

# HW3

P1) Let  $w_1 = \frac{P_1}{P_1 + P_2}$  ,  $w_2 = \frac{P_2}{P_1 + P_2}$

Then

trivially

$$w_1 + w_2 = 1$$

Now

$$D_1 = -\frac{(1+r)}{P_1} \frac{dP_1}{dr}$$

$$D_2 = -\frac{(1+r)}{P_2} \frac{dP_2}{dr}$$

(Macaulay duration)  
but also time of  
liability see  
slide 6.

$$D = -\frac{(1+r)}{P} \frac{dP}{dr} = -\frac{(1+r)}{P_1 + P_2} \frac{d(P_1 + P_2)}{dr}$$

$$\left( \text{where } P = P_1 + P_2 \right)$$

We Calculate

$$w_1 D_1 + w_2 D_2 = \frac{P_1}{P_1 + P_2} \left( -\frac{(1+r)}{P_1} \frac{dP_1}{dr} \right) + \frac{P_2}{P_1 + P_2} \left( -\frac{(1+r)}{P_2} \frac{dP_2}{dr} \right)$$

$$= -\frac{(1+r)}{P_1 + P_2} \left( \frac{dP_1}{dr} + \frac{dP_2}{dr} \right) = -\frac{(1+r)}{P_1 + P_2} \frac{d(P_1 + P_2)}{dr} = -\frac{(1+r)}{P} \frac{dP}{dr} \quad P$$