Peering Through The Fog – Using Ring Seismology to Determine Saturn's Composition and Rotation Profile

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An accurate picture of the structure and composition of the gas giant Saturn is key to unlocking a complete narrative of how the Solar System we observe today was established. And will answer questions such as: are the gas giants convective throughout their interiors? And did the gas giants form around solid planetesimal cores? During the last five years accurate limits have been placed on the mass distributions and rotation profile of Saturn. Primarily through measurements made by the Cassini spacecraft and using a technique known as ring seismology [1].

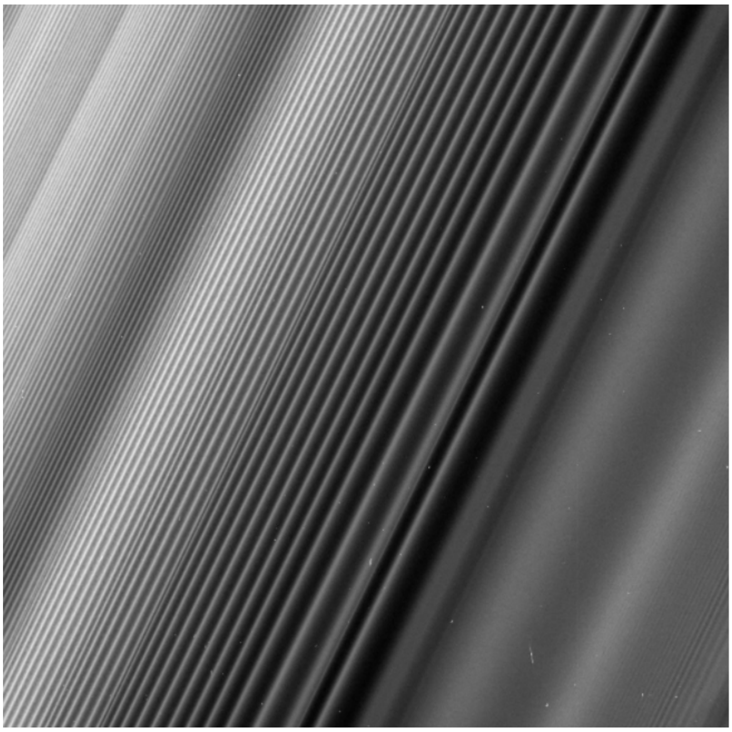

Figure 1: Density waves represented by alternating light and dark bands within an image of Saturn's C Ring taken by Cassini. Observable in the lower right hand corner of the image is a bending wave exhibiting a vertical perturbation of ring particles [3].

During its orbit of Saturn Cassini observed regular structures of matter density, known as spiral waves, within Saturn's rings. Whilst Voyager had observed spiral waves produced by Saturn's larger satellites, Cassini was able to pick out similar structures within Saturn's C-ring [1]. These spiral patterns are generated by periodic gravitational forcing of ring material, as a result of oscillations of the planet itself [2].

Just as earthquakes occurring on Earth have been used to study the planet's interior, so the vibrations of Saturn can be used to glean information about its core. The most likely source of Saturn's vibrations is heat driven convection in the planet's interior [4]. Spiral waves observed in Saturn's C-ring are not close enough to any known moons to be generated by resonance with their gravitational pull. Therefore, their generation is attributed to gravitational forcing caused by low order normal mode oscillations of Saturn [2, 5]. These planetary oscillations cause periodic variations in Saturn's gravitational field, which in turn disturb the usually well-ordered orbits of particles within the planet's rings. Using the observed ring patterns the full spectrum of allowed planetary oscillation frequencies can be determined. As this spectrum is dictated by the planet's mean density and rotation speed, a picture of Saturn's interior can be built. This process forms the foundation of ring seismology [1].

Two types of spiral wave exist in the form of bending and density waves. Density waves manifest as ring-plane compressions and rarefactions in the density of ring material. Bending waves are produced where resonance induces particle deviations orthogonal to the ring plane. Thus producing out of plane, sinusoidal distributions of ring density [5]. Observations of both types of waves are shown in figure 1. Each spiral wave consists of an integer number of arms that become more tightly wrapped with increasing distance from the point of exact generation. The entire wave rotates at a single pattern speed Ω_p [2].

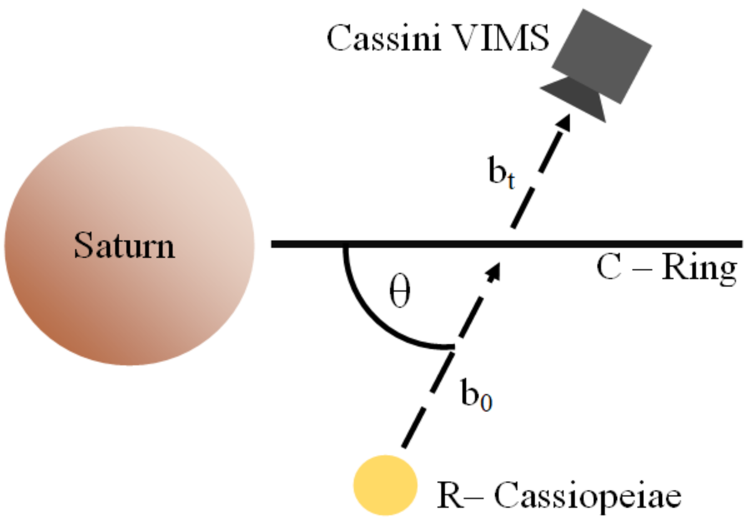

Figure 2: Geometry of the VIMS measurements performed by Cassini during it's orbit around Saturn. The attenuation of the light (transmission T) emitted by the reference star R-Cassiopeiae due to interception by the ring, is demonstrated via the ratio of the incident and transmitted star brightness b_t/b_0. The plane of the ring is shown using a solid black line and the direction of emitted light via dashed black lines

In order to study spiral waves, occultation profiles are generated using Cassini's onboard Visual and Infrared Mapping Spectrometer (VIMS) [5]. These profiles map the radial change in density across a ring. Using the apparent change in brightness of a reference star located behind the rings, as a result of a change in ring opacity (density change) [2]. Cassini was able to cal-

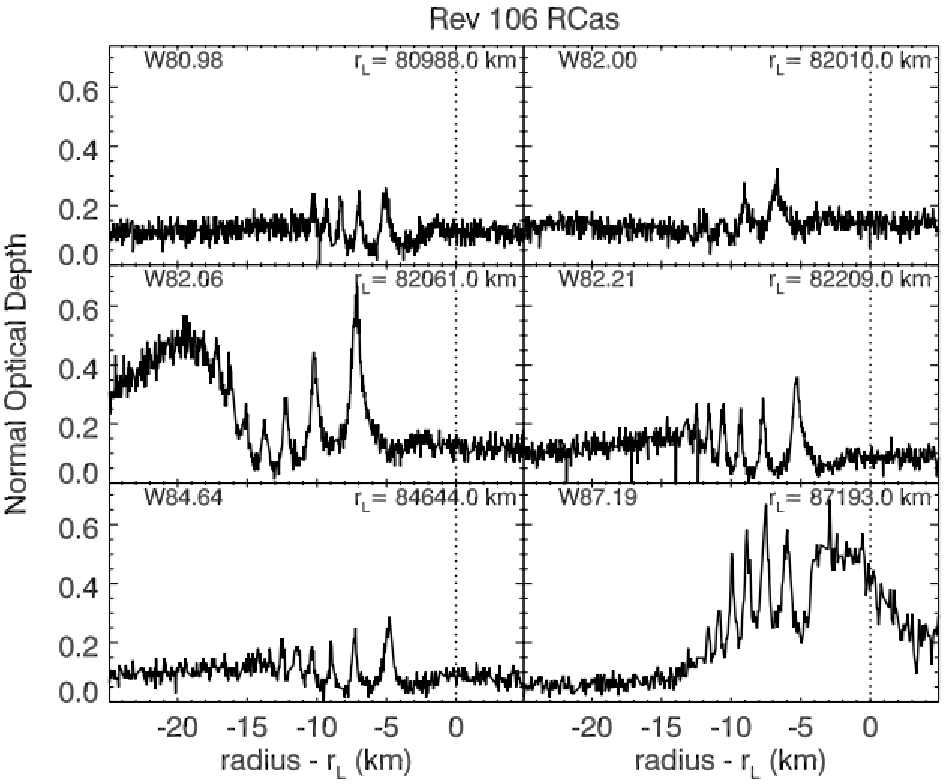

Figure 3: Occultation profiles extracted from [2] for six newly identified spiral waves. Each figure shows the change in the normal optical depth of Saturn's C-ring with increasing distance from the exact point of resonance r_L. The angle of the star relative to the ring plane (θ) is 56.04°

culate the transmission *T* through the rings, as a ratio of the observed reference star brightness through a given ring location to the brightness observed outside of the ring region. The optical depth is then calculated as τ = -$ln(T)$. Using the observation geometry demonstrated in figure 2, the normal optical depth is expressed as $\tau_n = \tau sin(\theta)$. Where θ is the angle between the ring plane and the incident star light [6]. Considering the example occultation profiles included in figure 3, the observed localized periodic fluctuations in the optical depth are attributed to spiral waves manipulating the density of ring material.

The perturbation of Saturn's density $\rho^{'}$ as a result of normal mode oscillations, can be described using a time dependent Eulerian perturbation. Which is expressed as

$$\rho^{'}_{lmn}(r, \theta, \psi, t) = \rho^{'}_{lmn}(r) Y^{m}_{l}(\theta, \psi) e^{-i\sigma_{lmn}t}. \tag{1}$$

The integers *l*, *m* and *n* indicate the specific harmonic degree and order. The variable *t* denotes time. The eigenfrequency of a particular mode is expressed as σ_{lmn}. The spherical coordinates: r, ψ and θ are used to indicate positions on the surface of the planet. The variable *Y* indicates the possible spherical harmonics, defined using the associated Legendre polynomial $P^{m}_{l}(cos\theta)$. An example of how spherical harmonics perturb the density is demonstrated in figure 4. Perturbation patterns evolve with time as they rotate around the planet, with magnitude changes shown using a colour map [7].

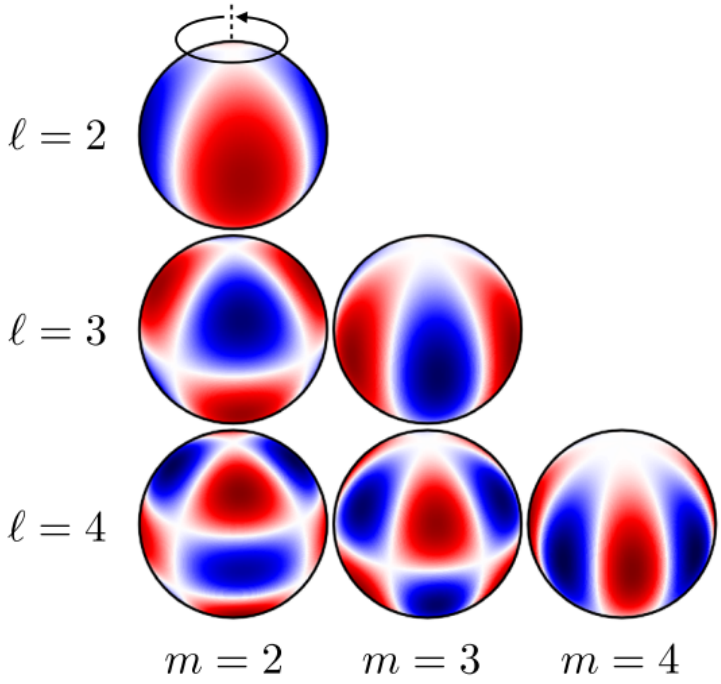

Figure 4: Spherical harmonic representations of the possible perturbations of Saturn. Described using their angular degree (l) and azimuthal order (m). The magnitude of the density perturbations is portrayed using colour density with blue equal to a region of greater perturbation and red a smaller perturbation. The axial rotation of the patterns is indicated. Adapted from [1].

The perturbation of the planet's density, combined with the rotation of the planet, generates a varying gravitational field $U(\theta, \lambda, t)$, which may be expressed as

$$U(\theta, \lambda, t) = P_l^m(cos\theta)e^{i[m(\lambda - \Omega_p t)]}, \tag{2}$$

where λ is the inertial longitude and Ω_{pat} is the pattern speed of the oscillation given as

$$\Omega_{pat} = \frac{1}{m}\sigma_{lmn}. \tag{3}$$

With all other terms as previously defined [8].

The interaction of the gravitational field with ring material may be described using Lindbald and Vertical resonances. During such resonances the varying gravitational potential exerts a torque on the ring material (gravitational forcing) [4]. For a Lindbald resonance the torque exerted during an interaction is positive and hence angular momentum is added to ring particles causing them to propagate outwards as a density wave. This type of Lindbald resonance generated by a planetary mode is known as an Outer Lindbald Resonance. Vertical resonances have the opposite effect, causing a wave to propagate towards the planet and generating bending wave structures [9]. As discussed the type of spiral wave produced is dependent on the generating oscillation mode and associated spherical harmonic. Modes with even *l-m* values stimulate radial oscillations and density waves. Whilst modes with odd *l-m* stimulate vertical oscillations and drive bending waves [4].

Most parts of the ring experience a negligible response to the gravitational forcing. As most material experiences opposite extrema of the planet's oscillation at random locations within their orbits. Therefore cancelling any overall response [1]. In order for an overall non-cancellation condition to be met the resonance may only occur at certain radii from the planet known as resonance locations r_l. At these locations the following condition is satisfied

$$m(n - \Omega_p) = \kappa, \tag{4}$$

where *n* is the orbital mean motion of the ring particles and κ is their epicyclic frequency. All other terms are as previously defined. This convenient requirement allows us to pick out individual spiral waves structures to study, within an otherwise evenly distributed ring. Hence the rings act like a natural seismograph of Saturn's vibrational history [7, 10].

The variation of the optical depth of the ring is dependent on the phase ϕ of the spiral wave at a radius *r* from Saturn. And is expressed as

$$\phi(r, \lambda, t) \approx |m|(\lambda - \Omega_p t) + \phi_r(r), \tag{5}$$

where $\phi_r(r)$ is the radial component of the phase (which later cancels). Using occultation data gathered by Cassini at different times and longitudes, we can compute the phase difference between profiles gathered. This difference is expressed as

$$\delta\phi = |m|(\delta\lambda - \Omega_p \delta t), \tag{6}$$

with all terms as previously defined. By comparing measured phase differences to theoretical predictions we can subsequently obtain two important unknown parameters. Firstly $|m|$ is equal to the number of spiral wave arms and and the generating mode's azimuthal order. Secondly, Ω_p is the spiral pattern rotation speed and is equal to the generating mode's propagation speed around the planet. These two parameters allow important conclusions to be made about the interior of the planet [2, 10].

The number of waves predicted by Marley and Porco within their original paper for the technique differed from the data recorded by Cassini and Voyager [8]. The expectation was the observation of spiral waves at distinct resonance locations, corresponding to fundamental mode oscillations (f-modes) due to standing surface gravity waves. Subsequent work uncovered clusters of waves in the regions of the strongest f-mode resonances [2]. The smaller clusters were later explained through an inclusion into the seismological model, that allowed Saturn to accommodate trapped internal gravity waves (g-modes). This new addition requires parts of Saturn's interior to be stably stratified in a gradient of molecular weight. A distinct deviation from the previously accepted model of a fully convective environment and a crucial aspect of successive models [11, 12].

Ring seismology also delivers a second major Saturnian insight, in the form of accurate constraints on Saturn's rotation rate. The incommensurability of the rotation of Saturn's meteorological systems with the rotation rate of the planet's solid interior makes the fundamentally important quantity difficult to measure [1]. Additionally, the parallel alignment of Saturn's magnetic dipole axis with its rotation axis restricts the measurement of Saturn's rotation from ground-based instruments [13]. Prior to the application of ring seismology Saturn's rotation rate was only known to an accuracy of $\pm$ 20 minutes. Using the value for Ω_p obtained through occultation data has lead to a rotation rate calculation of 10 hr 33 min 28 s $\pm \approx 1$ min. A faster value than previously measured, leading to a growing consensus that Saturn's rotation is coupled poorly to measured perturbations of Saturn's magnetosphere. This suggests additional field generating mechanisms have yet to be uncovered [1, 14].

Ring Seismology is therefore a powerful technique in the arsenal of astrophysicists due to its ability to penetrate into the inner workings of ringed planets. And whilst a complete model of Saturn's interior currently eludes us, using insights gained from ring seismology as a compliment to the data sets gathered during Cassini's final mission has provided clear indicators of certain model features [13, 14]. The next step in uncovering the planetary origins within our Solar System is to undertake missions to the ice giants Uranus and Neptune. Where is it likely that Neptune's rings will provide an opportunity to apply ring seismology techniques [15, 16].

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