Homework 2

CMPT333N
Due on Feb 14, 2020

Predicates

Problem 1

Identify the following as constants, variables, ground atomic formulas, or non-ground atomic formulas, using the conventions of this section.

a) CMPT333

b) cmpt333

c) 333

d) "cmpt333"

e) p(X,x)

f) p(3,4,5)

g) "p(3,4,5)"

Problem 2

$$(csg(\text{"}CMPT333\text{"}, S, G) \texttt{ AND } snap(S, \text{"}C.Brown\text{"}, A, P)) \rightarrow \; answer(G) \quad (1)$$

Write an expression similar to (1) for the question "What grade did L. Van Pelt get in CMPT220?" What substitution for variables did you make to demonstrate the truth of this answer? (See Example 3)

Problem 3

Remove redundant pairs of parentheses from the following expressions.

a) $(\forall X)((\exists Y)(\texttt{NOT}(p(X) \texttt{ OR } (p(Y) \texttt{ AND } q(X)))))$

b) $(\exists X)((\texttt{NOT}p(X)) \texttt{ AND } ((\exists Y)(p(Y)) \texttt{ OR } (\exists X)(q(X, Z))))$

Problem 4

Draw expression trees for the expressions of Problem 3. Indicate for each occurrence of a variable to which quantifier, if any, it is bound.

Problem 5

Rewrite the expression of Problem 3(b) so that it does not quantify the same variable twice.

Problem 6

Using the *csg* predicate of our running example, write expressions that assert the following.

1. C. Brown is an A student (i.e., he gets A's in all his courses).
2. C. Brown is not an A student.

Problem 7

For each of the following expressions, give one interpretation that makes it true and one interpretation that makes it false.

a) $(\forall X)(\exists Y)(loves(X,Y))$

b) $p(X) \rightarrow \texttt{NOT}p(X)$

c) $(\exists X)p(X) \rightarrow (\forall X)p(X)$

d) $(p(X,Y)\ \texttt{AND}\ p(Y,Z)) \rightarrow p(X,Z)$

Problem 8

Explain why each of the following are tautologies. That is, what expression(s) of predicate logic did we substitute into which tautologies of propositional logic?

a) $(p(X)\ \texttt{OR}\ q(Y)) \equiv (q(Y)\ \texttt{OR}\ p(X))$

b) $(p(X,Y)\ \texttt{AND}\ p(X,Y)) \equiv p(X,Y)$

c) $(p(X) \rightarrow \texttt{FALSE}) \equiv \texttt{NOT}\ p(X)$

Problem 9

Transform the following expressions into rectified expressions, that is, expressions for which no two quantifier occurrences share the same variable.

a) $(\exists X)(\texttt{NOT}\ p(X))\ \texttt{AND}((\exists Y)p(Y)))\ \texttt{OR}\ ((\exists X)q(X,Z))))$

b) $(\exists X)(\exists X)p(X)\ \texttt{OR}\ (\texttt{X})q(X)\ \texttt{OR}\ r(X))$

Problem 10

Turn the following into closed expressions by universally quantifying each of the free variables. If necessary, rename variables so that no two quantifier occurrences use the same variable.

a) $p(X,Y)\ \texttt{AND}\ (\exists Y)q(Y)$

b) $(\exists X)(p(X,Y)\ \texttt{OR}\ (\exists X)p(Y,X))$

Problem 11

Does law $(E\ \texttt{AND}\ (QX)F) \rightarrow (QX)(E\ \texttt{AND}\ F)$ imply that $p(X,Y)\ \texttt{AND}\ (\exists X)q(X)$ is equivalent to

$$(\exists X)(p(X,Y)\ \texttt{AND}\ q(X))$$

explain your answer.

Problem 12

Transform the expressions of Problem 9 into prenex form.

Problem 13

Show how to move quantifiers through an $\rightarrow$ operator. That is, turn the expression $((Q_1X)E) \rightarrow (Q_2Y)F)$ into a prenex form expression. What constraints on free variables in E and F do you need?

Problem 14

$$(\texttt{NOT}\ ((\forall X)E)) \equiv ((\exists X)(\texttt{NOT}\ E))$$

$$(\texttt{NOT}\ ((\exists X)E)) \equiv ((\forall X)(\texttt{NOT}\ E))$$

We can use the two tautologies above to move `NOT`'s inside quantifiers as well as to move them outside. Using these laws, plus DeMorgan's laws, we can move all `NOT`'s so they apply directly to atomic formulas. Apply this transformation to the following expressions.

1. `NOT` $((\exists X)(\exists Y)p(X, Y))$
2. `NOT` $((\exists X)p(X)$ `OR` $(\exists Y)q(X, Y)))$

Problem 15

Is it true that E is a tautology whenever $(\exists X)E$ is a tautology?