p + pq$ — Idempotence of OR

② $p = p$ — reflexive

③ $pp + p$ — distributive

④ p — ②

⑤ p — Idempotence of AND

b. $p(p+q) \equiv p$

① $pp + pq$ — distributive

② $p + pq$ — Idempotence of and

③ $(p+p)(p+q)$ — distributive

④ $p(p+q) \equiv p$ — Idempotence of OR

⑤ p — law of OR

Problem 22.

a. NOT $pq + \bar{p}\bar{r}$

NOT$(\bar{p}\bar{q}r)$

$pq\,p\bar{r}$

b. NOT(NOT $p + q$(NOT$(r+\bar{s})$))

NOT($p + q$(NOT$(r+s)$)))

NOT($+q(\bar{r}\bar{s})$))

$\boxed{\bar{p}\bar{q}(\bar{r}\bar{s})}$ $\bar{p}\,\bar{q}\,(\bar{r}\,\bar{s})$

Problem 23:

(1)

~~1. $W\bar{X} + W\bar{X}Y + \bar{Z}\bar{X}W$~~

1. $W\bar{X} + W\bar{X}Y + \bar{Z}\bar{X}W$ Base

2. $W\bar{X} + \bar{Z}\bar{X}W$ Subsumption

3. $\bar{X}W + \bar{Z}\bar{X}W$ Commutative

4. $W\bar{X}$ Subsumption

(2)

1. $(W+\bar{X})(W+Y+\bar{Z})(\bar{W}+X+\bar{Y})(\bar{X})$ Base

2. $(\bar{X})(W+Y+\bar{Z})(\bar{W}+X+\bar{Y})$ Subsumption

3. $(W\bar{X})+(\bar{X}\bar{Y})+(\bar{X}\bar{Z})$ Reflexive subsumption