

(94)

Last Period 1

Problem 1:

P	q	$P \text{ NAND } q$	$P \text{ NOR } q$	$P \geq q$
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

Problem 2:

a.)

P	q	$P \rightarrow q$	$\text{NOT } P \text{ OR } q$	E
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

b.)

P	q	r	$\overbrace{r \text{ OR NOT } p}^a$	$\overbrace{\overline{q} \rightarrow a}^b$	$P \rightarrow b$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

(c)

P	Q	$P \text{ OR } Q$	$P \text{ AND } Q$	$(P \text{ OR } Q) \rightarrow (P \text{ AND } Q)$
0	0	0	0	1
0	1	1	0	0
1	0	1	0	0
1	1	1	1	1

Union

intersection
Except

Problem 3:

exclusion



Problem 4:

$$(P \overset{\circ}{\text{NAND}} \overset{\circ}{Q}) \overset{\circ}{\text{NAND}} R = \overset{\circ}{F} \quad \text{NAND} \neq \text{associative}$$

$$\overset{\circ}{P} \overset{\circ}{\text{NAND}} (\overset{\circ}{Q} \overset{\circ}{\text{NAND}} R) = \overset{\circ}{F}$$

$$(P \rightarrow Q) \rightarrow R = \overset{\circ}{F} \quad \rightarrow \neq \text{associative}$$

$$P \rightarrow (Q \rightarrow R) = \overset{\circ}{F}$$

$$(P \overset{\circ}{\text{NOR}} \overset{\circ}{Q}) \overset{\circ}{\text{NOR}} R = \overset{\circ}{F}$$

$$\overset{\circ}{P} \overset{\circ}{\text{NOR}} (\overset{\circ}{Q} \overset{\circ}{\text{NOR}} R) = \overset{\circ}{F}$$

NOR \neq associative

1111
0000

Problem 5

TRUE

Fals

NOT p

NOT q φ, q

(1)

Problem 6

Raise AND $\rightarrow P \text{ NOT}(q \rightarrow p) \wedge \text{ XOR OR NOR} \equiv \overline{q} \wedge q \rightarrow p \wedge \overline{p} \rightarrow q \text{ NAND TRUE}$

8	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
9	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	1
2	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

Problem 7

$$\begin{array}{c|cc|c} p & q & \otimes & \\ \hline 0 & 0 & 0 & \\ 0 & 1 & 1 & \\ 1 & 0 & 1 & \\ 1 & 1 & 0 & \end{array} \quad p \otimes q = q \otimes p$$
$$(p \otimes q) \otimes r = 0$$
$$p \otimes (q \otimes r) = 0$$

commutative And associative

Problem 8

$$a = \bar{p}\bar{q}r + \bar{p}\bar{q}\bar{r} + \bar{p}q\bar{r} + p\bar{q}\bar{r} + pq\bar{r}$$

$$b = \bar{p}q\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r}$$

✓

Problem 9:

$$a = (P+q+r)(P+\bar{q}+r)(P+\bar{q}+\bar{r}) \quad \checkmark$$

$$b = (P+\bar{q}+\bar{r})(P+q+\bar{r})(P+\bar{q}+\bar{r})(P+\bar{q}+r)(P+\bar{q}+\bar{r}) \quad \checkmark$$

$$c = (x+y+z)(x+\bar{y}+\bar{z})(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z) \quad \checkmark$$

Problem 10:

(a)

			rs	11	10
00	0	1	1	1	
01	1	1	1	1	
11	1	1	0	1	
10	1	1	1	1	

(b)

			rs	11	10
00	0	1	0	1	
01	1	0	1	0	
11	0	1	1	1	
10	1	0	1	0	

(c)

			rs	11	10
00	1	1	1	1	
01	1	1	0	1	
11	1	0	0	0	
10	1	1	0	1	

(d)

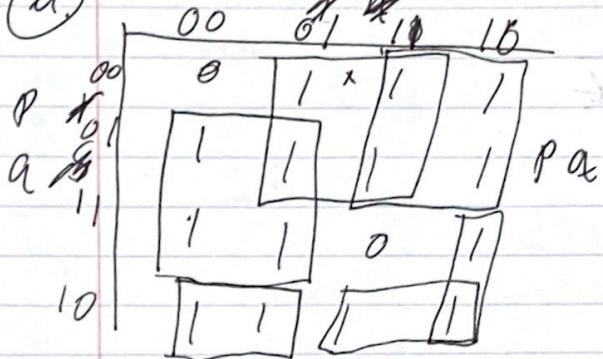
			rs	11	10
00	1	1	1	1	
01	1	1	1	1	
11	1	1	1	1	
10	1	1	1	1	

(e)

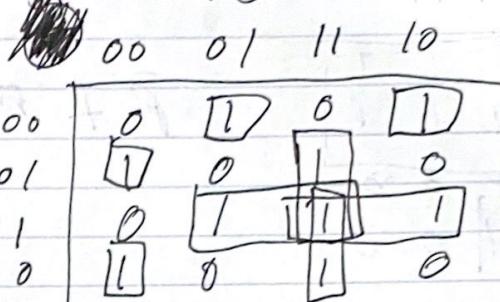
			rs	11	10
00	1	1	1	1	
01	1	1	1	1	
11	0	0	0	0	
10	1	1	0	0	

problem 11/12

(a)



rs



$$r\bar{s}p\bar{a} + r\bar{s}\bar{p}\bar{a} + \bar{r}s\bar{p}a + \bar{r}s\bar{p}\bar{a} + \dots$$

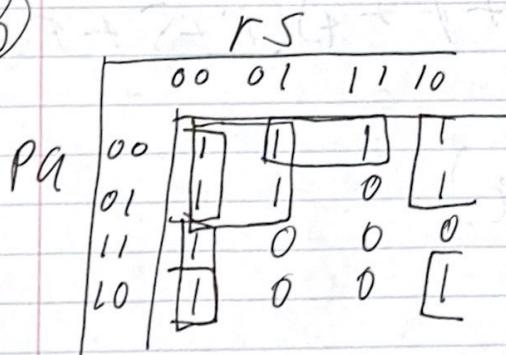
~~... + SP\bar{a} + RS\bar{a} + RS\bar{p} + RP\bar{a}~~

~~PAR + PRS + RP\bar{a} +~~ no prime implicants

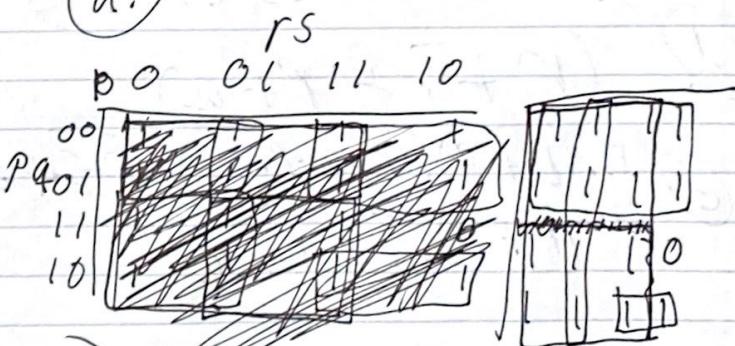
~~C\bar{P} + S\bar{P} + R\bar{P}~~

Prime Implicants

(b)



(d.)



$$\bar{s}p\bar{a} + r\bar{s} + \bar{s}\bar{p}\bar{a} + \dots$$

~~+ \bar{P}S + \bar{R}P~~

Prime Implicants

$$\bar{P} + \bar{P}R + \bar{P}S + \bar{P}aR$$

Q. rs

00	0	1	1	1	0
01	1	1	1	1	
11	0	0	0	0	
10	1	1	0	0	

$$\bar{P} + \bar{r}\bar{q}$$

✓

Problem 13/14:

13/14 (a) $(\bar{r} + \bar{s} + \bar{p} + \bar{q})(rs + q)$

13/14 (b) ~~$(r + s + q)(r + s + p)(p + q + s)(p + q + r)$~~

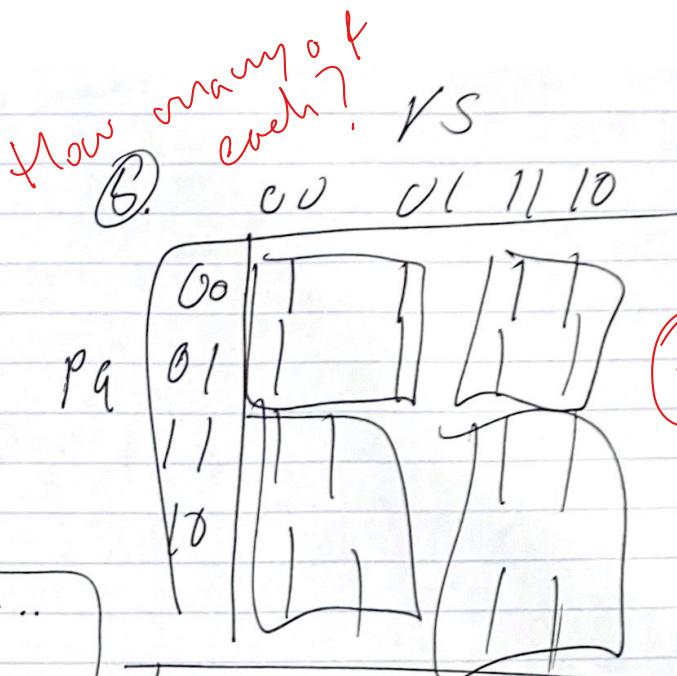
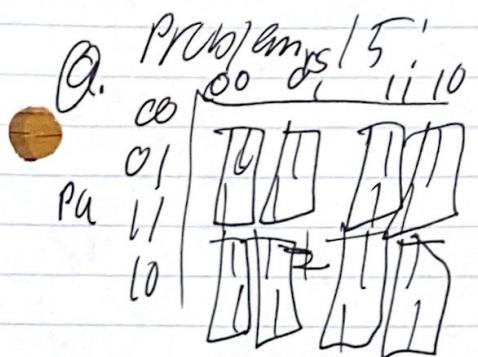
~~$(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p}) (\bar{r} + \bar{s} + \bar{p}) (\bar{r} + \bar{s} + \bar{p} + \bar{q})$~~

13/14 (c) $(\bar{p}a + \bar{r}s)(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})$
 $\cdot (\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})(\bar{r} + \bar{s} + \bar{p} + \bar{q})$

13/14 (d) $(r + \bar{s} + p + q)$

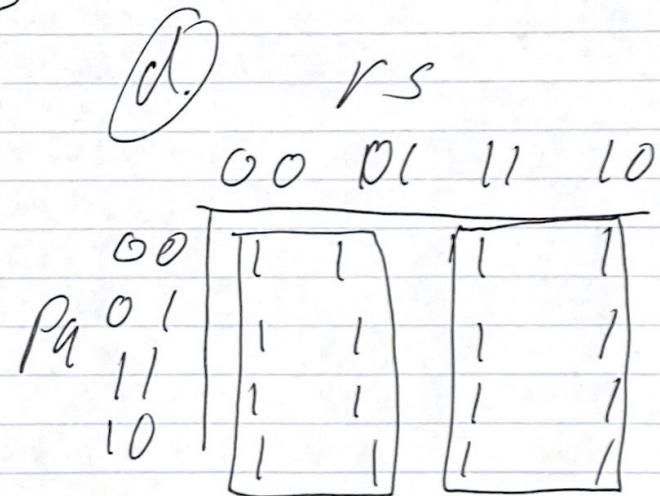
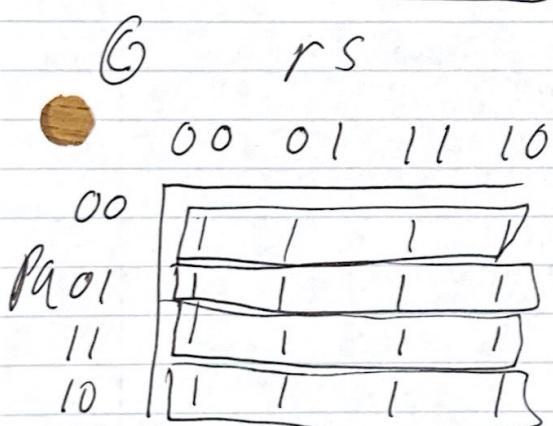
13. $(r + p)(\bar{r} + s + p + q)(\bar{r} + \bar{s} + \bar{p} + \bar{q})$

14. $(r + p)(\bar{r} + p + q)$



$$\begin{aligned} & \bar{R} \bar{S} \bar{P} + \bar{R} \bar{S} \bar{P} + \dots \\ & \dots + \bar{R} S \bar{P} + \bar{R} S \bar{P} + \dots \\ & \dots + R S P + R S \bar{P} + \dots \\ & \dots + R \bar{S} P + R \bar{S} \bar{P} + \dots \end{aligned}$$

$$\bar{R} \bar{P} + R P + \bar{R} P + R \bar{P}$$



$$\bar{P} \bar{A} + P \bar{A} + \bar{P} A + P A$$

$$T \bar{R} + R$$

Problem 16:

a.

pqr	$\overline{p \wedge q}$	$\overline{par} \rightarrow \overline{porq}$	$(\exists)(p \rightarrow q) (a \rightarrow r) \rightarrow (p \rightarrow r)$
0 0 0	0	1	1
0 0 1	0	1	1
0 1 0	1	1	1
0 1 1	1	1	1
1 0 0	1	1	1
1 0 1	1	1	1
1 1 0	1	1	1
1 1 1	1	1	1

Tautology C. Tautology

$p q$	$(p \rightarrow q) \rightarrow p$	$(p \equiv (q \vee r)) \rightarrow q \rightarrow p \wedge r$
0 0	0	1
0 1	0	1
1 0	1	1
1 1	1	1

not tautology D. not tautology

Program 17:

Both expressing one each to true

$P \wedge q$	$P \equiv P$	$P \vee Q \wedge \emptyset$	$P \vee Q \wedge \emptyset \equiv P$	$(P \equiv P) \equiv (P \vee Q \wedge \emptyset \equiv P)$
0 0	* 1	0	1	1
0 1	* 1	0	1	1
1 0	* 1	1	1	1
1 1	* 1	1	1	1

Problem 18:

<u>P</u>	<u>$P \equiv P$</u>
0	1
1	1

✓ $\stackrel{a}{\equiv} \stackrel{b}{\equiv}$

<u>p</u>	<u>q</u>	<u>$(P \equiv q)$</u>	<u>$(q \equiv P)$</u>	<u>$a \equiv b$</u>
0	0	1	1	1
0	1	0	0	1
1	0	0	0	1
1	1	1	1	1

<u>$p \wedge r$</u>	<u>$(P \equiv q)$</u>	<u>$(A \wedge D)$</u>	<u>$(a \equiv r)$</u>	<u>$(F \rightarrow)$</u>	<u>$(P \equiv r)$</u>
0 0 0	1	1	1	1	1
0 0 1	1	0	0	1	0
0 1 0	0	0	0	1	1
0 1 1	0	0	1	1	1
1 0 0	0	0	1	1	0
1 0 1	0	0	0	1	0
1 1 0	1	0	0	1	1
1 1 1	1	1	1	1	1

$\Rightarrow (P \equiv q) \text{ And } (a \equiv r) \rightarrow (P \equiv r)$

7.

<u>p</u>	<u>q</u>	<u>$\neg p$</u>	<u>$\neg q$</u>	<u>$(P \equiv q)$</u>	<u>$(\bar{P} \equiv \bar{q})$</u>	<u>$(P \equiv q) \equiv (\bar{P} \equiv \bar{q})$</u>
0	0	1	1	1	1	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
1	1	0	0	1	1	1

5.

P	q	$P \wedge q$	$\neg P$	$P \vee q$
0	1	0	1	1
0	0	0	1	0
1	1	1	0	1
1	0	0	0	1

6.

P	q	$P \wedge 1$	$(q \wedge 1)$	\neg	$(P \wedge q) \wedge 1$	1	E
0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	1
1	0	1	0	0	0	0	1
1	1	1	1	0	0	0	1
1	0	0	0	1	0	0	1
0	1	0	0	1	0	0	1
1	1	1	1	0	0	0	1

7.

$P \neg q$	$P \vee q$	\neg	$\neg P \vee q$	E
0 0	0	1	0	1
0 1	1	1	1	1
1 0	1	1	1	1

$P \text{ AND } Q$	$(P \text{ AND } Q) \text{ OR } (Q \text{ AND } P)$	$((P \text{ AND } Q) \text{ OR } (Q \text{ AND } P)) \equiv ((P \text{ OR } Q) \text{ AND } (Q \text{ OR } P))$
0 0 0	0 0	0 0 0
0 0 1	0 1	0 1 0
0 1 0	0 1	1 0 1
0 1 1	0 1	1 1 1
1 0 0	1 0	0 1 0
1 0 1	1 1	1 1 1
1 1 0	1 1	1 1 1
1 1 1	1 1	1 1 1

$P \text{ AND } Q$	Q OR P	$P \text{ AND } Q$					
0 0 0	0	0	0	0	0	0	1
0 0 1	1	0	0	0	0	0	1
0 1 0	0	0	0	0	0	0	1
0 1 1	1	0	0	0	0	0	1
1 0 0	0	0	0	0	0	0	1
1 0 1	1	1	0	1	1	1	1
1 1 0	1	1	1	1	0	1	1
1 1 1	1	1	1	1	1	1	1

⑩ $P \mid \overbrace{P \text{ AND } Q}^a \mid a = P$

P	$\overbrace{P \text{ AND } Q}^a$	$a = P$
0	0	1
1	1	1

⑪ $P \mid \overbrace{P \text{ OR } Q}^a \mid a = P$

P	$\overbrace{P \text{ OR } Q}^a$	$a = P$
0	0	1
1	1	1

⑫ $P \mid \overbrace{P \text{ AND } Q}^a \mid a = P$

P	$\overbrace{P \text{ AND } Q}^a$	$a = P$
0	0	1
1	0	1

⑬ $P \mid \overbrace{\neg P}^a \mid a = P$

P	$\overbrace{\neg P}^a$	$a = P$
0	1	1
1	0	1

14.

<u>PQR</u>	<u>ar</u>	<u>$P+Q R$</u>	<u>$P \oplus Q R$</u>	<u>$P \otimes R Q$</u>	<u>$b and d$</u>	<u>$d \equiv a$</u>
0 0 0	0	0	0	0	0	1
0 0 1	0	0	0	0	0	1
0 1 0	0	1 0	1	0	0	1
0 1 1	1	1	1	1	1	1
1 0 0	0	1	1	1	1	1
1 0 1	0	1	1	1	1	1
1 1 0	0	1	1	1	1	1
1 1 1	1	1	1	1	1	1

<u>P</u>	<u>$1 OR P$</u>	<u>$1 OR P = 1$</u>
0	1	1
1	1	1

<u>P</u>	<u>$P and P$</u>	<u>$P P = P$</u>
0	0	0
1	1	1

<u>P</u>	<u>$P OR P$</u>	<u>$P + P = P$</u>
0	0	0
1	1	1

<u>P</u>	<u>q</u>	<u>$P and q$</u>	<u>$Pq + P$</u>	<u>$E P = q$</u>
0 0	0	0	0	0
0 1	0	0	0	0
1 0	0	0	1	0
1 1	1	1	1	1

<u>P</u>	<u>q</u>	<u>$P and q$</u>	<u>$a and p$</u>	<u>$b = p$</u>
0 0	0	0	0	0
0 1	1	0	0	0
1 0	0	0	1	1
1 1	1	1	1	1

19. (a) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \neg p & \neg p \text{ OR } q & p \wedge q & p \wedge q & E \\ \hline 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ \hline \end{array}$

(b) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \bar{p} & \bar{p} \wedge q & \bar{p} \vee q & p \text{ OR } q & E \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \hline \end{array}$

20. (a) $\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline p & q & \bar{p} \text{ and } q & \neg q \wedge r & \neg p & \neg q & \neg p \vee \neg q & a \equiv b \\ \hline 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(b) $\begin{array}{|c|c|c|c|c|c|c|} \hline p & q & \neg p & \neg q & \text{NOT}(p \wedge q) & \neg p \vee \neg q & E \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(c) $\begin{array}{|c|c|c|c|c|c|} \hline p & \neg p & \neg p \text{ and } p & \text{not } p \vee \text{not } p & E \\ \hline 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ \hline \end{array}$

(d)

P	$\neg P$	$P \wedge \neg P$	$\text{not}(a) \neg P \text{ or } \neg P$	E
0	1	0	1	
1	0	0	0	

(21.)

p	q	$\neg p \rightarrow p$	$\neg p \rightarrow q$	$a \text{ and } b$	$p \equiv q$	E
0	0	1	1	0	0	
0	1	0	1	0	0	
1	0	1	0	0	0	
1	1	1	1	1	1	

(22.)

p	q	$P \equiv q$	$p \rightarrow q$	$a \rightarrow b$
0	0	1	1	1
0	1	0	1	1
1	0	0	0	1
1	1	1	1	1

(23.)

pqr	$P \rightarrow a$	$q \rightarrow r$	$a \text{ and } b$	$P \rightarrow R$	E
000	1	1	1	1	
001	1	1	1	1	
010	1	0	0	0	
011	1	1	1	1	
100	0	1	0	0	
101	0	1	0	1	
110	1	0	0	0	
111	1	1	1	1	

28

(A)

P	q	$P \rightarrow q$	$\neg P$	$\neg P \text{ OR } q$	E
0	0	1	1	1	
0	1	1	1	1	
1	0	0	0	0	
1	1	1	0	1	

(B)

P	q	$\frac{P \text{ and } P}{\neg q}$	$\neg q \rightarrow q$	$\neg P \text{ OR } P$	$\neg q \text{ OR } q$	E
0	0	0	1	0	0	
0	1	0	1	0	0	
1	0	1	0	1	1	
1	1	1	0	1	1	

problem 1g : $x+y = p$ $yz = q$ $r = (x)$ ✓

1. $x+y \equiv x+y$

2. $(x+y \equiv yz) \equiv (yz \equiv x+y)$

3. $((x+y \equiv yz) \text{ AND } (yz \not\equiv x)) \rightarrow (x+y) \equiv (x)$

4. $(x+y \equiv yz) \equiv (\text{Not}(x+y) \equiv \text{Not}(yz))$

5. $(x+y)(y) \equiv (x)(y+y)$

6. $(x+y)((yz)(x)) \equiv ((x+y)(yz))(x)$

7. $((x+y) + (yz)) \equiv ((yz) + (x+y))$

8. $((x+y) + (yz + x)) \equiv (((x+y) + yz) + x)$

9. $(x+y)(yz) + (x) \equiv (x+y)(yz) + ((x+y)(yz) + (x+y)(x))$

10. $(x+y) \text{ AND } 1 \equiv x+y$

11. $(x+y) \text{ OR } 0 \equiv x+y$

12. $((x+y) \text{ AND } 0) \equiv 0$

13. $(\text{Not}(\text{Not}(x+y))) \equiv x+y$

14. $((x+y) + (yz))(x) \equiv (((x+y) + (xy))((x+y) + (x)))$

15. $(1 \text{ OR } (x+y)) \equiv 1$

16. $(x+y)(\bar{x}+y) \equiv x+y$

$$17. (X+Y) + (X+Y) \equiv X+Y$$

18.

$$a. (X+Y) + (X+Y) \equiv X+Y$$

$$b. (X+Y)((X+Y) + YZ) \equiv X+Y$$

~~$(X+Y) + (X+Y)(YZ)$~~

19.

$$a. (X+Y)(\text{NOT}(X+Y) + YZ) \equiv (X+Y)(YZ)$$

$$b. (X+Y) + \text{NOT}(X+Y)(YZ) \equiv (X+Y) + (YZ)$$

$$20. a. \text{NOT}((X+Y)(YZ)) \equiv \text{NOT}(X+Y) + \text{NOT}(YZ)$$

$$b. \text{NOT}(X+Y) + YZ \equiv \text{NOT}((X+Y)YZ)$$

$$c. (\text{NOT}(X+Y)(X+Y)) \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$$

$$d. (\text{NOT}(X+Y) + (X+Y)) \equiv (\text{NOT}(X+Y) \text{ OR } \text{NOT}(X+Y))$$

$$21. ((X+Y) \rightarrow YZ) \text{ AND } (YZ \rightarrow (X+Y)) \equiv (X+Y) \equiv YZ$$

$$22. ((X+Y) \equiv YZ) \rightarrow ((X+Y) \rightarrow YZ)$$

$$23. ((X+Y) \rightarrow YZ) \text{ AND } (YZ \rightarrow (X+Y)) \rightarrow (X+Y \rightarrow X)$$

24.

$$a. (X+Y \rightarrow YZ) \equiv (\text{NOT}(X+Y) + YZ)$$

$$b. ((X+Y)(X+Y)) \rightarrow YZ \equiv (\text{NOT}(X+Y) + \text{NOT}(X+Y))$$

Problem 20:

(a) The d.istributive law

$$\begin{array}{ll} P(a+r+s) & \\ \text{distributive law} = & \cancel{(P(a+r))} \\ \text{distributive law} = & \cancel{(P(a)+P(r+s))} \end{array}$$

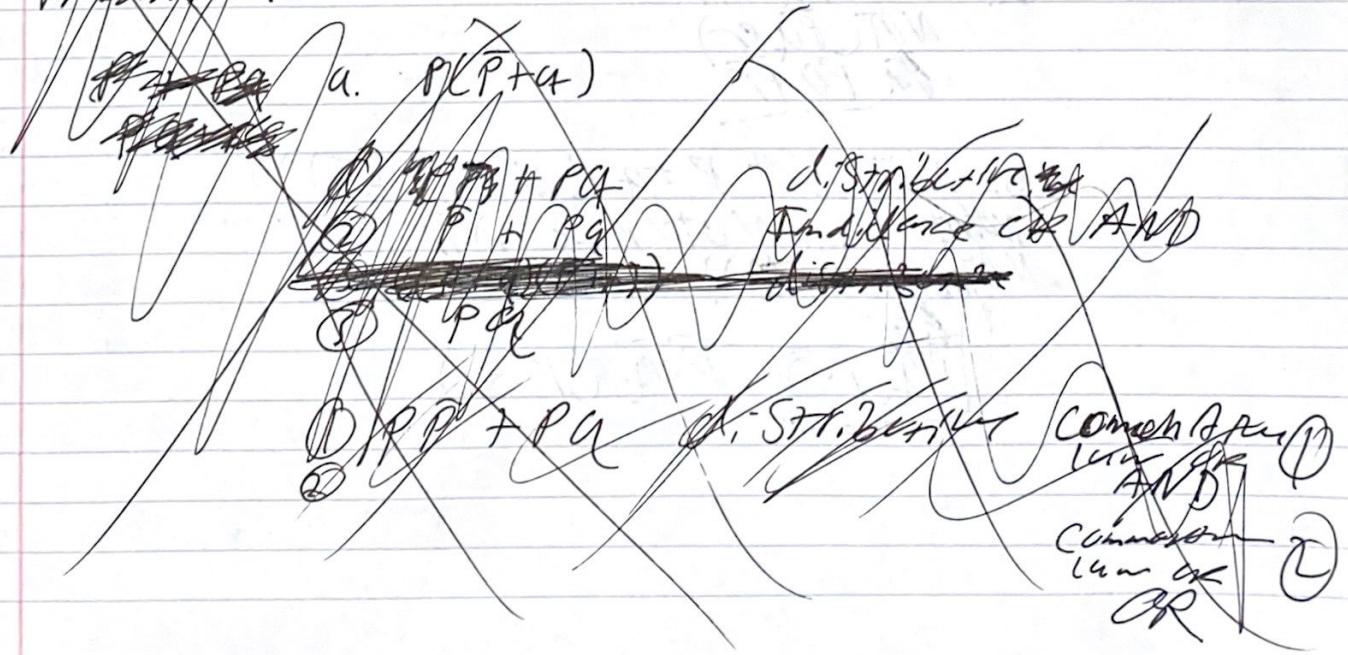
P(a+r+s) via distributive law

$$\boxed{\cancel{(P+r)(P+s)}(a+r)(a+s)}$$

(b) $\cancel{P(a+Pqr)} \rightarrow P(a+r)$

1. $P(a+\cancel{qr}) \neq$ distributive
2. $\cancel{P(a+r)}$ Elimination of certain negation

Problem 21:



Problem 21:

a. $P + pq \equiv P$

① $P + Pa$

② $P = P$

③ $Pp + Pa$

④ ~~Pa~~

⑤ P

Independence of OR

reflexive

distributive

②

Indulgence or AND

b. $P(P+q) \equiv P$

① $PP + Pq$

② $P + Pa$

③ $(P+P)(P+q)$

④ $P(P+q) \equiv P$

⑤ P

distributive

Indulgence of and
distributive

Indulgence of OR

law of OR

Problem 22.

a. $\text{NOT } Pa + \bar{P}\bar{r}$

$\text{NOT}(\overline{Pa} \cap \bar{r})$

~~$\overline{Pa} \overline{P} \bar{r}$~~

②

b. $\text{NOT}(\text{NOT } P + q \cap \text{NOT}(r + \bar{s}))$

$\text{NOT}(P + q \cap \text{NOT}(r + s)) \Rightarrow$

$\text{NOT}(P + q \cap \bar{r}s)$

~~$\overline{Pa}(\bar{r}s)$~~ $\bar{P} \bar{q} (r\bar{s})$

Problem 23:

①

$$\cancel{w\bar{x} + w\bar{x}\gamma + \bar{z}\bar{x}w}$$

①. ~~w \bar{x}~~ $w\bar{x} + w\bar{x}\gamma + \bar{z}\bar{x}w$ Base

② $w\bar{x} + \bar{z}\bar{x}w + \cancel{\bar{z}\bar{x}w}$ Subsumption

③

$$\cancel{\bar{x}w} + \bar{z}\bar{x}w$$

Commut.

~~④~~ $w\bar{x}$

Subsumption

②

$$① (w+x)(w+\gamma+\bar{z})(\bar{w}+x+\bar{\gamma})(\bar{x}) \quad \text{Base}$$

② $(\bar{x})(w+\gamma+\bar{z})(\bar{w}+\bar{x}+\bar{\gamma})$ Subsumption

③ $(w\bar{x}) + (\bar{x}\gamma) + (\bar{x}\bar{z})$

Replacement

