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Lab Period 1

Problem 1:

p	q	p NAND q	p NOR q	p ≡ q
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

Problem 2:

a).

p	q	p -> q	NOT p OR q	≡
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

b).

p	q	r	$\overbrace{\text{r OR NOT p}}^{a}$	$\overbrace{\text{q -> a}}^{b}$	p -> b
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

c.)

p	q	p OR q	p AND q	(p OR q) -> (p And q)
0	0	0	0	1
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

Union
intercut
Except

Problem 3:

exclusion

Problem 4:

$(\overset{0}{p}\ \text{NAND}\ \overset{0}{q})\ \text{NAND}\ \overset{1}{r} = 0$ $\neq$

$\overset{0}{p}\ \text{NAND}\ (\overset{0}{q}\ \text{NAND}\ \overset{1}{r}) = 1$

NAND $\neq$ associtave

$(\overset{0}{p} \to \overset{0}{q}) \to \overset{0}{r} = 0$ $\neq$

$\overset{0}{p} \to (\overset{0}{q} \to \overset{0}{r}) = 1$

$\to \neq$ assoitive

$(\overset{0}{p}\ \text{NOR}\ \overset{0}{q})\ \text{NOR}\ \overset{1}{R} = 0$

$\overset{0}{p}\ \text{NOR}\ (\overset{0}{q}\ \text{NOR}\ \overset{1}{R}) = 1$

NOR $\neq$ associtive

1111
0000

Problem 5

True
Fals
NOT p
NOT q p, q

-1

Problem 6

	False	AND	$\to$	P	NOT($q \to p$)	q	XOR	OR	NOR	$\equiv$	$\bar{q}$	$q \to p$	$\bar{p}$	$p \to q$	NAND	True
8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
4	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
2	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Problem 7

p	q	$\otimes$
0	0	0
0	1	1
1	0	1
1	1	0

$$p \otimes q \equiv q \otimes p$$

$$(\overset{1}{p} \otimes \overset{1}{q}) \otimes \overset{0}{r} = 0$$

$$\overset{1}{p} \otimes (\overset{1}{q} \otimes \overset{0}{r}) = 0$$

Commutitive And Associtive

Problem 8

$$a = \bar{p}\bar{q}r + p\bar{q}\bar{r} + \bar{p}qr + pq\bar{r} + pqr$$

$$b = \bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r}$$

Problem 9:

$a = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})$

$b = (p+\bar{q}+\bar{r})(\bar{p}+q+\bar{r})(\bar{p}+q+\bar{r})(\bar{p}+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r})$

$c = (x+y+c)(x+\bar{y}+\bar{c})(\bar{x}+\cancel{y}y+\bar{c})(\bar{x}+\bar{y}+c)$

Problem 10:

(a.)

pq \ rs	0 0	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

(c.)

pq \ rs	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

(b.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	1

(d.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

(e.)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

Problem 11/12

(a.)

	rs 00	01	11	10
pq 00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

$p\bar{q}r + pr\bar{s} + rp\bar{q} +$

$\bar{q}r + s\bar{p} + r\bar{p}$

Prime Implicants

(c.)

	rs 00	01	11	10
pq 00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

$r\bar{s}\bar{p}\bar{q} + \bar{r}s\bar{p}\bar{q} + \bar{r}spq + \bar{r}\bar{s}p\bar{q} + \ldots$

$\ldots + spq + rsq + rsp + rpq$

no prime implicants

(b)

	rs 00	01	11	10
pq 00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	0	0	1

$\bar{s}p\bar{q} + pr\bar{s} + s\bar{p}\bar{q} + \ldots$

$\ldots + \bar{p}\bar{s} + \bar{r}\bar{p}$

Prime implicants

(d.)

	rs 00	01	11	10
pq 00	1	1	1	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

$\bar{p} + p\bar{r} + prs + p\bar{q}r$

(e)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

$\bar{p} + \bar{r}\bar{q}$

Problem 13/14:

13/14 (a) $(\bar{r}+\bar{s}+\bar{p}+\bar{q})(r+s+p+q)$

(b) 14. $(r+s+q)(r+s+p)(p+q+s)(p+q+r)$

13. $(\bar{r}+s+p+q)(r+s+\bar{p}+q)(r+s+p+q)(r+s+p+\bar{q})(r+\bar{s}+p+\bar{q})$

13/14 (c) $(\bar{p}+\bar{q}+\bar{r}+\bar{s})(\bar{r}+\bar{s}+p+q)(\bar{r}+s+\bar{p}+q)(\bar{r}+s+p+\bar{q})\ldots$

$\ldots(r+s+\bar{p}+\bar{q})(r+\bar{s}+\bar{p}+q)(r+\bar{s}+p+\bar{q})$

13/14 (d) $(r+\bar{s}+p+q)$

(e) 13. $(r+p)(\bar{r}+s+p+q)(\bar{r}+\bar{s}+p+q)$

14. $(r+p)(\bar{r}+p+q)$

How many of each?

a. Problems 15:

Pa \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$\boxed{\begin{array}{l}\bar r\bar s\bar p + \bar r\bar s p + \ldots \\ \ldots + \bar r s p + \bar r s \bar p + \ldots \\ \ldots + r s p + r s \bar p + \ldots \\ \ldots + r\bar s p + r \bar s \bar p\end{array}}$$

b.

Pa \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$\bar r\bar p + rp + \bar r p + r\bar p$$

−2

c.

Pa \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$\boxed{\bar p\bar a + p\bar a + \bar p a + pa}$$

d.

Pa \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$\boxed{\bar r + r}$$

Problem 16:

A.

p	q	r	p OR q	p∧r -> p OR q
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Tautology

B. $((p \to q)\ (q \to r)) \to (p \to r)$

1
1
1
1
1
1
1
1

Tautology

C.

p	q	$(p \to q) \to p$
0	0	0
0	1	0
1	0	1
1	1	1

not tautology

D. $(p = (q \vee r)) \to q \to p \wedge r$

1
1
1
1
1
0
1

Not tautology

Problem 17:

Both expressions are equal to true

P	Q	$P \equiv P$	P OR Q	P OR Q $\equiv$ P	$(P \equiv P) \equiv (P \text{ OR } Q \equiv P)$
0	0	~~0~~ 1	0	1	1
0	1	~~0~~ 1	0	1	1
1	0	~~1~~ 1	1	1	1
1	1	~~1~~ 1	1	1	1

Problem 18:

1.

p	$p \equiv p$
0	0
1	1

2.

p	q	$\overbrace{(p \equiv q)}^{a}$	$\overbrace{(q \equiv p)}^{b}$	$a \equiv b$
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	1	1	1

3.

p	q	r	$(p \equiv q)$	(AND)	$(q \equiv r)$	$(\rightarrow)$	$(p \equiv r)$
0	0	0	1	1	1	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	0	1	1
0	1	1	0	0	1	1	0
1	0	0	0	0	1	1	0
1	0	1	0	0	0	1	1
1	1	0	1	0	0	1	0
1	1	1	1	1	1	1	1

$= (p \equiv q)$ And $(q \equiv r) \rightarrow (p \equiv r)$

4.

p	q	$\neg p$	$\neg q$	$(p \equiv q)$	$(\bar{p} \equiv \bar{q})$	$(p \equiv q) \equiv (\bar{p} \equiv \bar{q})$
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

5.

p	q	p ∧ q	q ∧ p	pq ≡ qr
0	1	0	0	1
0	0	0	0	1
1	1	1	1	1
1	0	0	0	1
				E

6.

r	p	q		p ∧	(q ∧ r)	≡	(p ∧ q)	∧	r	E
0	0	0	0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0	0	0	1
0	1	0	1	0	0	1	0	0	0	1
0	1	1	1	0	0	1	1	0	0	1
1	0	0	0	0	0	1	0	0	1	1
1	0	1	0	0	1	1	0	0	1	1
1	1	0	1	0	0	1	0	0	1	1
1	1	1	1	1	1	1	1	1	1	1

7.

p	q	p OR q	≡	q OR p	E
0	0	0	1	0	1
0	1	1	1	1	1
1	0	1	1	1	1
1	1	1	1	1	1

8.

p	q	r	(p	OR	(q or r))	≡	((p OR q	OR	r
0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	1	1	0	1	1
0	1	0	0	1	1	1	1	1	0
0	1	1	0	1	1	1	1	1	1
1	0	0	1	1	0	1	1	1	0
1	0	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1

tautology

9.

p	q	r	a: q or r	d: p and a	b: p and q	c: p and r	e: b OR c	d ≡ e
0	0	0	0	0	0	0	0	1
0	0	1	1	0	0	0	0	1
0	1	0	1	0	0	0	0	1
0	1	1	1	0	0	0	0	1
1	0	0	0	0	0	0	0	1
1	0	1	1	1	0	1	1	1
1	1	0	1	1	1	0	1	1
1	1	1	1	1	1	1	1	1

10.

p	a: p AND 1	a ≡ p
0	0	1
1	1	1

11.

p	a: p OR 0	a ≡ p
0	0	1
1	1	1

12.

p	a: p AND 0	a ≡ p
0	0	1
1	0	1

13.

p	¬p	¬¬p	a ≡ p
0	1	0	1
1	0	1	1

14.

pqr	qr	p+qr (a)	p ORq (b)	p OR r (c)	b and c (d)	d ≡ a
0 0 0	0	0	0	0	0	1
0 0 1	0	0	0	1	0	1
0 1 0	0	0	1	0	0	1
0 1 1	1	1	1	1	1	1
1 0 0	0	1	1	1	1	1
1 0 1	0	1	1	1	1	1
1 1 0	0	1	1	1	1	1
1 1 1	1	1	1	1	1	1

15.

p	1 OR p	1ORp ≡ 1
0	1	1
1	1	1

16.

p	p and p	pp ≡ p
0	0	1
1	1	1

17.

p	p OR p	p+p ≡ p
0	0	1
1	1	1

18.

a.

p q	p and q	pq+p (a)	p ≡ a
0 0	0	0	
0 1	0	0	
1 0	0	1	
1 1	1	1	

b.

p q	p or q (a)	a and p (b)	b ≡ p
0 0	0	0	1
0 1	1	0	1
1 0	1	1	1
1 1	1	1	1

19. a)

p	q	¬p	$\overbrace{\neg p \text{ OR } q}^{a}$	p ∧ q	p ∧ q	E
0	0	1	1	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	0	1
1	1	0	1	1	1	1

b)

p	q	$\bar{p}$	$\bar{p} \wedge q$	$\bar{p} \wedge q \vee p$	p OR q	E
0	0	1	0	0	0	1
0	1	1	1	0	1	1
1	0	0	0	0	1	1
1	1	0	0	0	1	1

20. a)

p	q	$\overbrace{p \text{ and } q}^{a}$	¬ q	¬ p	¬q	$\overbrace{\neg p \text{ OR } \neg q}^{b}$	a ≡ b
0	0	0	1	1	1	1	1
0	1	0	1	1	0	1	1
1	0	0	1	0	1	1	1
1	1	1	0	0	0	0	1

b)

p	q	¬p	¬q	NOT(p OR q)	¬p and ¬q	E
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	0	0	1

c)

p	¬p	not p and p...	not p OR not p	E
0	1	0	1	1
1	0	0	0	1

d.

		a			
p	$\neg p$	$p \land p$	not(a)	$\neg p$ or $\neg p$	E
0	1	0	1	1	1
1	0	1	0	0	1

21.

		a	b			
p	q	$q \to p$	$p \to q$	a and b	$p \equiv q$	E
0	0	1	1	1	1	1
0	1	0	1	0	0	1
1	0	1	0	0	0	1
1	1	1	1	1	0	1

22.

		a	b	E
p	q	$p \equiv q$	$p \to q$	$a \to b$
0	0	1	1	1
0	1	0	1	1
1	0	0	0	1
1	1	1	1	1

23.

			a	b	c	d	$c \to d$
p	q	r	$p \to q$	$q \to r$	a and b	$p \to r$	E
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

2a

(a.)

p	q	p -> q	¬p	¬p OR q	E
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

(b.)

p	q	$\underline{a}$ p and p	a -> q	$\underline{b}$ ¬p OR ¬p	b OR q	E
0	0	0	1	0	0	1
0	1	0	1	0	1	1
1	0	1	0	0	0	1
1	1	1	1	0	1	1

Problem 1a: $x+y = p$ $yz = q$ $r = (x)$

1. $x+y \equiv x+y$
2. $(x+y \equiv yz) \equiv (yz \equiv x+y)$
3. $((x+y \equiv yz) \text{ AND } (yz \equiv (x))) \to (x+y) \equiv (x)$
4. $(x+y \equiv yz) \equiv (\text{Not}(x+y) \equiv \text{Not}(yz))$
5. $(x+y)(\cancel{yz}) \equiv (\cancel{yz})(x+y)$
6. $(x+y)((yz)(x)) \equiv ((x+y)(yz))(x)$
7. $((x+y) + (yz)) \equiv ((yz) + (x+y))$
8. $((x+y) + (yz + (x)) \equiv (((x+y) + yz) + (x)$
9. $(x+y)((yz) + (x)) \equiv (x+y)(yz) + ((x+y)(yz) + (x+y)(x))$
10. $(x+y) \text{ AND } 1 \equiv x+y$
11. $(x+y) \text{ OR } 0 \equiv x+y$
12. $((x+y) \text{ AND } 0) \equiv 0$
13. $(\text{Not Not }(x+y)) \equiv x+y$
14. $((x+y) + (yz)(x) \equiv (((x+y) + (xy))((x+y) + (x)))$
15. $(1 \text{ OR } (x+y)) \equiv 1$
16. $(x+y)(x+y) \equiv x+y$

17. (X+Y) + (X+Y) ≡ X+Y

18.

a. ((X+Y) + (X+Y) ≡ X+Y

b. (X+Y)((X+Y) + YZ) ≡ X+Y

~~19 ((X+Y) + (X+Y)(YZ) ≡~~

19.

a. (X+Y)(NOT (X+Y) + YZ) ≡ (X+Y)(YZ)

b. (X+Y) + NOT (X+Y)(YZ) ≡ (X+Y) + (YZ)

20. a NOT ((X+Y)(ZY)) ≡ NOT (X+Y) + NOT (YZ)

b. NOT ((X+Y) + YZ) ≡ NOT((X+Y) YZ)

c. (NOT(X+Y)(X+Y)) ≡ (NOT (X+Y) + NOT (X+Y))

d. (NOT(X+Y) + (X+Y)) ≡ (NOT (X+Y) OR ~~NOT~~ (X+Y))

21. (((X+Y) -> YZ) AND (YZ -> (X+Y))) ≡ ((X+Y) ≡ YZ)

22. ((X+Y) ≡ YZ) -> ((X+Y) -> YZ)

23. (((X+Y) -> YZ) AND (YZ -> (X))) -> (X+Y -> (X))

24.

a. (X+Y -> YZ) ≡ (NOT (X+Y) + YZ)

b. ((X+Y)(X+Y)) -> YZ) ≡ (NOT(X+Y) + NOT (X+Y))

Problem 20:

(a.) The distributive law

$pq + rs$

1. $pq + r$ distributive law $= \cancel{(p+q)}(p+r)$
2. $pq + s$ distributive law $= \cancel{(p+q)(p+s)}$

$\therefore pq + rs$ via $\boxed{\text{distributive law}}$

$$= \boxed{(p+r)(p+s)(q+r)(q+s)}$$

(b.) $pq + p\bar{q}r \to p(q+r)$

conclusion	reason
1. $p(q+\bar{q}r)$	distributive
2. $p(q+r)$	Elimination of certain negation

~~Problem 21:~~

~~a. $p(\bar{p}+q)$~~

~~(1) $p\bar{p} + pq$ distributive~~

~~(2) $p + pq$ Annihilation of AND~~

~~(3) pq~~

~~(1) $p\bar{p} + pq$ distributive~~

~~(2)~~ ~~commutative law of AND (1)~~

~~commutative law of OR (2)~~

Problem 21:

a. $p + pq \equiv p$

① $p + pq$ — Idempotence of OR

② $p = p$ — reflexive

③ $pp + p\cancel{q}$ — distributive

④ $p\cancel{q}$

⑤ p — Idempotence of AND

②

b. $p(p+q) \equiv p$

① $pp + pq$ — distributive

② $p + pq$ — Idempotence of and

③ $(p+p)(p+q)$ — distributive

④ $p(p+q) \equiv p$ — Idempotence of OR

⑤ p — law of OR

Problem 22:

a. NOT $pq + \bar{p}\bar{r}$

NOT$(\bar{p}\bar{q}r)$

$\underline{pq\,\bar{p}\bar{r}}$

-2

b. NOT(NOT $p + q$(NOT$(r + \bar{s})$))

not(not $p + q$(NOT$(r+s)$)))

NOT(not $p + q(\bar{r}\bar{s})$))

$\boxed{\bar{p}\bar{q}(r\bar{s})}$ $\bar{p}\,\bar{q}\,(r\bar{s})$

Problem 23:

1

~~1. $W\bar{X} + W\bar{X}Y + \bar{Z}\bar{X}W$~~

1. $W\bar{X} + W\bar{X}Y + \bar{Z}\bar{X}W$ Base

2 $W\bar{X} + \bar{Z}\bar{X}W$ Subsumption

3 $\bar{X}W + \bar{Z}\bar{X}W$ Commutative

4 $W\bar{X}$ Subsumption

2

1 $(W+\bar{X})(W+Y+\bar{Z})(\bar{W}+X+\bar{Y})(\bar{X})$ Base

2 $(\bar{X})(W+Y+\bar{Z})(\bar{W}+\bar{X}+\bar{Y})$ Subsumption

3 $(W\bar{X})+(\bar{X}\bar{Y})+(\bar{X}\bar{Z})$ [illegible]

-1