1.1)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft*

From Fig. P1.1, the maximum Mach number with afterburner ("Max" thrust curve) is $M \approx 1.075$. From Table 1.2b, the standard day speed of sound at 20,000 *ft* is 1036.94 *ft/s*. Using the definition of the Mach number as $M \equiv V / a$ (the instructor may have to give this definition to the students since it is not found in Chapter 1), the velocity is:

$$V = Ma = (1.075)(1036.94 ft/s) = 1114.7 \ ft/s$$

The maximum lift-to-drag ratio, $(L/D)_{\max}$, occurs at the point of minimum drag, $D_{\min} = 850 \ lbf$. Assuming that the airplane is in steady, level, unaccelerated flight, $L = W = 10{,}000 \ lbf$, and:

$$(L/D)_{\max} = 10{,}000 lbf / 850 lbf = 11.76$$

The Mach number where this occurs is $M \approx 0.53$. The minimum velocity of the aircraft under the given conditions is $M \approx 0.34$ and is due to the buffet (or stall) limit.

1.2)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft* with "Mil" thrust at $M = 0.65$

From Eqn. 1.1, the total energy of the aircraft is:

$$E = 0.5mV^2 + mgh$$

The mass of the airplane is given by Eqn. 1.2:

$$m = W / g = 10{,}000 lbf / 32.174 ft/s^2 = 310.81 \ slugs$$

As in Problem 1.1, the velocity of the airplane can be found as:

$$V = Ma = (0.65)(1036.94 ft/s) = 674.01 \ ft/s$$

which yields a total energy of:

$$\begin{aligned} E &= 0.5mV^2 + mgh \\ &= 0.5(310.81 slugs)(674.01 ft/s)^2 + (310.81 slugs)(32.174 ft/s)(20{,}000 ft) \\ &= 270.60 \times 10^6 \ ft\text{-}lbf \end{aligned}$$

1.2) contd.

The energy height is given by Eqn. 1.3:

$$H_e = E/W = 270.60 \times 10^6 \ ft - lbf\ /10{,}000lbf = 27{,}060 \ ft$$

The specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T-D)V}{W}$$

At the given conditions, $T = 2{,}500 \ lbf$ and $D = 1{,}000 \ lbf$ from Fig. P1.1, and the specific excess power is:

$$P_s = \frac{(T-D)V}{W} = \frac{(2{,}500lbf - 1{,}000lbf)(674.01 ft/s)}{10{,}000lbf} = 101.1 \ ft/s$$

1.3)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft* with "Mil" thrust at $M = 0.65$

The acceleration possible is given by Eqn. 1.5 as:

$$\frac{(T-D)V}{W} = \frac{V}{g}\frac{dV}{dt}$$

For the conditions of Problem 1.2, the acceleration would be:

$$\frac{dV}{dt} = \frac{(T-D)V}{W}\frac{g}{V} = P_s \frac{g}{V} = (101.1 ft/s)\frac{32.174 ft/s^2}{674.01 ft/s} = 4.826 ft/s^2$$

The rate of climb is given by Eqn. 1.7 as:

$$\frac{dh}{dt} = P_s = 101.1 ft/s = 6{,}066 ft/m$$

1.4)

Given:

T-38A flying at 20,000 *ft* with weight of 8,000, 10,000, and 12,000 *lbf*

As in Problem 1.1, the maximum lift-to-drag ratio, $(L/D)_{\max}$, occurs at the minimum drag, $D_{\min}$. Also, assuming that the airplane is in steady, level, unaccelerated flight, $L = W$. For the three weights you can generate a table for finding $(L/D)_{\max}$ using values from Fig. P1.1 and Tab. 1.2b:

$W = L$ (*lbf*)	$D_{\min}$ (*lbf*)	$(L/D)_{\max}$	$M_{(L/D)_{\max}}$	$V_{(L/D)_{\max}}$ (*ft/s*)
8,000	650	12.31	0.47	487.3
10,000	850	11.76	0.53	549.6
12,000	1050	11.43	0.60	622.2

As can be seen, higher weight requires higher velocities to maintain aerodynamic efficiency, but also results in a reduction of that efficiency.

1.5)

Given:

10,000 *lbf* T-38A flying at 20,000 *ft* at $M = 0.35$, $M_{(L/D)_{\max}}$, and 0.70

As in Problem 1.2, the specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T - D)V}{W}$$

For the three velocities you can generate a table for specific excess power using values from Fig. P1.1:

M	T (*lbf*)	D (*lbf*)	V (*ft/s*)	P_s (*ft/s*)
0.35	2250	1600	362.93	23.6
0.53	2400	850	549.58	85.2
0.70	2600	1100	725.86	108.9

Notice the large increase in specific excess power from $M = 0.35$ to 0.53 as the airplane becomes more aerodynamically efficient. While the specific excess power continues to increase as the speed increases to $M = 0.70$, the increase in P_s is not as dramatic due to the increase in drag.

1.6)

Given:

Air and nitrogen with associated gas and Sutherland's law constants at $p = 150\ psia$ and $T = 350^\circ F$.

For a thermally perfect gas, the equation of state is given by Eqn. 1.10:

$$p = \rho RT$$

The fluid density can be found as:

$$\rho = \frac{p}{RT}$$

where R, the gas constant, is given in the problem statement, and $T = 350 + 459.67 = 809.67^\circ R$. The resulting densities are:

$$\rho_{Air} = \frac{p_{Air}}{R_{Air}T_{Air}} = \frac{(150 lbf/in^2)(144 in^2/ft^2)}{\left(53.34\frac{ft-lbf}{lbm-^\circ R}\right)(809.67^\circ R)} = 0.50014\ lbm/ft^3 = 0.01554\ slug/ft^3$$

$$\rho_N = \frac{p_{N_2}}{R_{N_2}T_{N_2}} = \frac{(150 lbf/in^2)(144 in^2/ft^2)}{\left(55.15\frac{ft-lbf}{lbm-^\circ R}\right)(809.67^\circ R)} = 0.48373\ lbm/ft^3 = 0.01503\ slug/ft^3$$

These values are very close to each other due to the very similar gas constants. The viscosity is given by Sutherland's law in English units, Eqn. 1.12b:

$$\mu = C_1\frac{T^{1.5}}{(T + C_2)}$$

where the constants are also given in the problem statement. The resulting viscosities are:

$$\mu_{Air} = 2.27\times10^{-8}\frac{lbf-s}{ft^2-^\circ R^{0.5}}\frac{(809.67^\circ R)^{1.5}}{(809.67^\circ R + 198.6^\circ R)} = 5.18694\times10^{-7}\frac{lbf-s}{ft^2}$$

$$\mu_{N_2} = 2.16\times10^{-8}\frac{lbf-s}{ft^2-^\circ R^{0.5}}\frac{(809.67^\circ R)^{1.5}}{(809.67^\circ R + 183.6^\circ R)} = 5.01012\times10^{-7}\frac{lbf-s}{ft^2}$$

1.6) contd.

Finally, the kinematic viscosity is given by Eqn. 1.6 as:

$$\nu = \frac{\mu}{\rho}$$

and the resulting kinematic viscosities are:

$$\nu_{Air} = \frac{\mu_{Air}}{\rho_{Air}} = \frac{5.18694 \times 10^{-7} \dfrac{lbf - s}{ft^2}}{0.01553 \dfrac{slug}{ft^3}} = 3.33995 \times 10^{-5} \; ft^2/s$$

$$\nu_{N_2} = \frac{\mu_{N_2}}{\rho_{N_2}} = \frac{5.01012 \times 10^{-7} \dfrac{lbf - s}{ft^2}}{0.01502 \dfrac{slug}{ft^3}} = 3.33563 \times 10^{-5} \; ft^2/s$$

1.7)

Given:

Air and nitrogen with associated gas and Sutherland's law constants at $p = 586 \; N/m^2$ and $T = 54.3K$.

For a thermally perfect gas, the equation of state is given by Eqn. 1.10:

$$p = \rho RT$$

The fluid density can be found as:

$$\rho = \frac{p}{RT}$$

where R, the gas constant, is given in the problem statement. The resulting densities are:

$$\rho_{Air} = \frac{p_{Air}}{R_{Air} T_{Air}} = \frac{58.6 N/m^2}{\left(287.05 \dfrac{N - m}{kg - K}\right)(54.3K)} = 0.03760 \; kg/m^3$$

1.7) contd.

$$\rho_N = \frac{p_{N_2}}{R_{N_2} T_{N_2}} = \frac{(58.6 N/m^2)}{\left(297 \dfrac{N-m}{kg-K}\right)(54.3K)} = 0.03634\, kg/m^3$$

These values are very close to each other due to the very similar gas constants. The viscosity is given by Sutherland's law in English units, Eqn. 1.12a:

$$\mu = C_1 \frac{T^{1.5}}{(T + C_2)}$$

where the constants are also given in the problem statement. The resulting viscosities are:

$$\mu_{Air} = 1.458 \times 10^{-6} \frac{kg}{s-m-K^{0.5}} \frac{(54.3K)^{1.5}}{(54.3K + 110.4K)} = 5.18694 \times 10^{-7} \frac{lbf-s}{ft^2}$$

$$\mu_{N_2} = 1.458 \times 10^{-6} \frac{kg}{s-m-K^{0.5}} \frac{(54.3K)^{1.5}}{(54.3K + 102K)} = 5.01012 \times 10^{-7} \frac{lbf-s}{ft^2}$$

Finally, the kinematic viscosity is given by Eqn. 1.6 as:

$$\nu = \frac{\mu}{\rho}$$

and the resulting kinematic viscosities are:

$$\nu_{Air} = \frac{\mu_{Air}}{\rho_{Air}} = \frac{5.18694 \times 10^{-7} \dfrac{lbf-s}{ft^2}}{0.01553 \dfrac{slug}{ft^3}} = 3.33995 \times 10^{-5}\, ft^2/s$$

$$\nu_{N_2} = \frac{\mu_{N_2}}{\rho_{N_2}} = \frac{5.01012 \times 10^{-7} \dfrac{lbf-s}{ft^2}}{0.01502 \dfrac{slug}{ft^3}} = 3.33563 \times 10^{-5}\, ft^2/s$$

1.8)

Given:

A perfect gas process which doubles the pressure and the density is decreased by three-quarters. Initial temperature $T_1 = 200°F$.

The pressure and density changes during the process may be represented as:

$$\frac{p_2}{p_1} = 2 \text{ and } \frac{\rho_2}{\rho_1} = \frac{3}{4}$$

The perfect gas law is given by Eqn. 1.10:

$$p = \rho RT$$

So each state may be represented by:

$$R_1 = \frac{p_1}{\rho_1 T_1} \text{ and } R_2 = \frac{p_2}{\rho_2 T_2}$$

Assuming that the gas constant does not change significantly due to changes in temperature (i.e. $R_1 = R_2$), the equations can be combined to yield:

$$\frac{T_2}{T_1} = \frac{p_2/p_1}{\rho_2/\rho_1} = \frac{2}{3/4} = \frac{8}{3}$$

Remembering that temperatures in the perfect gas law must be in absolute values, the final temperature can then be found as:

$$T_2 = \frac{8}{3}T_1 = \frac{8}{3}(200 + 459.67)^\circ R = 1759^\circ R = 1299^\circ F = 703.9^\circ C$$

1.9)

Given:

Isentropic expansion of perfect air $\frac{p}{\rho^{1.4}}$ = constant and $\frac{p_2}{p_1} = 0.5$

Initial temperature $T_1 = 20°$ C

For a perfect gas $\rho = \frac{p}{RT}$.

1.9) contd.

$$\frac{p_1}{\rho_1^{1.4}} = \frac{p_2}{\rho_2^{1.4}} \text{ so } p_1\left(\frac{RT_1}{p_1}\right)^{1.4} = p_2\left(\frac{RT_2}{p_2}\right)^{1.4}$$

Simplifying yields:

$$p_1^{-0.4}T_1^{1.4} = p_2^{-0.4}T_2^{1.4} \text{ or } \left(\frac{T_2}{T_1}\right)^{1.4} = \left(\frac{p_2}{p_1}\right)^{0.4}$$

Find the temperature ratio as:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.4/1.4}$$ = 0.820, so the temperature decreases by about 18.0%.

Remember that temperature ratios must always be calculated using absolute temperatures. So use $T_1 = 293.15\ K$.

$$T_2 = 0.820\,T_1 = 240.481\ K = -32.67°C.$$

1.10)

Given:

Pressure and temperature for the standard atmosphere at 20 *km* altitude.

From Table 1.2a:

$$p = \frac{p}{p_{SL}}p_{SL} = \left(5.4570\times10^{-2}\right)\left(1.01325\times10^{5}\,N/m^2\right) = 5.5293\times10^{3}\ N/m^2$$

$$T = 216.65K$$

The density can be found using the perfect gas law, Eqn. 1.10:

$$\rho = \frac{p}{RT} = \frac{5.5293\times10^{3}\,N/m^2}{\left(287.05\dfrac{N-m}{kg-K}\right)(216.65K)} = 0.08891\ kg/m^3$$

1.10) contd.

The value for density from Table 1.2a is:

$$\rho = \frac{\rho}{\rho_{SL}}\rho_{SL} = \left(7.2580\times10^{-2}\right)\left(1.2250 kg/m^3\right) = 0.08891\ kg/m^3$$

which is the same as the calculated value from above. The viscosity is given by Eqn. 1.12a:

$$\mu = 1.458\times10^{-6}\frac{T^{1.5}}{\left(T+110.4K\right)}$$
$$= 1.458\times10^{-6}\frac{\left(216.65^{\circ} R\right)^{1.5}}{\left(216.65K+110.4K\right)} = 1.422\times10^{-5}\ kg/s\text{-}m$$

The value for viscosity from Table 1.2b is:

$$\mu = \frac{\mu}{\mu_{SL}}\mu_{SL} = \left(0.79447\right)\left(1.7894\times10^{-5} kg/s-m\right) = 1.422\times10^{-5}\ kg/s\text{-}m$$

which is the same as the calculated value from above.

1.11)

Given:

Pressure and temperature for the standard atmosphere at 35,000 *ft* altitude.

From Table 1.2b:

$$p = \frac{p}{p_{SL}}p_{SL} = \left(0.23617\right)\left(2116.22 lbf/ft^2\right) = 499.79\ lbf/ft^2$$

$$T = 394.07^{\circ}R$$

The density can be found using the perfect gas law, Eqn. 1.10:

$$\rho = \frac{p}{RT} = \frac{499.79 lbf/ft^2}{\left(53.34\frac{ft-lbf}{lbm-^{\circ}R}\right)\left(394.07^{\circ}R\right)} = 0.02378\ lbm/ft^3 = 0.000739\ slug/ft^3$$

1.11) contd.

The value for density from Table 1.2b is:

$$\rho = \frac{\rho}{\rho_{SL}}\rho_{SL} = (0.31075)(0.002377 slug/ft^3) = 0.000739 \ slug/ft^3$$

which is the same as the calculated value from above. The viscosity is given by Eqn. 1.12b:

$$\mu = 2.27\times10^{-8}\frac{T^{1.5}}{(T+198.6^\circ R)}$$

$$= 2.27\times10^{-8}\frac{(394.07^\circ R)^{1.5}}{(394.07^\circ R+198.6^\circ R)} = 2.996\times10^{-7} \ lbf\text{-}s/ft^2$$

The value for viscosity from Table 1.2b is:

$$\mu = \frac{\mu}{\mu_{SL}}\mu_{SL} = (0.801425)(3.740\times10^{-7} lbf-s/ft^2) = 2.997\times10^{-7} \ lbf\text{-}s/ft^2$$

which is the same as the calculated value from above.

1.12)

$$p_{t1} = 5.723\times10^6 \ N/m^2 \ (= 56.48 \text{ atmospheres})$$

$$T_t = 750K$$

$$\rho_{t1} = \frac{p_{t1}}{R\,T_t} = \frac{5.723\times10^6 \ \frac{N}{m^2}}{(287.05 \ \frac{N\cdot m}{kg\cdot K})(750K)} = 26.583 \ \frac{kg}{m^3}$$

$$\mu_t = 1.458\times10^{-6}\frac{T_t^{\ 1.5}}{T_t+110.4} = 3.481\times10^{-5} \ \frac{kg}{s\cdot m}$$

1.13) $p_\infty = 586\ N/m^2$; $T_\infty = 54.3K$; $M_\infty = 8$

Using the perfect-gas law: $\rho_\infty = \dfrac{586\ N/m^2}{\left(287.05\ \frac{N \cdot m}{kg\,K}\right)(54.3K)}$

$$\rho_\infty = 0.03760\ \frac{kg}{m^3}$$

$$\mu_\infty = 1.458 \times 10^{-6} \frac{(54.3)^{1.5}}{54.3 + 110.4} = 3.542 \times 10^{-6}\ \frac{kg}{s \cdot m}$$

$$U_\infty = M_\infty\, a_\infty = 8.0\left[20.047\sqrt{54.3}\right] = 8.0(147.72) = 1181.8\ \frac{m}{s}$$

Note: that (in a conventional hypersonic wind tunnel) the speed of sound is very low (only 147.72 m/s in this problem) Thus, hypersonic flows are achieved at relatively low velocities (and, therefore, at relatively low total enthalpies). See Chapter 12. Although the information was not requested, the unit Reynolds number is:

$$Re_\infty/\text{length} = \frac{\rho_\infty U_\infty}{\mu_\infty} = 12.543 \times 10^6/m$$

1.14) $p_{t1} = 24.5$ psia ; $T_t = 1660°R$

$$\rho_{t1} = \frac{p_{t1}}{R\,T_t} = \frac{\left(24.5\ \frac{lbf}{in^2}\right)\left(144\ \frac{in^2}{ft^2}\right)}{\left(53.34\ \frac{ft\,lbf}{lbm\,°R}\right)(1660°R)} = 0.03984\ \frac{lbm}{ft^3}$$

Alternative English units for density are:

$$\rho_{t1} = \frac{\left(24.5\frac{lbf}{in^2}\right)\left(144\ \frac{in^2}{ft^2}\right)}{\left(1716.16\frac{ft^2}{s^2 \cdot °R}\right)(1660°R)} = 0.001238\ \frac{lbf \cdot s^2}{ft^4}\ \left[\equiv \frac{slugs}{ft^3}\right]$$

$$\mu_t = 2.27 \times 10^{-8}\ \frac{(1660)^{1.5}}{1660 + 198.6} = 8.260 \times 10^{-7}\ \frac{lbf \cdot s}{ft^2}$$

1.15) $M_\infty = 4$; $p_\infty = 23.25 \frac{\text{lbf}}{\text{ft}^2}$; $T_\infty = -65°\text{F}$

Since the equation of state, the viscosity equation, and the definition for the speed of sound all require the use of the temperature in absolute units, $T_\infty = 394.67°\text{R}$

Since $p_\infty = 23.25 \frac{\text{lbf}}{\text{ft}^2} = 0.01099\, p_{SL}$, the pressure altitude is approximately 100,000 ft.

$$\rho_\infty = \frac{23.25 \frac{\text{lbf}}{\text{ft}^2}}{\left(1716.16 \frac{\text{ft}^2}{\text{s}^2 \cdot °\text{R}}\right)(394.67°\text{R})} = 3.433 \times 10^{-5} \frac{\text{slugs}}{\text{ft}^3}$$

$$\mu_\infty = 2.27 \times 10^{-8} \frac{(394.67)^{1.5}}{394.67 + 198.6} = 3.000 \times 10^{-7} \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}$$

$$U_\infty = M_\infty \left[49.02 \sqrt{T_\infty}\right] = 3895.4 \frac{\text{ft}}{\text{s}}$$

Although it was not sought, the unit Reynolds number is:

$$Re_{\infty/\text{ft}} = \frac{\rho_\infty U_\infty}{\mu_\infty} = 0.4457 \times 10^6 \ /\text{ft}$$

1.16) $U_\infty = 470$ Knots $= 470 \frac{\text{nm}}{\text{h}}$; Altitude $= 35{,}000$ ft

$$U_\infty = 470 \text{ knots} \ \frac{1.15155 \frac{\text{mi}}{\text{h}}}{1 \text{ Knot}} = 541.23 \frac{\text{mi}}{\text{h}}$$

$$U_\infty = 541.23 \frac{\text{mi}}{\text{h}} \ \frac{5280 \frac{\text{ft}}{\text{mi}}}{3600 \frac{\text{s}}{\text{h}}} = 793.98 \frac{\text{ft}}{\text{s}}$$

The results are independent of altitude

1.17)
$$T = T_0 - Bz$$
$$= 288.15 - 0.0065(7000)$$
$$= 242.65\ K$$

as compared with 242.700 K in Table 1.2.

$$p = p_0 \left[1 - \frac{Bz}{T_0}\right]^{g/RB}$$

$$p = p_0 \left[1 - \frac{(0.0065)(7000)}{288.15}\right]^{5.26} = 0.40495\ p_0$$

Since $p_0 = p_{SL}$, this value compares favorably with the value of $p = 0.40567\, p_{SL}$ for 7 km in Table 1.2

1.18) Using equation (1.14):

$$T = T_0 - Bz$$
$$= 288.15 - (0.0065)(11000) = 216.650K$$

The temperature is constant at this value (216.650K) between altitudes of 11,000 m and 20,000 m.

From the surface to 11,000 m, the pressure is given by equation (1.15):

$$p = p_0 \left[1 - \frac{Bz}{T_0}\right]^{g/RB}$$

$$= p_0 \left[1 - \frac{(0.0065)(11{,}000)}{288.15}\right]^{5.26} = 0.22310\ p_0$$

(which compares favorably with the value of $0.22403\, p_{SL}$ for 11 km in Table 1.2). Thus, $p = 22605.6\ N/m^2$

Using the equation for constant temperature, i.e., equation (1.13), the pressure variation from 11,000 m

1.18 contd. to 20,000 m is given by:

$$p = p_{11,000} \exp\left\{\frac{g(11,000 - z)}{RT}\right\}$$

$$\frac{g}{RT} = \frac{9.8066 \ m/s^2}{\left(287.05 \frac{N \cdot m}{kg \cdot K}\right)(216.650 \ K)} = 0.0001577/m$$

$$p = \{0.22310 \ p_{SL}\} \exp\{1.7347 - 0.0001577 z\}$$

1.19 Recall that the temperature is constant at 216.650K from 11,000 m to 20,000 m. Using the equation developed in Problem 1.9, the pressure at 18,000 m is:

$$p = \{0.22310 \ p_{SL}\} \exp\{1.7347 - 2.8386\} = 0.07397 \ p_{SL}$$

This compares favorably with the tabulated value of $0.074663 \ p_{SL}$.

$$\rho = \frac{p}{RT} = \frac{7495.5 \ N/m^2}{\left(287.05 \frac{N \cdot m}{kg \cdot K}\right)(216.65 K)} = 0.1205 \ \frac{kg}{m^3}$$

$$\mu = 1.458 \times 10^{-6} \frac{T^{1.5}}{T + 110.4} = 1.422 \times 10^{-5} \ \frac{kg}{s \cdot m}$$

which is equal to the value at 18,000 m in Table 1.2.

$$a = 20.047\sqrt{T} = 295.073 \ \frac{m}{s}$$

1-20 At 10,000 ft

$$T = 518.67 - 0.003565 z = 484.75 \,^\circ R$$

$$\frac{p}{p_0} = \left[1 - 6.873 \times 10^{-6} z\right]^{5.26} = [0.93127]^{5.26} = 0.68759$$

1-20 contd. $\frac{\rho}{\rho_0} = \left[1 - 6.873\times10^{-6} z\right]^{4.26} = [0.93127]^{4.26} = 0.73834$

The corresponding values from Table 1.2 are

$T = 483.03\,^\circ R$; $\frac{p}{p_0} = 0.68783$; $\frac{\rho}{\rho_0} = 0.7386$

At 30,000 ft:

$T = 518.67 - 0.003565 z = 411.72\,^\circ R$

$$\frac{p}{p_0} = \left[1 - 6.873\times10^{-6} z\right]^{5.26} = [0.79380]^{5.26} = 0.29681$$

$$\frac{\rho}{\rho_0} = \left[1 - 6.873\times10^{-6} z\right]^{4.26} = [0.79380]^{4.26} = 0.37391$$

The corresponding values from Table 1.2 are

$T = 411.84\,^\circ R$; $\frac{p}{p_0} = 0.29754$; $\frac{\rho}{\rho_0} = 0.3747$

At 65,000 ft:

$T = 389.97\,^\circ R$

$$\frac{p}{p_0} = 0.2231 \exp\left(1.7355 - 4.8075\times10^{-5} z\right)$$

$$\frac{p}{p_0} = 0.2231\,(0.24923) = 0.05560$$

$$\frac{\rho}{\rho_0} = 0.2967 \exp\left(1.7355 - 4.8075\times10^{-5} z\right)$$

$$\frac{\rho}{\rho_0} = 0.2967\,(0.24923) = 0.07395$$

The corresponding values in Table 1.2 are:

$T = 389.97\,^\circ R$; $\frac{p}{p_0} = 0.05620$; $\frac{\rho}{\rho_0} = 0.0747$

1.21)

Given:

An aircraft required to perform a 9g turn at 15,000 *ft* MSL.

In order to evaluate these performance parameters you might need to know Standard Day values for pressure, density, temperature, viscosity, and speed of sound. These values would be used as free-stream values for various calculations and would be found in Table 1.2b. At 15,000 *ft* (which requires straight-forward interpolation) the values are:

$$p_\infty = \frac{p}{p_{SL}} p_{SL} = (0.564955)(2116.22 lbf/ft^2) = 1195.57 \ lbf/ft^2$$

$$\rho_\infty = \frac{\rho}{\rho_{SL}} \rho_{SL} = (0.630705)(0.002377 slug/ft^3) = 0.001499 \ slug/ft^3$$

$$\mu_\infty = \frac{\mu}{\mu_{SL}} \mu_{SL} = (0.917845)(3.740 \times 10^{-7} lbf - s/ft^2) = 3.433 \times 10^{-7} \ lbf\text{-}s/ft^2$$

$T_\infty = 465.22 \ ^oR$

$a_\infty = 1057.345 \ ft/s$

1.22)

Given:

Aircraft flying at an altitude of 20,000 *ft* with $p = 900 \ lbf/ft^2$ and $T = 460^\circ R$

a) The density altitude will require calculating the density using the perfect gas law:

$$\rho = \frac{p}{RT} = \frac{900 lbf/ft^2}{\left(53.34 \frac{ft - lbf}{lbm - ^\circ R}\right)(32.174 lbm/slug)(460^\circ R)} = 0.00140 \ slug/ft^3$$

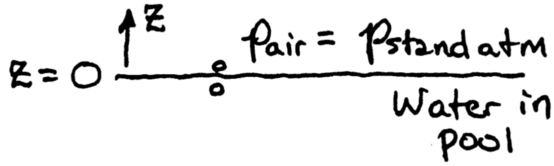
Using Table 1.12b and interpolating to the nearest 500 *ft*:

$$h_p = 21.86 kft \approx 22.0 kft$$
$$h_T = 16.47 kft \approx 16.5 kft$$
$$h_\rho = 17.69 kft \approx 17.5 kft$$

b) In a standard atmosphere an aircraft flying at 20,000 *ft* would experience pressure, temperature, and density altitudes of 20,000 *ft*.

1.23]

$$\frac{dp}{dz} = -\rho g$$



$$dp = -\rho g\, dz$$

$$p - p_{surface} = -\rho g (z - z_{surface})$$

If we chose the surface as our reference value,

$$z_{surface} = 0$$

The pressure of the air just above the surface of the water is the standard atmospheric value, and the pressure is constant across a fluid/fluid interface, the pressure at the surface of the water:

$$p_{surface} = p_{stand\ atm} = 101325 \frac{N}{m^2}$$

We want to find the depth where pressure in the water is 2 atmospheres, or $202650 \frac{N}{m^2}$. Thus,

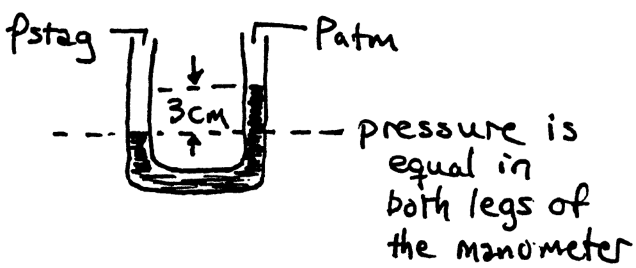
$$p - p_{surface} = -\rho g (z - z_{surface})$$

$$101325 \frac{N}{m^2} = -\left(1000 \frac{kg}{m^3}\right)\left(9.8066 \frac{m}{s^2}\right) z$$

$$z = -10.332\ m$$

Thus, to reach a point where the pressure in the water is two atmospheres, we must go 10.332 m below the surface.

1.24] The pressure at the surface of the mercury in the left leg is p_{stag}.



1.24 contd. Since the pressure at a given level in the mercury is equal in both legs,

$$p_{stag} = p_{atm} + \rho_{Hg}\, g(\Delta h)$$

$$p_{stag} - p_{atm} = \left[13595.1 \frac{Kg}{m^3}\right]\left[9.8066 \frac{m}{s^2}\right]\left[0.03\,m\right]$$

$$= 3999.65 \frac{N}{m^2}$$

Since the difference $p_{stag} - p_{atm}$ is the gage pressure

$$p_{stag} = 3999.65 \frac{N}{m^2}, \text{gage}$$

1.25 Refer to Figure 12.5, "Thermodynamic properties of air in chemical equilibrium"

When dissociation occurs, the equation of state can be written

$$p = z\rho RT$$

If $z = 1$, $m_o = m$ and the equation of state matches that for part (a). There is a line in Fig. 12.5 for which $z = 1.01$

(b) Referring to Fig. 12.5, the line designated 1000K (or 1800°R) is "horizontal", independent of pressure. Thus, within our ability to read these lines, a calorically perfect gas is one where the temperature is "1000K", or less.

1.26)

Given: $z_2 - z_1 = 30.0$ cm $= 0.30$ m

Constants: $\rho = 13.5951$ g/cm^3 = 13,595.1 kg/m^3

$g = 9.80665$ m/s^2

a) Following the procedure outlined in Problem 1.24, the pressure difference is:

$$\Delta p = p_1 - p_2 = \rho g (z_2 - z_1).$$

$$\Delta p = \frac{13{,}595.1\,\text{kg}}{\text{m}^3} \times \frac{9.80665\,\text{m}}{\text{s}^2} \times 0.30\,\text{m} = 39{,}997\,\frac{\text{kg}}{\text{s}^2\text{-m}}$$

Therefore, $\Delta p = 39{,}997\ N/m^2$.

b) For a total surface area of 0.5 m^2, and assuming that the pressure is constant over the surface, the net force acting on the surface is:

$$F = (39{,}997 N/m^2)(0.5 m^2) = 19{,}998 N$$

2.1 $$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$$

Although it is not stated, the flow is assumed to be steady so that $\frac{\partial \rho}{\partial t} = 0$.

$$\nabla\cdot(\rho\vec{V})$$

$$\left[\hat{e}_r \frac{\partial}{\partial r} + \frac{\hat{e}_\theta}{r}\frac{\partial}{\partial \theta} + \hat{e}_z \frac{\partial}{\partial z}\right]\cdot\left[\rho\left(v_r\hat{e}_r + v_\theta\hat{e}_\theta + v_z\hat{e}_z\right)\right]$$

$$= \hat{e}_r\cdot\hat{e}_r \frac{\partial(\rho v_r)}{\partial r} + \hat{e}_r\cdot\rho v_r\frac{\partial\hat{e}_r}{\partial r}$$

$$+ \hat{e}_r\cdot\hat{e}_\theta \frac{\partial(\rho v_\theta)}{\partial r} + \hat{e}_r\cdot\rho v_\theta\frac{\partial\hat{e}_\theta}{\partial r}$$

$$+ \hat{e}_r\cdot\hat{e}_z \frac{\partial(\rho v_z)}{\partial r} + \hat{e}_r\cdot\rho v_z\frac{\partial\hat{e}_z}{\partial r}$$

$$+ \frac{\hat{e}_\theta}{r}\cdot\hat{e}_r \frac{\partial(\rho v_r)}{\partial \theta} + \frac{\hat{e}_\theta}{r}\cdot\rho v_r\frac{\partial\hat{e}_r}{\partial \theta}$$

$$+ \frac{\hat{e}_\theta}{r}\cdot\hat{e}_\theta \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\hat{e}_\theta}{r}\cdot\rho v_\theta\frac{\partial\hat{e}_\theta}{\partial \theta}$$

$$+ \frac{\hat{e}_\theta}{r}\cdot\hat{e}_z \frac{\partial(\rho v_z)}{\partial \theta} + \frac{\hat{e}_\theta}{r}\cdot\rho v_z\frac{\partial\hat{e}_z}{\partial \theta}$$

$$+ \hat{e}_z\cdot\hat{e}_r \frac{\partial(\rho v_r)}{\partial z} + \hat{e}_z\cdot\rho v_r\frac{\partial\hat{e}_r}{\partial z}$$

$$+ \hat{e}_z\cdot\hat{e}_\theta \frac{\partial(\rho v_\theta)}{\partial z} + \hat{e}_z\cdot\rho v_\theta\frac{\partial\hat{e}_\theta}{\partial z}$$

$$+ \hat{e}_z\cdot\hat{e}_z \frac{\partial(\rho v_z)}{\partial z} + \hat{e}_z\cdot\rho v_z\frac{\partial\hat{e}_z}{\partial z}$$

To evaluate the terms in the left-hand column,

2.1 contd. we note that:

$$\hat{e}_r \cdot \hat{e}_r = \hat{e}_\theta \cdot \hat{e}_\theta = \hat{e}_z \cdot \hat{e}_z = 1 \quad \text{and}$$

$$\hat{e}_r \cdot \hat{e}_\theta = \hat{e}_\theta \cdot \hat{e}_z = \hat{e}_z \cdot \hat{e}_r = 0$$

To evaluate the terms in the right-hand column, we note that a vector can change in magnitude and/or in direction. Obviously, a unit vector cannot change in magnitude (or length). From vector calculus, the derivatives of the unit vectors in cylindrical coordinates are:

$$\frac{\partial \hat{e}_r}{\partial \theta} = \hat{e}_\theta \ ; \ \frac{\partial \hat{e}_\theta}{\partial \theta} = -\hat{e}_r$$

All other derivatives are zero. Thus, the equation becomes:

$$\nabla \cdot (\rho \vec{V}) = 0$$

$$= \frac{\partial(\rho v_r)}{\partial r} + \frac{\rho v_r}{r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z}$$

$$= \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} \quad QED$$

2.2 (a) We have a radial flow in which ρ = constant. Therefore, we can use the result from problem 2.1

$$\frac{1}{r}\frac{\partial(r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \stackrel{?}{=} 0$$

Since $\vec{V} = \frac{K}{2\pi r}\hat{e}_r$, we see that $v_r = \frac{K}{2\pi r}$; $v_\theta = 0$; and $v_z = 0$. Substituting these components into the continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{K}{2\pi r}\right) = 0! \quad \text{Continuity is satisfied.}$$

2.2 Contd.) (b) Let us use the continuity equation for a three-dimensional flow, i.e., equation (2.1):

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \stackrel{?}{=} 0$$

For constant density flow, this equation becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Thus,
$$\frac{\partial}{\partial x}\left\{-\frac{2xyz}{(x^2+y^2)^2} U_\infty L\right\} + \frac{\partial}{\partial y}\left\{\frac{(x^2-y^2)z}{(x^2+y^2)^2} U_\infty L\right\}$$
$$+ \frac{\partial}{\partial z}\left\{\frac{y}{x^2+y^2} U_\infty L\right\} = 0$$

Since U_∞ and L are constants and since they appear in every term, they can be divided out leaving:

$$-\frac{2yz}{(x^2+y^2)^2} - \frac{2xyz(-2)(2x)}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} + \frac{(x^2-y^2)z(-2)(2y)}{(x^2+y^2)^3}$$

$$= -\frac{4yz}{(x^2+y^2)^2} - \frac{-8x^2yz + 4x^2yz - 4y^3z}{(x^2+y^2)^3}$$

$$= \frac{-4x^2yz - 4y^3z + 8x^2yz - 4x^2yz + 4y^3z}{(x^2+y^2)^3} = 0$$

Therefore, the continuity equation is satisfied.

2.3)

Given: Two of three velocity components for an incompressible flow:

$$u = x^2 + 2xz \qquad v = y^2 + 2yz$$

The velocity components must satisfy the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

2.3) contd.

For incompressible flow this becomes:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Find the derivatives of the given velocity components:

$$\frac{\partial u}{\partial x}=2(x+z) \qquad \frac{\partial v}{\partial y}=2(y+z)$$

Therefore:

$$\frac{\partial w}{\partial z}=-2(x+y+2z)$$

Integrating yields:

$$w=-2z(x+y+z)=f(x,y,t)$$

Where $f(x,y,t)$ is an arbitrary function (x,y,t). Since the first two velocity components are not a function of time, it may be possible to assume the flow is steady and drop the time function from the arbitrary constant.

2.4)

Given: Velocity components for a 2D incompressible flow:

$$u=-\frac{Ky}{\left(x^2+y^2\right)} \qquad v=+\frac{Kx}{\left(x^2+y^2\right)}$$

For 2D incompressible flow the continuity equation is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Taking the required derivatives yields:

$$\frac{\partial u}{\partial x}=\frac{2xy}{\left(x^2+y^2\right)^2} \qquad \frac{\partial v}{\partial y}=-\frac{2xy}{\left(x^2+y^2\right)^2}$$

2.4) contd.

which shows that the flowfield satisfies continuity. Now convert to cylindrical coordinates for simplicity using:

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

Resulting in velocity components of:

$$u = -\frac{K\sin\theta}{r} \qquad\qquad v = +\frac{K\cos\theta}{r}$$

Now some vector information is required:

$$\vec{V} = u\hat{i} + v\hat{j} = v_r\hat{e}_r + v_\theta\hat{e}_\theta$$

$$v_r = \vec{V}\cdot\hat{e}_r = u\hat{i}\cdot\hat{e}_r + v\hat{j}\cdot\hat{e}_r \qquad \hat{i}\cdot\hat{e}_r = \cos\theta \qquad \hat{j}\cdot\hat{e}_r = \sin\theta$$

$$v_\theta = \vec{V}\cdot\hat{e}_\theta = u\hat{i}\cdot\hat{e}_\theta + v\hat{j}\cdot\hat{e}_\theta \qquad \hat{i}\cdot\hat{e}_\theta = -\sin\theta \qquad \hat{j}\cdot\hat{e}_\theta = \cos\theta$$

Resulting in:

$$v_r = u\hat{i}\cdot\hat{e}_r + v\hat{j}\cdot\hat{e}_r = -\frac{K\sin\theta\cos\theta}{r} + \frac{K\sin\theta\cos\theta}{r} = 0$$

$$v_\theta = u\hat{i}\cdot\hat{e}_\theta + v\hat{j}\cdot\hat{e}_\theta = \frac{K\sin^2\theta}{r} + \frac{K\cos^2\theta}{r} = \frac{K}{r}$$

This represents a counter-clockwise vortex flow about the origin with a velocity singularity at the origin and a circular velocity about the origin proportional to $1/r$.

2.5)

Given: Velocity components for a 2D incompressible flow:

$$u = \frac{C\left(y^2 - x^2\right)}{\left(x^2 + y^2\right)^2} \qquad\qquad v = -\frac{2Cxy}{\left(x^2 + y^2\right)^2}$$

2.5) contd.

Assume 2D incompressible flow and that C is a constant. For 2D incompressible flow the continuity equation is:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Taking the required derivatives yields:

$$\frac{\partial u}{\partial x}=C\left(y^2-x^2\right)(-2)\left(x^2+y^2\right)^{-3}(2x)+C(-2x)\left(x^2+y^2\right)^{-2}$$

$$\frac{\partial v}{\partial y}=-2Cxy(-2)\left(x^2+y^2\right)^{-3}(2y)+C(-2x)\left(x^2+y^2\right)^{-2}$$

$$\frac{-4Cx\left(y^2-x^2\right)}{\left(x^2+y^2\right)^3}-\frac{2Cx}{\left(x^2+y^2\right)^2}+\frac{8Cxy^2}{\left(x^2+y^2\right)^3}-\frac{2Cx}{\left(x^2+y^2\right)^2}=0$$

after some algebra and patience!

2.6) Referring to the continuity equation for a two-dimensional, incompressible flow:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\frac{\partial v}{\partial y}=-\frac{\partial u}{\partial x}=\frac{1}{2}\frac{a_1 y}{x^{1.5}}-\frac{3}{2}\frac{a_2 y^3}{x^{2.5}}$$

Integrating with respect to y

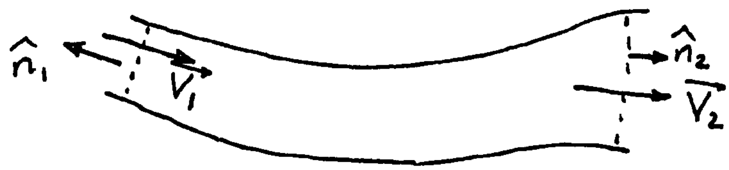
$$v=+\frac{1}{4}\frac{a_1 y^2}{x^{1.5}}-\frac{3}{8}\frac{a_2 y^4}{x^{2.5}}+C$$

To evaluate the constant of integration C, we note that $v=0$ when $y=0$. Thus, $C=0$ and

$$v=\frac{1}{4}\frac{a_1 y^2}{x^{1.5}}-\frac{3}{8}\frac{a_2 y^4}{x^{2.5}}$$

2.7 The integral form of the continuity equation [equation (2.5)] for steady, one-dimensional flow in a streamtube yields:

$$-\iint \rho_1 V_1 \, dA_1 + \iint \rho_2 V_2 \, dA_2 = 0$$



Since the flow properties (e.g., ρ and V) are uniform across the area (the one-dimension for which the flow properties vary is the streamwise coordinate:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \rho V A = \text{constant}$$

Differentiating:

$$(d\rho) V A + \rho (dV) A + \rho V (dA) = 0$$

Dividing by $\rho V A$, we obtain:

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

If the flow is incompressible, $d\rho = 0$, and

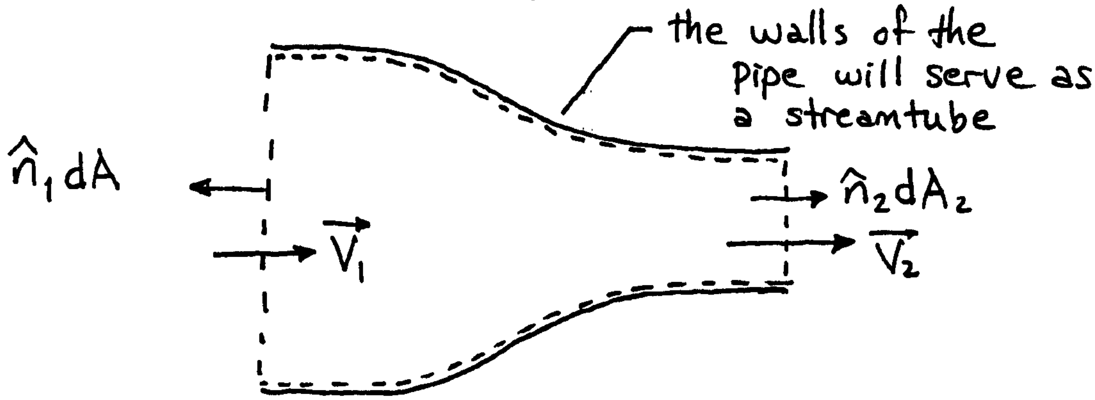
$$\frac{dV}{V} = -\frac{dA}{A}$$

Thus, if the cross-sectional area between streamlines (or of a streamtube, or of the walls of a wind tunnel) decreases, the flow accelerates. When the cross-sectional-area between streamlines increases, the velocity of the fluid particles decreases. These relations between dV and dA are not true if the flow is supersonic as will be discussed in Chapter 8.

2.8] Using the integral form of the continuity equation for steady flow, we can use equation (2.5)

$$\frac{\partial}{\partial t}\iiint_{Vol} \rho \, d(vol) + \oiint_{Area} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

to solve this problem. Let us draw a control volume between stations 1 and 2,



The vectors representing the areas ($\hat{n} dA$) are directed outward from the control volume, as shown in the sketch. The velocities represent an assumed flow from left to right. Using the vector dot products and noting that the flow properties do not vary across the cross-section, we obtain:

$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$$

where V_1 and V_2 are the magnitudes of the velocity vectors,

$\vec{V}_1$ and $\vec{V}_2$, respectively. Thus,

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \text{ (and by extension)} = \rho_3 V_3 A_3$$

(Often we see the expression for steady, one-dimensional flow in a streamtube as:

$$\rho V A = \text{constant}$$

2.8 Contd.

The duct need not be straight, providing the flow is approximately one-dimensional. Thus, the equation is often applied to flow in curved pipes and elbows.)

For this flow, water can be assumed to be of constant density. Thus,

$$\rho_1 = \rho_2 = \rho_3$$

As a result,

$$V_1 A_1 = V_2 A_2 = V_3 A_3 = 0.5 \frac{m^3}{s}$$

$$V_1 \left[\frac{\pi}{4}(0.4)^2\right] = V_2 \left[\frac{\pi}{4}(0.2)^2\right] = V_3 \left[\frac{\pi}{4}(0.6)^2\right] = 0.5$$

Solving,

$$V_1 = 3.979 \frac{m}{s} \; ; \; V_2 = 15.915 \frac{m}{s} \; ; \; V_3 = 1.768 \frac{m}{s}$$

2.9 Following the logic of Problem 2.8,

$$\rho_s V_s A_s = \rho_1 V_1 A_1 = \rho_2 \iint u_2 \, dA_2 = 10 \frac{kg}{s}$$

Note that u_2 is left in the integral, since it is not constant over the cross section, i.e.,

$$u_2 = U_0 \left[1 - \frac{r^2}{R^2}\right]$$

$$V_s \left[\frac{\pi}{4}(500\,cm)^2\right] = V_1 \left[\frac{\pi}{4}(20\,cm)^2\right] = \frac{10^4 \frac{gm}{s}}{0.85 \frac{gm}{cm^3}}$$

2.9 Contd. Thus, $V_s = 0.0599 \frac{cm}{s}$; $V_1 = 37.448 \frac{cm}{s}$

Note that the velocity at which a fluid particle at the free surface moves (V_s) is very small compared to the velocity in the drain pipe. Since the flow is axisymmetric at station 2:

$$dA = 2\pi r\, dr$$

$$\rho_2 \int_0^{R_2} U_0 \left[1 - \frac{r^2}{R_2^2}\right] 2\pi r\, dr = 10^4 \frac{gm}{s}$$

$$2\pi U_0 \rho_2 \left[\frac{R_2^2}{2} - \frac{R_2^4}{4R_2^2}\right] = 10^4 \frac{gm}{s}$$

$$U_0 = \frac{10^4 \frac{gm}{s}}{2\pi(\rho_2)\left(\frac{R_2^2}{4}\right)\frac{gm}{cm}} = 299.586 \frac{cm}{s}$$

2.10 Let us use the integral form of the continuity equation. Note that the effects of viscosity are such that there is a significant reduction of the velocity in the wake of the airfoil (at station 2). Thus, for this rectangular control volume, a significant fraction of the mass influx at station 1 does not leave the control volume through station 2. Thus, some fluid must exit through planes 3 and 4. Thus, they are obviously not streamlines.

$$\frac{\partial}{\partial t}\iiint \rho\, d(vol) + \oiint \rho \vec{V} \cdot \hat{n}\, dA = 0$$

By continuity, we know that the is a v-component of velocity in the wake of the airfoil and that $v(x,y)$ in 2. Along surface 3

2.10 Contd. $\vec{V}_3 = U_\infty \hat{i} + v_\infty(x)\hat{j}$

and along surface (4)

$$\vec{V}_4 = U_\infty \hat{i} - v_\infty(x)\hat{j}$$

Since the flow is steady, the mass fluxes per unit depth in the continuity equation can be written:

$$\underbrace{\rho\int_{-H}^{+H}\left[U_\infty \hat{i}\right]\cdot\left[-\hat{i}\,dy\right]}_{(1)} + \underbrace{\rho\int_{-H}^{0}\left[-\frac{U_\infty y}{H}\hat{i} - v\hat{j}\right]\cdot\left[\hat{i}\,dy\right]}_{(2a)}$$

$$+ \underbrace{\rho\int_{0}^{H}\left[\frac{U_\infty y}{H}\hat{i} + v\hat{j}\right]\cdot\left[\hat{i}\,dy\right]}_{(2b)} + \underbrace{\rho\int_{0}^{L}\left[U_\infty\hat{i} + v_\infty\hat{j}\right]\cdot\left[\hat{j}\,dy\right]}_{(3)}$$

$$+ \rho\int_0^L\left[U_\infty\hat{i} - v_\infty\hat{j}\right]\cdot\left[-\hat{j}\,dx\right] = 0$$

Note that the vertical component of velocity does not transport fluid across the surface at station (2) and that the horizontal component of velocity does not transport fluid across the surface at stations (3) and (4). This is because these velocity components are perpendicular to the area "vectors" at the station. Thus,

$$-\rho U_\infty 2H + \rho\frac{U_\infty}{H}\left(-\frac{y^2}{2}\right|_{-H}^{0} + \rho\frac{U_\infty}{H}\left(+\frac{y^2}{2}\right|_{0}^{H}$$

$$+ \rho\int_0^L v_\infty\,dx + \rho\int_0^L v_\infty\,dx = 0$$

The last two terms represent the total mass flow across the surfaces (3) and (4). The density is common to every term. We can divide by the density to get the volumetric flow across (3) and (4) $\left[2\int_0^L v_\infty\,dx\right] = U_\infty H$

2.11) Let us apply the integral form of the continuity equation. Note that, since surfaces ③ and ④ are streamlines, flow passes through only surfaces ① and ②.

$$\cancel{\frac{\partial}{\partial t}\iiint \rho \, d(vol)}^{\text{steady}} + \oiint \rho \vec{V} \cdot \hat{n} \, dA = 0$$

Since the flow is incompressible and steady, we can write the continuity equation as

$$\underbrace{-\rho U_\infty \int_{-H_u}^{+H_u} dy}_{①} + \underbrace{\rho U_\infty \int_{-H_D}^{0} \left(-\frac{y}{H_D}\right) dy}_{②a}$$

$$+ \underbrace{\rho U_\infty \int_{0}^{H_D} \left(\frac{y}{H_D}\right) dy}_{②b} = 0$$

(Refer to Problem 2.10 to see how to handle the u-component of velocity at station ②.)

$$-\rho U_\infty (2H_u) - \frac{\rho U_\infty}{H_D}\left(\frac{y^2}{2}\right|_{-H_D}^{0} + \frac{\rho U_\infty}{H_D}\left(\frac{y^2}{2}\right|_{0}^{H_D} = 0$$

Rearranging and dividing through by ρU_∞ (which is a common factor to every term), we obtain:

$$H_u = \frac{1}{2} H_D$$

2.12) Let us apply the integral form of the continuity equation. Note that the effects of viscosity have caused a significant reduction of velocity in the wake of the airfoil (at station ②). As a result, there is a v-component of velocity which produces a mass flux across planes ③ and ④, because they are horizontal (perpendicular to the v-component).

2.12 Contd. The flow is steady and incompressible. As a result, the integral continuity equation becomes.

$$\oiint \vec{V} \cdot \hat{n}\, dA = 0$$

$$\underbrace{\int_{-H}^{+H} \left[U_\infty \hat{i}\right] \cdot \left[-\hat{i}\, dy\right]}_{1} + \underbrace{\int_{-H}^{+H} \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2H}\right)\hat{i} + v\hat{j}\right] \cdot \left[\hat{i}\, dy\right]}_{2}$$

$$+ \underbrace{\int_0^L \left[U_\infty \hat{i} + v_\infty \hat{j}\right] \cdot \left[\hat{j}\, dx\right]}_{3} + \underbrace{\int_0^L \left[U_\infty \hat{i} - v_\infty \hat{j}\right] \cdot \left[-\hat{j}\, dx\right]}_{4} = 0$$

Note that the vertical component of velocity does not transport fluid across the surface at station 2 and that the horizontal component of velocity does not transport fluid across stations 3 and 4. This is because these velocity components are perpendicular to the area "vectors". Thus,

$$-U_\infty (2H) + U_\infty \left[y - 0.5 \frac{2H}{\pi} \sin \frac{\pi y}{2H} \right] \Bigg|_{-H}^{+H}$$

$$+ \int_0^L v_\infty\, dx + \int_0^L v_\infty\, dx = 0$$

The last two terms represent the total volumetric flow across surfaces 3 and 4. Since the flow is planar symmetric at stations 1 and 2, we'll assume that the volumetric flow rate across 3 is equal to that across 4.

$$\left[2 \int_0^L v_\infty\, dx\right] = 2HU_\infty - 2HU_\infty + \frac{HU_\infty}{\pi}\left[1 - (-1)\right]$$

$$\left[2 \int_0^L v_\infty\, dx\right] = \frac{2HU_\infty}{\pi}$$

2.13) Let us apply the integral form of the continuity equation. Note that, since surfaces 3 and 4 are streamlines, fluid can cross only surfaces 1 and 2. Since the flow is steady,

$$\xcancel{\frac{\partial}{\partial t}\iiint \rho\, d(\text{vol})}^{\text{steady}=0} + \oiint \rho \vec{V}\cdot\hat{n}\, dA = 0$$

Because the flow is incompressible (i.e., ρ = constant),

$$-\rho U_\infty \int_{-H_u}^{+H_u} dy + \rho U_\infty \int_{-H_D}^{+H_D} \left(1 - 0.5\cos\frac{\pi y}{2H_D}\right) dy = 0$$

Note that we have eliminated the v-component of velocity at station 2, since it doesn't contribute to the mass flux. See the discussion of terms 2a and 2b in Problem 2.10.

We can divide through by ρU_∞ and obtain:

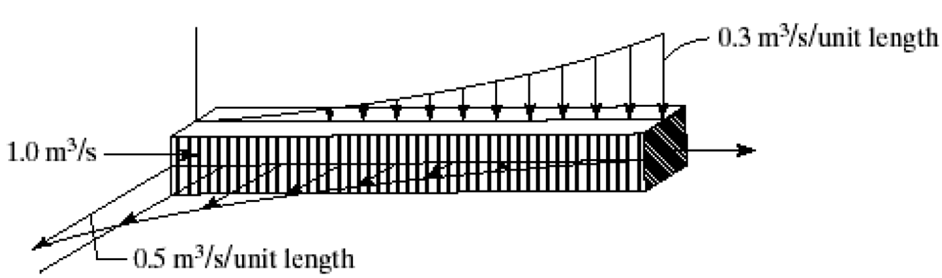
$$-2H_u + \left[y - 0.5\,\frac{2H_D}{\pi}\sin\frac{\pi y}{2H_D}\right]_{-H_D}^{+H_D} = 0$$

$$\therefore -2H_u + \left[2H_D - \frac{H_D}{\pi}(1+1)\right] = 0$$

$$H_u = H_D\left[1 - \frac{1}{\pi}\right] = 0.6817\,H_D$$

2.14)

Given: A rectangular duct as shown below with two porous surfaces. What is the average velocity of water leaving the duct if it is 1.0 m long and has a cross section of 0.1 m^2?



2.14) contd.

Conservation of mass requires:

$$\dot{m}_{out} = \dot{m}_{in}$$

Or, since the flow is incompressible:

$$\dot{q}_{out} = \dot{q}_{in}$$

where $\dot{q}$ is the volume flow rate. This yields:

$$\dot{q}_{in_{end}} + \dot{q}_{in_{top}} = \dot{q}_{out_{side}} + \dot{q}_{out_{end}}$$

$$1.0\frac{m^3}{s} + \int_0^1 0.3x^2 dx = \int_0^1 0.5(1-x)dx + \dot{q}_{out_{end}}$$

$$\dot{q}_{out_{end}} = 1.0\frac{m^3}{s} + 0.3\frac{x^3}{3}\Big|_0^1 - 0.5x\Big|_0^1 + 0.5\frac{x^2}{2}\Big|_0^1$$

$$\dot{q}_{out_{end}} = 0.85\frac{m^3}{s}$$

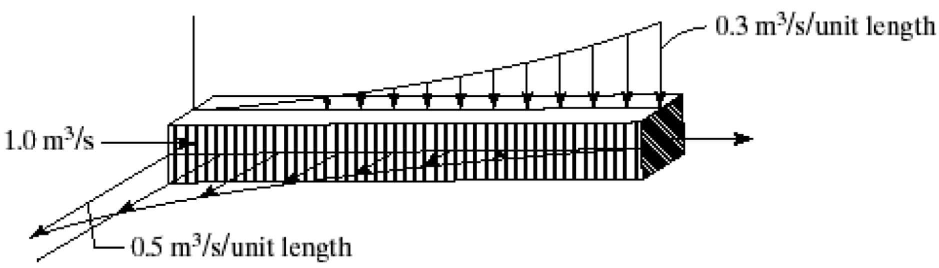
and the velocity at the outflow is:

$$V_{out_{end}} = \dot{q}_{out_{end}} / \rho A = 0.85 m^3/s \big/ \left(977.8 kg/m^3 \cdot 0.1 m^2\right)$$

$$V_{out_{end}} = 0.087 m/s$$

2.15)

Given: The same duct as in Problem 2.14.



2.15) contd.

Using the development presented in the solution for Prob. 2.14, the volume flow rate at any station along the duct is:

$$\dot{q}_{out_{end}} = 1.0\frac{m^3}{s} + \int_0^x 0.3\xi^2 dx - \int_0^x 0.5(1-\xi)dx$$

$$\dot{q}_{out_{end}} = 1.0\frac{m^3}{s} + 0.3\frac{\xi^3}{3}\Big|_0^x - 0.5\xi\Big|_0^x + 0.5\frac{\xi^2}{2}\Big|_0^x$$

$$\dot{q}_{out_{end}} = 1.0\frac{m^3}{s} + 0.1x^3 - 0.5x + 0.25x^2$$

and the velocity at the outflow is:

$$V_x = \dot{q}_x / \rho A = \left(1.0 - 0.5x + 0.25x^2 + 0.1x^3\right) / \left(977.8kg/m^3 \cdot 0.1m^2\right)$$

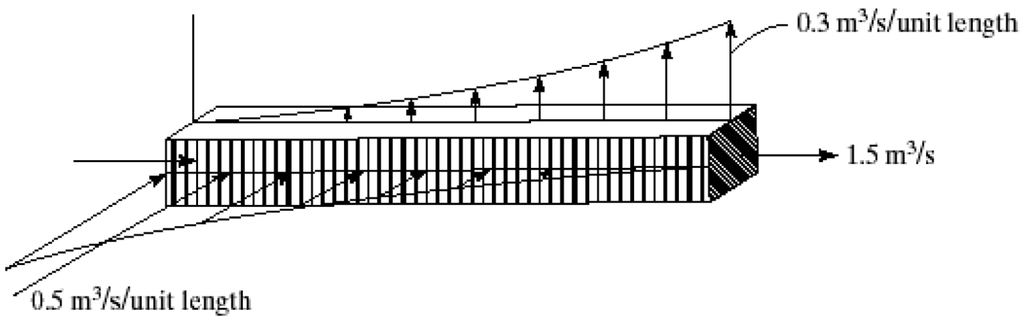
The minimum velocity is found by:

$$dV_x / dx = 3x^2 + 5x - 5 = 0$$

$$x = -\frac{5 \pm \sqrt{85}}{6} = 0.7m$$

2.16)

Given: A rectangular duct as shown below with two porous surfaces. What is the average velocity of water leaving the duct if it is 1.0 *m* long and has a cross section of 0.1 m^2?



2.16) contd.

Following the same procedure used in solving Prob. 2.14:

$$\dot{q}_{in_{end}} + \dot{q}_{in_{side}} = \dot{q}_{out_{top}} + \dot{q}_{out_{end}}$$

$$\dot{q}_{in_{end}} = 1.5\frac{m^3}{s} + \int_0^1 0.3x^2 dx - \int_0^1 0.5(1-x)dx$$

$$\dot{q}_{in_{end}} = 1.5\frac{m^3}{s} + 0.3\frac{x^3}{3}\Big|_0^1 - 0.5x\Big|_0^1 + 0.5\frac{x^2}{2}\Big|_0^1$$

$$\dot{q}_{out_{end}} = 1.35\frac{m^3}{s}$$

and the velocity at the outflow is:

$$V_{out_{end}} = \dot{q}_{out_{end}} / \rho A = 1.35m^3/s \big/ \left(977.8kg/m^3 \cdot 0.1m^2\right)$$

$$V_{out_{end}} = 0.014m/s$$

$\boxed{2.17}$ $\vec{V} = -\frac{x}{2t}\hat{i}\ ;\ \rho = \rho_0 x t$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

$$\vec{a} = +\frac{x}{2t^2}\hat{i} + \left[-\frac{x}{2t}\right]\left[-\frac{1}{2t}\hat{i}\right]$$

$$\vec{a} = \frac{x}{2t^2}\hat{i} + \frac{x}{4t^2}\hat{i} = \frac{3x}{4t^2}\hat{i}$$

$\boxed{2.18}$ $\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$

$$\vec{a} = 2t\hat{i} - 10\hat{j} + \left[6 + 2xy + t^2\right]\left[2y\hat{i} - y^2\hat{j}\right]$$
$$- \left[xy^2 + 10t\right]\left[2x\hat{i} - 2xy\hat{j}\right] + 25\left[0\right]$$

when (x, y, z) is $(3, 0, 2)$ and $t = 1$

$$\vec{a} = \hat{i}\left[2 - 60\right] + \hat{j}\left[-10\right] = -58\hat{i} - 10\hat{j}$$

2.19] It will be shown that the velocity function

$$\vec{V}(r,\theta) = U_\infty\left(1-\frac{R^2}{r^2}\right)\cos\theta\,\hat{e}_r - U_\infty\left(1+\frac{R^2}{r^2}\right)\sin\theta\,\hat{e}_\theta$$

represents an inviscid, steady flow around a cylinder of radius R. To develop the expression for the acceleration:

$$\frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}$$

$$= v_r \frac{\partial}{\partial r}\vec{V} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta}\vec{V}$$

$$= \left[U_\infty\left(1-\frac{R^2}{r^2}\right)\cos\theta\right]\left[U_\infty\cos\theta\,\hat{e}_r\left(+\frac{2R^2}{r^3}\right) - U_\infty\sin\theta\,\hat{e}_\theta\left(-\frac{2R^2}{r^3}\right)\right]$$

$$+\left[U_\infty\left(1+\frac{R^2}{r^2}\right)\frac{\sin\theta}{r}\right]\left\{\left[U_\infty\left(1-\frac{R^2}{r^2}\right)\hat{e}_r(-\sin\theta)\right.\right.$$

$$\left.+U_\infty\left(1-\frac{R^2}{r^2}\right)\cos\theta\,\hat{e}_\theta\right]+\left[-U_\infty\left(1+\frac{R^2}{r^2}\right)\hat{e}_\theta(\cos\theta)\right.$$

$$\left.\left.+U_\infty\left(1+\frac{R^2}{r^2}\right)\sin\theta\,\hat{e}_r\right]\right\}$$

In developing this expression, we have used the fact that

$$\frac{\partial \hat{e}_r}{\partial \theta} = \hat{e}_\theta \quad \text{and} \quad \frac{\partial \hat{e}_\theta}{\partial \theta} = -\hat{e}_r$$

Thus,

$$\frac{d\vec{V}}{dt} = \hat{e}_r\left[U_\infty^2\cos^2\theta\left(\frac{2R^2}{r^3}-\frac{2R^4}{r^5}\right)+U_\infty^2\sin^2\theta\left(\frac{1}{r}-\frac{R^4}{r^5}-\frac{1}{r}-\frac{2R^2}{r^3}-\frac{R^4}{r^5}\right)\right]$$

$$+\hat{e}_\theta\left[+U_\infty^2\sin\theta\cos\theta\left(+\frac{2R^2}{r^3}-\frac{2R^4}{r^5}-\frac{1}{r}+\frac{R^4}{r^5}+\frac{1}{r}+\frac{2R^2}{r^3}+\frac{R^4}{r^5}\right)\right]$$

2.19 Contd.

Note that when $r = R$, i.e., at points on the surface of the cylinder:

$$\frac{d\vec{V}}{dt} = \hat{e}_r \left[\frac{-4U_\infty^2 \sin^2\theta}{R} \right] + \hat{e}_\theta \left[\frac{4U_\infty^2 \sin\theta \cos\theta}{R} \right]$$

Note further that when $\theta = 0$ and when $\theta = \pi$

$$\frac{d\vec{V}}{dt} = 0$$

Thus, when $r = R$ and $\theta = 0$ and when $r = R$ and $\theta = \pi$, $\vec{V} = 0$ (these two points are stagnation points) and $\frac{d\vec{V}}{dt} = 0$ (the fluid particles are not accelerating at these two points). Note that $\theta = 0$ and $\theta = \pi$ represent points on the x-axis, which corresponds to the plane of symmetry for this flow.

2.20 From the integral form of the continuity equation:

$$uA = \text{constant} = Q$$

The cross-sectional area for a unit depth is $A = 2y\,(1)$

Using the boundary condition that $u = 2$ m/s and $h = 1$ m at $x = 0$. Thus, at the initial station:

$$Q \left(\text{the volumetric flow/unit depth}\right) = (u)(2h) = 4.0 \text{ m}^2/\text{s}$$

Thus,

$$Q/\text{depth} = (u)(2y) = u\left[2h - h \sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right] = 4.0 \quad (a)$$

2.20 Contd.

Differentiating: $u \frac{dA}{dx} + \frac{du}{dx} A = 0$

or $u \frac{d(2y)}{dx} + \frac{du}{dx}(2y) = 0$

$$u\left\{-\frac{\pi}{2}\frac{h}{L}\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\right\} + \frac{du}{dx}\left\{2h - h\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right\} = 0$$

$$\frac{du}{dx} = \frac{u\left\{\frac{\pi}{2}\frac{h}{L}\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\right\}}{\left\{2h - h\left(\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right)\right\}} \qquad \text{(b)}$$

The acceleration $\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$

reduces to $\vec{a} = \frac{d\vec{V}}{dt} = u\frac{\partial u}{\partial x}\hat{i}$ for this one-dimensional, steady flow.

$$\vec{a} = u\left\{u \frac{\left[\frac{\pi}{2}\frac{h}{L}\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\right]}{\left[2h - h\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right]}\right\}$$

Substituting the expression for the velocity, i.e., (a),

$$\vec{a} = \frac{16}{\left[2h - h\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right]^2}\left\{\frac{\frac{\pi}{2}\frac{h}{L}\cos\left(\frac{\pi}{2}\frac{x}{L}\right)}{\left[2h - h\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\right]}\right\}\hat{i}$$

At $x=0$: $\vec{a} = \frac{8\pi h}{L}\frac{1}{8h^3}\hat{i} = \frac{\pi}{Lh^2}\hat{i} = \pi\hat{i}$

At $x = 0.5L$ $\quad \vec{a} = \frac{\frac{8\pi h}{L}\cos\frac{\pi}{4}}{\left[2h - h\sin\left(\frac{\pi}{4}\right)\right]^3}\hat{i} = \frac{8\pi h}{Lh^3}(0.3272)\hat{i}$

$$\vec{a} = 8.223\,\hat{i} \ \ m/s^2$$

2.21)

Given: A mass flow rate for the cabin air of:

$$\dot{m}_c = -0.040415\frac{p_c}{\sqrt{T_c}}\left[A_{hole}\right]$$

Using the Ideal Gas Law and the definition of density:

$$p = \rho RT \qquad \rho = \frac{m}{\forall}$$

The pressure becomes:

$$p = \frac{m}{\forall}RT$$

And the mass flow rate equation can be rewritten as:

$$\dot{m}_c = -0.040415\frac{m_c}{\forall}R_c\sqrt{T_c}\left[A_{hole}\right]$$

and:

$$\frac{\dot{m}_c}{m_c} = -\frac{0.040415}{\forall}R_c\sqrt{T_c}\left[A_{hole}\right]$$

But the mass flow rate is defined as $\dot{m} = dm/dt$ and the relationship can be integrated as:

$$\int_{m_{c_i}}^{m_{c_f}}\frac{dm}{m} = -\frac{0.040415}{\forall}R_c\sqrt{T_c}\left[A_{hole}\right]\int_0^{t_f}dt$$

where *i* represents an initial value and *f* represents a final value. Solving for the final time:

$$t_f = \frac{-\forall}{0.040415R_c\sqrt{T_c}A_{hole}}\ln\left(\frac{m_{c_f}}{m_{c_i}}\right)$$

Since $T_c = 22^\circ C$ we see that $m_{c_f}/m_{c_i} = p_{c_f}/p_{c_i}$ and:

$$t_f = \frac{-\forall}{0.040415R_c\sqrt{T_c}A_{hole}}\ln\left(\frac{p_{c_f}}{p_{c_i}}\right)$$

Using $\forall = 71\ m^3$ and consistent units, we get:

$$t_f = 5589s = 1.55hours$$

2.22)

Given: A mass flow rate for the cabin air of:

$$\dot{m}_c = -0.5318 \frac{p_c}{\sqrt{T_c}} [A_{hole}]$$

Using the Ideal Gas Law and the definition of density:

$$p = \rho R T \qquad \rho = \frac{m}{V\!\!\!-}$$

The pressure becomes:

$$p = \frac{m}{V\!\!\!-} RT$$

And the mass flow rate equation can be rewritten as:

$$\dot{m}_c = -0.5318 \frac{m_c}{V\!\!\!-} R_c \sqrt{T_c} [A_{hole}]$$

and:

$$\frac{\dot{m}_c}{m_c} = -\frac{0.5318}{V\!\!\!-} R_c \sqrt{T_c} [A_{hole}]$$

But the mass flow rate is defined as $\dot{m} = dm/dt$ and the relationship can be integrated as:

$$\int_{m_{c_i}}^{m_{c_f}} \frac{dm}{m} = -\frac{0.5318}{V\!\!\!-} R_c \sqrt{T_c} [A_{hole}] \int_0^{t_f} dt$$

where *i* represents an initial value and *f* represents a final value. Solving for the final time:

$$t_f = \frac{-V\!\!\!-}{0.5318 R_c \sqrt{T_c} A_{hole}} \ln\left(\frac{m_{c_f}}{m_{c_i}}\right)$$

Since $T_c = 22^\circ C$ we see that $m_{c_f} / m_{c_i} = p_{c_f} / p_{c_i}$ and:

$$t_f = \frac{-V\!\!\!-}{0.5318 R_c \sqrt{T_c} A_{hole}} \ln\left(\frac{p_{c_f}}{p_{c_i}}\right)$$

Using $V\!\!\!- = 2513\ ft^3$ and consistent units, we get:

$$t_f = 5385 s = 1.50 hours$$

2.23)

Given: A mass flow rate for Oxygen of:

$$\dot{m}_{O_2} = -0.6847 \frac{p_{O_2}}{\sqrt{R_{O_2} T_{O_2}}} [A_{hole}]$$

Using the Ideal Gas Law and the definition of density:

$$p = \rho R T \qquad \rho = \frac{m}{\forall}$$

The pressure becomes:

$$p = \frac{m}{\forall} R T$$

And the mass flow rate equation can be rewritten as:

$$\dot{m}_{O_2} = -0.6847 \frac{m_{O_2}}{\forall} \sqrt{R_{O_2} T_{O_2}} [A_{hole}]$$

and:

$$\frac{\dot{m}_{O_2}}{m_{O_2}} = -\frac{0.6847}{\forall} \sqrt{R_{O_2} T_{O_2}} [A_{hole}]$$

But the mass flow rate is defined as $\dot{m} = dm/dt$ and the relationship can be integrated as:

$$\int_{m_{O_{2i}}}^{m_{O_2 f}} \frac{dm}{m} = -\frac{0.6847}{\forall} \sqrt{R_{O_2} T_{O_2}} [A_{hole}] \int_0^{t_f} dt$$

where *i* represents an initial value and *f* represents a final value. Solving for the final time:

$$t_f = \frac{-\forall}{0.6847 \sqrt{R_{O_2} T_{O_2}} A_{hole}} \ln\left(\frac{m_{O_{2f}}}{m_{O_{2i}}} \right)$$

Since $T_{O_2} = 18^\circ C$ we see that $m_{O_{2f}} / m_{O_{2i}} = p_{O_{2f}} / p_{O_{2i}}$ and:

$$t_f = \frac{-\forall}{0.6847 \sqrt{R_{O_2} T_{O_2}} A_{hole}} \ln\left(\frac{p_{O_{2f}}}{p_{O_{2i}}} \right)$$

Using $\forall = 1\ m^3$ and consistent units, we get:

$$t_f = 445859 s = 7431 hours = 310 days$$

For $\forall = 0.1\ m^3$ and consistent units, we get:

$$t_f = 31 days$$

2.24) As was done in Example 2.2, we can write that:

$$v = 0 \; ; \; w = 0; \quad \frac{dp}{dx} = \mu \frac{d^2u}{dy^2}$$

Integrating twice: $u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 y + C_2$ (i)

Which is subject to the two boundary conditions:

(a) $y = 0$: $u = 0$ (the lower plate is stationary)

(b) $y = h$: $u = U_0$ (the upper plate moves with constant speed)

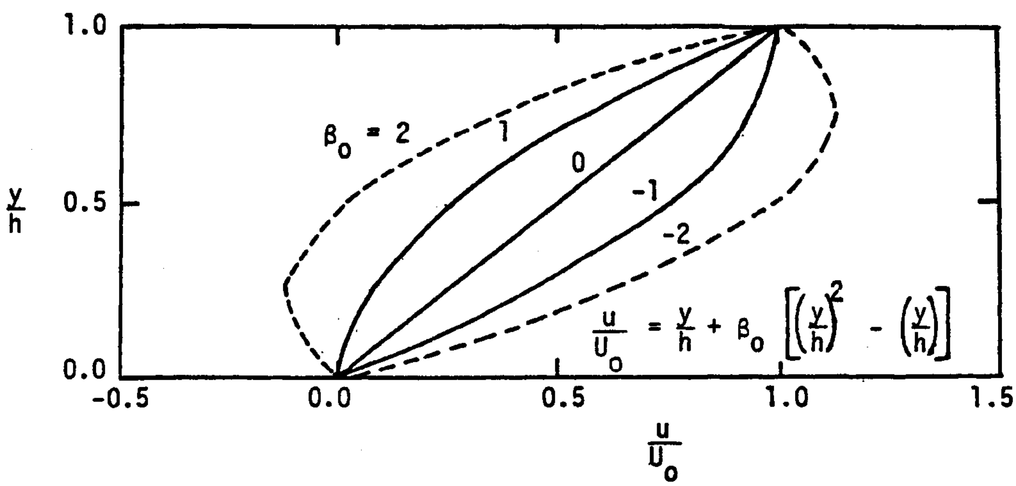
Applying these two boundary conditions:

(a) $0 = C_2$; (b) $U_0 = \frac{1}{2\mu}\frac{dp}{dx}h^2 + C_1 h$

Thus, $C_1 = \frac{U_0}{h} - \frac{1}{2\mu}\frac{dp}{dx}h$

Substituting these constants into (i)

$$u = \underbrace{\frac{U_0}{h}y}_{\text{linear variation due to movement of the upper plate}} + \underbrace{\frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh)}_{\text{velocity variation due to the existence of the pressure gradient}}$$



2.24 Contd. The factor $\frac{h^2}{2\mu U_0}\frac{dp}{dx}$ is a constant for a given problem, which we shall call β_0. The velocity profiles (u/U_0) are presented in the sketch as a function of (y/h) for various values of β_0. Note that the profile is "fuller" when the pressure decreases in the x-direction, i.e., β_0 is negative, which is known as a favorable pressure gradient.

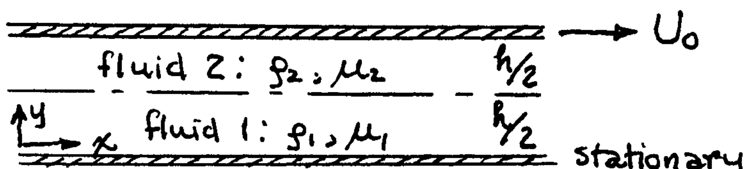
If $u=0$, when $y=\frac{h}{2}$

$$0 = \frac{U_0}{h}\frac{h}{2} + \frac{1}{2\mu}\frac{dp}{dx}\left[\frac{h^2}{4} - \frac{h^2}{2}\right]; \quad 0 = \frac{U_0}{2} + \frac{1}{2\mu}\frac{dp}{dx}\left[-\frac{h^2}{4}\right]$$

Solving: $\frac{dp}{dx} = \frac{4\mu U_0}{h^2}$ (>0, an adverse pressure gradient)

Also, we can write: $$\frac{u}{U_0} = \frac{y}{h} + \frac{h^2}{2\mu U_0}\frac{dp}{dx}\left[\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right)\right]$$

2.25)



(a) τ must be constant across the fluid (including across the fluid/fluid interface)

(b) (i) $y=0$: $u_1=0$ (the lower plate is stationary)

(ii) $y=h$; $u_2=U_0$ (the upper plate moves to the right)

(iii) $y=\frac{h}{2}$: $\tau_1 = \mu_1(du_1/dy) = \mu_2(du_2/dy) = \tau_2$

(the shear is constant across the interface)

(iv) $y=\frac{h}{2}$: $u_1=u_2$

(the velocity is continuous across the interface)

(c) For this fully-developed flow with no pressure gradient

$$\mu_1\frac{d^2u_1}{dy^2} = 0; \quad u_1 = C_1 y + C_2$$

2.25 Contd. $\mu_2 \dfrac{d^2 u_2}{dy^2} = 0$; $u_2 = C_3 y + C_4$

Applying boundary condition (i): $y = 0 : u_1 = 0 \Rightarrow C_2 = 0$

Applying boundary condition (ii): $y = h : u_2 = U_0$

$$U_0 = C_3 h + C_4 \quad \text{or} \quad C_4 = U_0 - C_3 h$$

Applying boundary condition (iii): $y = \dfrac{h}{2} : \tau_1 = \tau_2$

Thus, $\mu_1 C_1 = \mu_2 C_3$ or $C_3 = \dfrac{\mu_1}{\mu_2} C_1$

Applying boundary condition (iv): $y = \dfrac{h}{2} : u_1 = u_2$

$$C_1 \frac{h}{2} = C_3 \frac{h}{2} + U_0 - C_3 h$$

Rearranging: $(C_1 + C_3)\dfrac{h}{2} = U_0$

Substituting the fact that: $C_3 = \dfrac{\mu_1}{\mu_2} C_1$

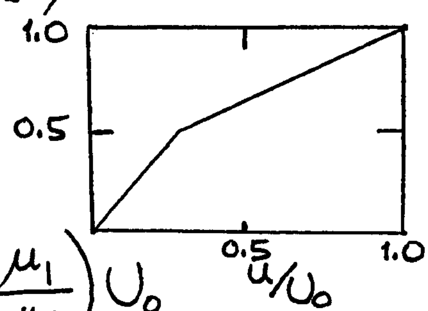
$$C_1\left(1 + \frac{\mu_1}{\mu_2}\right)\frac{h}{2} = U_0$$

Therefore: $C_1 = \dfrac{\mu_2}{\mu_2 + \mu_1}\dfrac{2}{h} U_0$ and $C_3 = \dfrac{\mu_1}{\mu_2 + \mu_1}\dfrac{2}{h} U_0$

Then: $C_4 = U_0 - \dfrac{\mu_1}{\mu_2 + \mu_1} 2 U_0 = \left(\dfrac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right) U_0$

Thus,

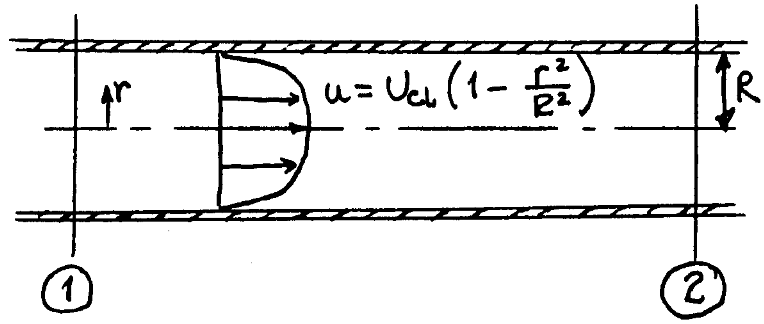
$$u_1 = \frac{\mu_2}{\mu_2 + \mu_1}\frac{2y}{h} U_0$$



$$u_2 = \frac{\mu_1}{\mu_2 + \mu_1}\frac{2y}{h} U_0 + \left(\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right) U_0$$

(d) At $y = 0$: $\tau = \tau_1 = \dfrac{\mu_1 \mu_2}{\mu_1 + \mu_2}\dfrac{2U_0}{h}$

2.26



$$\sum \vec{F} = \frac{\partial}{\partial t}\iiint \rho \vec{V}\, d(vol) + \oiint \left(\rho \vec{V}\cdot\hat{n}\, dA\right)\vec{V}$$

The first term on the right-hand side is zero for steady flow. The second term on the right-hand side is zero for fully-developed flow, since the efflux of momentum through the surface of the control volume (i.e., crossing station 2) is of equal magnitude but opposite sign to the influx of momentum through the surface of the control volume (i.e., crossing station 1). To see that this is true, let us evaluate the second term on the right-hand side:

$$\oiint \left(\rho \vec{V}\cdot\hat{n}\, dA\right)\vec{V}$$

$$= \underbrace{\int_0^R \rho \left[U_{CL}\left(1-\frac{r^2}{R^2}\right)\right]\left[2\pi r\, dr\right]\left[U_{CL}\left(1-\frac{r^2}{R^2}\right)\right]}_{\text{evaluated at station } 2}$$

$$- \underbrace{\int_0^R \rho \left[U_{CL}\left(1-\frac{r^2}{R^2}\right)\right]\left[2\pi r\, dr\right]\left[U_{CL}\left(1-\frac{r^2}{R^2}\right)\right]}_{\text{evaluated at station } 1}$$

The opposite signs result because the unit vector for area ($\hat{n}$) is directed outward for the control volume. Therefore, $\vec{V}\cdot\hat{n}\, dA > 0$ for station 2 and $\vec{V}\cdot\hat{n}\, dA < 0$ for station 1.

2.26 Contd. Since the magnitudes of the integrands are equal,

$$\oiint (\rho \vec{V} \cdot \hat{n}\, dA)\vec{V} = 0$$

Thus, $\sum F_x = p_1 A_1 - p_2 A_2 + 2\pi R(\Delta x)\tau_w = 0$

$$\tau_w = \mu\left(\frac{du}{dr}\right)_{r=R} = \mu U_{CL}\left(-\frac{2R}{R^2}\right) = -\frac{2\mu U_{CL}}{R}$$

$A_1 = A_2 = \pi R^2$. Note, that with the velocity distribution known, i.e., $u(r) = U_{CL}\left(1 - \frac{r^2}{R^2}\right)$, evaluating τ_w using the $\tau_w = \mu\left(\frac{du}{dr}\right)$ produces a negative shear force term in the momentum equation. Combining:

$$\frac{p_2 - p_1}{\Delta x} = -\frac{2}{R}\left(\frac{2\mu U_{CL}}{R}\right)$$

Thus, $\frac{dp}{dx} = -\frac{4\mu U_{CL}}{R^2}$

For U_{CL} in the direction shown, $\frac{dp}{dx} < 0$. Thus, the pressure decreases in the streamwise direction, i.e., a favorable pressure gradient exists, because of the presence of viscous forces. It is to compensate for this pressure decrease (termed a "head loss" in civil engineering terms) due to the viscous forces that pumps are needed in a pipeline. If the flow were inviscid, there would be no pressure gradient for the flow in a constant-area pipe.

The mass flow rate through the pipe is:

$$\dot{m} = \int_0^R \rho U_{CL}\left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr = \rho U_{CL} 2\pi \int_0^R \left(r - \frac{r^3}{R^2}\right) dr$$

$$\dot{m} = 2\pi \rho U_{CL}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right] = 2\pi \rho U_{CL}\frac{R^2}{4} = \frac{\rho U_{CL} \pi R^2}{2}$$

2.26 Contd. Thus, $U_{CL} = \dfrac{2\dot{m}}{\rho \pi R^2}$

If we are to maintain the same mass flow rate (i.e., $\dot{m}_1 = \dot{m}_2$) while doubling the radius of the pipe (i.e., $R_2 = 2R_1$), then

$$\left.\frac{dp}{dx}\right|_1 = -\frac{4\mu}{R_1^2}\left(\frac{2\dot{m}_1}{\rho\pi R_1^2}\right) \quad \text{and} \quad \left.\frac{dp}{dx}\right|_2 = -\frac{4\mu}{R_2^2}\left(\frac{2\dot{m}_2}{\rho\pi R_2^2}\right)$$

Dividing one by the other and noting that $\dot{m}_1 = \dot{m}_2$:

$$\left.\frac{dp}{dx}\right|_2 = \frac{R_1^4}{R_2^4}\left.\frac{dp}{dx}\right|_1 = \frac{1}{16}\left.\frac{dp}{dx}\right|_1$$

2.27 Let us apply the integral form of the momentum equation. Since we are interested in the drag, we only need to consider the x-component of this vector equation. Refer to the solution for Problem 2.7 for the discussion of the continuity equation of this flow.

$$\Sigma F_x = \frac{\partial}{\partial t}\iiint \rho V_x \, d(\text{vol}) + \oiint (\rho \vec{V} \cdot \hat{n}\, dA)\, V_x$$

Since the pressure is constant over the external surface of the control volume, the only force for the left-hand side is the force of the airfoil on the fluid within the control volume, which is the negative of the drag per unit span.

$$-d = \underbrace{\rho \int_{-H}^{+H} (U_\infty \hat{i})(-\hat{i}\, dy) U_\infty}_{1}$$

$$+ \underbrace{\rho \int_{-H}^{0} \left[\left(-U_\infty \frac{y}{h}\hat{i} - v\hat{j}\right)\cdot(\hat{i}\, dy)\right] \frac{-U_\infty y}{h}}_{2}$$

$$+ \underbrace{\rho \int_{0}^{H} \left[\left(U_\infty \frac{y}{h}\hat{i} + v\hat{j}\right)\cdot(\hat{i}\, dy)\right] \frac{U_\infty y}{h}}_{2}$$

2.27 Contd.

$$+\rho\int_0^L\left[(U_\infty\hat{i}+v_\infty\hat{j})\cdot(\hat{j}\,dx)\right]U_\infty$$

$$\longleftarrow \quad 3 \quad \longrightarrow$$

$$+\rho\int_0^L\left[(U_\infty\hat{i}-v_\infty\hat{j})\cdot(-\hat{j}\,dx)\right]U_\infty$$

$$\longleftarrow \quad 4 \quad \longrightarrow$$

Note that because of the approximations that we have employed, the velocity at the boundaries 3 and 4 actually exceeds U_∞, while the static pressure remains unchanged. These are "second-order inconsistencies" introduced by our flow model approximations.

Note also that v_∞ is some unspecified function of x. The exact functional relationship is not important. Using the result from the application of the continuity equation in Problem 2.10:

$$2\int_0^L v_\infty\,dx = U_\infty H$$

Thus,
$$-d = -\rho U_\infty^2(2H) + \rho\frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Big|_{-H}^{0}$$
$$+\rho\frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Big|_{0}^{H} + \rho U_\infty\left[2\int_0^L v_\infty\,dx\right]$$

can be written:

$$-d = -\rho U_\infty^2(2H) + \rho U_\infty^2\frac{H}{3} + \rho U_\infty^2\frac{H}{3} + \rho U_\infty^2 H$$

$$d = \frac{1}{3}\rho U_\infty^2 H$$

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\frac{1}{3}\rho U_\infty^2 H}{\frac{1}{2}\rho U_\infty^2 c} = \frac{1}{60} = 0.0167$$

2.28

This is very similar to Problem 2.27, except that the side boundaries of the control volume are streamlines. Thus, instead of using the continuity equation to determine the flow through sides 3 and 4 as was done for Problem 2.27, the continuity equation must be used to determine the relation between H_U and H_D.

Again, the pressure is constant over the external surface of the control volume for this steady, incompressible flow. Thus, the only force acting on the system of the fluid particles within the control volume is the negative of the drag.

$$-d = \underbrace{\rho \int_{-H_U}^{+H_U} \left[(U_\infty \hat{i}) \cdot (-\hat{i}\, dy) \right] U_\infty}_{1}$$

$$+ \underbrace{\rho \int_{-H_D}^{0} \left[\left(-U_\infty \frac{y}{H_D} \hat{i} - v \hat{j} \right) \cdot (\hat{i}\, dy) \right] \left(-U_\infty \frac{y}{H_D} \right)}_{2}$$

$$+ \underbrace{\rho \int_{0}^{H_D} \left[\left(U_\infty \frac{y}{H_D} \hat{i} + v \hat{j} \right) \cdot (\hat{i}\, dy) \right] \left(U_\infty \frac{y}{H_D} \right)}_{2}$$

There is no momentum transport across boundaries 3 and 4, since they are streamlines.

$$-d = \rho U_\infty^2 \left[2 H_U \right] + \rho U_\infty^2 \left[\frac{y^3}{3 H_D^2} \right]_{-H_D}^{0} + \rho U_\infty^2 \left[\frac{y^3}{3 H_D^2} \right]_{0}^{+H_D}$$

$$d = \rho U_\infty^2 \left[2 H_U - \frac{2}{3} H_D \right]$$

2.28 Contd. We can use the integral continuity equation to determine the relation between H_U and H_D for this steady, incompressible flow;

$$+\underbrace{\int_{-H_U}^{+H_U}\left[(U_\infty \hat{i})\cdot(-\hat{i}\,dy)\right]}_{①} + \underbrace{\int_{-H_D}^{0}\left[\left(-\frac{U_\infty y}{H_D}\hat{i} - v\hat{j}\right)\cdot(\hat{i}\,dy)\right]}_{②}$$

$$+\underbrace{\int_{0}^{H_D}\left[\left(+\frac{U_\infty y}{H_D}\hat{i} + v\hat{j}\right)\cdot(\hat{i}\,dy)\right]}_{②} = 0$$

Thus, $$U_\infty 2H_U - \left.\frac{U_\infty y^2}{2H_D}\right|_{-H_D}^{0} + \left.\frac{U_\infty y^2}{2H_D}\right|_{0}^{H_D} = 0$$

Therefore, $U_\infty 2H_U = U_\infty H_D$

as was shown in Problem 2.11, $H_U = \frac{1}{2}H_D$

$$d = \rho U_\infty^2\left[H_D - \frac{2}{3}H_D\right] = \frac{1}{3}\rho U_\infty^2 H_D$$

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\frac{1}{3}\rho U_\infty^2\left(\frac{1}{40}c\right)}{\frac{1}{2}\rho_\infty U_\infty^2 c} = \frac{1}{60} = 0.0167$$

As one would expect, we have gotten the same result as was obtained in Problem 2.28. Therefore, the result is not dependent on the control volume.

2.29) This is the third problem in this trilogy to illustrate that the drag coefficient is not dependent on the control volume chosen in the formulation of the problem, providing the viscous boundary layer is within the bounds of the control volume.

Applying the integral momentum equation for the steady, incompressible flow with the static pressure constant over the external surface of the control volume.

$$
\begin{aligned}
-d = &\underbrace{\rho \int_{-2H}^{+2H} \left[(U_\infty \hat{i})\cdot(-\hat{i}\,dy)\right] U_\infty}_{1} \\
&+ \underbrace{\rho \int_{-2H}^{-H} \left[(U_\infty \hat{i} - v_0 \hat{j})\cdot(\hat{i}\,dy)\right] U_\infty}_{2} \\
&+ \underbrace{\rho \int_{-H}^{0} \left[\left(-\frac{U_\infty y}{H}\hat{i} - v\hat{j}\right)\cdot(\hat{i}\,dy)\right]\left(-\frac{U_\infty y}{H}\right)}_{2} \\
&+ \underbrace{\rho \int_{0}^{H} \left[\left(\frac{U_\infty y}{H}\hat{i} + v\hat{j}\right)\cdot(\hat{i}\,dy)\right]\left(\frac{U_\infty y}{H}\right)}_{2} \\
&+ \underbrace{\rho \int_{H}^{2H} \left[(U_\infty \hat{i} + v_0 \hat{j})\cdot(\hat{i}\,dy)\right] U_\infty}_{2} \\
&+ \underbrace{\rho \int_{0}^{L} \left[(U_\infty \hat{i} + v_0 \hat{j})\cdot(\hat{j}\,dx)\right] U_\infty}_{3} \\
&+ \underbrace{\rho \int_{0}^{L} \left[(U_\infty \hat{i} - v_0 \hat{j})\cdot(-j\,dx)\right] U_\infty}_{4}
\end{aligned}
$$

Note the similarities between this expression and that of Problem 2.27. We are using v_0 in this problem, instead

2.29 Contd.

of v_∞, to represent the y-component of velocity outside of the viscous regim. Note also, v_0 is some unspecified function of x. However, since we are using the integral technique, the specifics of the function will not matter. Integrating,

$$-d = -\rho V_\infty^2(4H) + \rho U_\infty^2(H) + \rho \frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Big|_{-H}^{0}$$

$$+ \rho \frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Big|_{0}^{H} + \rho U_\infty^2(H) + \rho U_\infty\left[2\int_0^L v_0\, dx\right]$$

Note that the sum of the first, second, and fifth terms on the right-hand side is $-\rho U_\infty^2(2H)$, which is the first term on the right-hand side of the corresponding equation in the solution of Problem 2.18. Thus, as we might expect, the momentum exiting the control volume between $-2H \le y \le -H$ and $H \le y \le 2H$ at station ② is exactly balanced by the influx of momentum between $-2H \le y \le -H$ and $H \le y \le 2H$ at station ①. Thus,

$$-d = -\rho U_\infty^2(2H) + \rho U_\infty^2 \frac{H}{3} + \rho U_\infty^2 \frac{H}{3} + \rho U_\infty\left[2\int_0^L v_0\, dx\right]$$

Using the continuity equation

$$\frac{\partial}{\partial t}\iiint \rho\, d(vol) + \oiint \rho \vec{V}\cdot\hat{n}\, dA = 0$$

The first term is zero for steady flow. The second term is:

$$-\rho U_\infty(4H) + \rho U_\infty(H) + \rho U_\infty \frac{-y^2}{2H}\Big|_{-H}^{0} + \rho U_\infty \frac{y^2}{2H}\Big|_{0}^{H}$$

$$+ \rho U_\infty(H) + 2\rho\int_0^L v_0\, dx = 0$$

2.29 Contd.

As a result:

$$2\int_0^L v_o\,dx = U_\infty H$$

As we found in Problem 2.18. Substituting this into the momentum equation,

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\frac{1}{3}\rho U_\infty^2\left(\frac{1}{40}c\right)}{\frac{1}{2}\rho U_\infty^2 c} = \frac{1}{60} = 0.0167$$

Comparing the results of Problems 2.27, 2.28, and 2.29, we see that the resulting drag coefficient is the same for all three control volumes (which all enclose the viscous wake).

2.30 Let us apply the integral form of the momentum equation. Since we are interested in the drag acting on the airfoil, which is aligned with the x-axis, we need only consider the x-component of this vector equation.

$$\sum F_x = \frac{\partial}{\partial t}\iiint \rho V_x\, d(vol) + \oiint \rho(\vec{V}\cdot\hat{n}\,dA)V_x$$

Since atmospheric pressure acts over the entire external surface of the control volume, the only force acting on the fluid particles within the control volume (i.e., the left-hand side of this equation) is the negative of the drag. Furthermore, the flow is steady and the first term on the right-hand side is zero. The flow is incompressible (or the density is constant). Thus,

$$-d = +\rho\int_{-H}^{+H}\left[(U_\infty\hat{i})\cdot(-\hat{i}\,dy)\right]U_\infty$$

2.30 Contd.

$$+ \rho \int_{-H}^{+H} \left\{ \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2H}\right) \hat{i} \pm v \hat{j} \right] \cdot (\hat{i}\, dy) \right\} \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2H}\right) \right] + 2\rho \int_0^L \left[U_\infty \hat{i} + v_\infty \hat{j} \right] \cdot (\hat{j}\, dx)\, U_\infty$$

$$-d = \rho U_\infty^2 y \Big|_{-H}^{+H} + \rho U_\infty^2 \int_{-H}^{+H} \left[1 - \cos \frac{\pi y}{2H} + 0.25 \cos^2 \frac{\pi y}{2H} \right] dy + 2\rho U_\infty \int_0^L v_\infty\, dx$$

We can use the integral form of the continuity equation to find $\int_0^L v_\infty\, dx$

$$\rho \int_{-H}^{+H} (U_\infty \hat{i}) \cdot (-\hat{i}\, dy) + \rho \int_{-H}^{+H} \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2H}\right) \hat{i} + v \hat{j} \right] \cdot (\hat{i}\, dy) + 2\rho \int_0^L \left[U_\infty \hat{i} + v_\infty \hat{j} \right] \cdot \hat{j}\, dx = 0$$

$$-\rho U_\infty y \Big|_{-H}^{+H} + \rho U_\infty \int_{-H}^{+H} \left(1 - 0.5 \cos \frac{\pi y}{2H}\right) dy + 2\rho \int_0^L v_\infty\, dx = 0$$

$$-\rho U_\infty (2H) + \rho U_\infty (2H) - \rho U_\infty \frac{1}{2} \frac{2H}{\pi} \left[1 - (-1) \right] + 2\rho \int_0^L v_\infty\, dx = 0$$

Thus, $\quad 2\rho \int_0^L v_\infty\, dx = \rho U_\infty \dfrac{2H}{\pi}$

Substituting this result into the momentum equation

$$-d = -\rho U_\infty^2 (2H) + \rho U_\infty^2 \left(y \Big|_{-H}^{+H} - \frac{2H}{\pi} \sin \frac{\pi y}{2H} \Big|_{-H}^{+H} \right.$$

2.30 Contd.

$$+\frac{1}{4}\left(\frac{y}{2}+\frac{2H}{4\pi}\sin\frac{\pi y}{H}\right|_{-H}^{+H} + \rho U_\infty^2\frac{2H}{\pi}$$

$$-d = -\rho U_\infty^2(2H) + \rho U_\infty^2(2H) - \rho U_\infty^2\frac{2H}{\pi}\left[1-(-1)\right]$$

$$+ \rho U_\infty^2\frac{1}{4}\left[\frac{H}{2}-\left(-\frac{H}{2}\right)\right] + \rho U_\infty^2\frac{2H}{\pi}$$

$$-d = -\rho U_\infty^2 H\left[\frac{4}{\pi}-\frac{1}{4}-\frac{2}{\pi}\right] = -\rho U_\infty^2\frac{c}{40}\left[\frac{2}{\pi}-\frac{1}{4}\right]$$

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = 0.01933$$

2.31 As with problem 2.30, let us apply the integral equations to solve this problem. Again, since atmospheric pressure acts over the external surface of the control volume, the only force acting on the fluid particles within the control volumes i.e., the left-hand side of the integral momentum equation, is the negative of the drag. Furthermore, the flow is steady, so that the first term on the right-hand side of the momentum equation is zero; incompressible, so that the density is constant; and surfaces 3 and 4 are streamlines. Thus, there is no flux of momentum across these streamlines.

$$-d = -\rho U_\infty^2\int_{-H_U}^{+H_U}dy + \rho\int_{-H_D}^{+H_D}\left[U_\infty\left(1-0.5\cos\frac{\pi y}{2H_D}\right)\right.$$

$$\left.\text{times } dy\right]U_\infty\left(1-0.5\cos\frac{\pi y}{2H_D}\right)$$

$$-d = -\rho U_\infty^2(2H_U) + \rho U_\infty^2\int_{-H_D}^{+H_D}\left(1-1.0\cos\frac{\pi y}{2H_D}\right.$$

$$\left.+0.25\cos^2\frac{\pi y}{2H_D}\right)dy$$

2.31 Contd.

$$-d = -\rho U_\infty^2(2H_U) + \rho U_\infty^2\left[y - \frac{2H_D}{\pi}\sin\frac{\pi y}{2H_D} + \frac{1}{4}\left(\frac{y}{2} + \frac{2H_D}{4\pi}\sin\frac{\pi y}{2H_D}\right)\right]\Bigg|_{-H_D}^{+H_D}$$

$$-d = -\rho U_\infty^2(2H_U) + \rho U_\infty^2(2H_D) - \rho U_\infty^2\frac{2}{\pi}H_D(1+1) + \rho U_\infty^2\frac{1}{4}\left(\frac{H_D}{2} + \frac{H_D}{2}\right)$$

Using the results from Problem 2.13 that

$$H_U = \left(1 - \frac{1}{\pi}\right)H_D$$

$$-d = -\rho U_\infty^2 H_D\left[2 - \frac{2}{\pi} - 2 + \frac{4}{\pi} - \frac{1}{4}\right]$$

$$d = \rho U_\infty^2 H_D(0.3866)$$

The drag coefficient is:

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\rho U_\infty^2(0.025c)(0.3866)}{\frac{1}{2}\rho U_\infty^2 c} = 0.01933$$

Again, comparison of the results from Problems 2.30 and 2.31 shows that the drag coefficient does not depend on the control volume chosen.

2.32 (a) Referring to Table 1.2, we find that, at sea level,

$$\rho_\infty = 0.002376\ \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}^4}\ ;\ \mu_\infty = 3.740\times10^{-7}\ \frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}$$

Thus, $$Re_{\infty,d} = \frac{\rho_\infty U_\infty d}{\mu_\infty} = \frac{(0.002376)(200)\left(\frac{1.7}{12}\right)}{3.740\times10^{-7}} = 1.8\times10^5$$

Referring to the discussion of the drag on cylinders and

2.32 Contd. spheres in Chapter 3 (see the data presented in Fig. 3.30), we see that this Reynolds number is below the critical value. Therefore, if the golf ball were smooth, the forebody boundary layer would be laminar and the drag would be relatively high. Form drag dominates for the laminar flow over a smooth golf ball. Roughening the surface of the golf ball (such as through the use of dimples) would cause the forebody boundary layer to be turbulent, resulting in significantly delayed separation and reduced form drag.

$$M_\infty = \frac{U_\infty}{a_\infty} = \frac{200}{49.02\sqrt{519}} = 0.179$$

Problem 2.32b Solution

Given: An aircraft flying at a velocity of 1810 m/s at an altitude of 30 km with a length of 32.8 m.

The properties of air at 30 km are given in Table 1.2:

$$\rho_\infty = 0.018411 kg/m^3 \qquad \mu_\infty = 1.4753\times10^{-5} kg/s\cdot m \qquad a_\infty = 301.71 m/s$$

The Reynolds number is found from:

$$\mathrm{Re}_{\infty,L} = \frac{\rho_\infty U_\infty L}{\mu_\infty} = \frac{(0.018411 kg/m^3)(1810 m/s)(32.8 m)}{1.4753\times10^{-5} kg/s\cdot m} = 7.405\times10^7$$

Notice that the Reynolds number for this hypersonic transport is relatively large.

The Mach number is found from:

$$M_\infty = \frac{U_\infty}{a_\infty} = \frac{1810 m/s}{301.71 m/s} \approx 6.0$$

2.33] (a) $M_\infty = 3.0$ at an altitude of 20km. Using Table 1.2

$$a_\infty = 295.069\ \frac{m}{s}\ ;\ \mu_\infty = 1.4216\times10^{-5}\ \frac{kg}{s\cdot m}\ ;\ \rho_\infty = 0.0889\ \frac{kg}{m^3}$$

$$Re_{\infty,L} = \frac{(0.0889)\left[(3.0)(295.069)\right](10.4)}{1.4216\times10^{-5}} = 5.757\times10^{7}$$

Again, the Reynolds number for a high-speed airplane is in excess of 10^7

(b) Referring to the previous problem, we found the English unit values for the density and for the viscosity at sea level. Thus,

$$Re_{\infty,L} = \frac{\left(2.376\times10^{-3}\ \frac{lbf\cdot s^2}{ft^4}\right)\left[\left(160\ \frac{mi}{h}\right)\frac{5280\ \frac{ft}{mi}}{3600\ \frac{s}{h}}\right](4.0\ ft)}{3.740\times10^{-7}\ \frac{lbf\cdot s}{ft^2}}$$

$$Re_{\infty,L} = 5.963\times10^{6}$$

2.34] Since both cycles are reversible

$$\delta q = T\,ds \quad \text{and} \quad \delta w = p\,dv$$

Let us first compute the changes which occur during each portion of the cycle.

For process (i)

(a) Segment AB: $q_{AB} = \int_A^B \delta q = \int_A^B T\,ds$

where $ds = c_v \frac{dT}{T} + R\frac{dv}{v}$ (a)

For a perfect gas: $p = \frac{RT}{v}$

Thus, if we differentiate:

$$dp = -\frac{RT}{v^2}\,dv + \frac{R}{v}\,dT$$

2.34 Contd.

So that: $\frac{dT}{T} = \left[dp + \frac{RT\,dv}{v^2}\right]\frac{v}{RT} = \frac{dp}{p} + \frac{dv}{v}$

Substituting this expression into equation(a), we obtain:

$$ds = c_v \frac{dp}{p} + (c_v + R)\frac{dv}{v} = c_v \frac{dp}{p} + c_p \frac{dv}{v}$$

or $$T ds = c_v T \frac{dp}{p} + c_p T \frac{dv}{v}$$

Thus, $$q_{AB} = \int_A^B T ds = \int_A^B c_v T \frac{dp}{p} + \int_A^B c_p T \frac{dv}{v}$$

$$= 0 + \int_A^B c_p \frac{pv}{R}\frac{dv}{v} = \frac{c_p p}{R}\int_A^B dv$$

$$= \frac{c_p p_A}{R}(v_B - v_A)$$

And $$w_{AB} = \int_A^B \delta w = \int_A^B p\,dv = p_A(v_B - v_A)$$

Segment BC: $q_{BC} = \int_B^C \delta q = \int_B^C T ds$

$$q_{BC} = \int_B^C c_v T \frac{dp}{p} + \int_B^C c_p T \frac{dv}{v} = c_v \frac{v}{R}\int_B^C dp + 0$$

Thus, $q_{BC} = \frac{c_v v_B}{R}(p_c - p_B)$; $w_{BC} = \int_B^C \delta w = \int_B^C p\,dv = 0$

Segment CD:

$$q_{CD} = \int_c^D \delta q = \int_c^D T ds = \int_c^D c_v T \frac{dp}{p} + \int_c^D c_p T \frac{dv}{v}$$

Thus, $q_{cD} = c_p \frac{p}{R}\int_c^D dv = c_p \frac{p_c}{R}(v_D - v_c)$ and

2.34 Contd.

$$w_{CD} = \int_C^D \delta w = p_C (v_D - v_C)$$

Segment DA:

$$q_{DA} = \int_D^A \delta q = \int_D^A T ds = \int_D^A c_v T \frac{dp}{p} + \int_D^A c_p T \frac{dv}{v}$$

$$q_{DA} = c_v \frac{v}{R} \int_D^A dp = c_v \frac{v_A}{R} (p_A - p_D)$$

and $$w_{DA} = \int_D^A \delta w = \int_D^A p\, dv = 0$$

Let us now add up the values for each of the segments:

$$\oint_{ABCDA} \delta q = q_{AB} + q_{BC} + q_{CD} + q_{DA}$$

$$= \frac{c_p p_A}{R}(v_B - v_A) + \frac{c_v v_B}{R}(p_C - p_B)$$

$$+ \frac{c_p p_C}{R}(v_D - v_C) + \frac{c_v v_A}{R}(p_A - p_D)$$

Noting that $v_A = v_D$ and $v_B = v_C$; that $p_A = p_B$ and $p_C = p_D$

$$\oint_{ABCDA} \delta q = c_p \frac{p_A}{R}(v_B - v_A) + c_v \frac{v_B}{R}(p_C - p_A)$$

$$+ c_p \frac{p_C}{R}(v_A - v_B) + c_v \frac{v_A}{R}(p_A - p_C)$$

$$= -\frac{c_p}{R}(p_C - p_A)(v_B - v_A) + \frac{c_v}{R}(p_C - p_A)(v_B - v_A)$$

$$= (p_C - p_A)(v_B - v_A)\frac{(c_v - c_p)}{R} = (p_A - p_C)(v_B - v_A)$$

since $c_p - c_v = R$. Note also that, since $(v_B - v_A) > 0$ and since $(p_A - p_C) < 0$, $\oint_{ABCD} \delta q < 0$. Thus, heat is

2.34 Contd.

transferred to the surroundings from the air in the system.

$$\oint_{ABCDA} \delta w = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= p_A(v_B - v_A) + 0 + p_C(v_D - v_C) + 0$$

$$= p_A(v_B - v_A) - p_C(v_B - v_A) = (p_A - p_C)(v_B - v_A)$$

Note that $\delta w < 0$ also. Thus, work is done by the surroundings on the air in this system.

Finally, note that

$$\oint_{ABCDA} \delta q - \oint_{ABCDA} \delta w = 0$$

as should be the case for a closed cycle.

(b) The process represented by AB is a constant pressure process in which heat is added to the system. The process represented by BC is a constant volume process in which heat is added to the system. The process represented by CD is a constant-pressure, cooling process; while DA represents a constant-volume, cooling process.

(c) and (d)

$$\oint_{ABCDA} \delta q - \oint_{ABCDA} \delta w = (p_A - p_C)(v_B - v_A) - (p_A - p_C)(v_B - v_A)$$

$= 0$. As would be expected, the first law of thermodynamics is satisfied for this process.

Process (ii)

(a) For process (ii):

$$p = C_1(v - v_A) + p_A \text{ ; and } v = C_2(p - p_A) + v_A$$

2.34 Contd.

where $C_1 = \dfrac{p_c - p_A}{v_c - v_A}$ and $C_2 = \dfrac{v_c - v_A}{p_c - p_A}$

Evaluating the expression for the heat flux for segment AC:

$$q_{AC} = \int_A^C \delta q = \int_A^C T ds = \int_A^C c_v T \frac{dp}{p} + \int_A^C c_p T \frac{dv}{v}$$

$$= \frac{c_v}{R}\int_A^C v\,dp + \frac{c_p}{R}\int_A^C p\,dv$$

$$= \frac{c_v}{R}\int_A^C \left[C_2(p-p_A) + v_A\right]dp + \frac{c_p}{R}\int_A^C \left[C_1(v-v_A)+p_A\right]dv$$

Examining this expression, it is clear that:

$$q_{CA} = \int_C^A \delta q = -\int_A^C \delta q$$

Thus, $\quad q_{ACA} = \int_A^C \delta q + \int_C^A \delta q = 0$

Similarly,

$$w_{AC} = \int_A^C \delta w = \int_A^C p\,dv = \int_A^C \left[C_1(v-v_A) + p_A\right] dv$$

Examining this expression, it is clear that:

$$w_{CA} = \int_C^A \delta w = -\int_A^C \delta w; \text{ so that}$$

$$w_{ACA} = \int_A^C \delta w + \int_C^A \delta w = 0$$

(b) For the process designated AC, heat is added to the system; while the system is cooled for the segment designated CA.

2.34 Contd.

(c) and (d)

$$\oint_{ACA} \delta q - \oint_{ACA} \delta w = 0 - 0 = 0$$

consistent with the first law of thermodynamics for this process.

Note that the net heat transferred from the system to the surroundings during process (i), which is $(p_A - p_C)(v_B - v_A)$, is not equal to the net heat transferred from the system to the surroundings during process (ii), which is zero. Since the first law of thermodynamics must be satisfied (and has been shown to be satisfied), the same is true for the work done during the two processes. Both the heat transfer and the work done are path dependent phenomena.

2.35 (a) Yes, entropy is a property and is, therefore, independent of the path for the process.

(b) Even if the processes were irreversible, $s_C - s_A$ would be the same as determined in Problem (2.25). Entropy is a property and its change, therefore, depends only on the gas properties at the end points of the process. Recall that, once any two properties of a gas (which is in equilibrium) are known, the remaining properties of the gas can be determined. Thus, (p_C and v_C) and (p_A and v_A) are the same whether the process is reversible or irreversible, $s_C - s_A$ does not depend on the path of the process.

2.36) When deriving equation (2.32), we used the definitions for τ_{ij} which were given on pages 37 and 38. Thus, the fluid must satisfy the criteria given on page 37 that the stress components are a linear function of the components of the rate of strain, that the relations between the stress components and the rate-of-strain components are invariant to coordinate transformations, and that the stress components reduce to the hydrostatic pressure when all velocity gradients are zero.

In addition, we ignored effects associated with very high gas temperatures, which result in dissociation, ionization, and chemical reactions. E.g., the nitrogen molecules of air begin to dissociate at approximately 4000K. Thus, when the temperature of the gas is extremely high, one must consider additional energy transfer mechanisms, such as radiative heat transfer

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2.37) For the adiabatic, inviscid flow, the terms of the right-hand side of equation (2.32a) are zero. Thus, equation (2.32a) becomes:

$$\rho \frac{dh}{dt} - \frac{dp}{dt} = 0$$

But $T ds = du_e + p dv$

which can be rewritten using the definition for the enthalpy ($u_e = h - pv$) and the definition for the specific volume ($v = \frac{1}{\rho}$). Thus,

$$\rho T \frac{ds}{dt} = \rho \left(\frac{dh}{dt} - v \frac{dp}{dt} \right) = \rho \frac{dh}{dt} - \frac{dp}{dt}$$

which is equal to zero for this flow. Hence, $\frac{ds}{dt} = 0$.

Note that, if this flow is initially isentropic and if

2.37 Contd. the fluid along each streamline undergoes adiabatic, reversible changes, the flow is everywhere isentropic. The requirement of reversible flow implies that the flow is inviscid. Hence, the results obtained using the thermodynamic relations in the problem are consistent with those obtained using Kelvin's Theorem.

In the boundary layer near the surface, the effects of viscosity and of heat transfer produce variations in the entropy and in the stagnation enthalpy between neighboring streamlines.

2.38 At 10,000 feet, the free-stream static temperature is 483.03 °R. $U_1 = 130 \text{ mi/h} = 190.67 \text{ ft/s}$. Thus, we can use these to calculate the total enthalpy:

$$H_t = h_1 + \frac{1}{2}U_1^2$$

Assuming that the flow is a perfect gas: $H_t = c_p T_t$ and $h_1 = c_p T_1$, we can write:

$$T_t = T_1 + \frac{U_1^2}{2c_p}$$

So that:

$$T_t = 483.03\,°R + \frac{\left(190.67 \frac{ft}{s}\right)^2}{2\left(0.2404 \frac{Btu}{lbm\,°R}\right)\left(778.2 \frac{ft\,lbf}{Btu}\right)\left(32.174 \frac{ft\,lbm}{lbf\,s^2}\right)}$$

$$T_t = 483.03\,°R + 3.02\,°R = 486.05\,°R$$

The kinetic energy term is relatively small. As a result, the total (or stagnation) temperature is not much greater than the static temperature. Therefore, convective heating would not be a problem for aircraft flying at this speed.

2.39) For an airplane flying at 80,000 feet, $a_1 = 977.62 \frac{ft}{s}$ and $T_1 = 397.69\,°R$. Thus, $U_1 = M_1 a_1 = 2932.86 \frac{ft}{s}$.

Following the relations developed for the last problem:

$$T_t = T_1 + \frac{U_1^2}{2c_p} = 397.69 + \frac{(2932.86)^2}{2(0.2404)(778.2)(32.174)}$$

Thus, $T_t = 397.69 + 714.53 = 1112.22\,°R$

Convective heating could be a significant problem for aircraft flying at these speeds. The total temperature is in excess of 650 °F, which could affect the strength of many materials subjected to this environment.

The total temperature could have been calculated using the relation:

$$T_t = T_\infty\left(1 + \frac{\gamma-1}{2} M_\infty^2\right) \text{ or } T_t = T_1\left(1 + \frac{\gamma-1}{2} M_1^2\right)$$

which will be developed in the next problem.

2.40) We start with the integral form of the energy equation for a one-dimensional, steady, adiabatic flow:

$$H_t = h + \frac{U^2}{2}$$

For a perfect gas: $H_t = c_p T_t$ and $h = c_p T$. Thus, we can rewrite this equation as:

$$c_p T_t = c_p T + \frac{1}{2} U^2$$

or

$$\frac{T_t}{T} = 1 + \frac{1}{2} \frac{U^2}{c_p T}$$

But we also know that: $c_p = \frac{\gamma R}{\gamma - 1}$ for a perfect gas, 2.40 Contd. Thus, $T_t = \left(1 + \frac{\gamma-1}{2} M^2\right) T$

which defines the relation between the total temperature (T_t), the static temperature (T), and Mach number (M) for the adiabatic flow of a perfect gas.

The entropy change equation is:

$$s - s_r = c_p \ln \frac{T}{T_r} - R \ln \frac{p}{p_r}$$

For the isentropic flow of a perfect gas:

$$\frac{\gamma R}{\gamma - 1} \ln \frac{T}{T_r} - R \ln \frac{p}{p_r} = 0$$

Thus,

$$\ln \left(\frac{T}{T_r}\right)^{\gamma/(\gamma-1)} = \ln \left(\frac{p}{p_r}\right)$$

Designating the stagnation condition of a gas which is brought to rest through an isentropic process (i.e., one which is both adiabatic and reversible) represented by the symbol "t" as the reference condition "r".

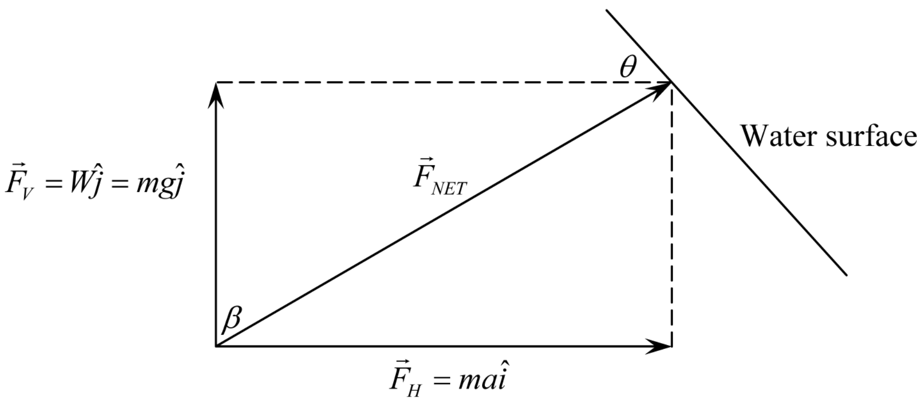
$$\frac{p}{p_{t1}} = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}} = \left[\frac{1}{1 + \frac{\gamma-1}{2} M^2}\right]^{\frac{\gamma}{\gamma-1}}$$

This equation can be used to calculate the static pressure of a perfect gas which has undergone an isentropic expansion from a stagnant gas (whose pressure is p_{t1} and whose temperature is T_t) to a Mach number M_1. Conditions are those for the isentropic flow of a perfect gas.

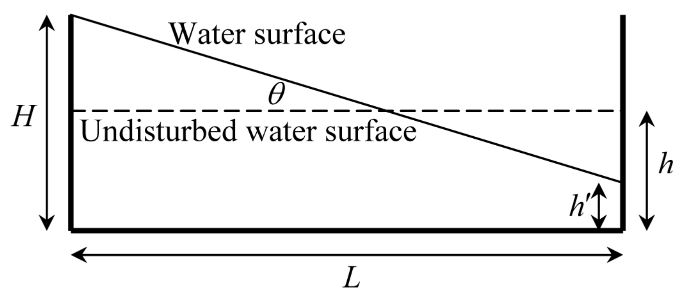
3.1)

Given: A truck carrying an open container of water.

The surface of the water will be normal to the net force exerted by the truck on the body of water. The truck bed exerts an upward force on the water to counteract the weight of the water. The back of the truck's container exerts a forward force on the water due to the truck's acceleration.



The water surface orientation in the container is also shown below:



We need to find h so that the water depth at the rear of the bed is H (the container height) under maximum acceleration. We are given the dimensions of the truck container as: $H = 3\text{m}$, $L = 6\text{m}$, W (width) = 2m (which is irrelevant for this problem, so we can assume unit dimensions for the width).

The mass of water in the tank is given by:

$$m = \rho_{H_2O}\cancel{V} = \rho_{H_2O}hL$$

and the forces acting on the water are:

$$\vec{F}_V = \rho_{H_2O}hLg\hat{j}$$
$$\vec{F}_H = \rho_{H_2O}hLa\hat{i}$$

3.1) contd.

yielding a net force of:

$$\vec{F}_{NET} = \left(g\hat{j} + a\hat{i}\right)\rho_{H_2O} hLg$$

From the geometry in the first figure, $\theta = \beta$, and $\tan\theta = F_H / F_V = a/g$. Since the volume of water does not change:

$$\begin{aligned} V\!\!\!/ &= hL = (H - h')L/2 + h'L \\ h' &= 2h - H \end{aligned}$$

Therefore:

$$\begin{aligned} \frac{a}{g} &= \tan\theta = \frac{H - h'}{L} = 2(H - h)/L \\ h &= H - \frac{La}{2g} \end{aligned}$$

Substituting gives:

$$h = 3 - \left(\frac{2}{9.81}\right)\left(\frac{6}{2}\right) = 2.388m$$

3.2)

Given: A truck carrying an open container of water.

Following the approach of Prob. 3.1:

$$h = H - \frac{La}{2g}$$

$$h = 10 - \left(\frac{6.3}{32.2}\right)\left(\frac{20}{2}\right) = 8.043\,ft$$

3.3) The conditions that must be satisfied before Bernoulli's equation can be applied are:
(1) steady flow, (2) incompressible flow, (3) inviscid flow, (4) conservative body forces, and (5) either irrotational flow or application along a streamline.

3.4 (a) Since the water at the surface of the tank (point 1) is immediately adjacent to the air, the static pressure for the water at this point equals that of the atmosphere. Rule: the pressure is constant across a fluid/fluid interface. Also, since the water exits from the tank to the atmosphere (at subsonic speeds), the static pressure of the water at point 3 is the atmospheric value.

We are told that the velocity at points more than 10.0 feet from the opening can be neglected. Thus, $U_1 \approx U_2 \simeq 0$.

To calculate the static pressure at point 2, we can use Bernoulli's equation:

$$p_1 + \rho g z_1 + \frac{1}{2}\rho U_1^2 = p_2 + \rho g z_2 + \frac{1}{2}\rho U_2^2$$

If we assume that the surface of the water is the reference height, then $z_1 = 0$ and $z_2 = -15$ feet. Furthermore, $U_1 \approx U_2 \approx 0$ and $p_1 = p_{atm}$. Thus,

$$p_2 = -\left(1.940\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(32.174\ \frac{\text{ft}}{\text{s}^2}\right)(-15\text{ ft}) + p_{atm}$$

$$p_2 = 936.3\ \frac{\text{lbf}}{\text{ft}^2}\text{, gage}$$

To calculate the velocity U_3, we apply Bernoulli's equation to the inviscid flow between points 1 and 3:

$$p_1 + \rho g z_1 + \frac{1}{2}\rho U_1^2 = p_3 + \rho g z_3 + \frac{1}{2}\rho U_3^2$$

Note that: $p_1 = p_3 = p_{atm}$; $z_1 = 0$; $z_3 = -15.0$ ft; and $U_1 \approx 0$. Thus,

$$U_3 = \sqrt{\frac{2}{\rho}(-\rho g z_3)} = \sqrt{-2\left(32.174\ \frac{\text{ft}}{\text{s}^2}\right)(-15.0\text{ feet})}$$

$$U_3 = 31.07\ \frac{\text{ft}}{\text{s}}$$

3.4 Contd.

(b) Having found the value of U_3, we can use the integral form of the continuity equation, i.e., equation (2.5), $\rho_1 U_1 A_1 = \rho_3 U_3 A_3$. Furthermore, since $\rho_1 = \rho_3$,

$$U_1 = U_3 \frac{A_3}{A_1} = \left(31.07 \frac{ft}{s}\right)\left[\frac{\pi (0.5\,in)^2}{\pi (120\,in)^2}\right] = 5.4 \times 10^{-4} \frac{ft}{s}$$

Thus, the assumption that the velocity at point 1, i.e., $U_1 \approx 0$, is valid

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3.5 Since we know the velocity and the altitude at which the vehicle is flying, we know p_1, ρ_1, and U_1.

Outside of the boundary layer, the flow is inviscid and irrotational. Thus, we can apply Bernoulli's equation to relate flows at points 2, 3, and 6 to the free-stream flow, i.e., 1. However, since points 4 and 5 are located within the boundary layer, we cannot use Bernoulli's equation to determine the local velocity, even though the local pressure is known. This is true whether the boundary layer is laminar or turbulent.

3.6 The airfoil is moving at 300 $\frac{km}{h}$ at an altitude of 3 km.

(a) $$U_1 = 300\,\frac{km}{h} \times \frac{1000\,\frac{m}{km}}{3600\,\frac{s}{h}} = 83.33\,\frac{m}{s}$$

Using Table 1.2 to obtain the free-stream properties at 3 km: $\rho_1 = 0.9092\,\frac{kg}{m^3}$; $a_1 = 328.583\,\frac{m}{s}$; $\mu_1 = 1.6938 \times 10^{-5}\,\frac{kg}{s \cdot m}$; and $p_1 = 7.012 \times 10^4\,\frac{N}{m^2}$

Thus,
$$M_1 = \frac{U_1}{a_1} = \frac{83.33\,\frac{m}{s}}{328.583\,\frac{m}{s}} = 0.254$$

and $$Re_c = \frac{\rho_1 U_1 c}{\mu_1} = \frac{(0.9092)(83.33)(1.5)}{1.6938 \times 10^{-5}} = 6.71 \times 10^6$$

(b) $$q_1 = \frac{1}{2}\rho_1 U_1^2 = \frac{1}{2}(0.9092)(83.33)^2 = 3.157 \times 10^3\,\frac{N}{m^2}$$

and $p_1 = 7.012 \times 10^4\,\frac{N}{m^2}$

Rearranging the definition for C_p, $p_{local} = p_1 + C_p q_1$

At point 2: $p_{local} = 7.328 \times 10^4\,\frac{N}{m^2} = 10.63$ psi

$$\frac{p_{local} - p_\infty}{p_\infty} = +0.045$$

At points 3 and 4: $p_{local} = 6.065 \times 10^4\,\frac{N}{m^2} = 8.797$ psi

$$\frac{p_{local} - p_\infty}{p_\infty} = -0.135$$

At points 5 and 6: $p_{local} = 7.063 \times 10^4\,\frac{N}{m^2} = 10.24$ psi

$$\frac{p_{local} - p_\infty}{p_\infty} = +0.007$$

(the problem continues)

3.6 Contd.] Although the changes in pressure are as much as 13%, there is a corresponding change in temperature. Thus, the changes in density are not even as much as the changes in pressure. Thus, the assumption of constant density is reasonable.

(c) As discussed on pp. 49-52, the pressure variation across the boundary layer is usually negligible. Thus, the static pressures at points 3 and 4 are equal. Similarly, the static pressures at points 5 and 6 are equal. However, because the velocity varies across the boundary layer, the total pressure is not constant across the boundary layer. Recall that one of the assumptions related to Bernoulli's equation is "inviscid flow".

(d) We cannot use Bernoulli's equation to calculate the local velocity at points within the boundary layer, i.e., points 4 and 5. Rearranging eq. (3.13),

$$U = U_\infty \left[1 - C_p\right]^{0.5}$$

Thus,

	point 2	point 3	point 6
$U\left(\frac{m}{s}\right)$	0.00	166.66	76.38

3.7] At an altitude of 6000 m, $p_1 = 4.722 \times 10^4\ N/m^2$ and $\rho_1 = 0.6601\ kg/m^3$ (see Table 1.2). Using Bernoulli's equation:

$$U_1 = \sqrt{\frac{2}{\rho}(p_t - p_1)} = 63.30\ \frac{m}{s} = 227.9\ \frac{km}{h}$$

The pressure recorded by a gage which measures the difference between the stagnation pressure and the "free-stream" static pressure, i.e., $(p_t - p_1)$ which is what a

3.7 Contd. pitot-static probe senses, is 1322.6 N/m^2 = 0.192 psi. To produce this same reading on this gage, if the airplane were flying at sea level where the density (ρ) is 1.2250 $\frac{kg}{m^3}$:

$$U_1 = \sqrt{\frac{2}{1.2250}\left(1.3225\times10^3\right)} = 46.47\ \frac{m}{s} = 167.3\ \frac{km}{h}$$

This is the equivalent airspeed, which is defined on pp. 79-80.

3.8 At an altitude of 1 mile (which is 5280 ft)

$$\frac{p}{p_{SL}} = 0.8240\ ,\quad p = 1743.79\ \frac{lbf}{ft^2}$$

$$\frac{\rho}{\rho_{SL}} = 0.8548\ ,\quad \rho = 0.002031\ \frac{slugs}{ft^3}$$

$$U_\infty = \left(40\ \frac{mi}{h}\right)\left(5280\ \frac{ft}{mi}\right)\left(\frac{h}{3600\ s}\right) = 58.667\ \frac{ft}{s}$$

$$q_\infty = \frac{1}{2}\rho_\infty U_\infty^2 = 3.4952\ \frac{lbf}{ft^2}$$

$$p_{ext} = p_\infty + q_\infty C_p = p_\infty + (3.4952)(-5) = p_\infty - 17.476$$

The pressure differential across the window is:

$$\Delta p = p_{ext} - p_{int} = (p_\infty - 17.476) - p_\infty = -17.476$$

(the minus sign indicates that the pressure outside of the window is less than the pressure inside). The total force is:

$$F = (\Delta p)(A) = 419.424\ lbf$$

3.9)

Given: A high rise office building at sea level with the wing blowing at 75 km/h.

For standard day sea-level conditions from Table 1.2A:

$$p = 1.01325 \times 10^5 N/m^2$$

$$\rho = 1.2250 kg/m^3$$

$$U_\infty = 75 km/h = 20.8 m/s$$

$$q_\infty = \rho_\infty U_\infty^2 / 2 = (1.2250)(20.8)^2 / 2 = 264.99 N/m^2$$

The maximum pressure acting on the building would occur at a stagnation point where the pressure would equal the total pressure. From Bernoulli's equation (Eqn. 3.10):

$$p_t = p_\infty + q_\infty = 1.01325 \times 10^5 + 264.99 = 1.0159 \times 10^5 N/m^2$$

For the definition of the pressure coefficient:

$$C_p = \frac{p - p_\infty}{q_\infty}$$

Solving for the local static pressure yields:

$$p = p_\infty + C_p q_\infty = 1.01325 \times 10^5 + (-4)(264.99) = 1.00265 \times 10^5 N/m^2$$

The pressure differential across the window is:

$$F = (\Delta p)(A) = (1.01325 \times 10^5 - 1.00265 \times 10^5)(3m^2) = 3180 N$$

which would be a force toward the outside since the pressure outside is less than the pressure inside.

3.10 The aircraft is flying at an altitude of 8km. Thus, using Table 1.2, $\rho_\infty = 0.5258 \ kg/m^3$

(a) Since gage 1 is known to be at the stagnation point:

$$p_1 = p_t = 1550 + p_\infty \text{ (which is gage pressure)}$$

From Bernoulli's equation:

$$p_t = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2$$

Comparing the two expressions: $\frac{1}{2}\rho_\infty U_\infty^2 = 1550 \frac{N}{m^2}$. Thus,

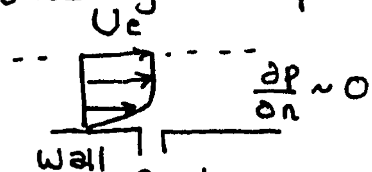
$$U_\infty = \sqrt{\frac{2(1550 \ N/m^2)}{0.5258 \ kg/m^3}} = 76.78 \ \frac{m}{s}$$

(b) As noted, when comparing the expressions in part (a),

$$1500 \frac{N}{m^2} = \frac{1}{2}\rho_\infty U_\infty^2 = q_\infty \text{ (the dynamic pressure)}$$

(c) At gage 2,

$$p_2 = -3875 + p_\infty$$



p_2 is a measure of p_e

Applying Bernoulli's equation:

$$p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 = p_t = p_1 = p_2 + \frac{1}{2}\rho_\infty U_e^2$$

$$p_\infty + 1550 = -3875 + p_\infty + 0.5(0.5258)U_e^2$$

Solving for U_e: $U_e = 143{,}65 \ \frac{m}{s}$ (relative to the vehicle)

Relative to the ground: $U = 143.65 - 76.78 = 66.87 \ \frac{m}{s}$

To calculate C_p: $C_p = \dfrac{p_2 - p_\infty}{q_\infty}$

$$C_p = \frac{(-3875 + p_\infty) - p_\infty}{1550} = -2.5$$

3.11) $p_{t1} = p_{t2}$

$$p_1 + \frac{1}{2}\rho U_1^2 = p_2 + \frac{1}{2}\rho U_2^2$$ (since the density is constant)

Comparing the two expressions:

$$p_{t1} = p_2 + \frac{1}{2}\rho U_2^2 \quad (\text{where } \rho = \rho_{air})$$

But from the principles of manometry:

$$p_{t1} = p_2 + \rho_{oil}\, g\, \Delta h$$

Equating the two expressions for p_{t1}:

$$\frac{1}{2}\rho_{air} U_2^2 = \rho_{oil}\, g\, \Delta h$$

$$U_2^2 = \frac{2\left(1.9404 \frac{slugs}{ft^3}\right)\left(32.174 \frac{ft}{s^2}\right)\left(\frac{5}{12} ft\right)}{\left(0.00238 \frac{slugs}{ft^3}\right)}$$

$$U_2 = 147.85 \frac{ft}{s}\ ;\ Q = U_2 A_2 = 7.258 \frac{ft^3}{s}$$

3.12) The problem statement does not identify the altitude where the in-draft is located. Let us assume that the wind tunnel is at the U.S. Air Force Academy, which is at an altitude of (approximately) 2000m. Thus,

$$p_{atm} = 79{,}501 \frac{N}{m^2}\ ;\ \rho_{atm} = 1.0066 \frac{kg}{m^3}$$

For an in-draft wind tunnel, $p_t = p_{atm}$ (the ambient atmosphere serves as the reservoir for the wind tunnel). Applying Bernoulli's equation: $p_t = p_{atm} = p_{ts} + \frac{1}{2}\rho U_{ts}^2$

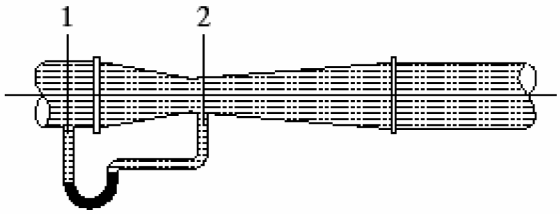
$$p_{ts} = p_{atm} - \frac{1}{2}(1.0066)(50)^2 = 79{,}501 - 1258$$

Thus, the static pressure in the test section (p_{ts}) is $78{,}243 \frac{N}{m^2}$

The stagnation-point pressure is $p_t = p_{atm} = 79{,}501 \frac{N}{m^2}$

3.13)

Given: A venturi meter with two static pressure ports as shown below.



Conservation of mass requires:

$$\dot{m}_1 = \dot{m}_2$$

Where $\dot{m}$ is the mass flow rate $\left(\dot{m} = \rho U A\right)$. Since the flow is incompressible ($\rho_1 = \rho_2$):

$$Q_1 = Q_2$$

where Q is the volume flow rate, which is given by:

$$Q = A_1 U_1 = A_2 U_2$$

Solving for the velocity at Station 1 yields:

$$U_1 = \frac{A_2}{A_1} U_2$$

Bernoulli's equation (Eqn. 3.10) applied between Station 1 and Station 2 yields:

$$p_1 + \rho U_1^2 / 2 = p_2 + \rho U_2^2 / 2$$

$$p_1 - p_2 = \rho \left\{ U_2^2 - U_1^2 \right\} / 2$$

Replacing U_1 with the relationship from the volume flow rate gives:

$$p_1 - p_2 = \rho \left\{ U_2^2 - \left(\frac{A_2}{A_1} \right)^2 U_2^2 \right\} / 2$$

$$p_1 - p_2 = \frac{\rho U_2^2}{2} \left\{ 1 - \left(\frac{A_2}{A_1} \right)^2 \right\}$$

3.13) contd.

Solving for U_2 yields:

$$U_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left\{1 - (A_2/A_1)^2\right\}}}$$

Now calculate the volume flow rate at Station 2 using U_2:

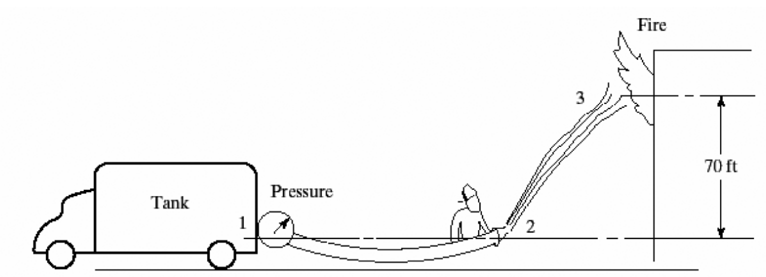
$$Q = A_2 U_2 = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left\{1 - (A_2/A_1)^2\right\}}} = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2(p_1 - p_2)}{\rho}}$$

Using the definition of specific weight, $\gamma = \rho g$, and the coefficient of discharge, C_d (an empirical correction to the theoretical relationship above) yields:

$$Q = C_d \left[\frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{\frac{2g(p_1 - p_2)}{\gamma}} \right]$$

3.14)

Given: A fire truck pumping water to a height of 70 ft.



Pumping from Station 2 to Station 3 requires a pressure high enough to overcome the potential energy change required to shoot the water 70 ft. high. Start with Bernoulli's equation as given in Eqn. 3.9:

$$\frac{U^2}{2} + gz + \frac{p}{\rho} = C$$

Multiplying through by the density yields:

$$\rho U^2/2 + \rho g z + p = C$$

3.14) contd.

Which can be re-written as:

$$\rho U_2^2/2 + \rho g z_2 + p_2 = \rho U_3^2/2 + \rho g z_3 + p_3$$

Between these two stations the following information is assumed:

- $U_3 = 0$ in order to find the minimum pressure required
- $z_2 = 0$ since the ground will be assumed to be the reference plane
- $p_2 = p_3 = p_{atm}$
- $\rho = C$ since the water is incompressible

The above equation then simplifies to:

$$\rho U_2^2/2 = \rho g z_3$$

Solving for the velocity at Station 2:

$$U_2 = \sqrt{2gz_3} = \sqrt{2(32.17)(70)} = 67.1 ft/s$$

Now work back to Station 1 from Station 2 to find the pressure required in the tank to create this flow.

$$\rho U_1^2/2 + \rho g z_1 + p_1 = \rho U_2^2/2 + \rho g z_2 + p_2$$

Between these two stations the following information is assumed:

- $U_1 = 0$ in the tank
- $z_1 = z_2$ since the ground will be assumed to be the reference plane
- $p_2 = p_{atm}$
- $\rho = C$ since the water is incompressible

The above equation then simplifies to:

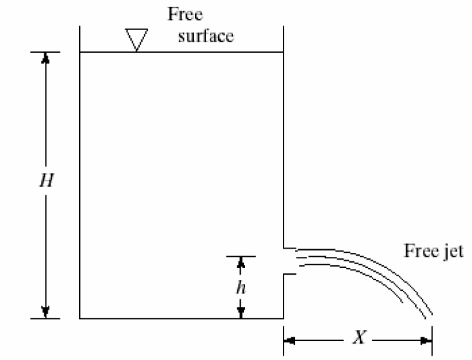
$$p_1 = p_2 + \rho\left(U_2^2 - U_1^2\right)/2 = p_2 + \rho U_2^2/2$$
$$= 2116.22 + (0.002377)(67.1)^2/2 = 2121.6 lb/ft^2$$

The hose has a diameter of $d = 3in = 0.25 ft$ and a cross-sectional area of $\pi d^2/4 = 0.0491 ft^2$. The volume flow rate through the hose is:

$$Q = AU = (0.0491)(67.1) = 3.294 ft^3/s = 1478 gal/m$$

3.15)

Given: A free jet of water exiting a tank as shown below.



Bernoulli's equation (Eqn. 3.10) applied between the upper free surface (Station 1) and the outflow of the free jet (Station 2) yields:

$$p_1 + \rho U_1^2/2 + \rho g z_1 = p_2 + \rho U_2^2/2 + \rho g z_2$$

Between these two stations the following information is assumed:

- $U_1 = 0$ in the pressure tank
- $z_1 = H$ and $z_2 = h$
- $p_1 = p_2 = p_{atm}$ since the jet is free and the upper surface is free
- $\rho = C$ since the water is incompressible

The equation simplifies to:

$$U_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2g(H-h)}$$

Once the water exits the tank it starts to accelerate downward due to gravity. It takes the water a certain time to reach the ground due to the acceleration of gravity:

$$x = at^2/2 \qquad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2h}{g}}$$

During this time the water is traveling laterally at the velocity found above, so the distance traveled is:

$$X = U_2 t = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}} = \sqrt{4h(H-h)} = 2\sqrt{h(H-h)}$$

Different fluids do not affect the results since the impact of density is cancelled out.

3.16)

Given: Various flow models and their basic assumptions.

a) A stream function requires the following assumptions (see pp. 88-89): steady, incompressible, and inviscid flow (rotational flow is allowed). As described in the text, a stream function does not have to be 2D, and it is even possible to define a stream function for steady, compressible flow.
b) A velocity potential only exists for irrotational flow, and the theory derived in the text (p. 88) also assumes incompressible and inviscid flow. Unsteady and/or 3D velocity potentials are also possible, so the assumptions of steady and 2D flow are not necessary.
c) Bernoulli's equation can be used to relate two points in a flow field if the flow is steady, inviscid, incompressible, irrotational flow with conservative body forces. These assumptions yield that that total pressure anywhere in the field is constant. See p. 76.
d) The circulation around a closed fluid line is constant with time if the flow is inviscid, baratropic, with conservative body forces. See p. 86.

3.17 $\Gamma = -\oint \vec{V} \cdot \vec{ds} = -\int_0^{2\pi} \left(\frac{\Gamma}{2\pi R_1}\hat{e}_\theta\right) \cdot \left(R_1\, d\theta\, \hat{e}_\theta\right)$

Thus, $\Gamma = -\frac{\Gamma}{2\pi}\int_0^{2\pi} d\theta = -\Gamma$

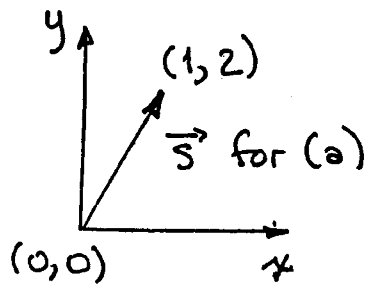
3.18 $$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - \frac{h^2}{4}\right) & 0 & 0 \end{vmatrix}$$

where μ, $\frac{dp}{dx}$, and $\frac{h^2}{4}$ are constants. Thus,

$$\nabla \times \vec{V} = -\hat{k}\left[\frac{1}{\mu}\frac{dp}{dx}\,y\right]$$

The flow is rotational because of the effects of viscosity.

3.19 $\vec{V} = (x^2+y^2)\hat{i} + 2xy^2\hat{j}$



(a) For a straight line connecting (0,0) and (1,2)

$$y = 2x \quad \text{and} \quad dy = 2dx$$

For the differential displacement along the $\vec{s}$ vector

$$\vec{ds} = dx\,\hat{i} + dy\,\hat{j}$$

Therefore, $\int \vec{V}\cdot\vec{ds} = \int [u\hat{i} + v\hat{j}]\cdot[dx\,\hat{i} + dy\,\hat{j}]$

so that $\int \vec{V}\cdot\vec{ds} = \int u\,dx + \int v\,dy$

For the current velocity field:

$$\int \vec{V}\cdot\vec{ds} = \int (x^2+y^2)\,dx + \int 2xy^2\,dy$$

In order to be able to evaluate these integrals, we must write each integral in terms of a single variable. Thus, since $y = 2x$ or $x = \frac{y}{2}$

$$\int_a \vec{V}\cdot\vec{ds} = \int_0^1 (x^2+4x^2)\,dx + \int_0^2 2\left(\frac{y}{2}\right)y^2\,dy$$

$$= \frac{5x^3}{3}\Big|_0^1 + \frac{y^4}{4}\Big|_0^2 = \frac{5}{3} + \frac{16}{4} = \frac{17}{3}$$

We could have made other substitutions, such as:

$$\int_a \vec{V}\cdot\vec{ds} = \int_0^2 \left(\frac{y^2}{4} + y^2\right)\frac{dy}{2} + \int_0^2 y^3\,dy$$

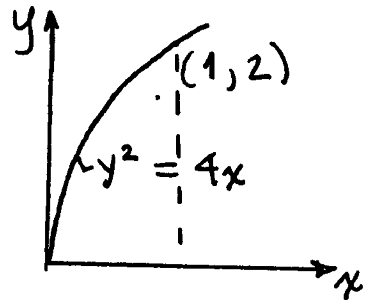
$$= \frac{1}{2}\left(\frac{y^3}{12} + \frac{y^3}{3}\right)\Big|_0^2 + \frac{y^4}{4}\Big|_0^2 = \frac{1}{2}\left(\frac{8}{12} + \frac{8}{3}\right) + \frac{16}{4} = \frac{17}{3}$$

which (as we should expect) is the same result.

3.19 Contd.

(b) The equation for a parabola whose vertex is at the origin and which opens to the right is:

$$y^2 = 4x$$



As was shown in part (a)

$$\int \vec{V} \cdot \vec{ds} = \int u\,dx + \int v\,dy = \int (x^2+y^2)\,dx + \int 2xy^2\,dy$$

Again, we must write each integral in terms of a single variable. For the first integral, we use the fact that:

$$y^2 = 4x$$

while for the second integral: $x = \frac{y^2}{4}$

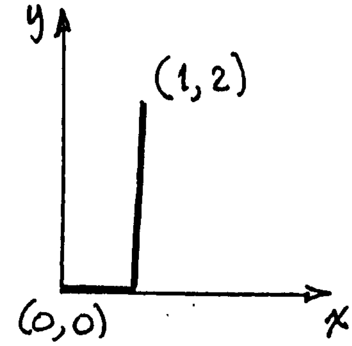
Thus,

$$\int_b \vec{V} \cdot \vec{ds} = \int_0^1 (x^2 + 4x)\,dx + \int_0^2 2\left(\frac{y^2}{4}\right) y^2\,dy$$

$$= \int_0^1 (x^2 + 4x)\,dx + \int_0^2 \frac{y^4}{2}\,dy$$

$$= \left(\frac{x^3}{3} + 2x^2\right)\Big|_0^1 + \frac{y^5}{10}\Big|_0^2 = \frac{1}{3} + 2 + \frac{32}{10} = \frac{83}{15}$$

(c) Finally, let us calculate $\int \vec{V} \cdot \vec{ds}$ along the two segment path connecting the origin (0,0) and the point (1,2). Along the first segment



$$x = x \quad \text{and} \quad y = 0$$

Thus, $\vec{V} = x^2\,\hat{i}$ and $\vec{ds} = dx\,\hat{i}$

So that $\int_{C1} \vec{V} \cdot \vec{ds} = \int_0^1 x^2\,dx = \frac{1}{3}$

3.19 Contd.

For the second segment

$$x = 1;\ y = y;\ dx = 0;\ \text{and } dy = dy$$

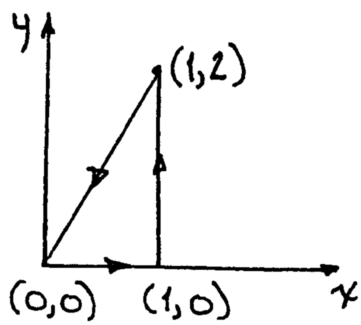
Thus, $\vec{V} = (1+y^2)\hat{i} + 2y^2\hat{j}$ and $\vec{ds} = dy\,\hat{j}$

So that $\int_{C_2} \vec{V}\cdot\vec{ds} = \int_0^2 2y^2\,dy = \frac{2}{3}y^3\Big|_0^2 = \frac{16}{3}$

The total integral is the sum of those for the two segments:

$$\int_C \vec{V}\cdot\vec{ds} = \int_{C1}\vec{V}\cdot\vec{ds} + \int_{C2}\vec{V}\cdot ds = \frac{1}{3} + \frac{16}{3} = \frac{17}{3}$$

3.20 In Problem 3.19, we examined the integral of the velocity component along the various segments connecting the origin to point (1,2). In this problem, we work with a closed path and the area it encloses. The velocity field is again:



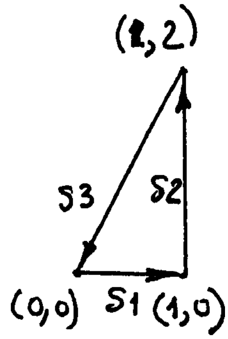
$$\vec{V} = (x^2+y^2)\hat{i} + (2xy^2)\hat{j}$$

Is the flow rotational or irrotational?

$$\nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2+y^2) & (2xy^2) & 0 \end{vmatrix} = \hat{k}\left[\frac{\partial}{\partial x}(2xy^2) - \frac{\partial}{\partial y}(x^2+y^2)\right]$$

$$= \hat{k}\left[2y^2 - 2y\right] \neq 0$$

3.20 Contd.



Thus, the flow is rotational.

Let us calculate the circulation around the closed triangle consisting of the three segments shown at right.

$$\oint \vec{V} \cdot \vec{ds} = \int_{S1} \vec{V} \cdot \vec{ds} + \int_{S2} \vec{V} \cdot \vec{ds} + \int_{S3} \vec{V} \cdot \vec{ds}$$

For S1: $y = 0$; $x = x$; $\vec{ds} = dx\,\hat{i}$; $\vec{V} = x^2\hat{i}$

Thus, $\int_{S1} \vec{V} \cdot \vec{ds} = \int_0^1 x^2 dx = \left.\frac{x^3}{3}\right|_0^1 = \frac{1}{3}$

For S2: $y = y$; $x = 1$; $\vec{ds} = dy\,\hat{j}$; $\vec{V} = (1+y^2)\hat{i} + 2y^2\hat{j}$

Thus, $\int_{S2} \vec{V} \cdot \vec{ds} = \int_0^2 2y^2 dy = \left.\frac{2}{3}y^3\right|_0^2 = \frac{16}{3}$

For S3: $y = 2x$; $dy = 2dx$; $\vec{ds} = dx\,\hat{i} + dy\,\hat{j}$

Thus, $\int_{S3} \vec{V} \cdot \vec{ds} = \int_1^0 u\,dx + \int_2^0 v\,dy = \int_1^0 5x^2 dx + \int_2^0 y^3 dy$

$$= \left.\frac{5}{3}x^3\right|_1^0 + \left.\frac{y^4}{4}\right|_2^0 = -\frac{5}{3} - \frac{16}{4} = -\frac{17}{3}$$

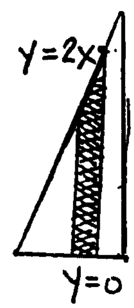
Note that $\int_{S3} \vec{V} \cdot \vec{ds}$ for this problem is equal in magnitude but opposite in sign to the value of the integral calculated in Problem 3.19a. But should this not be the case?

3.20 Contd.

Recall that $\int \vec{V}\cdot\vec{ds}$ is the component of the velocity vector $\vec{V}$ along the curve $\vec{ds}$ (which happens to be a straight line) integrated along the curve. In Problem (3.19a), the integration moved from the origin (0,0) to the point (1,2) along the straight line connecting those points. In this problem, the integration moved from the point (1,2) to the origin (0,0) along the same straight line for S3. Thus, we should expect to find that the integral is of equal magnitude but opposite in sign.

$$\oint \vec{V}\cdot\vec{ds} = \int_{S1}\vec{V}\cdot\vec{ds} + \int_{S2}\vec{V}\cdot\vec{ds} + \int_{S3}\vec{V}\cdot\vec{ds} = \frac{1}{3} + \frac{16}{3} - \frac{17}{3} = 0$$

To evaluate $\iint(\nabla\times\vec{V})\cdot\hat{n}\,dA$, we must carefully describe the limits on the differential area (the shaded region)



$$\iint(\nabla\times\vec{V})\cdot\hat{n}\,dA = \iint \hat{k}(2y^2-2y)\cdot\hat{k}\,dx\,dy$$

$$= \int_0^1 dx \int_0^{2x}(2y^2-2y)\,dy$$

$$= \int_0^1 dx\left[\left(2\frac{y^3}{3} - y^2\right)\Big|_0^{2x}\right] = \int_0^1\left(\frac{16x^3}{3} - 4x^2\right)dx$$

$$= \left(\frac{4x^4}{3} - \frac{4x^3}{3}\right)\Big|_0^1 = 0$$

Note that, although the flow is rotational, we have the situation where $\oint \vec{V}\cdot\vec{ds}$. Nevertheless, the results are consistent with Stokes's theorem, in that

$$\oint \vec{V}\cdot\vec{ds} = \iint(\nabla\times\vec{V})\cdot\hat{n}\,dA$$

3.21 In order that the flow can be represented by a potential function, it should be irrotational. Is it?

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & -2y & 0 \end{vmatrix} = 0$$

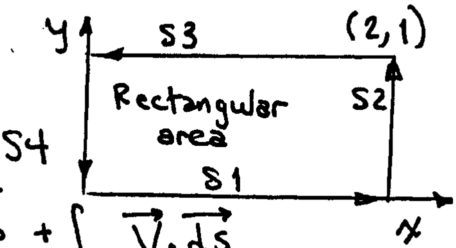
Thus, the flow is indeed irrotational!

$$u = \frac{\partial \phi}{\partial x} = 2x \,; \quad v = \frac{\partial \phi}{\partial y} = -2y \,; \text{ and } |\vec{V}| = \sqrt{u^2 + v^2}$$

so that $|\vec{V}| = \sqrt{4x^2 + 4y^2}$, as given in the problem.

Since $\nabla \times \vec{V} = 0$, $\iint (\nabla \times \vec{V}) \cdot \hat{n}\, dA = 0$

To calculate the circulation around the closed, rectangular path:



$$\oint \vec{V} \cdot \vec{ds} = \int_{S1} \vec{V} \cdot \vec{ds} + \int_{S2} \vec{V} \cdot \vec{ds} + \int_{S3} \vec{V} \cdot \vec{ds} + \int_{S4} \vec{V} \cdot \vec{ds}$$

For S1: $x = x \,; y = 0 \,; \vec{ds} = dx\,\hat{i}$

$$\int_{S1} \vec{V} \cdot \vec{ds} = \int_0^2 u\, dx = \int_0^2 2x\, dx = x^2 \Big|_0^2 = 4$$

For S2: $x = 2 \,; y = y \,; \vec{ds} = dy\,\hat{j}$

$$\int_{S2} \vec{V} \cdot \vec{ds} = \int_0^1 v\, dy = \int_0^1 (-2y)\, dy = -y^2 \Big|_0^1 = -1$$

For S3: $y = 1 \,; x = x \,; \vec{ds} = dx\,\hat{i}$

$$\int_{S3} \vec{V} \cdot \vec{ds} = \int_2^0 u\, dx = \int_2^0 2x\, dx = x^2 \Big|_2^0 = -4$$

For S4: $x = 0,\ y = y \,; \vec{ds} = dy\,\hat{j}$

$$\int_{S4} \vec{V} \cdot \vec{ds} = \int_1^0 v\, dy = \int_1^0 (-2y)\, dy = -y^2 \Big|_1^0 = +1$$

Thus, $\oint \vec{V} \cdot \vec{ds} = +4 - 1 - 4 + 1 = 0$

demonstrating the validity of Stokes's theorem for this irrotational flow.

3.22) The flow is incompressible, irrotational, and two dimensional. Thus, the flow can be described both by a potential function and a stream function. We are given:

$$\phi = K \ln\sqrt{x^2+y^2}$$

(a) Note that, if one converts from Cartesian coordinates to cylindrical coordinates where $r=\sqrt{x^2+y^2}$, we see that

$$\phi = K \ln r$$

Referring to Table 3.2, we see that this is the potential function for flow from a source. Comparing the current potential function with that given in Table 3.2, it is clear that the 2π is included in the constant K for this problem. We could, therefore, work this problem using cylindrical coordinates. However, let us work with Cartesian coordinates as would be expected based on what is given.

$$u = \frac{\partial \phi}{\partial x} = K \frac{1}{\sqrt{x^2+y^2}} \frac{\left(\frac{1}{2}\right)(2x)}{\sqrt{x^2+y^2}} = \frac{Kx}{x^2+y^2}$$

Note that along the x-axis ($y=0$), $u = \frac{K}{x}$, i.e., the velocity varies inversely with distance from the origin. Furthermore, $u>0$ for $x>0$ and $u<0$ for $x<0$, since the source flow is always directed away from the origin.

$$v = \frac{\partial \phi}{\partial y} = K \frac{1}{\sqrt{x^2+y^2}} \frac{\left(\frac{1}{2}\right)(2y)}{\sqrt{x^2+y^2}} = \frac{Ky}{x^2+y^2}$$

Thus, $\vec{V} = u\hat{i} + v\hat{j} = \frac{Kx}{x^2+y^2}\hat{i} + \frac{Ky}{x^2+y^2}\hat{j}$

Let us verify that the flow is irrotional.

Is $\nabla \times \vec{V} = 0$?

3.22 Contd.

$$\nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{Kx}{x^2+y^2} & \frac{Ky}{x^2+y^2} & 0 \end{vmatrix} = \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{Ky}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{Kx}{x^2+y^2}\right)\right]$$

$$= \hat{k}\left[\frac{Ky(-2x)}{(x^2+y^2)^2} - \frac{Kx(-2y)}{(x^2+y^2)^2}\right] = 0$$

Q.E.D. that the flow is irrotational. Let us now calculate the magnitude and the direction of the flow at specific points. Since

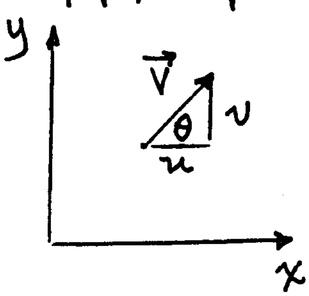
$$\vec{V} = \frac{Kx}{x^2+y^2}\hat{i} + \frac{Ky}{x^2+y^2}\hat{j}$$

Thus, at $x=2;\ y=0$: $\vec{V} = \frac{K}{2}\hat{i}$. Obviously, the magnitude of the velocity is $\frac{1}{2}K$ and the flow is in the direction of the positive x-axis.

At $x=\sqrt{2}\,;\ y=\sqrt{2}$: $\vec{V} = \frac{K\sqrt{2}}{4}\hat{i} + \frac{K\sqrt{2}}{4}\hat{j}$

The magnitude of the velocity is

$$|\vec{V}| = \sqrt{u^2+v^2} = \frac{K}{4}\sqrt{2+2} = \frac{K}{2}$$



The direction of the flow, θ, is given by:

$$\theta = \tan^{-1}\left\{\frac{\frac{K\sqrt{2}}{4}}{\frac{K\sqrt{2}}{4}}\right\} = \tan^{-1}1 = 45^\circ$$

At $x=0;\ y=2$: $\vec{V} = \frac{K}{2}\hat{j}$. Clearly, the magnitude of the velocity is $\frac{1}{2}K$ and it is in the direction of the y-axis.

3.22 Contd.] Note that the magnitude of the velocity is the same at all three points. This should not be surprising, since the three points are the same radial distance from the origin, i.e., $r=2$ for all three.

(b) For the stream function, recall that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

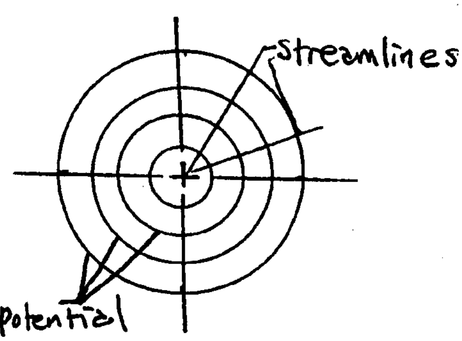
for a two-dimensional, incompressible flow. Thus, the stream function must satisfy both of the following expressions:

$$\psi = \int u\, dy + f(x) \quad \text{and} \quad \psi = -\int v\, dx + g(y)$$

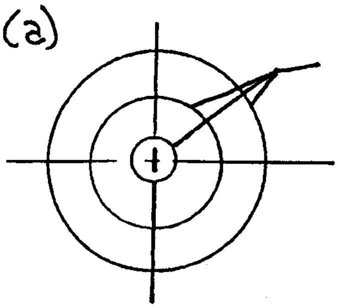
so that $\psi = \int \frac{Kx}{x^2+y^2}\, dy + f(x)$ and $\psi = -\int \frac{Ky}{x^2+y^2}\, dx + g(y)$

Thus, $\psi = K \tan^{-1} \frac{y}{x} = K\theta$

(c) The streamlines are perpendicular to the lines of constant potential.



3.23] (a)



Streamlines are lines of constant r

$$\psi = \frac{\Gamma}{2\pi} \ln r$$

(b) The velocity components in cylindrical coordinates may be written in terms of the derivatives of the stream function as:

3.23 Contd.

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 0 \quad \text{and} \quad v_\theta = -\frac{\partial \psi}{\partial r} = -\frac{\Gamma}{2\pi r}$$

These velocity components are consistent with the streamlines (i.e., concentric circles) shown above. Using the answer to problem (2.1),

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(-\frac{\Gamma}{2\pi}\right) = 0$$

Q.E.D. that the velocity field satisfies continuity.

(c) To calculate the circulation $\oint \vec{V} \cdot \vec{ds}$, if the path of integration is a circle of constant radius (say R_1)

$$\vec{ds} = R_1 \, d\theta \, \hat{e}_\theta$$

$$\oint \vec{V} \cdot \vec{ds} = \int_0^{2\pi} \left(-\frac{\Gamma}{2\pi R_1}\hat{e}_\theta\right) \cdot \left(R_1 \, d\theta \, \hat{e}_\theta\right) = -\frac{\Gamma}{2\pi}\int_0^{2\pi} d\theta = -\Gamma$$

Since the final integral does not contain the radius, it is clear that the circulation is independent of the radius. Indeed, the circulation is $-\Gamma$ for any closed path (regardless of shape) that encloses the origin.

3.24 The magnitude of the velocity is;

$$\vec{V} = \sqrt{2y^2 + x^2 + 2xy} \quad \text{(a)}$$

The stream function is given by $\psi = x^2 + 2xy = \text{constant}$

Differentiating:

$$u = \frac{\partial \psi}{\partial y} = 2y + 2x \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = -2y \quad \text{(b)}$$

Let us use these expressions to check the magnitude of the velocity:

$$|\vec{V}| = \sqrt{u^2 + v^2} = \sqrt{4y^2 + 8xy + 4x^2 + 4y^2} = 2\sqrt{2y^2 + x^2 + 2xy} \quad \text{(c)}$$

3.24 Contd.] Note that this magnitude is twice as large as that in the problem definition. Let us assume that the constant in the stream function expression is "2S", where the S allows us to correlate with the stream function's role in defining the volumetric flux. Thus,

$$\psi' = \frac{y^2}{2} + xy = S$$

Differentiating: $u = \frac{\partial \psi'}{\partial y} = y + x$ and $v = -\frac{\partial \psi'}{\partial x} = -y$ (d)

By reducing the stream function and, therefore, both velocity components by a factor of two, the flow direction at any point in space will be the same for either expression (b) or/and (d). Furthermore, we now have the correct magnitude for the velocity (a)

The modified velocity field satisfies the stream function given in the problem statement, as canbe seen in the following.

$$\psi' = \int u\,dy + f(x) = \frac{y^2}{2} + xy + f(x)$$

and $\psi' = -\int v\,dx + g(y) = xy + g(y)$

For these two expressions to be consistent, $f(x) = \text{constant}$ and $g(y) = \frac{y^2}{2} + \text{constant}$. The resultant expression for the stream function ψ' is equivalent to that in the problem statement, if we note that a factor of 2 has been absorbed into the constant.

Thus, the desired velocity components are those given in (d).

3.25) $u = \frac{\partial\psi}{\partial y} = x + 2y$ and $v = -\frac{\partial\psi}{\partial x} = -y$

To verify that the magnitude of the velocity corresponds to that given in the problem statement:

$$|\vec{V}| = \sqrt{u^2+v^2} = \sqrt{x^2 + 4xy + 4y^2 + y^2} = \sqrt{5y^2 + x^2 + 4xy}$$

Let us evaluate the two sides of the equation representing Stokes's Theorem:

$$\oint \vec{V}\cdot\vec{ds} = \int_0^2 x\,dx + \int_0^1 (-y)\,dy + \int_2^0 (x+2)\,dx + \int_1^0 (-y)\,dy$$

$$= \frac{x^2}{2}\Big|_0^2 - \frac{y^2}{2}\Big|_0^1 + \left(\frac{x^2}{2} + 2x\right\Big|_2^0 - \frac{y^2}{2}\Big|_1^0 = 2 - \frac{1}{2} - 2 - 4 + \frac{1}{2}$$

$$= -4$$

$$\nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y & -y & 0 \end{vmatrix} = \hat{k}\left[\frac{\partial}{\partial x}(-y) - \frac{\partial}{\partial y}(x+2y)\right]$$

$$= -2\hat{k}$$

$$\iint (\nabla\times\vec{V})\cdot\hat{n}\,dA = \int_0^2 dx \int_0^1 (-2)\,dy = -4$$

Q.E.D.

3.26) $\vec{V} = (x^2y - xy^2)\hat{i} + \left(\frac{y^3}{3} - xy^2\right)\hat{j}$

(a) For a steady, incompressible flow, $\nabla\cdot\vec{V}$ should be 0.

$$\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2xy - y^2 + y^2 - 2xy = 0$$

Thus, continuity is satisfied. Because the flow is two-dimensional and incompressible, a stream function does exist.

To find it:

3.26 Contd.] $\psi = \int u\,dy + f(x) = \frac{x^2y^2}{2} - \frac{xy^3}{3} + f(x)$

and $\psi = -\int v\,dx + g(y) = -\frac{xy^3}{3} + \frac{x^2y^2}{2} + g(y)$

For these two expressions to be consistent, $f(x) = g(y)$ = constant. Thus,

$$\psi = \frac{x^2y^2}{2} - \frac{xy^3}{3} + \text{constant}$$

(b) In order for a potential function to exist, the flow must be irrotational, i.e., $\nabla \times \vec{V} = 0$. Does it?

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2y - xy^2) & \left(\frac{y^3}{3} - xy^2\right) & 0 \end{vmatrix} = \hat{k}\,(-y^2 - x^2 + 2xy) \neq 0$$

Therefore, a potential function does not exist for this flow.

(c) To evaluate $\iint (\nabla \times \vec{V}) \cdot \hat{n}\,dA$ for the triangle shown:

$$\iint (\nabla \times \vec{V}) \cdot \hat{n}\,dA = \int_0^1 dx \int_0^x (-y^2 - x^2 + 2xy)\,dy$$

$$= \int_0^1 dx \left[-\frac{y^3}{3} - x^2y + xy^2\right|_0^x = \int_0^1 dx \left(-\frac{x^3}{3} - x^3 + x^3\right)$$

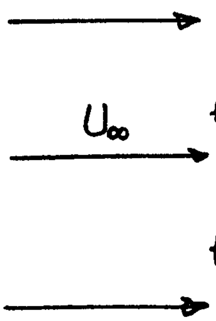
$$= -\int_0^1 \frac{x^3}{3}\,dx = -\frac{x^4}{12}\Big|_0^1 = -\frac{1}{12}$$

To evaluate the circulation around the closed path bounding the triangle

$$\oint \vec{V} \cdot \vec{ds} = \int_0^1 (0)\,dx + \int_0^1 \left(\frac{y^3}{3} - y^2\right) dy + \int_1^0 (x^3 - x^3)\,dx$$

$$+ \int_1^0 \left(\frac{y^3}{3} - y^3\right) dy = \left[\frac{y^4}{12} - \frac{y^3}{3}\right|_0^1 + \left[\frac{y^4}{12} - \frac{y^4}{4}\right|_0^1 = -\frac{1}{12}$$

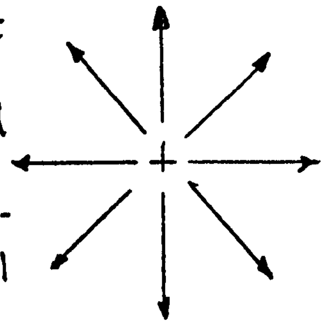
3.27



A stagnation point will occur where the velocity induced

$\oplus$

by the two elementary flows are equal in magnitude and opposite in direction.



Uniform flow from the left to the right: $\psi_{UF} = U_\infty r \sin\theta$

A source located at the origin of the r, θ coordinate system: $\psi_s = \dfrac{K\theta}{2\pi}$

Since each of these components satisfies the condition for irrotationality, they can be added together to produce a flow that itself is irrotational. To satisfy the boundary conditions, we adjust the strength of the source K.

The resultant stream function is the sum of the individual stream functions for a source and for a uniform flow:

$$\psi = \frac{K\theta}{2\pi} + U_\infty r \sin\theta$$

The velocity components are:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{K}{2\pi r} + U_\infty \cos\theta \,;\; v_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty \sin\theta$$

Note that $v_\theta = 0$ at any (and all) radial coordinate when $\theta = 0$ or when $\theta = \pi$. (This result could also be determined from the stream function, since

$$\psi = 0 \text{ when } \theta = 0 \text{ and } \psi = \frac{K}{2} \text{ when } \theta = \pi)$$

Since $v_\theta = 0$ for $\theta = 0$ and for $\theta = \pi$, there will be a stagnation point if $v_r = 0$ at any point on either of these

3.27 Contd. rays. For $\theta = 0$, both terms in the expression for v_r are positive and, therefore, the radial component of the velocity can never be zero. However, when $\theta = \pi$

$$v_r = \frac{K}{2\pi r} - U_\infty$$

Clearly, $v_r = 0$ when $r = \frac{K}{2\pi U_\infty} = R$

which will be designated as R, the radial coordinate of the stagnation point. Similarly, we can define the strength of the source K as:

$$K = 2\pi R U_\infty$$

Thus, the stream function can be written:

$$\psi = R U_\infty \theta + r U_\infty \sin\theta \qquad (a)$$

(a) Since the stagnation point occurs when $r = R$ and $\theta = \pi$, we can substitute these coordinates into the expression for the stream function (a) to identify the streamline:

$$\psi = R U_\infty \pi = \frac{K}{2}$$

Thus, the equation of the streamline that passes through the stagnation point is:

$$R U_\infty \pi = R U_\infty \theta + r U_\infty \sin\theta$$ which can be

rearranged as

$$\frac{r}{R} = \frac{(\pi - \theta)}{\sin\theta} \qquad (i)$$

Thus, when $\theta = 30° = \frac{\pi}{6}$, $\frac{r}{R} = 5.2360$

Noting that $U^2 = v_r^2 + v_\theta^2 = \left(\frac{R}{r} U_\infty + U_\infty \cos\theta\right)^2 + \left(U_\infty \sin\theta\right)^2$

3.27 Contd. Thus, $\left(\frac{U}{U_\infty}\right)^2 = \left(\frac{R}{r}\right)^2 + \frac{2R}{r}\cos\theta + 1$ (ii)

at $\theta = 30°$, $\frac{r}{R} = 5.2360$, so that $\frac{U}{U_\infty} = 1.1693$

Finally, $C_p = 1 - \left(\frac{U}{U_\infty}\right)^2 = -\left(\frac{R}{r}\right)^2 - \frac{2R}{r}\cos\theta$ (iii)

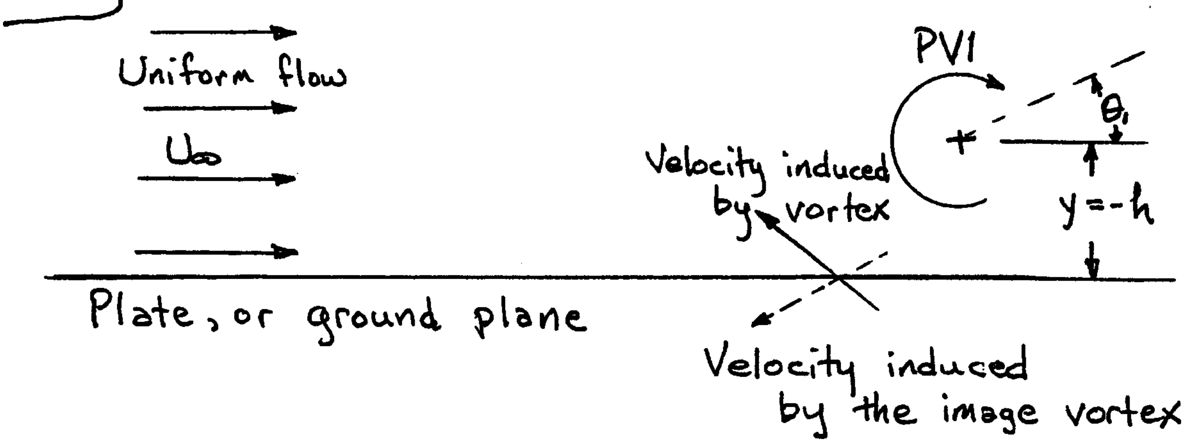
so that at $\theta = 30°$, $C_p = -0.3675$

The desired table can be completed using equations (i), (ii), and (iii). At the stagnation point, where $r = R$, we must use l'Hopital's rule.

θ	r/R	U/U_∞	C_p
30°	5.2360	1.1693	−0.3675
45°	3.3322	1.2306	−0.5145
90°	1.5708	1.1854	−0.4053
135°	1.1107	0.7330	+0.4624
150°	1.0472	0.5078	+0.7421
180°	1.0000	0.0000	+1.0000

Note that when $\theta = 180° = \pi$: $r = R$, $U = 0$, and $C_p = 1.0$, as one would expect at the stagnation point.

3.28



3.28 Contd. For the two-dimensional, free, potential vortex (PV1) which is located a distance h above the plate (or ground plane) in a uniform flow:

$$\phi = \phi_{UF} + \phi_{PV1} = U_\infty x - \frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)$$

where the x,y coordinate system is located at the origin of the vortex, as shown in the sketch. The velocity induced by the vortex at a representative point on the plate is represented by the solid arrow in the sketch. Clearly, the sum of the uniform flow and the single potential vortex (PV1) do not result in a flow parallel to the plate (which is a boundary condition: that is, the inviscid flow must be parallel to a plate surface). In order for the plate to be a streamline, we need to add an "image" vortex of equal strength, located a distance h below the plate. This image vortex induces a velocity which is represented by the broken arrow. The normal component of the velocity induced by the image vortex is equal and opposite to that induced by the original vortex. Thus, the resultant velocity induced by these three potential functions is parallel to the plate. To calculate the resultant velocity, let us calculate first the velocity induced by PV1:

$$u = \frac{\partial \phi_{PV1}}{\partial x} = \frac{\partial}{\partial x}\left[-\frac{\Gamma}{2\pi}\tan^{-1}\left(\frac{y}{x}\right)\right]$$

$$u = -\frac{\Gamma}{2\pi}\frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right) = +\frac{\Gamma y}{2\pi(x^2+y^2)}$$

We shall not worry about the y-component of velocity (v)

3.28 Contd. since that is cancelled by the image vortex. However, the horizontal velocity component (u) induced by the image vortex at the plate is equal to that induced by the original vortex (PV1), so that the velocity induced by the original vortex and its image at this plate is twice this value. Further, note that $y = -h$ for the plate. Thus, adding the free-stream velocity, the net velocity at the surface of the plate is:

$$U = U_\infty - \frac{\Gamma h}{\pi (x^2 + h^2)}$$

The pressure along the upper surface is:

$$p_u = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - \frac{1}{2}\rho_\infty U^2$$

Since the pressure acting on the underside of the plate is p_∞, the net pressure force acting on the plate is:

$$\Delta p = p_\ell - p_u = \frac{1}{2}\rho_\infty (U^2 - U_\infty^2)$$

$$= \frac{1}{2}\rho_\infty \left[\frac{\Gamma^2}{\pi^2} \frac{h^2}{(x^2+h^2)^2} - \frac{2U_\infty \Gamma}{\pi} \frac{h}{x^2+h^2} \right]$$

To find the net lift force acting on the plate per unit depth, we integrate the net pressure force from $x = -\infty$ to $x = +\infty$. Thus, the force per unit depth is:

$$\ell = \frac{\rho_\infty}{2} \int_{-\infty}^{+\infty} \left\{ \frac{\Gamma^2 h^2}{\pi^2} \frac{dx}{(x^2+h^2)^2} - \frac{2U_\infty \Gamma h}{\pi} \frac{dx}{(x^2+h^2)} \right\}$$

$$\ell = \frac{\rho_\infty}{2} \left\{ \frac{\Gamma^2 h^2}{\pi^2} \left[\frac{1}{2h^2} \frac{x}{(x^2+h^2)} + \frac{1}{2h^2} \frac{1}{h} \tan^{-1}\left(\frac{x}{h}\right) \right] - \frac{2U_\infty \Gamma h}{\pi} \frac{1}{h} \tan^{-1}\left(\frac{x}{h}\right) \right\} \Bigg|_{-\infty}^{+\infty}$$

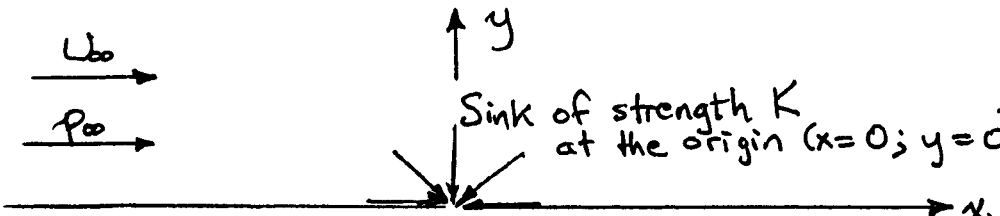
3.28 Contd.

$$\ell = \frac{\rho_\infty}{2}\frac{\Gamma^2}{2\pi^2 h}\left[\frac{\pi}{2} - \frac{3\pi}{2}\right] - \frac{\rho_\infty}{2}\frac{2U_\infty\Gamma}{\pi}\left[\frac{\pi}{2} - \frac{3\pi}{2}\right]$$

$$\ell = \rho_\infty U_\infty \Gamma - \frac{\rho_\infty \Gamma^2}{4\pi h}$$

Note that as $h \to \infty$, the lift per unit span becomes equal to $\rho_\infty U_\infty \Gamma$.

3.29



$$\phi_{sink} = -\frac{K}{2\pi}\ln r$$

For the component of the flow represented by the sink,

$$v_r = \frac{\partial \phi}{\partial r} = -\frac{K}{2\pi r}$$

Note that v_r is negative at all points, i.e., it is directed toward the origin. Since we are concerned only with the pressure distribution along the wall (where $y=0$), we can write that the component of the velocity parallel to the wall due to the sink alone is:

$$u_{sink} = -\frac{K}{2\pi x}$$

Note that u_{sink} is positive (directed to the right) when x is negative. Further, u_{sink} is negative (directed to the left) when x is positive. This is as it should be.

The total velocity parallel to the wall is the sum of that induced by the sink and that due to the uniform flow:

3.29 Contd.

$$u = u_{UF} + u_{sink} = U_\infty - \frac{K}{2\pi x} \qquad (i)$$

The stagnation point corresponds to the x location where $u = 0$. Thus,

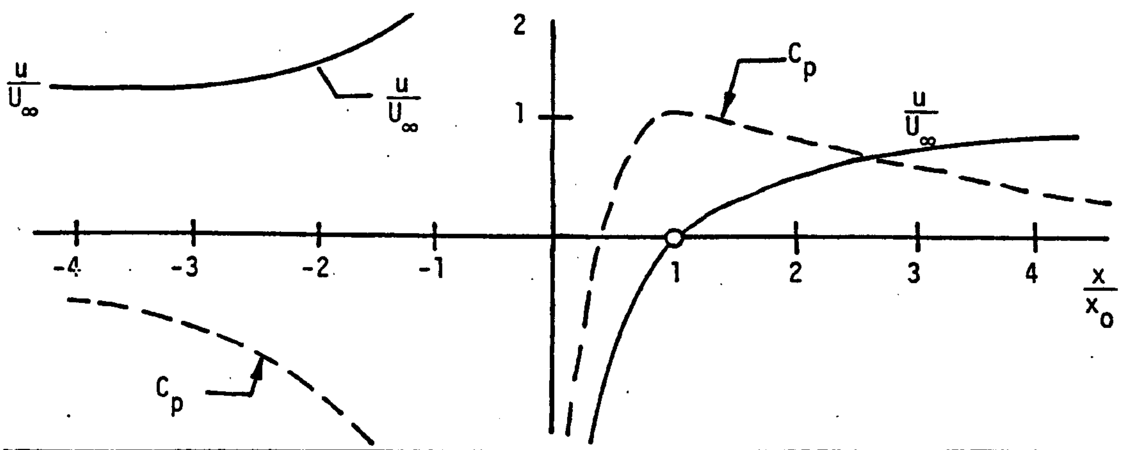
$$x_{stag} = x_o = \frac{K}{2\pi U_\infty}$$

where x_o is the x coordinate of the stagnation point. Note that the terms in the velocity expression [eq. (i)] are of opposite sign only for positive x. Thus, the stagnation point is located to the right of the origin (i.e., for positive x). We can write the velocity along the wall as:

$$u = U_\infty - \frac{K}{2\pi x} = U_\infty - \frac{2\pi x_o U_\infty}{2\pi x} = U_\infty\left(1 - \frac{x_o}{x}\right)$$

The pressure coefficient along the wall is:

$$C_p = 1 - \frac{u^2}{U_\infty^2} = 1 - \left[1 - \frac{2x_o}{x} + \frac{x_o^2}{x^2}\right] = \frac{2x_o}{x} - \frac{x_o^2}{x^2}$$



3.30] Since the potential flow around a cylinder is represented by the superposition of a uniform flow and a doublet, the stream function is:

$$\psi = U_\infty r \sin\theta - \frac{B}{r}\sin\theta$$

where $R = \sqrt{B/U_\infty}$. Thus, $B = R^2 U_\infty = 50 \, m^3/s$, so that:

$$\psi = 50 r \sin\theta - \frac{50}{r}\sin\theta$$

To calculate the change in pressure from the free-stream value to that at a point on the surface of the cylinder where $\theta = 90°$, use Eq. (3.44):

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1 - 4\sin^2\theta = -3$$

Thus, $$p - p_\infty = -\frac{3}{2}\rho_\infty U_\infty^2 = -\frac{3}{2}\left(1.225 \frac{kg}{m^3}\right)\left(50 \frac{m}{s}\right)^2$$

$$= -4.594 \times 10^3 \, N/m^2$$

3.31] $$\psi_{\text{uniform flow}} = U_\infty r \sin\theta$$

$$\psi_{\text{doublet}} = -\frac{B}{r}\sin\theta$$

$$\psi = U_\infty r \sin\theta - \frac{B}{r}\sin\theta$$

If $B = R^2 U_\infty$, then

$$\psi = U_\infty \sin\theta \left(r - \frac{R^2}{r}\right)$$

When $r = R$, $\psi = 0$, a constant. Thus, a cylinder of radius $r = R$ is a streamline in the flow field

3.32) $\psi_{\text{uniform flow}} = V_\infty r \sin\theta$

$\psi_{\text{doublet}} = -\frac{B}{r}\sin\theta$

$\psi_{\text{vortex}} = \frac{\Gamma}{2\pi}\ln r$

$$\psi = V_\infty r\sin\theta - \frac{B}{r}\sin\theta + \frac{\Gamma}{2\pi}\ln r$$

If $B = R^2 V_\infty$, then

$$\psi = V_\infty \sin\theta\left(r - \frac{R^2}{r}\right) + \frac{\Gamma}{2\pi}\ln r$$

When $r = R$, $\psi = \frac{\Gamma}{2\pi}\ln R$, a constant.

Thus, a cylinder of radius $r = R$ is a streamline in the flow field.

3.33) For a cylindrical yaw probe, the static pressure around the cylinder where the pressure orifices are may be approximated by Eq. (3.44):

$$C_p = \frac{p - p_\infty}{q_\infty} = 1 - 4\sin^2\theta$$

(a) For the orifices located where $p = p_\infty$, or $C_p = 0$. Thus, we can solve our expression for C_p to find that, if

$1 - 4\sin^2\theta = 0$, then $\theta = 30°$ or $150°$

(i.e., these orifices should be $\pm 30°$ from the stagnation point)

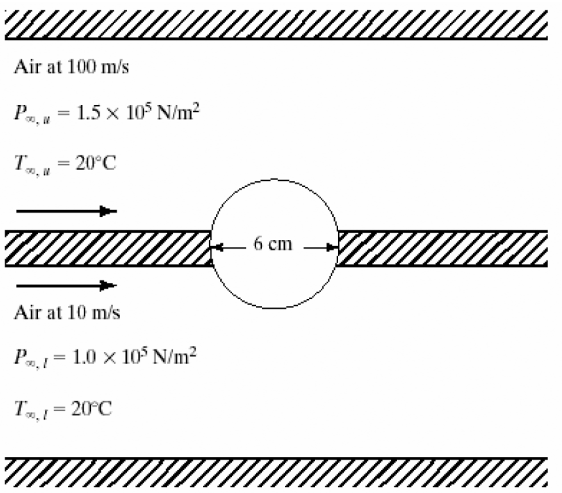
(b) To find $\frac{dp}{d\theta}$, note that $p = p_\infty + q_\infty(1 - 4\sin^2\theta)$

Thus, $\frac{dp}{d\theta} = -8q_\infty \sin\theta\cos\theta$

which, at $\theta = 30°$, becomes $\frac{dp}{d\theta} = -3.464\,q_\infty$

3.34)

Given: An infinite span cylinder serving as a plug as shown below.



From Eqn. 3.43, the pressure variation on a cylinder in a free stream flow is given by:

$$p = p_\infty + \rho_\infty U_\infty^2/2 - 2\rho_\infty U_\infty^2 \sin^2\theta$$

Since the pressure variation for each half of the cylinder will be given by the same relation, we only need to find which pressure field is higher at one point on the cylinder, say $\theta = 90^\circ$ (the top or bottom of the cylinder). The pressure variation on the upper portion of the cylinder is given by:

$$p = p_{\infty,u} + \rho_{\infty,u} U_{\infty,u}^2/2 - 2\rho_{\infty,u} U_{\infty,u}^2 \sin^2\theta$$

For $\theta = 90^\circ$:

$$p = p_{\infty,u} - 3\rho_{\infty,u} U_{\infty,u}^2/2 = 1.5\times10^5 - 3\left(\frac{1.5\times10^5}{(287.05)(293.15)}\right)(100)^2/2 = 1.23\times10^5 \text{ N/m}^2$$

Likewise, on the bottom of the cylinder

$$p = p_{\infty,l} - 3\rho_{\infty,l} U_{\infty,l}^2/2 = 1.0\times10^5 - 3\left(\frac{1.0\times10^5}{(287.05)(293.15)}\right)(10)^2/2 = 9.98\times10^4 \text{ N/m}^2$$

The pressure on the upper surface is greater than the pressure on the bottom surface, so the cylinder will move down.

3.35)

Given: A 3*m* diameter cylindrical smokestack which is 50*m* tall in a 4*m/s* wind.

The conditions provided for the atmosphere are non-standard, so the density and viscosity will have to calculated.

$$T = 30^\circ C = 303.15K$$

$$p = 99kPa = 99{,}000N/m^2$$

From the Ideal Gas Law:

$$p = \rho RT$$

$$\rho = \frac{p}{RT} = \frac{99{,}000}{(287.05)(300.15)} = 1.1377kg/m^3$$

Find the viscosity using Sutherland's Law (Eqn. 1.12a):

$$\mu = 1.458\times 10^{-6}\frac{(T)^{1.5}}{T+110.4} = 1.458\times 10^{-6}\frac{(303.15)^{1.5}}{303.15+110.4} = 1.86087\times 10^{-5}kg/s-m$$

The Reynolds number of the cylinder is given by:

$$\mathrm{Re}_d = \frac{\rho_\infty U_\infty d}{\mu_\infty} = \frac{(1.1377)(4)(3)}{1.86087\times 10^{-5}} = 7.34\times 10^5$$

From Fig. 3.31 for a smooth cylinder, $C_d \approx 0.32$, which is from the region where the critical Reynolds number has been exceeded. From Eqn. 3.52, the definition of the cylinder drag coefficient is:

$$C_d \equiv \frac{d}{q_\infty D}$$

$$q_\infty = \rho_\infty U_\infty^2/2 = (1.1377)(4)^2/2 = 9.102N/m^2$$

So the drag per unit height of the smokestack is:

$$d = C_d q_\infty D = (0.32)(9.102)(3) = 8.738N/m$$

Assuming the drag is constant along the height, the total drag on the smokestack is:

$$D = dL = (8.738)(50) = 436.9N$$

Since the drag is constant along the height of the smokestack, assume the drag acts at the centroid of the cylinder, so the moment is:

$$M = (436.9)(25) = 1.092\times 10^4 N-m$$

3.36)

Given: A 6*in* diameter cylindrical flag pole which is 15*ft* tall in a 45*mph* wind.

The conditions provided for the atmosphere are non-standard, so the density and viscosity will have to calculated.

$$T = 85^\circ F = 554.67^\circ R$$

$$p = 14.4 lb/in^2 = 2073.6 lb/ft^2$$

$$U_\infty = 45 mph = 66 ft/s$$

From the Ideal Gas Law:

$$p = \rho R T$$

$$\rho = \frac{p}{RT} = \frac{2073.6}{(1716.16)(554.67)} = 0.002178 slug/ft^3$$

Find the viscosity using Sutherland's Law (Eqn. 1.12b):

$$\mu = 2.27\times 10^{-8}\frac{(T)^{1.5}}{T+198.6} = 2.27\times 10^{-8}\frac{(554.67)^{1.5}}{554.67+198.6} = 3.93665\times 10^{-7} lb-s/ft^2$$

The Reynolds number of the cylinder is given by:

$$\mathrm{Re}_d = \frac{\rho_\infty U_\infty d}{\mu_\infty} = \frac{(0.002178)(66)(0.5)}{3.93665\times 10^{-7}} = 1.83\times 10^5$$

From Fig. 3.31 for a smooth cylinder, $C_d \approx 1.2$, which is from the region where the critical Reynolds number has not been reached. From Eqn. 3.52, the definition of the cylinder drag coefficient is:

$$C_d \equiv \frac{d}{q_\infty D}$$

$$q_\infty = \rho_\infty U_\infty^2/2 = (0.002178)(66)^2/2 = 4.744 lb/ft^2$$

So the drag per unit height of the flag pole is:

$$d = C_d q_\infty D = (1.2)(4.744)(0.5) = 2.846 lb/ft$$

Assuming the drag is constant along the height, the total drag on the smokestack is:

$$D = dL = (2.846)(15) = 42.69 lb$$

Since the drag is constant along the height of the smokestack, assume the drag acts at the centroid of the cylinder, so the moment is:

$$M = (42.69)(7.5) = 320 ft-lb$$

3.37)

Given: A rotating cylinder with the following information:

$$U_\infty = 120\, ft/s$$

$$D = 3 \text{ ft}$$

$$T = 68^\circ F = 527.67^\circ R$$

From Eqn. 3.59 the stagnation points on the cylinder are given by:

$$\theta = \sin^{-1}\left(-\frac{\Gamma}{4\pi R U_\infty}\right)$$

So the circulation is given by:

$$\Gamma = -4\pi R U_\infty \sin\theta$$

For a stagnation point at $\theta = 30^\circ$

$$\Gamma = -4\pi(1.5)(120)\sin 30^\circ = -1131 \text{ ft}^2/\text{s}$$

From Eqn. 3.57, the pressure coefficient variation for a rotating circular cylinder is:

$$C_p = 1 - \frac{1}{U_\infty^2}\left[4U_\infty^2 \sin^2\theta + \frac{2\Gamma U_\infty \sin\theta}{\pi R} + \left(\frac{\Gamma}{2\pi R}\right)^2\right]$$

For $\theta = 30^\circ$

$$C_p = 1 - \frac{1}{(120)^2}\left[4(120)^2 \sin^2 30^\circ + \frac{2(-1131)(120)\sin 30^\circ}{\pi(1.5)} + \left(\frac{-1131}{2\pi(1.5)}\right)^2\right]$$

$$C_p = 1 - [1 - 2 + 1] = 1 = \frac{p - p_\infty}{\rho_\infty U_\infty^2/2}$$

The density is given by the Ideal Gas Law:

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{2000}{(1716.16)(527.67)} = 0.00221 \text{ slug/ft}^3$$

3.37) contd.

and the pressure is:

$$p = C_p\left(\rho_\infty U_\infty^2/2\right) + p_\infty = 1(0.00221)(120)^2/2 + 2000 = 2015.9 \text{ lb/ft}^2$$

From Eqn. 3.56, the tangential velocity for $\theta = 90^\circ$ is:

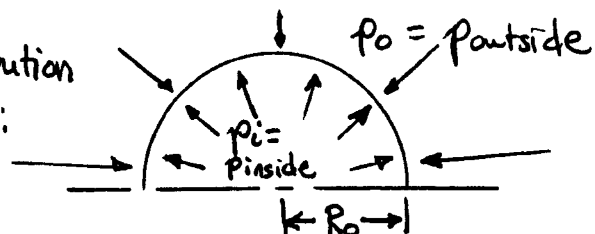
$$v_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi R} = -2(120)\sin 90 - \frac{-1131}{2\pi(1.5)} = -120 \text{ ft/s}$$

which is flow to the right at the top of the cylinder. The tangential velocity for $\theta = 270^\circ$ is:

$$v_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi R} = -2(120)\sin 270 - \frac{-1131}{2\pi(1.5)} = 360 \text{ ft/s}$$

which is also flow to the right, but at the bottom of the cylinder.

3.38) The pressure distribution over the outside surface is:



$$p = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 C_p$$
$$= p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 (1 - 4\sin^2\theta)$$

The lift force per unit span of the hut due to the pressure acting on the outer surface is:

$$l_o = -\int_0^\pi p_o \sin\theta\, R_o\, d\theta$$

$$l_o = -\int_0^\pi p_\infty \sin\theta\, R_o\, d\theta - \int_0^\pi \frac{1}{2}\rho_\infty U_\infty^2 \sin\theta\, R_o\, d\theta$$
$$+ \int_0^\pi 2\rho_\infty U_\infty^2 \sin\theta\, R_o\, d\theta$$

$$l_o = -p_\infty R_o(-\cos\theta\Big|_0^\pi - \frac{1}{2}\rho_\infty U_\infty^2 R_o(-\cos\theta\Big|_0^\pi$$
$$+ 2\rho_\infty U_\infty^2 R_o(-\cos\theta + \frac{1}{3}\cos^3\theta\Big|_0^\pi$$

3.38 Contd.

$$l_0 = -2p_\infty R_0 - \frac{1}{2}\rho_\infty U_\infty^2 2R_0 + 2\rho_\infty U_\infty^2 R_0\left(2-\frac{2}{3}\right)$$

$$l_0 = -2p_\infty R_0 + \frac{1}{2}\rho_\infty U_\infty^2 2R_0\left(-1+4-\frac{4}{3}\right)$$

$$l_0 = -2p_\infty R_0 + \frac{5}{3}\rho_\infty U_\infty^2 R_0$$

The lift force per unit span due to the pressure inside the hut is:

$l_i = p_i 2R_0$ (this results since the internal pressure is constant at p_i, the lift force per unit span is the pressure times the projected area)

Thus, the net lift per unit span acting acting on the hut is the sum of these two forces.

$$l = p_i 2R_0 - p_\infty 2R_0 + \frac{5}{3}\rho_\infty U_\infty^2 R_0$$

(a) If the door is located at ground level, the pressure inside the hut would be equal to the stagnation pressure.

$$p_i = p_t = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2$$

Thus, $l = p_\infty 2R_0 + \frac{1}{2}\rho_\infty U_\infty^2 2R_0 - p_\infty 2R_0 + \frac{5}{3}\rho_\infty U_\infty^2 R_0$

$$l = \left(\frac{1}{2}\rho_\infty U_\infty^2\right)(2R_0)\left(1+\frac{5}{3}\right) = \left(\frac{1}{2}\rho_\infty U_\infty^2\right)(2R_0)\left(\frac{8}{3}\right)$$

Thus, $C_l = \dfrac{l}{\frac{1}{2}\rho_\infty U_\infty^2\, 2R_0} = \dfrac{8}{3}$

Note that the characteristic dimension has been assumed to be the length across the floor of the hut, $2R_0$.

The net lift per unit span is:

$$l = \left[\frac{1}{2}(1.2250)(48.61)^2\right]\left[2(6)\right]\frac{8}{3} = 4.631\times10^4 \text{ N/m}$$

3.38 Contd.]

(b) The net force on the hut will vanish when $l = 0$.
Thus,

$$p_i 2R_o - p_\infty 2R_o + \frac{5}{3}\rho_\infty U_\infty^2 R_o = 0$$

Rearranging:
$$\frac{p_i - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = -\frac{5}{3}$$

Thus, to have zero net lift, the door should be located at a point where the local pressure coefficient is $-\frac{5}{3}$. We refer to the equation defining the pressure distribution and solve for θ. Assuming that $C_p = 1 - 4\sin^2\theta = -\frac{5}{3}$; $\sin^2\theta = \frac{2}{3}$
Therefore, $\theta = 54.74°$ from the ground (on either side of the building).

3.39] The lift is zero by symmetry. The drag per unit span (i.e., the section drag) is:

$$d = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} p_{wwd}\, R\, d\theta \cos\theta - p_b(2R)$$

p_{wwd} is the pressure acting on the windward surface. Since the base pressure is constant, it produces a force of $p_b(2R)$ in the negative drag direction. For $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ (which is the windward surface)

$$p_{wwd} = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - 2\rho_\infty U_\infty^2 \sin^2\theta$$

We are told that the base pressure is equal to the pressure at the separation point (which is the corner):

$$p_b = p_{wwd}(\theta = \tfrac{\pi}{2}) = p_\infty - \frac{3}{2}\rho_\infty U_\infty^2$$

3.39 Contd.

Thus, $$d = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - 2\rho_\infty U_\infty^2 \sin^2\theta\right)\cos\theta \, R\, d\theta$$

$$- \left(p_\infty - \frac{3}{2}\rho_\infty U_\infty^2\right) 2R$$

$$d = -p_\infty R \sin\theta \Big|_{\pi/2}^{3\pi/2} - \frac{1}{2}\rho_\infty U_\infty^2 R \sin\theta \Big|_{\pi/2}^{3\pi/2}$$

$$+ 2\rho_\infty U_\infty^2 R\left(\frac{1}{3}\sin^3\theta\right|_{\pi/2}^{3\pi/2} - \left(p_\infty - \frac{3}{2}\rho_\infty U_\infty^2\right) 2R$$

$$d = -p_\infty R(-1-1) - \frac{1}{2}\rho_\infty U_\infty^2 R(-1-1) + \frac{2}{3}\rho_\infty U_\infty^2 R(-1-1)$$

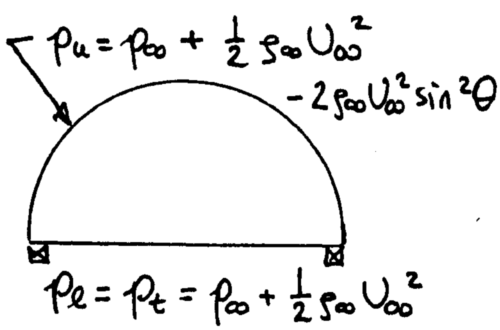
$$-2p_\infty R + 3\rho_\infty U_\infty^2 R = \frac{8}{3}\rho_\infty U_\infty^2 R = \frac{8}{3}\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R]$$

Thus, $$C_d = \frac{d}{q_\infty(2R)} = \frac{\frac{8}{3}\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R]}{\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R]} = \frac{8}{3}$$

3.40 $\quad l = 0;$

$$d = \frac{8}{3}\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R] = \frac{8}{3}\left[\frac{1}{2}(1.22)(75)^2\right]\left[2(0.3)\right] = 5490\ \frac{N}{m}$$

3.41 Since $v_r = \left(U_\infty - \frac{B}{r^2}\right)\cos\theta$, the tangency requirement that $v_r = 0$ when $r = R$ (i.e., the surface of the cylinder):



$$B = R^2 U_\infty$$

$$B = (225)(100) = 2.25\times 10^4 \ \frac{ft^3}{s}$$

$$l = -\int_0^\pi p_u \sin\theta\, R\, d\theta + p_l(2R)$$

$$l = -R\int_0^\pi \left[p_\infty \sin\theta + \frac{1}{2}\rho_\infty U_\infty^2 \sin\theta - 2\rho_\infty U_\infty^2 \sin^3\theta\right] d\theta$$

$$+ p_\infty 2R + \frac{1}{2}\rho_\infty U_\infty^2 2R$$

3.41 Contd.

$$\ell = + R p_\infty \cos\theta \Big|_0^\pi + \frac{1}{2}\rho_\infty U_\infty^2 R \cos\theta \Big|_0^\pi$$
$$+ 2\rho_\infty U_\infty^2 R \left(-\cos\theta + \frac{1}{3}\cos^3\theta \Big|_0^\pi + p_\infty 2R + \frac{1}{2}\rho_\infty U_\infty^2 2R\right.$$

$$\ell = R p_\infty (-1-1) + \frac{1}{2}\rho_\infty U_\infty^2 R(-1-1)$$
$$+ 2\rho_\infty U_\infty^2 R\left(1+1-\frac{1}{3}-\frac{1}{3}\right) + 2 p_\infty R + \rho_\infty U_\infty^2 R = \frac{8}{3}\rho_\infty U_\infty^2 R$$

$$C_\ell = \frac{\ell}{q_\infty 2R} = \frac{\frac{8}{3}\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R]}{\left[\frac{1}{2}\rho_\infty U_\infty^2\right][2R]} = \frac{8}{3}$$

Since we are working with the force acting on an entire, closed surface, we can add the effect of a constant pressure acting over this surface:

$\oint p_\infty \sin\theta R\, d\theta$ which equals zero.

Thus, $\ell = -\oint p \sin\theta R\, d\theta + \oint p_\infty \sin\theta R\, d\theta$

As noted, the second term is zero, since there is no net force in any direction due to a constant pressure acting over a closed surface. Combining the terms

$$\ell = -q_\infty \oint \frac{(p-p_\infty)}{q_\infty}\sin\theta R\, d\theta = -q_\infty \oint C_p \sin\theta R\, d\theta$$

$$\ell = -q_\infty R\int_0^\pi (1-4\sin^2\theta)\sin\theta\, d\theta + q_\infty (1) 2R$$

← the component due to flow over the upper surface →

← we are not using $R\,d\theta$, since the bottom is flat and not a circular arc →

3.41 Contd.

$$l = q_\infty R\cos\theta\Big|_0^\pi + 4q_\infty R\left[-\cos\theta + \frac{1}{3}\cos^3\theta\Big|_0^\pi\right. + q_\infty 2R$$

$$l = q_\infty R(-1-1) + 4q_\infty R\left[-(-1-1) + \frac{1}{3}(-1-1)\right] + q_\infty 2R$$

$$l = \frac{16}{3} q_\infty R = \frac{8}{3} q_\infty (2R)$$, as was found in the earlier work out for this problem

$$l = \frac{8}{3}\left[\frac{1}{2}(0.00238)(10^4)\right]\left[2(15)\right] = 952 \text{ lbf/ft}$$

$d = 0$, by symmetry

3.42] Since $v_\theta = -2U_\infty \sin\theta - \frac{\Gamma}{2\pi R}$, the local static pressure distribution is given by:

$$p = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - \frac{1}{2}\rho_\infty\left[4U_\infty^2\sin^2\theta + \frac{2\Gamma U_\infty \sin\theta}{\pi R} + \frac{\Gamma^2}{4\pi^2R^2}\right]$$

Thus, the lift per unit span is:

$$l = -\int_0^{2\pi} p(R\,d\theta)\sin\theta$$

$$l = - -p_\infty R\int_0^{2\pi}\cancel{\sin\theta\,d\theta} \xrightarrow{} 0 - \frac{1}{2}\rho_\infty U_\infty^2 R\cancel{\int_0^{2\pi}\sin\theta\,d\theta} \xrightarrow{} 0$$

$$+ 2\rho_\infty U_\infty^2\int_0^{2\pi}\sin^3\theta\,d\theta + \frac{\rho_\infty U_\infty \Gamma}{\pi}\int_0^{2\pi}\sin^2\theta\,d\theta + \frac{\rho_\infty\Gamma^2}{8\pi^2R^2}\cancel{\int_0^{2\pi}\sin\theta\,d\theta}$$

$$l = 2\rho_\infty U_\infty^2\left[-\cos\theta + \frac{1}{3}\cos^3\theta\Big|_0^{2\pi}\right. + \frac{\rho_\infty U_\infty\Gamma}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right|_0^{2\pi}$$

$$l = \rho_\infty U_\infty \Gamma \quad \text{(Q.E.D.)}$$

$$C_l = \frac{l}{q_\infty 2R} = \frac{\Gamma}{RU_\infty}$$

3.43) $C_p = 1 - \frac{1}{U_\infty^2}\left[4U_\infty^2 \sin^2\theta + \frac{2\Gamma U_\infty \sin\theta}{\pi R} + \frac{\Gamma^2}{4\pi^2 R^2}\right]$

Thus, the section lift coefficient is:

$$C_l = -\frac{1}{2}\int_0^{2\pi} C_p \sin\theta \, d\theta$$

$$C_l = -\frac{1}{2}\int_0^{2\pi} \cancel{\sin\theta\, d\theta}^{\,0} + 2\int_0^{2\pi} \sin^3\theta \, d\theta$$

$$+ \frac{\Gamma}{\pi R U_\infty}\int_0^{2\pi} \sin^2\theta \, d\theta + \frac{\Gamma^2}{8\pi^2 R^2 U_\infty}\int_0^{2\pi} \cancel{\sin\theta\, d\theta}^{\,0}$$

$$C_l = 2\left[-\cos\theta + \frac{1}{3}\cos^3\theta\right|_0^{2\pi} + \frac{\Gamma}{\pi R U_\infty}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right|_0^{2\pi}$$

$C_l = \frac{\Gamma}{R U_\infty}$, which is the same result that was obtained in Problem 3.29 using the local static pressure itself, rather than the pressure coefficient.

3.44) $R = 0.5\,m$; $b = 10.0\,m$; $U_\infty = 27.78\,\frac{m}{s}$; $\rho_\infty = 1.0066\,\frac{kg}{m^3}$

$\omega = 100\,\frac{rev}{min}\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{min}{60\,s}\right) = 10.47\,\frac{rad}{s}$! Assuming the no-slip condition for the air particles adjacent to the surface, the velocity at a point on the surface of the cylinder is:

$v_\theta = R\omega = 5.24\,\frac{m}{s}$. The circulation Γ is:

$$\Gamma = \oint \vec{V} \cdot \vec{ds} = \int_0^{2\pi} v_\theta R \, d\theta$$

Thus, $\Gamma = 16.45\,m/s$ and the lift is: $L = \rho_\infty U_\infty \Gamma b$

$$L = 4599.4\,N$$

3.45) Since this is very similar to Example 3.6, this problem is not worked out here.

3.46) The drag on the disk (D_f) is:

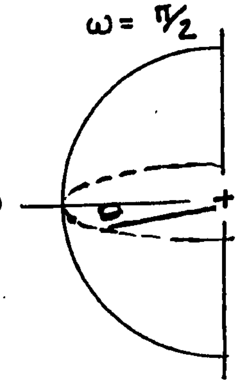
$$D_f = \left(\bar{p}_{fore} - \bar{p}_{lee}\right)\frac{\pi D^2}{4}$$ where $\bar{p}_{fore}$ is the average pressure acting on the forebody and $\bar{p}_{lee}$ is the average pressure acting on the leeward side (back side).

Converting to the drag coefficient and adding and subtracting p_∞:

$$C_D = \frac{D_f}{q_\infty\left(\frac{\pi D^2}{4}\right)} = \frac{(\bar{p}_{fore} - p_\infty) - (\bar{p}_{lee} - p_\infty)}{q_\infty}$$

Thus, $$C_D = \bar{C}_{p_{fore}} - \bar{C}_{p_{lee}} = 0.75 - (-0.40) = 1.15$$

3.47) $$u_\omega = \frac{3}{2}\sin\omega$$



for $0 \le \omega \le \frac{\pi}{2}$ in the coordinate system shown in the sketch.

On the windward side, the pressure is:

$$p_{wwd} = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - \frac{9}{8}\rho_\infty U_\infty^2 \sin^2\omega$$

On the leeward side (which is flat), the pressure is:

$$p_{lee} = p_\infty - \frac{5}{8}\rho_\infty U_\infty^2$$

$$D = \left[\text{Drag force on the windward side}\right] - (p_{lee})\pi R^2$$

and the drag force on the windward side (D_{wwd}) is:

$$D_{wwd} = 2\underbrace{\int_0^{\frac{\pi}{2}}}_{\omega \text{ limits}} \underbrace{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}}_{\theta \text{ limits}} p_{wwd}\cos\omega\,\cos\theta\,(R\cos\omega\, d\omega) R\, d\theta$$

3.47 Contd.

$$D_{wwd} = 2\int_0^{\pi/2} d\omega \left\{ \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \left(p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - \frac{9}{8}\rho_\infty U_\infty^2 \sin^2\omega\right)\cos^2\omega \cos\theta R^2 d\theta \right\}$$

$$D_{wwd} = 2R^2 \int_0^{\pi/2} d\omega \left\{ \left(p_\infty + \frac{1}{2}\rho_\infty U_\infty^2 - \frac{9}{8}\rho_\infty U_\infty^2 \sin^2\omega\right)\cos^2\omega \left[\sin\theta \Big|_{-\pi/2}^{+\pi/2}\right] \right\}$$

$$D_{wwd} = 4R^2 \left\{ \int_0^{\pi/2} \left(p_\infty + \frac{1}{2}\rho_\infty U_\infty^2\right)\cos^2\omega\, d\omega - \int_0^{\pi/2} \frac{9}{8}\rho_\infty U_\infty^2 \sin^2\omega \cos^2\omega\, d\omega \right\}$$

$$D_{wwd} = 4R^2 \left\{ \left(p_\infty + \frac{1}{2}\rho_\infty U_\infty^2\right)\left[\frac{\omega}{2} + \frac{\sin 2\omega}{4}\Big|_0^{\pi/2}\right] - \frac{9}{8}\rho_\infty U_\infty^2 \left[\frac{\omega}{8} - \frac{\sin 4\omega}{32}\Big|_0^{\pi/2}\right] \right\}$$

$$D_{wwd} = 4R^2\left[p_\infty \frac{\pi}{4} + \frac{1}{2}\rho_\infty U_\infty^2 \frac{\pi}{4} - \frac{9}{8}\rho_\infty U_\infty^2 \frac{\pi}{16}\right]$$

The net drag force is:

$$D = \left(p_\infty \pi R^2 + \frac{1}{2}\rho_\infty U_\infty^2 \pi R^2 - \frac{9}{32}\rho_\infty U_\infty^2 \pi R^2\right) - \left(p_\infty \pi R^2 - \frac{5}{8}\rho_\infty U_\infty^2 \pi R^2\right)$$

$$D = 0.8438\, \rho_\infty U_\infty^2 \pi R^2$$ $L = 0$, by symmetry

$$C_D = \frac{D}{\frac{1}{2}\rho_\infty U_\infty^2 \pi R^2} = 1.6875$$

$$C_L = 0$$

3.48

$$D = 0.8438\, \rho_\infty U_\infty^2 \pi R^2$$

$$D = 0.8438\left(0.002376\ \frac{lbf\, s^2}{ft^4}\right)\left(4\times10^4\ \frac{ft^2}{s^2}\right)(\pi)(1.0\ ft)^2$$

$$D = 251.92\ lbf$$

3.49) On the surface of the sphere, the local pressure is:

$p = p_\infty + \frac{1}{2}\rho_\infty U_\infty^2\left(1 - \frac{9}{4}\sin^2\omega\right)$. The net lift force due to the external pressure acting on the surface of the hemisphere is:

$$L = -R^2\int_0^{\pi}\left\{\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\left[p_\infty + \frac{1}{2}\rho_\infty U_\infty^2\left(1 - \frac{9}{4}\sin^2\omega\right)\right]\sin\omega\,\cos\theta\,(\sin\omega\,d\omega)\,d\theta\right.$$

$$L = -R^2\int_0^{\pi}\left[p_\infty + q_\infty\left(1-\frac{9}{4}\sin^2\omega\right)\right]\sin^2\omega\left[\sin\theta\Big|_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\right]d\omega$$

$$L = -2R^2\int_0^{\pi}\left[p_\infty + q_\infty\left(1-\frac{9}{4}\sin^2\omega\right)\right]\sin^2\omega\,d\omega$$

$$L = -2R^2\left\{\left[(p_\infty + q_\infty)\left(\frac{\omega}{2} - \frac{\sin 2\omega}{4}\right) - \frac{9}{4}q_\infty\left(\frac{3\omega}{8} - \frac{\sin 2\omega}{4} + \frac{\sin 4\omega}{32}\right)\right]\Big|_0^{\pi}\right\} = -2R^2\left[(p_\infty + q_\infty)\frac{\pi}{2} - \frac{9}{4}q_\infty\frac{3\pi}{8}\right]$$

$$L = -\pi R^2 p_\infty + \frac{11}{16}\pi R^2 q_\infty$$

If the net lift force on the hemispherical shell is to be zero:

$$p_i\,\pi R^2 - p_\infty\,\pi R^2 + \frac{11}{16}\pi R^2 q_\infty = 0$$

Thus, $p_i = p_\infty - \frac{11}{16}q_\infty$. Rearranging these terms in a form corresponding to the pressure coefficient:

$$\frac{p_i - p_\infty}{q_\infty}\,(= C_p) = -\frac{11}{16}\,.\quad \text{Since } C_p = 1 - \frac{9}{4}\sin^2\omega$$

we compare the two expressions to find $\sin^2\omega = \frac{3}{4}$; $\omega = 60°$

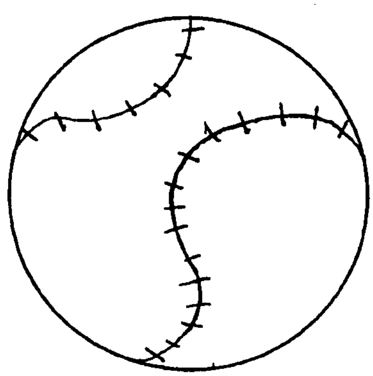
3.50) Let us first calculate Re_D to determine whether the Reynolds number is critical or subcritical.

$$Re_D = \frac{\left(0.002376\ \frac{lbf\,s^2}{ft^4}\right)\left(90\ \frac{mi}{h}\ \frac{5280\ \frac{ft}{mi}}{3600\ \frac{s}{h}}\right)\left(\frac{2.75}{12}\ ft\right)}{3.74\times10^{-7}\ \frac{lbf\,s}{ft^2}}$$

$$Re_D = 1.9222\times10^5$$

Clearly the Reynolds number is subcritical. If the ball were smooth, the forebody boundary layer. Scuffing the ball would promote boundary-layer transition downstream of the scuff. Thus, an asymmetric separation pattern results, producing a complex, asymmetric pressure field.

The seams of a baseball lead to a complex flow in any case. Large fluctuating forces are found to occur when the seam of the baseball coincide with the "point" where boundary-layer separation occurs (an angle of approximately 110°). The separation point jumps because of the transition-promoting roughness of the seam stiches, produces unsteady forces. This is the underlying principal of a Knuckleball.



3.51) Consider a source, located at the origin, so that the flow rate passing through a circle of radius r is proportional to K (see Example 3.4)

3.51 Contd.] Thus,

$$\vec{V} = \frac{K}{2\pi r}\hat{e}_r = v_r\hat{e}_r$$

The stream function is related to the velocity components:

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{K}{2\pi r}$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = 0$$

$$\psi = \int r\,v_r\,d\theta + f(r) = \int \frac{K}{2\pi}\,d\theta + f(r)$$

$$\psi = -\int v_\theta\,dr + g(\theta) = g(\theta)$$

Thus, for a source

$$\psi = \frac{K\theta}{2\pi}$$

as shown in Table 3.2. Similar approach would yield the other stream functions.

3.52) Velocity potential: if the flow is irrotational

Stream function: if the flow is two-dimensional, incompressible
(or two-dimensional, steady flow)

4.1 At standard sea-level conditions, we use Table 1.2 to find that $\rho_\infty = 1.225 \frac{kg}{m^3}$ and $\mu_\infty = 1.7894\times10^{-5}\frac{kg}{s\cdot m}$. Thus, the Reynolds number at the trailing edge of the wing is:

$$Re_c = \frac{(1.225)(15)(0.5)}{1.7894\times10^{-5}} = 5.1344\times10^5$$

This Reynolds number is only slightly above the 500,000 used as a "typical" transition Reynolds number. Thus, we shall assume that the flow is laminar over the entire wing. At the trailing edge:

$$\delta = \frac{5x}{(Re_x)^{0.5}} = \frac{5c}{(Re_c)^{0.5}} = \frac{5(0.5)}{(5.1344\times10^5)^{0.5}} = 3.489\times10^{-3}\,m$$

$$\delta^* = \frac{1.72c}{(Re_c)^{0.5}} = 1.2\times10^{-3}\,m;\; C_f = \frac{0.664}{(Re_c)^{0.5}} = 9.267\times10^{-4}$$

$$\tau = C_f\left(\frac{1}{2}\rho_\infty U_\infty^2\right) = \frac{0.664}{(Re_c)^{0.5}}\left[\frac{1}{2}(1.225)(15)^2\right] = 0.1277\,\frac{N}{m^2}$$

All of the above parameters are evaluated at a single streamwise location, the trailing edge.

The total drag can be calculated using Eq. (4.30)/(4.31)

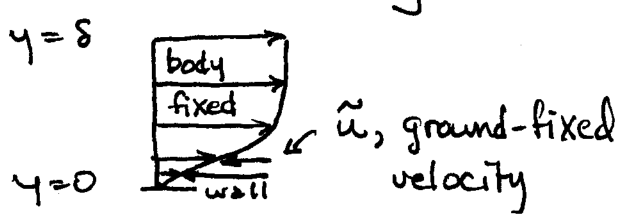
$$D = 1.328\,b\sqrt{c\,\rho_\infty \mu_\infty U_\infty^3}$$

$$D = 1.328(5)\sqrt{(0.5)(1.225)(1.7894\times10^{-5})(3375)} = 1.277\,N$$

To prepare a graph of $\tilde{u}$ as a function of y

$$\tilde{u} = U(1-f')$$

and $$y = \sqrt{\frac{2\nu x}{U_\infty}}\,\eta$$



4.2) The external, inviscid flow is given by: $u_e = Ax$

$$\beta = \frac{2s}{u_e}\frac{du_e}{ds} \quad . \quad s = \int u_e\,dx = \int Ax\,dx = 0.5Ax^2$$

$$\frac{du_e}{ds} = \frac{du_e}{dx}\frac{dx}{ds} = A\frac{1}{Ax} = \frac{1}{x}$$

Thus, $$\beta = \frac{2[0.5Ax^2]}{Ax}\frac{1}{x} = 1$$

Using Table 4.2, when $\beta = 1$, $f''(0) = 1.2326$

As given by Eqs. (4.18) and (4.19)

$$\tau = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0} = \mu\frac{u_e^2}{\sqrt{2\nu s}}f''(0)$$

Thus, in the presence of the pressure gradient represented by $\beta = 1$

$$\tau_{\beta=1} = \mu\frac{(Ax)^2}{\sqrt{2\nu(0.5Ax^2)}}\,1.2326 = \frac{1.2326\,\mu A^{1.5}x}{\sqrt{\nu}}$$

where A obviously has the units of s^{-1}. For a laminar boundary-layer flow over a flat plate, $u_e = U_\infty$. As a result, $\beta = 0$. For a flat plate:

$$\tau_{\beta=0} = \mu\frac{U_\infty^2}{\sqrt{2\nu(U_\infty x)}}\,0.4696 = \frac{0.332\,\mu U_\infty^{1.5}}{\sqrt{\nu x}}$$

As a result:

$$\frac{\tau_{\beta=1}}{\tau_{\beta=0}} = \frac{3.712\,(Ax)^{1.5}}{U_\infty^{1.5}}$$

4.3) Given: $u_e = 2U_\infty \sin\theta$ and $x = R\theta$

Thus, $dx = R\,d\theta$ and $s = \int u_e\,dx = \int 2U_\infty \sin\theta\, R\,d\theta$

so that $s = 2RU_\infty(-\cos\theta\big|_0^\theta = 2RU_\infty(1-\cos\theta)$

$$\frac{du_e}{ds} = \frac{du_e}{d\theta}\frac{d\theta}{ds} = \left[2U_\infty\cos\theta\right]\left[\frac{1}{2RU_\infty\sin\theta}\right] = \frac{\cos\theta}{R\sin\theta}$$

Thus, $\beta = \dfrac{2s}{u_e}\dfrac{du_e}{ds} = \dfrac{2\left[2RU_\infty(1-\cos\theta)\right]}{2U_\infty\sin\theta}\,\dfrac{\cos\theta}{R\sin\theta}$

$$\beta = \frac{2(1-\cos\theta)\cos\theta}{\sin^2\theta} = \frac{2(1-\cos\theta)\cos\theta}{(1-\cos\theta)(1+\cos\theta)} = \frac{2\cos\theta}{1+\cos\theta}$$

when	θ	=	0.0°	30°	45°	90°
	β	=	1.0	0.9282	0.8284	0.0

4.4) Using Eq. (4.14)

$$v = -u_e\sqrt{2\nu s}\left[\left(\frac{\partial\eta}{\partial s}\right)_y\left(\frac{\partial f}{\partial\eta}\right)_s + \left(\frac{\partial f}{\partial s}\right)_\eta + \frac{f}{2s}\right]$$

but $\left(\frac{\partial f}{\partial\eta}\right)_s = \frac{u}{u_e} = 0$ at the wall, because of the no-slip Condition. In order to generate similar solutions, $\left(\frac{\partial f}{\partial s}\right)_\eta$ must be zero. I.e., for similar solutions, f is a function of η only. Thus, Eq. (4.14) becomes:

$$v(0) = v_w = -u_e\sqrt{2\nu s}\,\frac{f(0)}{2s}$$

Rearranging: $\dfrac{v_w}{u_e} = -\sqrt{\dfrac{\nu}{2s}}\,f(0)$. For flow past a flat plate, $s = \int u_e\,dx = \int U_\infty\,dx = u_e x$. As a result:

$$\frac{v_w}{u_e} = -\sqrt{\frac{\nu}{2u_e x}}\,f(0) = -\frac{f(0)}{\sqrt{2\,Re_x}}$$

4.5] Since we want $v_w = -0.001 u_e$ for steady flow past a flat plate. Using the relation developed in the previous problem,

$$\frac{v_w}{u_e} = -\frac{f(0)}{\sqrt{2\,Re_x}}$$

For this flow, $Re_x = \dfrac{(1.225)(10)x}{1.7894\times10^{-5}} = 6.8459\times10^{5}\,x$

Thus,
$$-0.001 = \frac{-f(0)}{\sqrt{1.3692\times10^{6}\,x}}\;;\; f(0) = 1.170\sqrt{x}$$

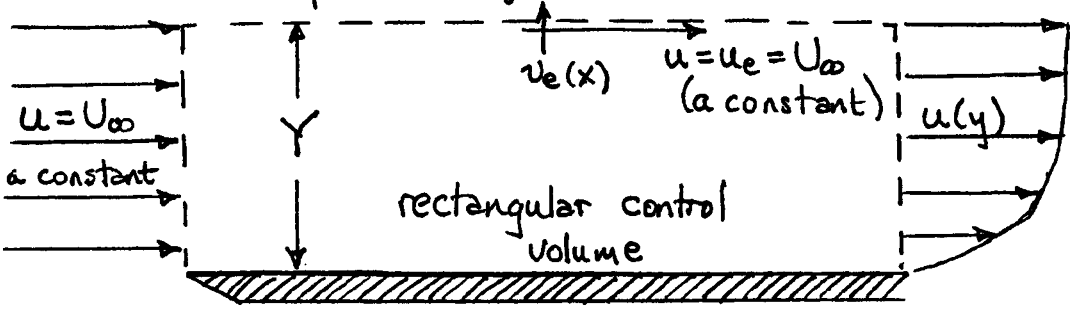
Also, $f'(0)=0$ and $f'(\infty)=1$ (as usual)

4.6] In this case, we are using blowing into the boundary layer to reduce skin friction. Thus, $v(0)=v_w>0$ and $f(0)<0$. In particular, $f(0) = -0.25$. Using the equation developed in Problem 4.4,

$$\frac{v_w}{u_e} = -\frac{f(0)}{\sqrt{2Re_x}}\;;\; Re_x = \frac{(0.00237)(50)x}{3.744\times10^{-7}} = 3.165\times10^{5}x$$

Thus, $$v_w = \frac{1.571\times10^{-2}}{\sqrt{x}}$$

4.7] We are to use the integral form of the equations of motion using a rectangular control volume.



The continuity equation for a steady flow is:

$$\oiint \rho\vec{V}\cdot\hat{n}\,dA = 0 \qquad (2.5)$$

4.7 Contd. Since we are considering a two-dimensional flow, the continuity equation becomes:

$$+\int_0^Y \rho U_\infty \hat{i}\cdot(-\hat{i}\,dy) + \int_0^Y \rho\left[u\hat{i} + v\hat{j}\right]\cdot(\hat{i}\,dy)$$

$$+\int_0^L \rho\left[u_e\hat{i} + v_e\hat{j}\right]\cdot(j\,dx) = 0$$

where Y is equal to (or greater than) the boundary-layer thickness at the downstream end of the control volume. Note that the third term in this equation represents the mass flowing through the top (horizontal) surface of the rectangular control volume. This term represents the difference between the influx at the vertical, left-end surface (the first term) and the efflux at the vertical, right-end surface (the second term) – which is less because of the streamwise growth of the boundary layer. Since the density is constant:

$$\int_0^Y (u - U_\infty)\,dy + \int_0^L v_e\,dx = 0$$

So that:

$$\int_0^L v_e\,dx = \int_0^Y (U_\infty - u)\,dy = U_\infty \int_0^Y \left(1 - \frac{u}{U_\infty}\right)dy$$

For a flat plate, $u_e = U_\infty$, a constant. Using the displacement thickness (for an incompressible boundary layer):

$$\delta^* = \int_0^Y \left(1 - \frac{u}{u_e}\right)dy \qquad (4.26)$$

We note that: $\int_0^L v_e\,dx = u_e\,\delta^*$

The integral momentum equation for steady flow:

$$\sum \vec{F} = \oiint (\rho\vec{V}\cdot\hat{n}\,dA)\vec{V} \qquad (2.13)$$

Let us evaluate the terms in this equation for the flat-plate flow (depicted in the sketch) in the

4.7 Contd.] x-direction, which includes the drag per unit span. To more clearly identify the terms, we will include the intermediate steps with the complete vector notation. Since the upper boundary of the control volume is outside of the boundary layer and since the pressure is the same at both ends of the control volume, the only force acting on the fluid within the control volume is the negative of the drag per unit span.

$$-d\hat{i} = \int_0^Y \rho U_\infty \hat{i} \cdot (\hat{i}\,dy)\, U_\infty \hat{i}$$

$$+ \int_0^Y \rho \left[u\hat{i} + v\hat{j}\right] \cdot (\hat{i}\,dy)\, u\hat{i}$$

$$+ \int_0^L \rho \left[u_e\hat{i} + v_e\hat{j}\right] \cdot (\hat{j}\,dx)\, u_e\hat{i}$$

Note that the last term is the streamwise component of the momentum which carried by the fluid passing through the top of our control volume ($\int_0^L v_e\,dx$). Thus,

$$-d = -\rho U_\infty^2 \int_0^Y dy + \rho \int_0^Y u^2 dy + \rho u_e \int_0^L v_e\,dx$$

From the continuity equation, we learned that:

$$\int_0^L v_e\,dx = \int_0^Y (u_e - u)\,dy$$

For a flat-plate flow, $u_e = U_\infty$

$$-\frac{d}{\rho} = \int_0^Y (u^2 - u_e^2)\,dy + u_e \int_0^Y (u_e - u)\,dy$$

Thus,

$$-\frac{d}{\rho} = \int_0^Y (u^2 - u u_e)\,dy = u_e^2 \int_0^Y \frac{u}{u_e}\left(\frac{u}{u_e} - 1\right) dy$$

4.7 Contd.] Recalling that the momentum thickness is defined as:

$$\theta = \int_0^{\delta} \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)dy = \int_0^{\infty} \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)dy \quad (4.28)$$

noting that the upper limit can be Y, δ, or ∞, since $1 - \frac{u}{u_e} = 0$ for $y \geq \delta$, the section drag coefficient is:

$$C_d = \frac{d}{\frac{1}{2}\rho u_e^2 L} = \frac{2\theta}{L}$$

which is exactly the same as Eq.(4.75).

4.8] We are told that the velocity profile for a laminar boundary layer is: $\frac{u}{u_e} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

Note that this approximation satisfies the boundary conditions that: $u = 0$, when $y = 0$; and $u = u_e$, when $y = \delta$.

Let us employ the definition for θ:

$$\theta = \int_0^{\delta} \frac{u}{u_e}\left[1 - \frac{u}{u_e}\right]dy$$

Substituting the given velocity distribution:

$$\theta = \int_0^{\delta} \left\{ \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 - \frac{9}{4}\left(\frac{y}{\delta}\right)^2 + \frac{3}{2}\left(\frac{y}{\delta}\right)^4 - \frac{1}{4}\left(\frac{y}{\delta}\right)^6 \right\} dy$$

$$\theta = \frac{3}{4}\frac{y^2}{\delta} - \frac{1}{8}\frac{y^4}{\delta^3} - \frac{9}{12}\frac{y^3}{\delta^2} + \frac{3}{10}\frac{y^5}{\delta^4} - \frac{1}{28}\frac{y^7}{\delta^6}\Big|_0^{\delta}$$

$$\theta = (0.750 - 0.125 - 0.750 + 0.300 - 0.035714)\,\delta$$

$$\theta = 0.13929\,\delta$$

At this point, we do not know how either θ (the momentum thickness) or δ (the boundary-layer thickness) depend either on x or on Re_x. Thus, we need another equation. Recall that:

$$C_d = 2\theta/L$$

4.8 Contd.) Since we know that, for a flat plate at zero angle of attack, the drag is due entirely to the viscous shear layer at the wall, we can write:

$$C_d = \frac{d}{\frac{1}{2}\rho u_e^2 L} = \frac{\int_0^L \tau \, dx}{\frac{1}{2}\rho u_e^2 L}$$

Recall that $\tau = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \mu \left\{\frac{3}{2}\frac{u_e}{\delta}\right\}$

Substituting this expression for τ into our expression for C_d and equating the two expressions for C_d:

$$\frac{2\theta}{L} = \frac{2}{L} \frac{\int_0^L \mu \left\{\frac{3}{2}\frac{u_e}{\delta}\right\} dx}{\rho u_e^2}$$

Simplifying and the differentiating:

$$d\theta = 0.13929\, d\delta = \frac{3}{2}\frac{\mu}{\rho u_e \delta}\, dx$$

Rearranging: $\delta \, d\delta = 10.7689 \frac{\mu}{\rho u_e} dx$

Integrating and noting that $\delta = 0$, when $x = 0$:

$$\frac{1}{2}\delta^2 = 10.7689 \frac{\mu}{\rho u_e} x$$

Thus, $\delta = 4.6409 \sqrt{\frac{\mu x}{\rho u_e}}$

or $\frac{\delta}{x}\sqrt{Re_x} = 4.6409$ (one of the relations sought)

Since we have already shown that $\theta = 0.13929\,\delta$,

$$\frac{\theta}{x}\sqrt{Re_x} = 0.64643$$

4.8 Contd.) To develop the relation for the displacement thickness:

$$\delta^* = \int_0^\delta \left[1 - \frac{u}{u_e}\right] dy = \int_0^\delta \left[1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy$$

$$\delta^* = \left[y - \frac{3}{4}\frac{y^2}{\delta} + \frac{1}{8}\frac{y^4}{\delta^3}\right|_0^\delta = (1.000 - 0.750 + 0.125)\,\delta$$

Thus, $\delta^* = 0.375\,\delta$ or $\frac{\delta^*}{x}\sqrt{Re_x} = 1.7403$

To calclulate C_d, we note that: $C_d = \frac{2\theta}{L}$

We note also that:

$$C_d = \frac{d}{\frac{1}{2}\rho u_e^2 L} = \frac{\int \tau\, dx}{\frac{1}{2}\rho u_e^2 L} = \frac{1}{L}\int C_f\, dx$$

Thus,
$$C_f = \frac{dC_d}{dx} L = 2\,\frac{d\theta}{dx}$$

Since
$$\theta = \frac{0.64643\sqrt{x}}{\sqrt{\frac{\rho u_e}{\mu}}},$$

$$C_f = 2\left\{\frac{0.64643\,(0.5\,x^{-0.5})}{\sqrt{\frac{\rho u_e}{\mu}}}\right\} = \frac{0.64643}{\sqrt{Re_x}}$$

Thus,
$$C_f\sqrt{Re_x} = 0.64643$$

Note, we could have obtained the same result by using the definition that:

$$C_f = \frac{\tau}{\frac{1}{2}\rho u_e^2} \quad \text{where} \quad \tau = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \mu\,\frac{3}{2}\,\frac{u_e}{\delta}$$

Thus, $C_f = 3\left(\frac{\mu}{\rho u_e \delta}\right)$. Since $\delta = \frac{4.6409\,x}{\sqrt{Re_x}}$

4.8 Contd.) $C_f = \dfrac{3\sqrt{Re_x}}{4.6409\left(\frac{\rho u_e x}{\mu}\right)}$. So that, $C_f\sqrt{Re_x} = 0.64643$.

The continuity equation for a rectangular control volume gives:

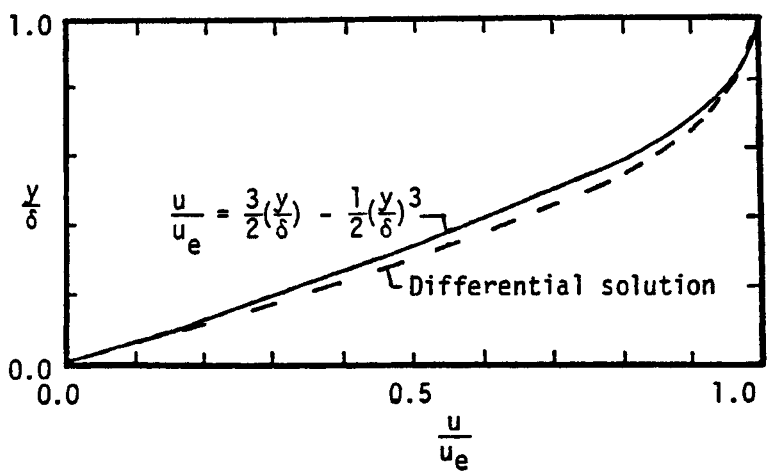
$$\int_0^L v_e\,dx = -\int_0^Y (u - u_e)\,dy = u_e\,\delta^*$$

Differentiating, we obtain $v_e = \dfrac{d}{dx}(u_e\,\delta^*) = u_e\dfrac{d\delta^*}{dx}$

But we have already shown that: $\delta^* = \dfrac{1.7403\sqrt{x}}{\sqrt{\frac{\rho u_e}{\mu}}}$

Comparison of the values for the integral solution using the assumed velocity profile with the values obtained solving the differential equation

	Integral Approach	Differential Approach
$\frac{\delta}{x}\sqrt{Re_x}$	4.6409	5.0
$\frac{\delta^*}{x}\sqrt{Re_x}$	1.7403	1.72
$\frac{v_e}{u_e}\sqrt{Re_x}$	0.8702	0.84
$C_f\sqrt{Re_x}$	0.64643	0.664
$C_d\sqrt{Re_x}$	1.2929	1.328



4.8 Contd.] Thus,

$$v_e = u_e \left\{ \frac{1.7403(0.5x^{-0.5})}{\sqrt{\frac{\rho u_e}{\mu}}} \right\}$$

So that:

$$\frac{v_e}{u_e}\sqrt{Re_x} = 0.8702$$

The results are summarized on the previous page's table.

4.9] For a linear profile: $\frac{u}{u_e} = \frac{y}{\delta}$. We can employ the definition for the momentum thickness for an incompressible boundary layer, Eq. (4.28):

$$\theta = \int_0^\delta \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\Big|_0^\delta$$

$\theta = \frac{1}{6}\delta$. At this point, neither the momentum thickness (θ) nor the boundary-layer thickness (δ) are known as a function of x or of Re_x. Recall that:

$$C_d = \frac{2\theta}{L} = \frac{d}{\frac{1}{2}\rho u_e^2 L} = \frac{\int_0^L \tau\, dx}{\frac{1}{2}\rho u_e^2 L}$$

But $\quad \tau = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u_e}{\delta}$

Thus, $\quad \theta = \frac{\int_0^L \tau\, dx}{\rho u_e^2} = \frac{\int_0^L \frac{\mu}{\delta} u_e\, dx}{\rho u_e^2} = \frac{\mu}{\rho u_e \delta}\int_0^L dx$

So that: $\quad d\theta = \frac{\mu}{\rho u_e \delta} dx = \frac{1}{6} d\delta$

$\delta\, d\delta = \frac{6\mu}{\rho u_e} dx$. Integrating and noting that $\delta = 0$, when $x = 0$:

4.9 Contd.

$$\delta^2 = \frac{12\mu x}{\rho u_e}$$

Thus, $\dfrac{\delta}{x} = \dfrac{3.464}{\sqrt{Re_x}}$ or $\dfrac{\delta}{x}\sqrt{Re_x} = 3.464$

Since $\theta = \frac{1}{6}\delta$, $\dfrac{\theta}{x}\sqrt{Re_x} = 0.5773$

$$C_f = \frac{\tau}{\frac{1}{2}\rho u_e^2} = \frac{2\mu}{\rho u_e^2}\frac{u_e}{\delta} = \frac{2\mu}{\rho u_e}\frac{\sqrt{Re_x}}{3.464x} = \frac{0.5773}{\sqrt{Re_x}}$$

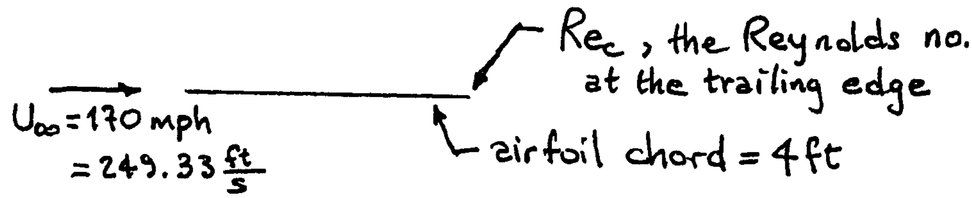
$$\delta^* = \int_0^\delta \left[1 - \frac{u}{u_e}\right] dy = y - \frac{y^2}{2\delta}\Big|_0^\delta = \frac{1}{2}\delta$$

Thus, $\delta^* = \dfrac{1.732x}{\sqrt{Re_x}}$; $\dfrac{\delta^*}{x}\sqrt{Re_x} = 1.732$

$C_d = \dfrac{2\theta}{L}$ and $\theta = \dfrac{0.5773x}{\sqrt{Re_x}}$

$$C_d\sqrt{Re_x} = 2(0.5773) = 1.1546$$

4.10



$$Re_c = \frac{\rho_\infty U_\infty c}{\mu_\infty} = \frac{\left(0.002376\ \frac{lbf\ s^2}{ft^4}\right)\left(249.33\ \frac{ft}{s}\right)(4.0\ ft)}{3.740\times 10^{-7}\ \frac{lbf\ s}{ft^2}}$$

$$Re_c = 6.336\times 10^6$$

Flow is clearly turbulent at the trailing edge. Let us calculate the transition location assuming $Re_{x,tr}$ is 500,000.

4.10 Contd.)

$$x_{tr} = \frac{500,000}{\frac{(0.002376)(249.33)}{3.740\times10^{-7}}} = 0.316 \text{ ft}$$

Since less than 10% of the wing experiences a laminar boundary layer, let us assume turbulent flow along the entire length.

$$C_f = \frac{0.0583}{(Re_x)^{0.2}} \quad \text{and} \quad \tau = C_f\left[\frac{1}{2}\rho u_e^2\right]$$

The total friction drag is:

$$D = 2(\text{span})\int_0^{\text{chord}} \tau\, dx$$

$$D = 2(28)\left[\frac{1}{2}(0.002376)(249.33)^2\right]\frac{0.0583}{17.376}\int_0^4 \frac{dx}{x^{0.2}}$$

$$D = 13.876 \left.\frac{x^{0.8}}{0.8}\right|_0^4 = 52.580 \text{ lbs}$$

$$C_D = \frac{D}{q_\infty c b} = \frac{52.580}{\left[\frac{1}{2}(0.002376)(249.33)^2(4)(28)\right]}$$

$$C_D = 0.00636$$

4.11) (a) The unit Reynolds number, i.e., the Reynolds number per meter, is:

$$Re_\infty/m = \frac{\rho_\infty U_\infty}{\mu_\infty} = \frac{\left(1.2250\,\frac{kg}{m^3}\right)\left(100\,\frac{m}{s}\right)}{1.7894\times10^{-5}\,\frac{kg}{m\cdot s}} = 6.8459\times10^6/m$$

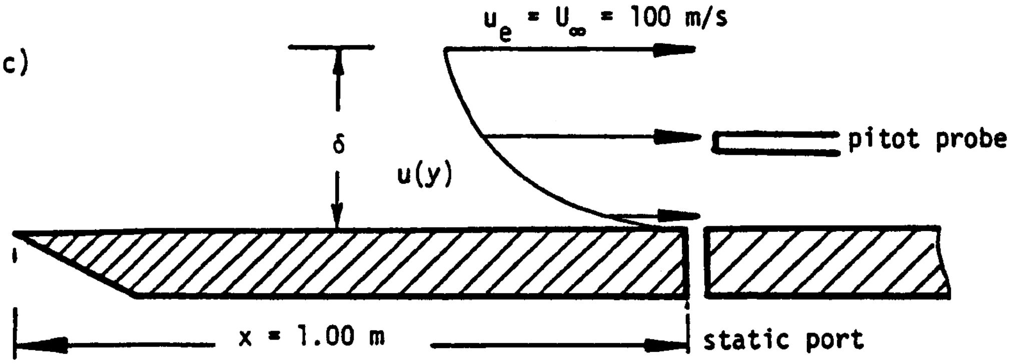
To calculate the location at which transition occurs:

$$x_{tr} = \frac{Re_{x_{tr}}}{Re_\infty/m} = \frac{500,000}{6.8459\times10^6/m} = 0.0730\text{ m}$$

4.11 Contd.) (b) Note that transition occurs well upstream of the location at which the pitot probe is used to determine the velocity profile. Transition occurs so near the leading edge of the plate, that we may use the relations for a boundary layer which is turbulent along its entire length. To calculate the thickness of the turbulent boundary layer at a point 1.00 m from the leading edge, we use Eq. (4.79):

$$\delta = \frac{0.3747\, x}{(Re_x)^{0.2}} = \frac{0.3747\,(1.00\text{ m})}{(6.8459\times10^6)^{0.2}} = 0.01609\text{ m}$$

(c)



Because the variation of the static pressure across the boundary layer is negligible, i.e., $\frac{\partial p}{\partial y} \approx 0$. $p_\infty = p_w = p_{static}$

Thus, we apply Bernoulli's equation along a streamline (in this rotational flow) over a distance too short to be affected by viscosity to relate the velocity at a point in the boundary layer and the pitot pressure measured by the probe at that "point"

$$p_{static} + \frac{1}{2}\rho_\infty [u(y)]^2 = p_t(y)$$

$$p_t(y) - p_{static} = \frac{1}{2}\rho_\infty [u(y)]^2 = 0.6125\,[u(y)]^2$$

where $u(y) = \left(\frac{y}{\delta}\right)^{1/7} u_e$. Thus,

4.11 Contd.

$$p_t(y) - p_{static} = 1.993 \times 10^4 \, y^{0.2857}$$

To calculate the pressure coefficient for the pitot probe:

$$C_p(y) = [p_t(y) - p_{static}] \Big/ \tfrac{1}{2} \rho_\infty U_\infty^2$$

where, since we are considering flow past a flat plate, $u_e = U_\infty$. We can combine these equations to get:

$$C_p(y) = \left[\frac{u(y)}{U_\infty}\right]^2 = \left[\frac{y}{\delta}\right]^2 = 3.2538 \, y^{0.2857}$$

As a result, we obtain the values in the following table:

y (m)	$\frac{y}{\delta}$	u (m/s)	$p_t(y)-p_{static}$ (N/m^2)	$c_p(y)$
0.0000	0.00	0.00	0.0	0.0000
0.0008	0.05	65.18	2602.5	0.4249
0.0016	0.1	71.97	3172.4	0.5179
0.0032	0.2	79.46	3867.2	0.6314
0.0048	0.3	84.20	4342.2	0.7089
0.0064	0.4	87.73	4714.2	0.7697
0.0080	0.5	90.57	5024.6	0.8203
0.0097	0.6	92.96	5293.2	0.8642
0.0113	0.7	95.03	5531.6	0.9031
0.0129	0.8	96.86	5746.7	0.9382
0.0145	0.9	98.51	5943.4	0.9703
0.0161	1.0	100.00	6125.0	1.0000

(d) Since we are dealing with the flow in a boundary layer, the velocity function should be rotational.

$$u = \left[\frac{y}{\delta}\right]^{1/7} u_e$$

Noting that $\delta = \dfrac{0.3747\,x}{(Re_x)^{0.2}}$

Thus, $u = C_1 \, y^{0.1428} \, x^{-0.1143}$

4.11 Contd. By continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial v}{\partial y} = +0.1143\, C_1 \frac{y^{0.1428}}{x^{1.1143}}$$

Since $v = 0$ when $y = 0$, $v = C_2 \frac{y^{1.1428}}{x^{1.1143}}$

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ C_1 \frac{y^{0.1428}}{x^{0.1143}} & C_2 \frac{y^{1.1428}}{x^{1.1143}} & 0 \end{vmatrix}$$

$$\nabla \times \vec{V} = \left[-1.1143\, C_2 \frac{y^{1.1428}}{x^{2.1143}} - 0.1428\, C_1 \frac{1}{x^{0.1143}\, y^{0.8572}} \right] \hat{k}$$

Since $\nabla \times \vec{V} \neq 0$, the flow is rotational (as expected)

4.12 For air at atmospheric pressure and 100°C

$$\rho_\infty = \frac{p_\infty}{R T_\infty} = \frac{1.01325 \times 10^5\ \text{N/m}^2}{\left(287.05\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(373.15\,\text{K})} = 0.9460\ \frac{\text{kg}}{\text{m}^3}$$

$$\mu_\infty = 1.458 \times 10^{-6} \frac{(373.15)^{1.5}}{373.15 + 110.4} = 2.173 \times 10^{-5}\ \frac{\text{kg}\cdot\text{s}}{\text{m}}$$

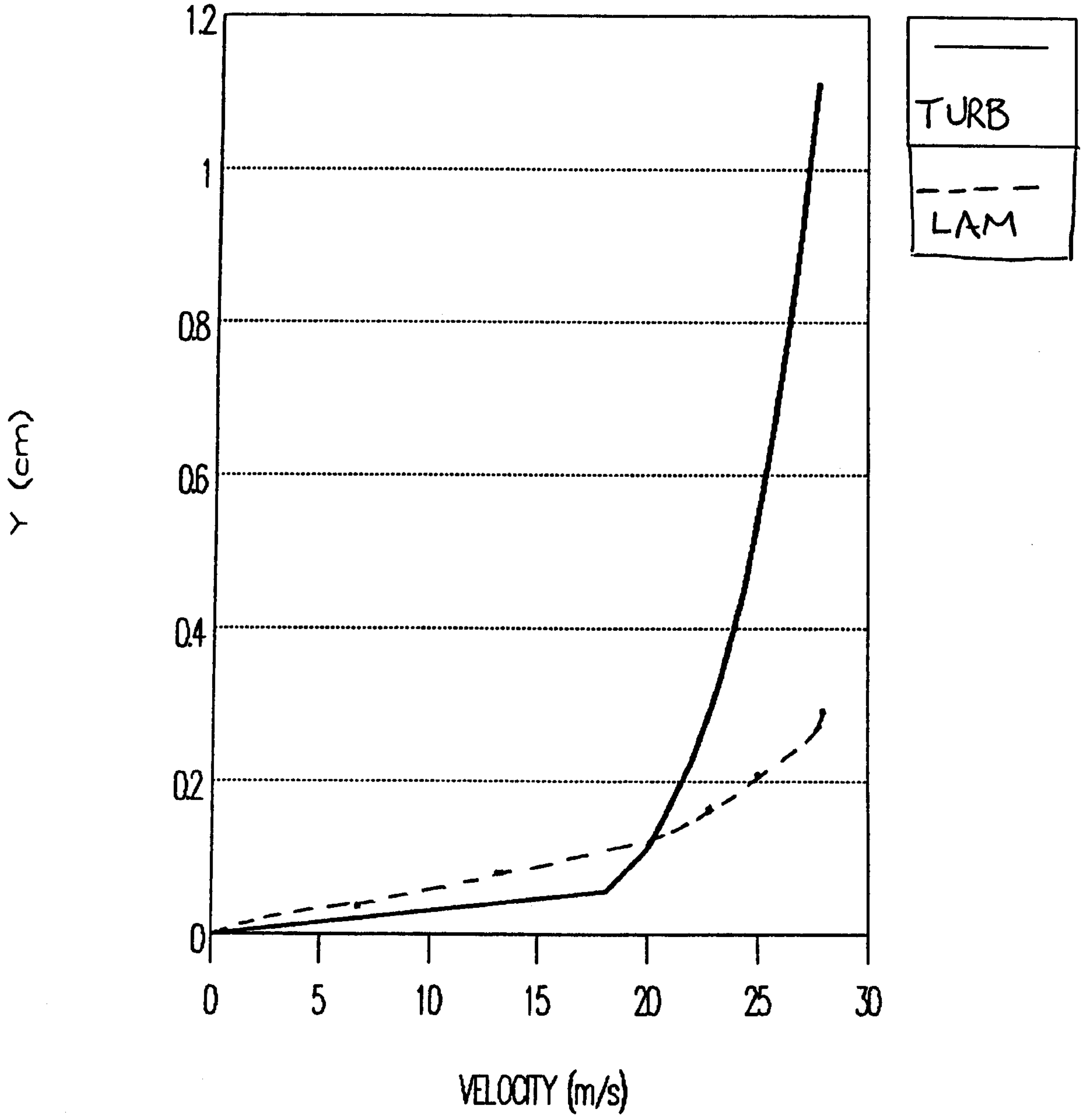
$$U_\infty = 100\ \frac{\text{km}}{\text{h}}\ \frac{1000\ \frac{\text{m}}{\text{km}}}{3600\ \frac{\text{s}}{\text{h}}} = 27.778\ \frac{\text{m}}{\text{s}}$$

Note that since the Mach number is less than 0.2, we can use the transition criteria for incompressible flow past a flat plate: $Re_{x,tr} = 500{,}000$.

Thus, $x_{tr} = Re_{x,tr} / (\rho_\infty U_\infty / \mu_\infty) = 0.4135\,\text{m}$

4.12 Contd.

BOUNDARY LAYER PROFILES



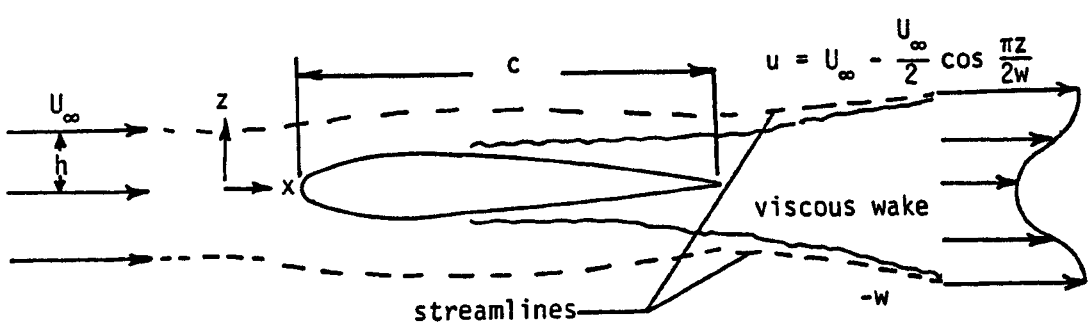
The thickness of the laminar boundary layer at this point, $\delta_{lam} = 5.0x/\sqrt{Re_x} = 0.002924\ m = 0.2924\ cm$

4.12 Contd.) For comparison, we will calculate the thickness of a turbulent boundary layer at this point for the same Reynolds number, assuming that the boundary layer is turbulent all the way from the leading edge:

$$\delta_{turb} = \frac{0.3747\,x}{(Re_x)^{0.2}} = 0.011230\,\text{m} = 1.1230\,\text{cm}$$

The results are presented on the graph on the previous page. Note that the streamwise component of velocity (u) increases much more rapidly with y near the wall for the turbulent boundary layer. Thus, the shear at the wall is greater for the turbulent boundary layer, even though the turbulent boundary layer is much thicker than the laminar boundary layer for the same edge conditions at a given x-station. The macroscopic transport of fluid in the y-direction causes the turbulent boundary layer to have both increased thickness (δ) and increased shear at the wall.

4.13)



The continuity equation for this steady, two-dimensional flow gives:

4.13 Contd.

$$-\rho_\infty U_\infty (2h) + \int_{-w}^{+w} \rho_\infty \left[U_\infty - \frac{U_\infty}{2}\cos\frac{\pi z}{2w}\right] dz = 0$$

Integrating:

$$-\rho_\infty U_\infty (2h) + \rho_\infty U_\infty \left[z - \frac{1}{2}\frac{2w}{\pi}\sin\frac{\pi z}{2w}\right|_{-w}^{+w} = 0$$

Dividing through by $\rho_\infty U_\infty$ and rearranging:

$$2h = 2w - \frac{w}{\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

So that

$$h = w\left(1 - \frac{1}{\pi}\right) = 0.61869w$$

Thus, we have used the continuity equation to establish the relation between the spacing of the streamlines of the undisturbed flow ahead of the airfoil (h) in terms of the spacing between the same two streamlines, which are just at the edge of the viscous wake downstream of the airfoil (w). Using the integral form of the momentum equation:

$$-d = -\rho_\infty U_\infty^2 2h + \int_{-w}^{+w} \rho_\infty \left(U_\infty - \frac{U_\infty}{2}\cos\frac{\pi z}{2w}\right)^2 dz$$

Note that the force of the airfoil on the fluid per unit span (which is the negative of the drag per unit span) is equal to the net efflux of momentum through the surface of our control volume. Since the control volume is bounded by streamlines on the top and on the bottom, there is neither mass flow nor momentum flux across these streamlines.

$$-d = -\rho_\infty U_\infty^2 2h + \int_{-w}^{+w} \rho_\infty U_\infty^2 dz - \int_{-w}^{+w} \rho_\infty U_\infty^2 \cos\frac{\pi z}{2w} dz$$

$$\frac{1}{4}\int_{-w}^{+w} \rho_\infty U_\infty^2 \cos^2\frac{\pi z}{2w} dz$$

$$-d = \rho_\infty U_\infty^2 \left\{-2h + 2w - \frac{2w}{\pi}\sin\frac{\pi z}{2w}\right|_{-w}^{+w} + \frac{1}{4}\left[\frac{z}{2} + \frac{\sin\frac{2\pi z}{2w}}{(4\pi/2w)}\right|_{-w}^{+w}$$

4.13 Contd.

$$-d = \rho_\infty U_\infty^2 \left\{ -2h + 2w - \frac{2w}{\pi}\left[1-(-1)\right] + \frac{1}{4}\left[\frac{w}{2} - \left(-\frac{w}{2}\right)\right] \right\}$$

$$-d = \rho_\infty U_\infty^2 \left\{ -2h + 2w - \frac{4w}{\pi} + \frac{w}{4} \right\}$$

$$-d = \rho_\infty U_\infty^2 \left\{ -1.36338w + 2.0000w - 1.27324w + 0.2500w \right\}$$

$$-d = 0.38662\, w \rho_\infty U_\infty^2$$

Noting that $w = 0.009c$, the section drag coefficient is given by:

$$C_d = \frac{d}{\frac{1}{2}\rho_\infty U_\infty^2 c} = \frac{0.38662(0.009c)\rho_\infty U_\infty^2}{\frac{1}{2}\rho_\infty U_\infty^2 c} = 0.00696$$

4.14)

Given: Standard day sea-level, "flat plate" wing:

$U_\infty = 15\ m/s$

chord $c = 0.5\ m$

span $b = 5\ m$

1976 atm: $\rho = 1.225$ kg/m^3

$\mu = 1.7894\times10^{-5}$ kg/m-s

Note: the problem statement should ask for the total skin friction drag coefficient and drag, although it is possible to find the total skin friction coefficient.

The drag is given by:

$$D = q_\infty S_{ref} C_D$$

$$S_{ref} = cb = (0.5m)(5.0m) = 2.5m^2$$

$$q_\infty = \rho U_\infty^2/2 = 137.8 \text{ Pa}$$

Reynolds number is give by $Re = \dfrac{\rho U_\infty c}{\mu} = 513{,}440 > 500{,}000$, so clearly a large part of the wing is laminar. In fact, transition takes place at $x_{tr} = \mathrm{Re}_{tr}/\mathrm{Re} = 500{,}000/513{,}440 = 97.4\%$ of the wing chord. According to Eqn. 4.86 the drag coefficient for one side of the flat plate wing is:

$$\overline{C}_f = \overline{C}_{f,turb}(c)\frac{cb}{S_{ref}} - \overline{C}_{f,turb}(x_{tr})\frac{x_{tr}b}{S_{ref}} + \overline{C}_{f,lam}(x_{tr})\frac{x_{tr}b}{S_{ref}}$$

4.14) contd.

where $\overline{C}_{f,turb}(x) = \dfrac{0.455}{\log(Re_x)^{2.58}}$ and $\overline{C}_{f,lam}(x) = \dfrac{1.328}{\sqrt{Re}}$

$\overline{C}_{f,turb}(c) = 0.005079$, $\overline{C}_{f,turb}(x_{tr}) = 0.004972$, $\overline{C}_{f,lam}(x_{tr}) = 0.001829$

therefore, $C_D = 0.001936$. For two sides of the wing, $C_D = 0.003872$, and the drag is given by

$$D = q_\infty bcC_D = 1.334 \text{ N}$$

Using the approximate method from Eqn. 4.87 the drag coefficient for two sides of the flat plate wing is:

$$C_D = \overline{C}_f \frac{S_{wet}}{S_{ref}} \qquad \text{where } S_{wet}/S_{ref} = 2cb/cb = 2$$

The total skin friction coefficient is $\overline{C}_f(c) = \dfrac{0.455}{\log_{10}(Re_c)^{2.58}} - \dfrac{1700}{Re_c}$

$\overline{C}_f(c) = 0.005079 - 0.003311 = 0.001768$ and $C_D = 0.003536$.

The drag is given by $D = q_\infty bcC_D = 1.218$ N. This is probably inaccurate since the empirical correction is being used for such a large region of the wing. In fact, using the laminar skin friction method from Eqn. 4.32 the drag coefficient for two sides of the flat plate wing is:

$$C_D = \overline{C}_{f,lam} \frac{S_{wet}}{S_{ref}}$$

where $\overline{C}_{f,lam}(x) = \dfrac{1.328}{\sqrt{Re_x}}$

$\overline{C}_{f,lam}(c) = 0.001853$ and $C_D = 0.003707$.

The drag is given by $D = q_\infty bcC_D = 1.277$ N. It would certainly be reasonable for this problem to assume that the plate was fully laminar.

4.15)

Given: Problem 4.11 conditions with $U_\infty = 80 m/s$

1976 atm: ρ = 1.225 kg/m^3
μ = 1.7894×10^{-5} kg/m-s

Assume the length of the plate is L = 1 m, as shown in Fig. P4.11. The drag is given by:

$$D = q_\infty S_{ref} C_D$$
$$q_\infty = \rho U_\infty^2 / 2 = 3920\ Pa$$

Reynolds number is give by $Re = \dfrac{\rho U_\infty L}{\mu} = 5.477 \times 10^6$. The Prandtl-Schlichting turbulent skin friction relation is:

$$\overline{C}_{f,turb}(x) = \frac{0.455}{\log(Re_x)^{2.58}} \text{ and } \overline{C}_{f,turb}(L) = 0.003314$$

therefore, $D = q_\infty S_{wet} \overline{C}_f$ and $D/b = q_\infty L \overline{C}_{f,turb} = 12.99\ N/m$.

Using the approximate method from Eqn. 4.87 the drag including transition:

$$\overline{C}_f(L) = \frac{0.455}{\log_{10}(Re_L)^{2.58}} - \frac{1700}{Re_L}$$

$\overline{C}_f(L)$ = 0.003314 – 0.00031 = 0.003004 and $D/b = q_\infty L \overline{C}_f$ = 11.77 N/m, which means the fully turbulent assumption is approximately 10% in error.

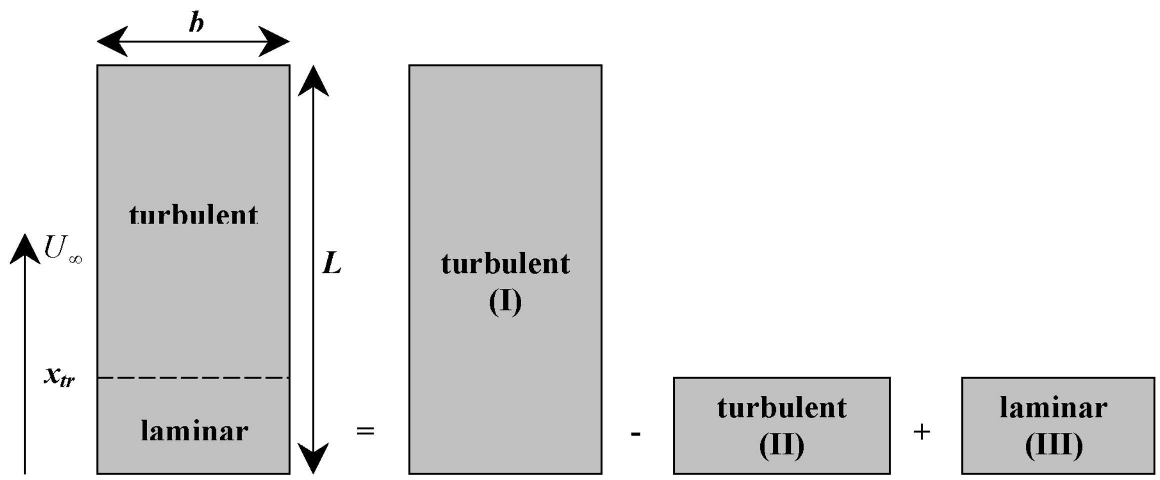
4.16)

Given: Equation 4.86 and Figure 4.18

Find: Derive the formulas for the drag coefficient and drag of the flat plate.

From Fig. 4.18, the drag coefficient for the flat plate (one side only) is found by determined by first finding the skin friction for turbulent flow over the entire plate, then finding the turbulent skin friction for the portion of the plate up to the transition location, and finally the laminar skin friction for the portion of the plate up to the transition location.

4.16) contd.



The drag coefficients for the three sections are:

$$C_{D_I} = \overline{C}_{f,turb}(L)\frac{S_{wet}}{S_{ref}} = \overline{C}_{f,turb}(L)\frac{Lb}{S_{ref}}$$

$$C_{D_{II}} = \overline{C}_{f,turb}(x_{tr})\frac{x_{tr}b}{S_{ref}}$$

$$C_{D_{III}} = \overline{C}_{f,lam}(x_{tr})\frac{x_{tr}b}{S_{ref}}$$

and the total drag coefficient is:

$$C_D = C_{D_I} - C_{D_{II}} + C_{D_{III}} = \overline{C}_{f,turb}(L)\frac{Lb}{S_{ref}} - \overline{C}_{f,turb}(x_{tr})\frac{x_{tr}b}{S_{ref}} + \overline{C}_{f,turb}(x_{tr})\frac{x_{tr}b}{S_{ref}}$$

The drag of the plate is:

$$D = C_D q_\infty S_{ref} = \overline{C}_{f,turb}(L)Lbq_\infty - \overline{C}_{f,turb}(x_{tr})x_{tr}bq_\infty + \overline{C}_{f,lam}(x_{tr})x_{tr}bq_\infty$$

$$D = bq_\infty\left[\overline{C}_{f,turb}(L)L - \overline{C}_{f,turb}(x_{tr})x_{tr} + \overline{C}_{f,lam}(x_{tr})x_{tr}\right] = Lbq_\infty\left[\overline{C}_{f,turb}(L) - \frac{x_{tr}}{L}\left(\overline{C}_{f,turb}(x_{tr}) - \overline{C}_{f,lam}(x_{tr})\right)\right]$$

4.17 $k = 4.76 \times 10^{-6} \dfrac{T^{1.5}}{T+112} = 4.76 \times 10^{-6} \dfrac{(2000)^{1.5}}{2112}$

$k = 2.0158 \times 10^{-4} \dfrac{\text{cal}}{\text{cm}\cdot\text{s}\cdot\text{K}}$; $\mu = 1.458 \times 10^{-6} \dfrac{T^{1.5}}{T+110.4} = 6.1792 \times 10^{-5}$ kg/s·m

$$Pr = \frac{\mu c_p}{\mu} = \frac{(6.1792\times10^{-5})(1004.7)}{(2.0158\times10^{-4})(4.187)(100)} = 0.738$$

Recall that it was stated in the text that the Prandtl number for air is essentially constant and approximately equal to 0.7 over a wide range of flow conditions. Note also, however, that the expressions used to calculate the thermal conductivity (k) and the specific heat (c_p) are for thermally perfect gas models. Therefore, they can be used only over a restricted temperature range.

4.18 From Eq. (4.109)

$$\theta = C \int_0^{\eta} (f'')^{Pr} d\eta + \theta_0$$

We are to apply the two boundary conditions:

(a) $\theta'(0) = 0$, i.e., the wall is adiabatic

(b) $\theta(\infty) = 1$, i.e., the edge temperature is T_e

Differentiating Eq. (4.109)

$$\theta' = C (f'')^{Pr}$$

Applying boundary condition (a): $C(f'')^{Pr} = 0$

But from the momentum equation, we know that

$f''(0) \neq 0$. Thus, C must equal 0.

4.18 Contd.]

Applying (b) to Eq. (4.109) $\theta(\infty) = 1 = \theta_0$

Thus, $\theta = \dfrac{T - T_w}{T_e - T_w} = 1$ everywhere. The only way that this can be true is if $T - T_w = T_e - T_w$ and, therefore $T_e = T_w = T$. Thus, for low-speed flow past an adiabatic wall, the temperature everywhere is constant and $T_w = T_e$

4.19] $U_\infty = 170 \frac{mi}{h} = 249.33 \frac{ft}{s}$

Let us find out where transition occurs:

$$x_{tr} = \frac{Re_{x,tr}}{\frac{\rho_\infty U_\infty}{\mu_\infty}} = \frac{500{,}000}{\frac{(0.002376)(249.33)}{3.74 \times 10^{-7}}} = 0.3157 \text{ ft}$$

A "rigorous analysis would treat part of the boundary layer as laminar and a subsequent region where the boundary layer is turbulent. However, since the flow is laminar for less than 0.1 of the wing chord, we will use turbulent relations for the entire length.

$k = 0.01466 \frac{Btu}{hr\ ft\ °F}$; $Pr = 0.712$

$$\dot{q} = \frac{(0.0292)\left[\frac{\rho_\infty U_\infty}{\mu_\infty}\right]^{0.8} x^{0.8} (Pr)^{0.333} k (T_e - T_w)}{x}$$

$$\dot{q} = \frac{(0.0292)(1.584 \times 10^6)(0.712)^{0.333} (0.01466)(9)}{x^{0.2}}$$

4.19 Contd.]

The total heating rate integrated over the wing is:

$$\dot{Q} = \int_{-14}^{+14}\left[\int_0^4 \dot{q}\,dx\right]dy = 28\left[3.1361\times10^2\int_0^4 \frac{dx}{x^{0.2}}\right]$$

$$\dot{Q} = 33\,274\ \frac{\text{Btu}}{\text{h}} = 9.2429\ \frac{\text{Btu}}{\text{s}}$$

4.20] $U = 70\ \frac{m}{s}$; $\rho = 1.225\ \frac{kg}{m^3}$; $\mu = 1.7894\times10^{-5}\ \frac{kg}{m\cdot s}$

$$Re_x = \frac{(1.225)(70)x}{1.7894\times10^{-5}} = 4.792\times10^6\,x$$

The Reynolds number indicates that the boundary layer on the walls of the tunnel is turbulent. Let us assume a $\frac{1}{7}$-th power law profile.

$$\delta^* = \int_0^\delta\left(1-\frac{u}{u_e}\right)dy \quad \text{with} \quad \frac{u}{u_e} = \left[\frac{y}{\delta}\right]^{1/7}$$

Thus, $$\delta^* = \int_0^\delta\left[1-\left(\frac{y}{\delta}\right)^{1/7}\right]dy = \left[y - \frac{7}{8}\left(\frac{y^{8/7}}{\delta^{1/7}}\right)\right]\Bigg|_0^\delta$$

So that $$\delta^* = \frac{1}{8}\delta = \frac{1}{8}\left[\frac{0.3747\,x}{(Re_x)^{0.2}}\right]$$

For $x = 1.5\,m$: $\delta^* = 2.988\times10^{-3}\ m$

$x = 6.0\,m$: $\delta^* = 9.058\times10^{-3}\ m$

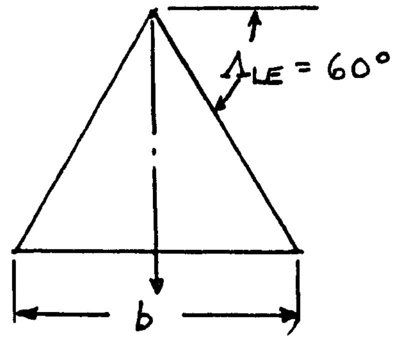
The walls must diverge by the angle θ, where

$$\theta = \tan^{-1}\left[\frac{0.009058 - 0.002988}{4.5}\right] = 0.0773^\circ$$

5.1

From Eq. (5.1)

$$AR = \frac{b^2}{S} = \frac{4}{\tan \Lambda_{LE}}$$



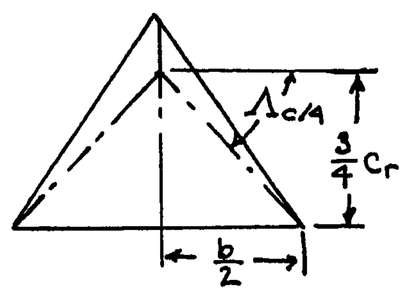
$$AR = \frac{4}{\tan 60°} = 2.309$$

$$\frac{b^2}{S} = AR = 2.309 \quad ; \text{Thus, } b^2 = (2.309)(58.65 m^2)$$

$$b = 11.637 \text{ m}$$

5.2

$$\tan \Lambda_{c/4} = \frac{\frac{3}{4} c_r}{\frac{1}{2} b}$$



$$\tan \Lambda_{c/4} = \frac{3 c_r}{2 b}$$

As with Example 5.1,

$$S = \frac{b c_r}{2} = \frac{b}{2}\left[\frac{2}{3} b \tan \Lambda_{c/4}\right] = \frac{b^2}{3} \tan \Lambda_{c/4}$$

Since $AR = \frac{b^2}{S}$ by definition; $AR = \frac{3}{\tan \Lambda_{c/4}}$

5.3 The equation for the drag of a finite wing is:

$$C_D = C_{D,0} + k C_L^2 \qquad (5.46)$$

Thus, the lift-to-drag ratio is: $\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D0} + k C_L^2}$

5.3 Contd. To find the lift coefficient for the maximum lift-to-drag ratio, we differentiate this expression with respect to C_L (noting that k and C_{Do} are constants).

$$\frac{d\left(\frac{L}{D}\right)}{dC_L} = \frac{1}{C_{Do}+kC_L^2} + \frac{C_L(-2kC_L)}{(C_{Do}+kC_L^2)^2}$$

When does this equal zero?

$$(C_{Do}+kC_L^2) - 2kC_L^2 = 0$$

$$C_{Do} = 2kC_L^2 - kC_L^2 = kC_L^2$$

Thus, $\dfrac{d\left(\frac{L}{D}\right)}{dC_L} = 0$, when $C_{Do} = kC_L^2$

or $C_L = \sqrt{\dfrac{C_{Do}}{k}}$

To verify that this corresponds to the maximum $\left(\frac{L}{D}\right)$, let us calculate the second derivative and determine its sign, when $C_{Do} = kC_L^2$.

$$\frac{d^2\left(\frac{L}{D}\right)}{dC_L^2} = \frac{-2kC_L}{(C_{Do}+kC_L^2)^2} - \frac{4kC_L}{(C_{Do}+kC_L^2)^2} - \frac{2kC_L^2(-2)(2kC_L)}{(C_{Do}+kC_L^2)^3}$$

$$\frac{d^2\left(\frac{L}{D}\right)}{dC_L^2} = -\frac{6kC_L}{(C_{Do}+kC_L^2)^2} + \frac{8k^2C_L^3}{(C_{Do}+kC_L^2)^3}$$

$$\frac{d^2\left(\frac{L}{D}\right)}{dC_L^2} = \frac{-6C_{Do}kC_L + 2k^2C_L^3}{(C_{Do}+kC_L^2)^3}$$

But we have shown that $C_{Do} = kC_L^2$ at this point,

5.3 Contd.] So that
$$\frac{d^2\left(\frac{L}{D}\right)}{dC_L^{\,2}} = \frac{-4k^2C_L^{\,3}}{(C_{D0}+kC_L^2)^3}$$

Since the second derivative is negative at the point where the first derivative is zero, this corresponds to the maximum (L/D) ratio. Thus,

$$\left(\frac{L}{D}\right)_{max} = \frac{C_{L(L/D)max}}{C_{D0}+k\left[C_{L(L/D)max}\right]^2} = \frac{\sqrt{\frac{C_{D0}}{k}}}{C_{D0}+k\left(\frac{C_{D0}}{k}\right)}$$

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{kC_{D0}}}$$

5.4] If the airplane is in level flight, its weight must be balanced by the lift acting on the vehicle, i.e., L = W. Since

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} = \frac{W}{\frac{1}{2}\rho_\infty U_\infty^2 S}$$

To find the reference area of the wing, we refer to Table 5.1. For a Cessna 172:

$$AR = 7.32 \quad \text{and} \quad b = 35.83 \text{ ft}$$

Using the definition for the aspect ratio: $AR = \frac{b^2}{S}$

so that $S = 175.38 \text{ ft}^2$. Note also that:

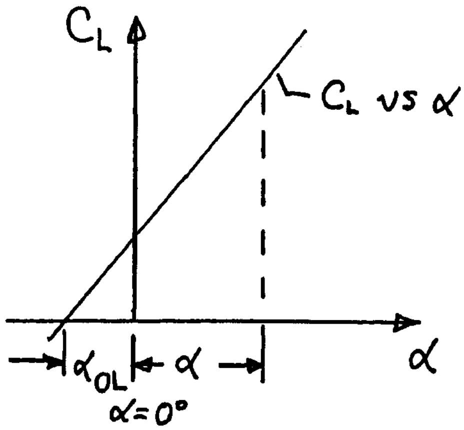
$$U_\infty = 130\frac{mi}{h} = 130\frac{mi}{h}\frac{\left(5280\frac{ft}{mi}\right)}{\left(3600\frac{s}{h}\right)} = 190.67\frac{ft}{s}$$

Thus,

$$C_L = \frac{2300 \text{ lbf}}{\frac{1}{2}\left[0.001756\frac{\text{lbf s}^2}{\text{ft}^4}\right]\left[190.67\frac{ft}{s}\right]^2\left[175.38 \text{ ft}^2\right]} = 0.4109$$

5.5]

For a linear lift curve whose slope is $C_{L,\alpha}$



$$C_L = C_{L,\alpha}(\alpha - \alpha_{OL})$$

But from Problem (5.3), the value of the lift coefficient at the maximum value of L/D is:

$$C_{L\,(L/D)max} = \sqrt{\frac{C_{Do}}{k}}$$

Thus,
$$\sqrt{\frac{C_{Do}}{k}} = C_{L,\alpha}\left[\alpha_{(L/D)max} - \alpha_{OL}\right]$$

$$\alpha_{(L/D)max} = \alpha_{OL} + \frac{1}{C_{L,\alpha}}\sqrt{\frac{C_{Do}}{k}}$$

5.6] Since $C_D = C_{Do} + kC_L^2$

and
$$C_{L\,(L/D)max} = \sqrt{\frac{C_{Do}}{K}}$$

Then, $C_{D(L/D)max} = C_{Do} + k\left[\frac{C_{Do}}{K}\right] = 2C_{Do}$

5.7] We are to consider a uniform stream approaching a flat plate at zero angle of attack. In this case, the velocity of the air particles outside the boundary layer remains constant along the entire length of the plate. Note that the span of the plate (the direction transverse to the flow) has been omitted from the problem statement. Thus, we will analyze the flow per unit span. Hence, we will use the symbol C_d, 5.7 contd.] the lower case d denoting the drag force per unit span.

Let us first calculate where "transition occurs", realizing that transition not occur at a point, but transition is a process that takes place over an extended length. To do this, we calculate the unit Reynolds number (or the Reynolds number per unit length). Referring to Table 1.2 for the properties of air at sea level:

$$\rho_\infty = 1.225\ \frac{kg}{m^3} \text{ and } \mu_\infty = 1.7894\times10^{-5}\ \frac{kg}{s\cdot m}$$

Thus, $$Re/m = \frac{\rho_\infty U_\infty}{\mu_\infty} = \frac{1.225(35)}{1.7894\times10^{-5}} = 2.396\times10^6/m$$

and $$x_{tr} = \frac{Re_{x,tr}}{Re/m} = \frac{500{,}000}{2.396\times10^6/m} = 0.20868\ m$$

If one assumes that transition occurs "instantaneously" at $x = 0.20868\ m$, then the laminar correlation for shear [Eq.(5.24)] is to be used upstream of this point; and the turbulent correlation for shear [Eq.(5.25)] is to be used from this point to the trailing edge. A more sophisticated analysis (and, in fact, a more realistic one) would account for a region of transitional flow. But such a treatment of the viscous flow is beyond the scope of this question. Since the flow acts on both sides of the plate, the drag force per unit span is twice the integral of $\tau\,dx$.

$$d = 2\int \tau\,dx = 2\left[\int_0^{x_{tr}} \frac{0.664}{(Re_x)^{0.5}}\,q_\infty\,dx + \int_{x_{tr}}^{c} \frac{0.0583}{(Re_x)^{0.2}}\,q_\infty\,dx\right.$$

$$d = 2q_\infty\left[\frac{0.664}{(Re/m)^{0.5}}\int_0^{x_{tr}}\frac{dx}{x^{0.5}} + \frac{0.0583}{(Re/m)^{0.2}}\int_{x_{tr}}^{c}\frac{dx}{x^{0.2}}\right.$$

5.7 Contd.

$$d = 2q_\infty \left\{ \frac{0.664}{1547.9} \left[2(x_{tr})^{0.5} \right] + \frac{0.0583}{18.8755} \left[1.25 (c^{0.8} - x_{tr}^{0.8}) \right] \right\}$$

$$d = 2q_\infty \{ 0.0003919 + 0.0042380 \} = 0.009260 q_\infty$$

$$C_d = \frac{d}{q_\infty c} = 0.006173$$

If we had assumed that the boundary layer is turbulent along the entire length:

$$d = 2q_\infty \frac{0.0583}{(Re/m)^{0.2}} \int_0^c \frac{dx}{x^{0.2}}$$

$$d = 2q_\infty \frac{0.0583}{18.8755} \left[1.25 c^{0.8} \right] = 0.01068 q_\infty$$

$$C_d = \frac{d}{q_\infty c} = 0.007120$$

The section drag coefficient calculated assuming a completely turbulent boundary layer is approximately 15% greater than that calculated "accounting" for the transition location.

5.8 If the airplane lift is to equal its weight:

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} = \frac{W}{\frac{1}{2}\rho_\infty U_\infty^2 S}$$

Since there are 0.27778 $\frac{m}{s}$ per $\frac{km}{h}$

$$C_L = \frac{50{,}000}{\frac{1}{2}\rho_\infty (0.27778)^2 U_\infty (21.5)}$$

5.8 Contd. where U_∞ is km/h. If the airport is at sea level, $\rho_\infty = 1.225\ \frac{kg}{m^3}$. If the airport is at 1600m, $\rho_\infty = 1.0486\ \frac{kg}{m^3}$ (see Table 1.2). Thus, for the airport at sea level:

$$C_L = \frac{49{,}207}{U_\infty^2}$$

Thus,

U_∞ (km/h)	300	270	240	210	180
C_L	0.5467	0.6750	0.8543	1.1158	1.5187

For the airport which is at 1600 m, $C_L = \dfrac{57{,}485}{U_\infty^2}$

U_∞ (km/h)	300	270	240	210	180
C_L	0.6387	0.7885	0.9980	1.3035	1.7742

Note that the lift coefficient required to balance the airplane's weight at a given velocity is greater at the airport which is at the higher elevation.

5.9 Since the lift-to-drag ratio is 30,

$$C_D = \frac{1}{30} C_L$$

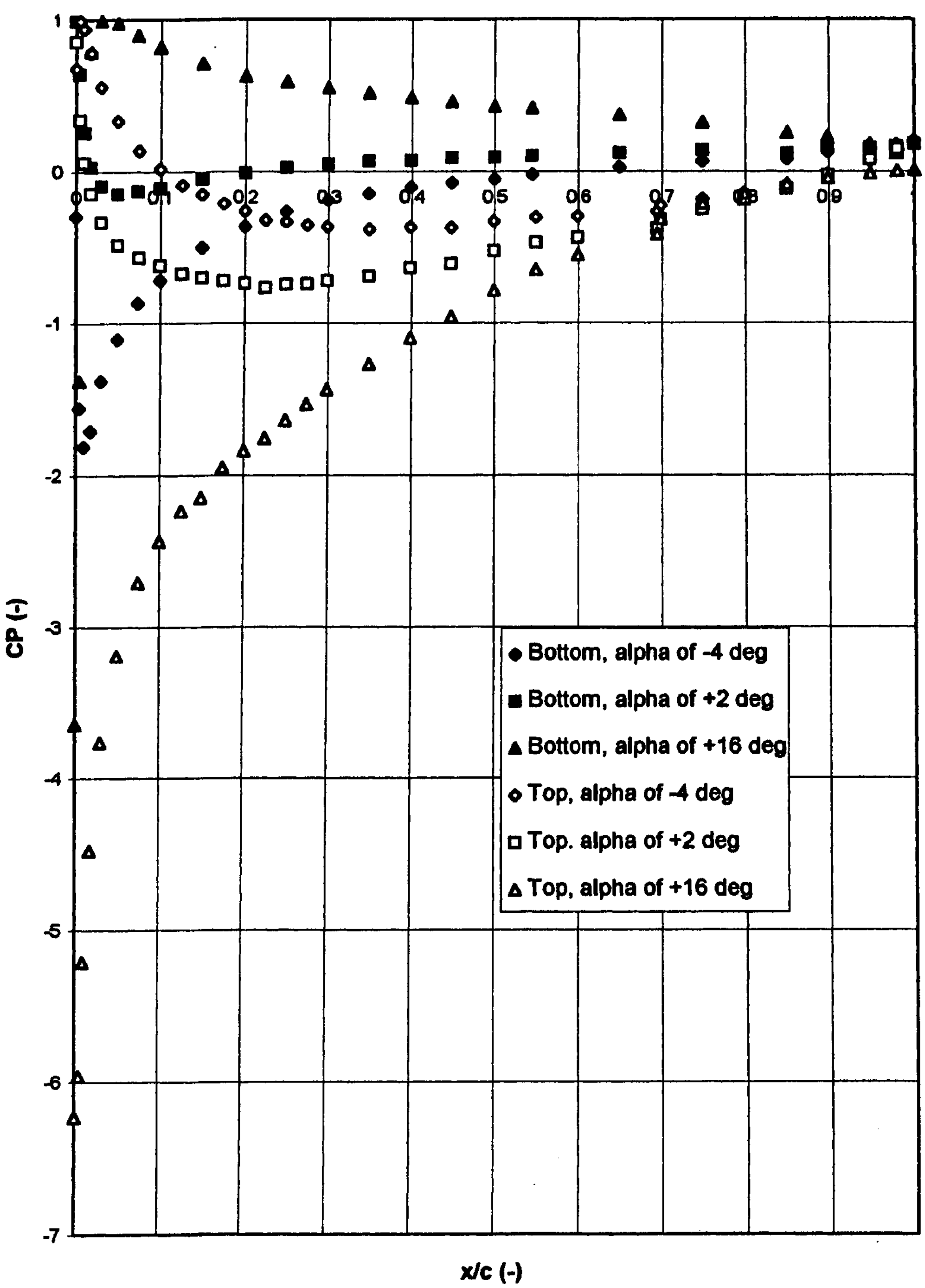
For these conditions: $U_\infty = 170\ \frac{km}{h}\ \frac{\left(1000\ \frac{m}{km}\right)}{\left(3600\ \frac{s}{h}\right)} = 47.22\ \frac{m}{s}$

Using Table 1.2, $\rho_\infty = 1.1117$ at 1 km. Thus,

$$C_D = \frac{1}{30}\ \frac{3150\text{N}}{\frac{1}{2}\left(1.1117\ \frac{kg}{m^3}\right)\left(47.22\ \frac{m}{s}\right)^2\left(10.0\ m^2\right)} = 0.00847$$

5.10) The pressure distributions for the three angles of attack are presented in the figure on the next page. The data for the bottom of the wing (the negative

Pressure coefficients for an NACA 4412 airfoil, as taken from Pinkerton (1936)



5.10 Contd. z-coordinates) are represented by the filled symbols. Note that when the airfoil is at $-4°$ angle of attack the bottom of the wing is the leeside and the pressure coefficients are negative near the leading edge, $(x/c) \sim 0$. The measurements for the top of the wing (the positive z-coordinates) are represented by the open symbols. The largest negative pressure coefficients are near the leading edge of top of the airfoil (open symbols for positive z-coordinates), when the airfoil is at $+16°$ angle of attack.

The stagnation point moves from the top of the airfoil (positive z-coordinates, open symbols) around the leading edge of the model, being essentially at $x=0$ for $\alpha = +2°$ onto the bottom (negative z-coordinates, filled symbols) to $x = 0.03c$ when $\alpha = +16°$.

The "clearest" constant-pressure separation pattern appears to be the leeward pressures downstream of $x \simeq 0.2c$ for $\alpha = -4°$ (the filled triangular symbols).

5.11 From Bernoulli's equation

$$C_p = 1 - \frac{U}{U_\infty^2}$$

Thus, when $C_p = 1$, $U = 0$ and we have a stagnation point. For positive values of C_p, $U < U_\infty$ and the flow has slowed from the free-stream velocity. For negative values of C_p, $U > U_\infty$ and the local velocity exceeds the free-stream velocity. Thus, the highest local velocity corresponds to the point where C_p has the greatest negative value.

5.11 Contd.

α	Maximum velocity for positive z-coordinates C_p	$\frac{U}{U_\infty}$	Maximum velocity for negative z-coordinates C_p	$\frac{U}{U_\infty}$
$+16°$	-6.230	2.689	-3.648	2.156
$+2°$	-0.769	1.330	-0.150	1.072
$-4°$	-0.381	1.175	-1.812	1.677

5.12

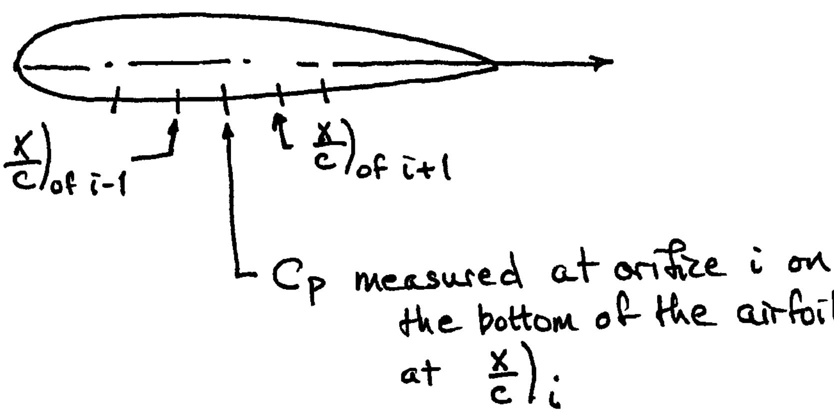
Following the notes in the chapter,

$$n = \oint p\, dx$$

$$n = \oint (p - p_\infty)\, dx$$

$$C_n = \frac{n}{q_\infty c} = \oint \frac{p - p_\infty}{q_\infty}\, d\left(\frac{x}{c}\right)$$

In problem 5.10, we have the pressure coefficients measured at discrete orifices.



5.12 Contd.] Using discrete pressure measurements let us say that the pressure acting at the ith orifice acts over the surface from half-way between the ith and i+1 orifices to half-way between the i-1th and ith orifices

for the bottom

$$\Delta n = p(i)\left[\frac{x_{i+1} + x_i}{2} - \frac{x_i + x_{i-1}}{2}\right]$$

or

$$\Delta C_n = C_p(i)\left[\frac{x/c\,(i+1) - x/c\,(i-1)}{2}\right]$$

note that a positive pressure coefficient produces a positive normal force (relative to p_∞ acting over the entire surface, top and bottom)

for the top

$$\Delta C_n = -C_p(i)\left[\frac{x/c\,(i+1) - x/c\,(i-1)}{2}\right]$$

So that a negative pressure coefficient for the top orifices produces a positive normal force. Using a spreadsheet code to integrate the pressures in the table, and noting that $C_\ell \approx C_n \cos\alpha$

The coefficients based on integrating the pressures are in the following table (in the first two columns). The last two columns are the measured values for C_ℓ as taken from Abbott and von Doenhoff (1949)

5.12 Contd.

α	Integration of the pressures presented in Ref. 5.17 yielded: C_n	C_l	The measured values of C_l from Ref. 5.1, based on the actual α: C_l	based on the effective α: α_{eff}	C_l
-4°	-0.02427	-0.0242	-0.025	-4°	-0.025
$+2^\circ$	$+0.4987$	$+0.4983$	$+0.6$	$+1.2^\circ$	$+0.5$
$+16^\circ$	$+1.5428$	$+1.4830$	$+1.4$	$+13^\circ$	$+1.51$

The lift coefficients determined through the small angle approximation integration of the pressure measurements (presented in Pinkerton (1936)) are presented in column 3. The experimental lift coefficients presented in Abbott and von Doenhoff (1949) are given for the actual angle of attack in column 4 and for the effective angle of attack in column 6. The agreement between the lift coefficients in column 6 (A&vD) and those in column 3 (P) appears to be very good.

5.13 The pitching moment about the quarter chord can be estimated by integrating the pressures:

$$m_{0.25c} = \oint p(x-0.25c)\,dx$$

Using the pressure coefficients and the pitching-moment coefficient:

$$C_{m_{0.25c}} = \left[\sum C_p\left(\frac{x}{c}-0.25\right) d\left(\frac{x}{c}\right)\right.$$

The moments calculated by integrating the pressures are:

$\alpha=-4$, $C_{m_{0.25c}}=0.3387$; $\alpha=+2$, $C_{m_{0.25c}}=0.0864$; $\alpha=+16^\circ$, $C_{m_{0.25c}}=0.0538$. Only the AvD measurement at $\alpha=-4^\circ$ is close

5.14 To sustain steady, level unaccelerated flight (SLUF)

$$T = D = 10{,}000 \text{ pounds}$$

Using either Fig. 5.31 or Table on page 248, the total drag acting on the aircraft at 20,000 is 10,000 pounds when the Mach number is just above 1 or just below Mach 0.2. Thus, the maximum Mach number is approximately 1.1. The minimum Mach number is approximately 0.19.

For stall,

$$L = W = C_{L,max}\left[\frac{1}{2}\rho_\infty V_{stall}^2\right] S$$

$$23{,}750 \text{ pounds} = 1.57\left[(0.5)(0.001267)\, V_{stall}^2\right] 300$$

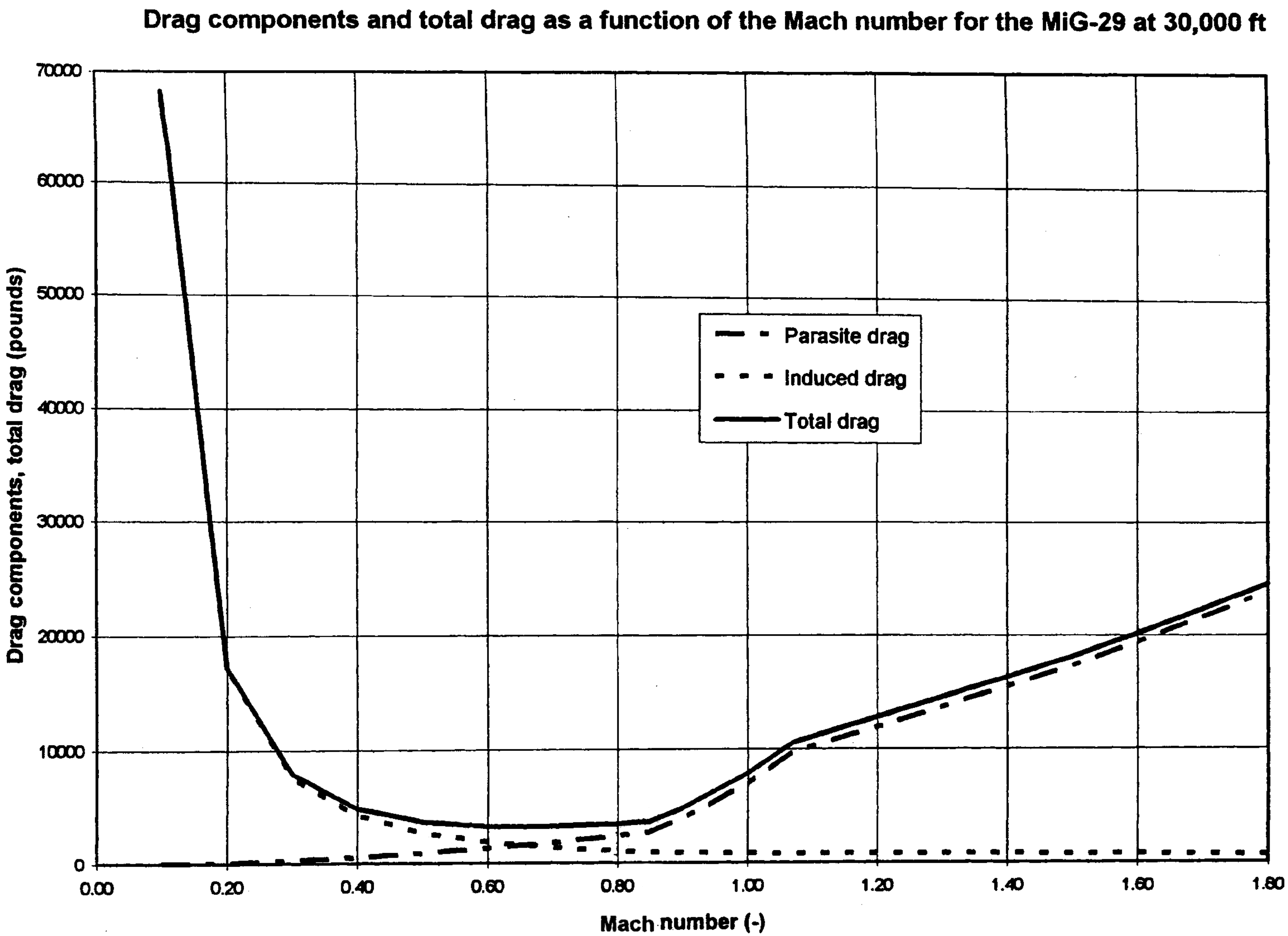
$$V_{stall} = 281.1 \text{ ft/s}$$

$$M_{stall} = \frac{V_{stall}}{a_{stall}} = \frac{281.1}{1036.94} = 0.271$$

5.15 and 5.16 are on the next three pages.

Problem 5.15 MiG-29

Aircraft	MiG-29										
S (sqft)	409										
span (ft)	37.3										
AR	3.401687										
e	0.75			a(fps)	rho(sl/ft3)		W(lbs)				
				994.8	0.000891		31000				
	Mach	CD,0	k	Vel	Pdyn	ParaD	CL	CD,ind	IndD	TotD	L/D
	(-)	(-)	(-)	(fps)	(psf)	(lbs)	(-)	(-)	(lbs)	(lbs)	(-)
	0.1000	0.0207	0.1279	99.48	4.41	37	17.1969	37.8241	68184	68221	0.4544
	0.2000	0.0207	0.1279	198.96	17.63	149	4.2992	2.3640	17046	17195	1.8028
	0.3000	0.0207	0.1279	298.44	39.67	336	1.9108	0.4670	7576	7912	3.9182
	0.4000	0.0207	0.1279	397.92	70.52	597	1.0748	0.1478	4261	4859	6.3805
	0.5000	0.0207	0.1279	497.40	110.19	933	0.6879	0.0605	2727	3660	8.4694
	0.6000	0.0207	0.1279	596.88	158.67	1343	0.4777	0.0292	1894	3237	9.5758
	0.7000	0.0207	0.1279	696.36	215.97	1828	0.3510	0.0158	1392	3220	9.6275
	0.8000	0.0207	0.1279	795.84	282.08	2388	0.2687	0.0092	1065	3454	8.9763
	0.8500	0.0207	0.1279	845.58	318.44	2696	0.2380	0.0072	944	3640	8.5171
	0.9000	0.0267	0.1391	895.32	357.00	3902	0.2123	0.0063	916	4818	6.4348
	1.0000	0.0388	0.1616	994.80	440.75	6989	0.1720	0.0048	861	7850	3.9491
	1.0700	0.0472	0.1773	1064.44	504.61	9741	0.1502	0.0040	826	10567	2.9337
	1.1000	0.0469	0.1881	1094.28	533.30	10224	0.1421	0.0038	829	11052	2.8048
	1.2000	0.0458	0.2240	1193.76	634.68	11883	0.1194	0.0032	829	12713	2.4385
	1.3000	0.0447	0.2599	1293.24	744.86	13614	0.1018	0.0027	820	14433	2.1478
	1.4000	0.0436	0.2958	1392.72	863.86	15402	0.0877	0.0023	805	16207	1.9128
	1.5000	0.0425	0.3317	1492.20	991.68	17238	0.0764	0.0019	786	18024	1.7199
	1.6000	0.0419	0.3625	1591.68	1128.31	19351	0.0672	0.0016	755	20106	1.5418
	1.7000	0.0414	0.3932	1691.16	1273.76	21551	0.0595	0.0014	725	22276	1.3916
	1.8000	0.0408	0.4240	1790.64	1428.02	23830	0.0531	0.0012	698	24527	1.2639



Drag components and total drag as a function of the Mach number for the MiG-29 at 30,000 ft
Drag components, total drag (pounds)
70000
60000
50000
40000
30000
20000
10000
0
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
Mach number (-)
Parasite drag
Induced drag
Total drag

Problem 5.16 Eurofighter 2000

Aircraft	Euro2000										
S (sqm)	50										
span (m)	10.5										
AR	2.205										
e	0.84			a(mps)	rho(kg/m3)		W(kg)				
				320.55	0.736433		17500				
	Mach	CD,0	k	Vel	Pdyn	ParaD	CL	CD,ind	IndD	TotD	L/D
	(-)	(-)	(-)	(mps)	(N/m2)	(N)	(-)	(-)	(N)	(N)	(-)
	0.1000	0.0131	0.1725	32.06	378	248	9.0693	14.1884	268411	268659	0.6386
	0.2000	0.0131	0.1725	64.11	1513	991	2.2673	0.8868	67103	68094	2.5196
	0.3000	0.0131	0.1725	96.17	3405	2230	1.0077	0.1752	29823	32054	5.3525
	0.4000	0.0131	0.1725	128.22	6054	3965	0.5668	0.0554	16776	20741	8.2720
	0.5000	0.0131	0.1725	160.28	9459	6195	0.3628	0.0227	10736	16932	10.1328
	0.6000	0.0131	0.1725	192.33	13621	8922	0.2519	0.0109	7456	16377	10.4760
	0.7000	0.0131	0.1725	224.39	18539	12143	0.1851	0.0059	5478	17621	9.7366
	0.8000	0.0131	0.1725	256.44	24214	15860	0.1417	0.0035	4194	20054	8.5552
	0.8600	0.0131	0.1725	275.67	27983	18329	0.1226	0.0026	3629	21958	7.8135
	0.9000	0.0161	0.1816	288.50	30646	24732	0.1120	0.0023	3488	28220	6.0798
	1.0000	0.0237	0.2043	320.55	37835	44910	0.0907	0.0017	3178	48088	3.5678
	1.1000	0.0313	0.2269	352.61	45780	71738	0.0750	0.0013	2918	74656	2.2981
	1.1100	0.0321	0.2292	355.81	46617	74820	0.0736	0.0012	2895	77714	2.2077
	1.2000	0.0314	0.2574	384.66	54483	85433	0.0630	0.0010	2782	88214	1.9449
	1.3000	0.0305	0.2888	416.72	63941	97642	0.0537	0.0008	2659	100301	1.7105
	1.4000	0.0297	0.3201	448.77	74157	110199	0.0463	0.0007	2542	112740	1.5218
	1.5000	0.0289	0.3515	480.83	85129	123011	0.0403	0.0006	2431	125442	1.3677
	1.6000	0.0285	0.3816	512.88	96858	138022	0.0354	0.0005	2319	140342	1.2225
	1.7000	0.0281	0.4116	544.94	109343	153628	0.0314	0.0004	2216	155844	1.1009
	1.8000	0.0277	0.4417	576.99	122586	169781	0.0280	0.0003	2121	171903	0.9981

5.17)

Given: A finless missile at standard day sea-level conditions:

$U_\infty = 450\ mph$
$L = 20\ ft$
$d = 2.4\ ft$

1976 atm: $\rho = 0.2377$ slug/ft^3
$\mu = 3.740\times10^{-7}$ lbf-s/ft^2

The zero-lift drag coefficient is given by:

$$C_{D_0} = K\overline{C}_f \frac{S_{wet}}{S_{ref}}$$

$$S_{wet} \approx \pi dL = 150.8\,ft^2$$

$$S_{ref} = \pi d^2/4 = 4.5239\,ft^2$$

$$q_\infty = \rho U_\infty^2/2 = 517.7\ lbf/ft^2$$

Following the fuselage method of pp. 230-232, the Reynolds number is given by $Re = \dfrac{\rho U_\infty c}{\mu} = 2.768\times10^{10}$, so clearly the vast majority of the fuselage is turbulent. The Prandtl-Schlichting skin friction relation is:

$$\overline{C}_f(x) = \frac{0.455}{\log(Re_x)^{2.58}} \text{ and } \overline{C}_f(L) = 0.001070$$

From Fig. 5.22 for $L/d = 20/2.4 = 8.33$, the body form factor is $K \approx 1.13$. The drag coefficient is given by

$$C_{D_0} = K\overline{C}_f \frac{S_{wet}}{S_{ref}} = (1.13)(0.001070)\left(\frac{150.8}{4.5239}\right) = 0.0403$$

The drag is given by $D = q_\infty S_{ref} C_{D_0} = 94.39$ lbf, neglecting the drag of the base.

5.18)

Given: A flying wing with

$S_{ref} = 3699\,ft^2$ $\quad c_r = 34.13\,ft$ $\quad \lambda = 0.275$ $\quad h = 37000\,ft$

$b = 170\,ft$ $\quad \Lambda_{c/4} = 36^\circ$ $\quad \left(\dfrac{t}{c}\right)_{avg} = 10.2\%$ $\quad \dfrac{W}{S} = 105 lb/ft^2$

$M_\infty = 0.23$

5.18) contd.

Find: skin friction drag coefficient (not stated but needed), pressure drag coefficient, induced drag coefficient, total drag coefficient

Atmospheric conditions at 17000 *ft* on a standard day:

$$\rho = 0.001401 slug / ft^3 \qquad a = 1049.2 ft/s \qquad \mu = 3.388 \times 10^{-7} slug / ft - s$$

and calculate the velocity and dynamic pressure for later use:

$$U_\infty = M_\infty a = (0.23)(1049.2 ft/s) = 241.3 ft/s$$
$$q_\infty = \frac{1}{2}\rho V_\infty^2 = 0.5(0.001401 slug/ft^3)(241.3 ft/s)^2 = 40.79 lb/ft^2$$

Now find the skin friction coefficient for the wing, assuming that the wing can be represented by an equivalent rectangular wing. First find the mean aerodynamic chord:

$$\bar{c} = \frac{2}{3} c_r \left(\frac{\lambda^2 + \lambda + 1}{\lambda + 1} \right) = \frac{2}{3}(34.13 ft)\left(\frac{1.351}{1.275} \right) = 24.1 ft$$

In order to find the skin friction coefficient we need to find the Reynolds number on the rectangular wing:

$$R_L = \frac{\rho V_\infty \bar{c}}{\mu} = \frac{(0.001401 slug/ft^3)(241.3 ft/s)(24.1 ft)}{3.388 \times 10^{-7} slug/ft - s} = 24.05 \times 10^6$$

Now find the skin friction coefficient assuming that there are both laminar and turbulent portions to the boundary layer:

$$C_f = \frac{0.455}{(\log R_L)^{2.58}} - \frac{1700}{R_L} = \frac{0.455}{(\log(24.05 \times 10^6))^{2.58}} - \frac{1700}{24.05 \times 10^6}$$
$$= 0.00262 - 0.00007 = 0.00255$$

Including the laminar portion of the boundary layer only changed the result by about 3%, so the laminar run could have been neglected and still yielded a reasonable estimate.

5.18) contd.

Now correct the skin friction coefficient for roughness (although the problem did not state that the wing surface was rough, it usually is!). Assuming the wing has a mass production, spray-painted surface, the equivalent sand grain roughness is $k = 2\times 10^{-3}\ in$ or $k = 1.67\times 10^{-4}\ ft$. In order to determine the impact of roughness at the Reynolds number calculated above use Fig. 5.19 in the text, which requires finding $k/L = 1.67\times 10^{-4}/24.1 = 6.92\times 10^{-6}$. At the flight Reynolds number of $R_L = 24.05\times 10^{6}$, the figure shows that the average skin friction coefficient is not impacted by the surface roughness, since the equivalent sand grain height would be "buried" in the viscous sublayer.

Now find the drag coefficient for skin friction drag as:

$$C_{D_{SF}} = C_f \frac{S_{wet}}{S_{ref}} = (0.00255)\frac{2\times 3699\, ft^2}{3699\, ft^2} = 0.00510$$

where we assumed that the wetted area included the top and the bottom of the wing. Now we want to find the pressure (form) drag due to flow separation for the wing. Using the wing form factor chart with the correct airfoil thickness and quarter-chord sweep angle, *K*=1.17, or in other words, separation adds 17% to the skin friction drag of the wing. If you want to just know the amount of drag due to pressure drag, it would be:

$$C_{D_P} = 0.17(0.00510) = 0.00087$$

or the total zero-lift drag coefficient of the wing is:

$$C_{D_0} = KC_{D_{SF}} = KC_f \frac{S_{wet}}{S_{ref}} = 1.17(0.00510) = 0.00597$$

Now we want to find the induced drag coefficient. This calculation requires us to know the lift coefficient of the airplane—we are given the wing loading, so the weight of the aircraft is:

$$W = \frac{W}{S_{ref}} S_{ref} = (105 lb/ft^2)(3699\, ft^2) = 388{,}395 lb$$

For SLUF, *L*=*W* and the lift coefficient is:

$$C_L = \frac{W}{q_\infty S_{ref}} = \frac{388{,}395 lb}{(40.79 lb/ft^2)(3699\, ft^2)} = 2.574$$

5.18) contd.

In order to find the induced drag coefficient we need to know the aspect ratio and the space efficiency factor, *e*, for the wing.

$$AR = \frac{b^2}{S_{ref}} = \frac{(170\,ft)^2}{3699\,ft^2} = 7.81$$

The span efficiency factor can be theoretically estimated, empirically estimated, or assumed. For the case of this example assume $e = 0.99$ (students would be free to make any reasonable assumption). Now find the induced drag coefficient from:

$$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{2.574^2}{\pi(0.99)(7.81)} = 0.2728$$

Finally, the wing total drag coefficient for incompressible flow is:

$$C_D = C_{D_0} + C_{D_i} = 0.00597 + 0.2728 = 0.2788$$

of which about 3% is due to skin friction/separation and 97% is due to wing-tip vortices. Why is there so much induced drag in this case?

How much total drag is this for the airplane (this wasn't asked in the question but should prove interesting)?

$$D = C_D q_\infty S_{ref} = (0.2788)(40.79\,lb/ft^2)(3699\,ft^2) = 42{,}061\,lb$$

which is a lot of drag! Do you think an airplane of this size could have enough thrust to overcome this drag? What airplane do you think this is? Hint: this is not really a flying wing, but this is the drag of the wing alone of an existing aircraft—these are the basic dimensions for the Boeing 767-400.

6.1 The vorticity distribution for a flat-plate (symmetrical) airfoil is:

$$\gamma(\theta) = 2\alpha U_\infty \frac{1+\cos\theta}{\sin\theta}$$

Is the Kutta condition for the trailing edge satisfied, i.e., is $\gamma(\pi)=0$? Since $\sin(\pi)=0$ and since $1+\cos(\pi)=0$, we must use l'Hopitel's rule to evaluate $\gamma(\pi)$. By l'Hopital's rule, if $f(a)=g(a)=0$ and if the limit of the ratio of $\frac{f'(t)}{g'(t)}$ exists as $t\to a$, then $\lim_{t\to a}\frac{f(t)}{g(t)} = \lim_{t\to a}\frac{f'(t)}{g'(t)}$. Translated to our problem:

$$\lim_{\theta\to\pi}\frac{2\alpha U_\infty(1+\cos\theta)}{\sin\theta} = \lim_{\theta\to\pi}\frac{2\alpha U_\infty(-\sin\theta)}{\cos\theta} = 0$$

Thus, the Kutta condition is satisfied.

Note that γ has the units of velocity. The incremental lift (per unit span) acting on an infinitesimal chordwise element is given by the Kutta-Joukowski theorem as:

$$dl = \rho_\infty U_\infty \gamma$$

But the increment of lift (per unit span) acting on an infinitesimal chordwise element is equal to the difference in the pressure acting on the lower surface and that acting on the upper surface:

$$dl = \Delta p = p_l - p_u = (p_l - p_\infty) - (p_u - p_\infty)$$

Equating the two expressions for the incremental lift:

$$\rho_\infty U_\infty \gamma = (p_l - p_\infty) - (p_u - p_\infty)$$

6.1 Contd. Dividing by the dynamic pressure ($q_\infty = 0.5 \rho_\infty U_\infty^2$):

$$\frac{2\gamma}{U_\infty} = C_{p_l} - C_{p_u} = 4\alpha\left[\frac{1+\cos\theta}{\sin\theta}\right]$$

Thus, the parameter $\frac{2\gamma}{U_\infty}$ represents the difference between the pressure coefficient for the lower surface and that for the upper surface at a given chordwise station.

Since $C_l = 2\pi\alpha$, the angle of attack required to develop a section lift coefficient of 0.5 is $4.56° = 0.0796$ radians. Thus, the load distribution for a flat-plate airfoil which develops a section lift coefficient of 0.5 is that given in the following table.

θ	$\frac{x}{c} = \frac{1}{2}(1-\cos\theta)$	$\frac{2\gamma}{U_\infty}$	θ	$\frac{x}{c} = \frac{1}{2}(1-\cos\theta)$	$\frac{2\gamma}{U_\infty}$
0°	0.000	∞	120°	0.750	0.1837
30°	0.067	1.1879	135°	0.854	0.1318
45°	0.146	0.7684	150°	0.933	0.0853
60°	0.250	0.5513	180°	1.000	0.0000
90°	0.500	0.3183			

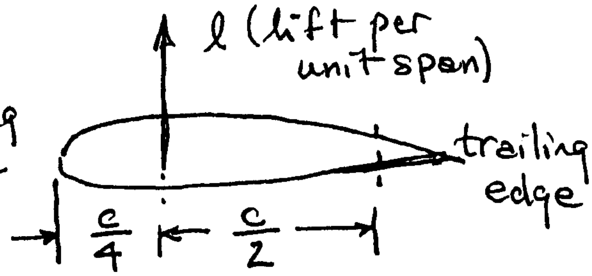
Let us calculate the section moment coefficient about a point 0.75 chord (0.75c) from the leading edge. A nose up pitching moment is positive.

$$m_{0.75c} = \int_0^{0.75c} \Delta p \, d\xi \, (0.75c - \xi) - \int_{0.75c}^{1.0c} \Delta p \, d\xi \, (\xi - 0.75c)$$

$$m_{0.75c} = 0.75c \int_0^c \Delta p \, d\xi - \int_0^c \Delta p \, \xi \, d\xi \qquad \text{(P. 6.1a)}$$

6.1 Contd. Recall that the lift per unit span for the section is:

$$\ell = \int_0^c \Delta p \, d\xi$$



and that the pitching moment about the leading edge is:

$$m_0 = -\int_0^c \Delta p \, \xi \, d\xi$$

Thus, equation (P.6.1a) can be written:

$$m_{0.75c} = 0.75c \, \ell + m_0 = (0.75c)(\pi \rho_\infty U_\infty^2 \alpha c) - \frac{\pi}{4} \rho_\infty U_\infty^2 \alpha c^2 = \frac{\pi}{2} \rho_\infty U_\infty^2 \alpha c^2$$

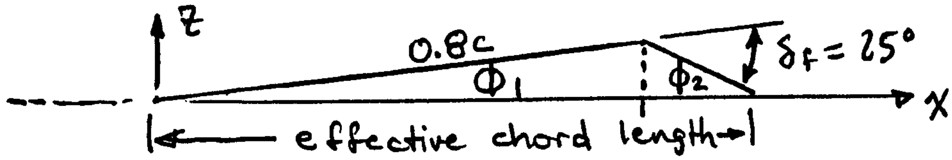
Since the theoretical location of the center of pressure for a flat-plate airfoil is at the quarter chord, we can calculate the moment about the three-quarter chord point as:

$$m_{0.75c} = \ell \, (0.5c)$$

where ℓ, the lift per unit span, is a force that acts effectively at the quarter chord and 0.5c is the moment arm. A positive (nose-up) moment results when the lift is positive. Thus,

$$m_{0.75c} = \frac{\pi}{2} \rho_\infty U_\infty^2 \alpha c^2$$

6.2] The NACA 0009 airfoil is a symmetric airfoil, whose chord is a straight line. Thus, when a plain flap whose length is 0.2c is deflected 25°, we can represent the configuration by the "equivalent cambered" airfoil section.



First, we need to calculate the slopes of the two segments of the "equivalent cambered" section.

$$z_{max} = 0.8c \sin\phi_1 = 0.2c \sin\phi_2 \quad \text{and} \quad \phi_2 = 25° - \phi_1$$

Thus, $4\sin\phi_1 = \sin(25° - \phi_1) = \sin 25° \cos\phi_1 - \cos 25° \sin\phi_1$

$4.9063 \sin\phi_1 = 0.4226 \cos\phi_1$; $\phi_1 = 4.923°$ and $\phi_2 = 20.077°$

Thus, the effective chord length (c_{eff}) is:

$$c_{eff} = 0.8c \cos\phi_1 + 0.2c \cos\phi_2 = c_1 + c_2$$

$$c_{eff} = 0.79705c + 0.18785c = 0.98490c$$

Equivalently, $c_1 = 0.80927 c_{eff}$ and $c_2 = 0.19073 c_{eff}$

Let us now determine the limits for the integration, i.e., what is the value of θ for which

$$0.80927 c_{eff} = \frac{c_{eff}}{2}(1 - \cos\theta)$$

Thus, $\theta = 128.21° = 2.2377$ radians

Note that:

$$\left.\frac{dz}{dx}\right)_{fore} = \tan\phi_1 = 0.08613 \; ; \; \left.\frac{dz}{dx}\right)_{aft} = -\tan\phi_2 = -0.36549$$

6.2 Contd. Thus,

$$A_0 = \alpha_{eff} - \frac{1}{\pi}\left\{\int_0^{2.2377}\left(\frac{dz}{dx}\right)_{fore} d\theta + \int_{2.2377}^{\pi}\left(\frac{dz}{dx}\right)_{aft} d\theta\right.$$

$$A_0 = \alpha_{eff} - \frac{1}{\pi}\left\{0.08613\,\theta\Big|_0^{2.2377} - 0.36549\,\theta\Big|_{2.2377}^{\pi}\right.$$

$$A_0 = \alpha_{eff} + 0.04381$$

Note that $\alpha = \alpha_{eff} - 4.923^\circ = \alpha_{eff} - 0.08592$

$$A_1 = \frac{2}{\pi}\left\{\int_0^{128.21^\circ}\left(\frac{dz}{dx}\right)_{fore}\cos\theta\, d\theta + \int_{128.21^\circ}^{180^\circ}\left(\frac{dz}{dx}\right)_{aft}\cos\theta\, d\theta\right\}$$

$$A_1 = \frac{2}{\pi}\left\{0.08613\sin\theta\Big|_0^{128.21^\circ} - 0.36549\sin\theta\Big|_{128.21^\circ}^{180^\circ}\right\}$$

$$A_1 = 0.22591$$

$$A_2 = \frac{2}{\pi}\left\{\int_0^{128.21^\circ}\left(\frac{dz}{dx}\right)_{fore}\cos 2\theta\, d\theta + \int_{128.21^\circ}^{180^\circ}\left(\frac{dz}{dx}\right)_{aft}\cos 2\theta\, d\theta\right\}$$

$$A_2 = \frac{2}{\pi}\left\{(0.08613)\frac{1}{2}\sin 2\theta\Big|_0^{128.21^\circ} - (0.36549)\frac{1}{2}\sin 2\theta\Big|_{128.21^\circ}^{180^\circ}\right\}$$

$$A_2 = -0.13974$$

Thus, we have calculated A_0, A_1, and A_2 for our "equivalent cambered" airfoil section.

$$A_0 = \alpha_{eff} + 0.4381$$
$$A_1 = 0.22591$$
$$A_2 = -0.13974$$

6.2 Contd.

$$C_l = 2\pi\left(A_0 + \frac{A_1}{2}\right) = 2\pi\left(\alpha_{eff} + 0.04381 + 0.11296\right)$$

$$C_l = 2\pi\left(\alpha_{eff} + 0.15677\right)$$

But this lift coefficient is based on c_{eff}. To convert to the standard reference length c,

$$l = C_l q_\infty c = 2\pi\left(\alpha_{eff} + 0.15677\right) q_\infty c_{eff}$$

$$C_l = 2\pi\left[\alpha + 0.08592 + 0.15677\right] 0.98490$$

Since $\frac{c_{eff}}{c} = 0.98490$

$$C_l = 6.18831\left[\alpha + 0.24269\right] = 6.18831\alpha + 1.50184$$

$$x_{cp} = \frac{c_{eff}}{4}\left[1 + \frac{\pi}{C_l}\left(A_1 - A_2\right)\right]$$

we will use C_l for the effective angle of attack

$$x_{cp} = \frac{c_{eff}}{4}\left[1 + \frac{0.36565}{2\left(\alpha_{eff} + 0.15677\right)}\right]$$

$$C_{m_{0.25c}} = \frac{\pi}{4}\left(-0.13974 - 0.22591\right) = -0.28718$$

which is based on the "effective cambered" airfoil section

$$m_{0.25c} = C_{m_{0.25c}} q_\infty c c = \left[-0.28718\right] q_\infty c_{eff} c_{eff}$$

Solving for $C_{m_{0.25c}}$ based on the actual chord length

$$C_{m_{0.25c}} = -0.27857$$

6.3 $$\left[x-\frac{1}{2}c\right]^2+\left[z-z_0\right]^2=R^2 \qquad (P.6.3a)$$

We have already used the symmetry condition to evaluate the x-coordinate of the center of the circle, 0.5c. To evaluate z_0, note that at $x = 0.5c$, $z = kc$ where k is a fraction (typically 0.02 to 0.04). At this point

$$(kc - z_0)^2 = R^2$$

At $x = 0$, $z = 0$: $\frac{1}{4}c^2 + z_0^2 = R^2 = (kc - z_0)^2$

Thus, $$2kcz_0 = k^2c^2 - \frac{1}{4}c^2$$

Solving for the other coordinate of the centes of our circle: $$z_0 = \frac{kc}{2} - \frac{1}{8}\frac{c}{k}$$

To solve for the slope of the mean camber line, Eq. (P.6.3a), let us reexamine the equation for the circular arc that represents the mean camber line.

$$\left[x-\frac{1}{2}c\right]^2+\left[z-\frac{kc}{2}+\frac{1}{8}\frac{c}{k}\right]^2=\left[\frac{kc}{2}-\frac{1}{8}\frac{c}{k}\right]^2$$

Differentiating:

$$2\left[x-\frac{1}{2}c\right]+2\left[z-\frac{kc}{2}+\frac{1}{8}\frac{c}{k}\right]\frac{dz}{dx}=0$$

Solving: $$\frac{dz}{dx}=\frac{(c-2x)}{2z-kc+\frac{1}{4}\frac{c}{k}} \qquad (P.6.3b)$$

Introducing the coordinate transformation: $x = \frac{c}{2}(1-\cos\theta)$

Thus, $$c - 2x = c\cos\theta \qquad (P.6.3c)$$

so that $$z = \left[R^2 - \frac{c^2}{4}\cos^2\theta\right]^{0.5} + \frac{kc}{2} - \frac{1}{8}\frac{c}{k}$$

6.3 Contd. Rearranging:

$$2z - kc + \frac{1}{4}\frac{c}{K} = 2\left[R^2 - \frac{c^2}{4}\cos^2\theta\right]^{0.5} \qquad \text{(P.6.3d)}$$

Note that the left-hand side is the denominator of Eq. (P.6.3b). Thus, combining Eqs. (P.6.3c) and (P.6.3d) with Eq. (P.6.3b), we see that:

$$\frac{dz}{dx} = \frac{c\cos\theta}{2\left[R^2 - \frac{c^2}{4}\cos^2\theta\right]^{0.5}}$$

Let us compare the size of the individual terms in the radical:

$$R^2 - \frac{c^2}{4}\cos^2\theta = \left[\frac{Kc}{2} + \frac{1}{8}\frac{c}{K}\right]^2 - \frac{c^2}{4}\cos^2\theta$$

First, let us note that, since $K \ll 1$, $Kc \ll \frac{c}{K}$ and that $[\cos\theta]^2 \leq 1$, we can introduce the approximation:

$$R^2 - \frac{c^2}{4}\cos^2\theta \approx \frac{1}{64}\frac{c^2}{K^2}$$

so that $$\frac{dz}{dx} \approx \frac{c\cos\theta}{2\left[\frac{1}{8}\frac{c}{K}\right]} = 4K\cos\theta$$

To estimate the coefficients used in the γ distribution, note that for $n > 1$

$$A_n = \frac{2}{\pi}\int_0^{\pi} 4K\cos\theta\cos n\theta\, d\theta$$

which is equal to zero for all $n \neq 1$.

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} 4K\cos\theta\, d\theta = \alpha$$

6.3 Contd.

$$A_1 = \frac{2}{\pi}\int_0^{\pi} (4\,k\cos\theta)\cos\theta\, d\theta$$

$$A_1 = \frac{8K}{\pi}\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right|_0^{\pi} = 4k$$

Thus, $\gamma(\theta) = 2U_\infty\left[\alpha\,\frac{1+\cos\theta}{\sin\theta} + 4k\sin\theta\right]$

So that $C_l = \pi(2A_0 + A_1) = 2\pi\alpha + 4\pi k$

In order that $C_l = 0$, $\alpha_{0l} = -2k$

Further, $C_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1) = -\pi k$

6.4 For $0.0c \le x \le 0.2025c$

$$\left(\frac{z}{c}\right)_{fore} = 2.6595\left[\left(\frac{x}{c}\right)^3 - 0.6075\left(\frac{x}{c}\right)^2 + 0.11471\left(\frac{x}{c}\right)\right]$$

Thus, for $0 \le \theta \le 53.487°$ (or 0.93352 radians)

$$\left(\frac{dz}{dx}\right)_{fore} = 7.9785\left(\frac{x}{c}\right)^2 - 3.2313\left(\frac{x}{c}\right) + 0.30507$$

$$= 1.9946\cos^2\theta - 2.3736\cos\theta + 0.6840$$

For $0.2025c \le x \le 1.0000c$

$$\left(\frac{z}{c}\right)_{aft} = 0.022083\left[1 - \frac{x}{c}\right]$$

Thus, for $53.487° \le \theta \le 180°$

$$\left(\frac{dz}{dx}\right)_{aft} = -0.022083$$

$$A_0 = \alpha + \frac{1}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^2\theta - 2.3736\cos\theta + 0.6840\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\right]d\theta\right.$$

6.4 Contd.] $$A_0 = \alpha - \frac{1}{\pi}\left\{\left[1.9946\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - 2.3736\sin\theta + 0.6840\,\theta\right|_0^{0.93352} + \left[-0.022083\,\theta\right|_{0.93352}^{\pi}\right\}$$

$$A_0 = \alpha - 0.02866$$

$$A_1 = \frac{2}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^3\theta - 2.3736\cos^2\theta + 0.6840\cos\theta\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\cos\theta\right]d\theta\right\}$$

$$A_1 = \left\{\left[1.9946\left(\sin\theta - \frac{1}{3}\sin^3\theta\right) - 2.3736\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + 0.6840\sin\theta\right|_0^{0.93352} + \left[-0.022083\sin\theta\right|_{0.93352}^{\pi}\right\}$$

$$A_1 = 0.09550$$

$$A_2 = \frac{2}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^2\theta\left(2\cos^2\theta - 1\right) - 2.3736\cos\theta\cos 2\theta + 0.6840\cos 2\theta\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\cos 2\theta\right]d\theta\right.$$

$$A_2 = \frac{2}{\pi}\left\{\left[1.9946\left(\frac{\theta}{4} + \frac{1}{8}\sin 2\theta + \frac{\cos^3\theta\sin\theta}{2}\right) - 2.3736\left(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6}\right) + 0.6840\,\frac{\sin 2\theta}{2}\right|_0^{0.93352} + \left[-0.022083\,\frac{\sin 2\theta}{2}\right|_{0.93352}^{\pi}\right\}$$

$$A_2 = 0.07914$$

6.4 Contd.] $C_l = \pi(2A_0 + A_1) = 2\pi\alpha + 0.11992$

when $\begin{cases} \alpha = & -1.0936° & 0.0° & 1.642° & 3.0° \\ C_l = & 0.0 & 0.11992 & 0.30 & 0.44891 \end{cases}$

These calculated values of C_l are in close agreement with the data from Abbott and von Doenhoff (1949)

The center of pressure is given by:

$$x_{cp} = \frac{c}{4}\left[1 + \frac{\pi}{4}(A_1 - A_2)\right]$$

Thus, when $\alpha = 3.0°$, $C_l = 0.44891$ and

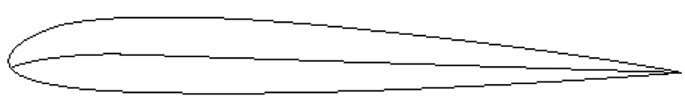
$$x_{cp} = 0.2786c$$

$$C_{mac} = \frac{\pi}{4}(A_2 - A_1) = -0.01285$$

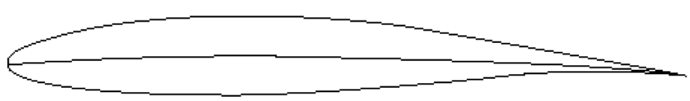
which also is in good agreement with the experimental values of Abbott and von Doenhoff (1949)

6.5)

Given: Three airfoils shown below from Fig. 6.12.



NACA 23012



NACA 63_1-412

NACA 66_1-212

6.5) contd.

The NACA 23012 airfoil shown at the top is a 5-digit airfoil and represents a fairly typical airfoil section. Notice that the camber is fairly high near the front of the airfoil and that the airfoil is fairly thick near the front. In fact, the 5-digit airfoils were tested because the 4-digit airfoils showed an increase in maximum lift coefficient when the position of maximum camber was shifted either forward or aft of the mid-chord position. The NACA 23012 airfoil has a design lift coefficient in tenths is three-halves of the first digit, which is 0.3. The next two digits represent twice the position of maximum camber in tenths of chord, which is 15%. This is far forward compared with most of the 4-digit airfoils. The last two digits represent the thickness of the airfoil in percent of chord, which is 12%.

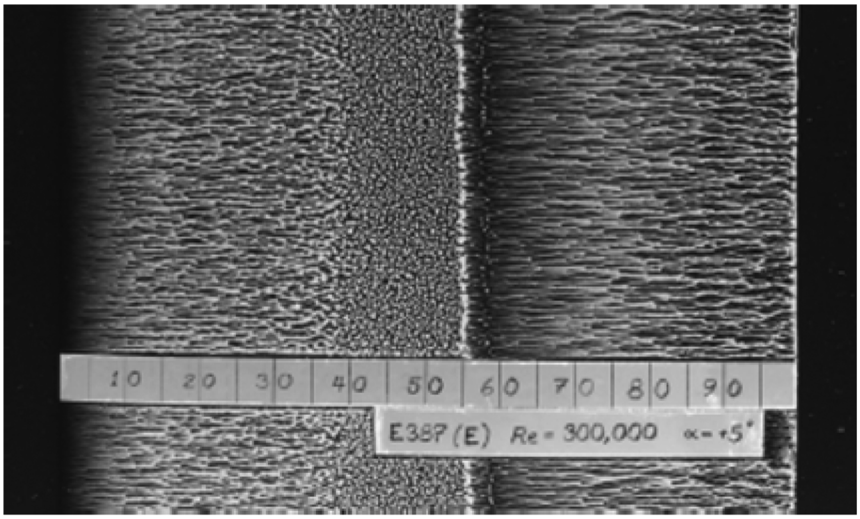
Compared with the NACA 23012 airfoil, the next two airfoils shown are quite different. Both the NACA 63_1-412 airfoil and the NACA 66_1-212 airfoil have the same thickness as the NACA 23012 (12%), but the location of maximum thickness is much further aft. The first number in the 6-digit series is always a 6, which designates the airfoil as a 6-digit airfoil. The second number represents the position on the airfoil of minimum pressure (or maximum velocity) at zero lift in tenths of chord. So the NACA 63_1-412 airfoils has its minimum pressure point at 30% of the chord and the NACA 66_1-212 airfoil has its minimum pressure point at 60% of the chord. This is easy to see in the figure, since the maximum thickness location of the NACA 63_1-412 airfoil is further forward than the NACA 66_1-212 airfoil, but further aft than the NACA 23012 airfoil. The 6-digit airfoils were designed this way to maintain laminar flow over a significant portion of the chord by maintaining favorable pressure gradients and having relatively low leading-edge radii. These airfoils showed the drag buckets at low angles of attack that make them interesting for reducing skin friction drag (see Fig. 6.13). Unfortunately, these airfoils did not work well when the surfaces were rough.

6.6)

Given: A laminar flow airfoil.

Laminar flow airfoils, such as those shown in Prob. 6.5, maintain laminar flow over the leading-edge of an airfoil, and even far beyond the leading-edge to approximately the mid-chord of the airfoil. Recall that laminar boundary layers have less skin friction drag when compared with turbulent boundary layers, but laminar boundary layers are also more prone to separation in the vicinity of an adverse pressure gradient. This leads to one of the primary difficulties in design laminar airfoils: maintaining laminar flow without allowing separation to occur. As can be seen in Fig. 6.16, however, the problem is even worse than that.

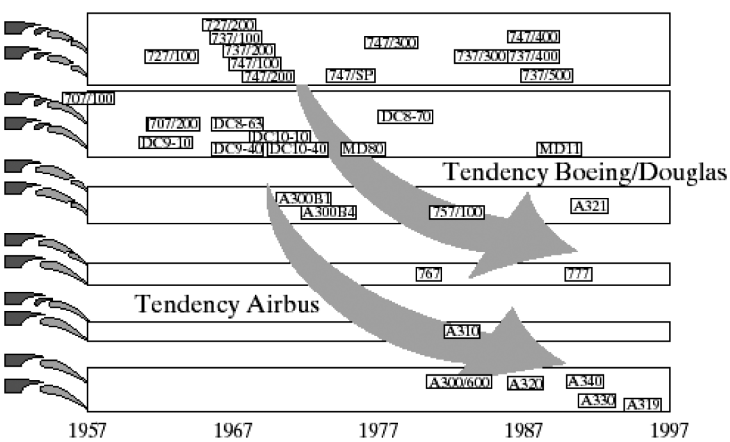
6.6) contd.



Once the laminar boundary layer does separate due to an adverse pressure gradient, the separated boundary layer can transition into a turbulent layer and then re-attach to the airfoil surface, creating a re-circulation bubble on the upper surface of the airfoil. This bubble reduces the lift of the airfoil and increases the drag, reducing the advantage of using a laminar airfoil (or making the laminar airfoil essentially useless). A great deal of experimentation has been done my M. Selig at the University of Illinois to find low Reynolds number airfoils that do not display laminar separation bubbles, primarily for use on small aircraft, gliders, and wind turbines.

6.7)

Given: The history of high-lift wing design since 1957 as shown below.



6.7) contd.

The first generation of jet-powered commercial aircraft, like the Douglas DC-8 and the Boeing 707, 727, and 737, used double- or triple-slotted flaps in order to obtain the maximum lift coefficient required for landing at reasonable speeds in reasonable runway distances. These wings were designed exclusively with experimental approaches, even though it is very difficult to match the wing geometry and flight conditions (Reynolds number and Mach number) in most wind tunnels. As designs progressed through the 1960s and 1970s, an increase in computational capabilities was taking place, first with inviscid panel methods and later with more advanced potential flow methods. It was not until the 1980s and 1990s that designers were able to incorporate viscous flow simulations into their design process. When this started happening designers were able to improve their design processes, which led to improved double-slotted flaps, and eventually even high quality single-slotted flaps. Thus both Airbus and Boeing now have the ability to design high-lift systems with single-slotted flaps for very large aircraft, including the Airbus A380 and the Boeing 787. This makes the flap system lighter, less mechanically complex, have less drag, and more dependable.

7.1) Since the airplane is flying at 3000 m, we can use Table 1.2 to find that: $\rho_\infty = 0.9092 \frac{kg}{m^3}$. The airspeed is:

$$U_\infty = 300 \frac{km}{h} \frac{(1000\ m/km)}{(3600\ s/h)} = 83.333 \frac{m}{s}$$

Thus, $q_\infty = \frac{1}{2}\rho_\infty U_\infty^2 = 3156.9 \frac{N}{m^2}$

Since $AR = \frac{b^2}{S}$; $b = \sqrt{(6.2)(17.0 m^2)} = 10.266\, m$

$$C_L = \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 S} = \frac{\pi b \Gamma_0}{2 U_\infty S} \qquad (7.12)$$

Thus,

$$\Gamma_0 = \frac{L}{q_\infty}\frac{2U_\infty}{\pi b} = \frac{(14,700)}{(3156.9)}\frac{2(83.333)}{\pi(10.266)} = 24.06 \frac{m^2}{s}$$

$$w_{y_1} = -\frac{\Gamma_0}{4s} = -\frac{\Gamma_0}{4(b/2)} = -1.172 \frac{m}{s}$$

Since $C_L = \frac{L}{q_\infty S} = 0.2739$; $C_{D_v} = \frac{C_L^2}{\pi AR} = 0.00385$

$$\varepsilon = \frac{C_L}{\pi AR} = 0.01406 \text{ rad} = 0.8057°$$

Referring to Fig. 7.12:

$$C_L = 2\pi(\alpha_e - \alpha_{0L}) = 0.2739$$

Thus, $\alpha_e - \alpha_{0L} = 0.04359 \text{ rad} = 2.4977°$

$\alpha_e = 1.2977°$ and $\alpha = \alpha_e + \varepsilon = 2.1034°$

7.2 The lift is given by:

$$L = \int_{-s}^{+s} \rho_\infty U_\infty \Gamma(y)\, dy \qquad (7.6)$$

Let us designate Γ_0 for the parabolic load distribution as $\Gamma_{0,p}$ and that for the elliptic load distribution as $\Gamma_{0,e}$. Thus, the total lift for the parabolic load distribution is:

$$L_p = \rho_\infty U_\infty \Gamma_{0,p} \int_{-s}^{+s} \left(1 - \frac{y^2}{s^2}\right) dy$$

$$L_p = \rho_\infty U_\infty \Gamma_{0,p} \left[y - \frac{y^3}{3s^2} \right|_{-s}^{+s} = \rho_\infty U_\infty \Gamma_{0,p} \frac{4s}{3}$$

If the total lift generated by the wing with the parabolic circulation distribution is to be equal to the total lift generated by a wing with an elliptic circulation distribution, i.e., $L_p = L_e$, then:

$$\rho_\infty U_\infty \Gamma_{0,p} \frac{4s}{3} = \rho_\infty U_\infty \Gamma_{0,e} \frac{\pi}{2} s$$

Thus, $$\Gamma_{0,p} = \frac{3\pi}{8} \Gamma_{0,e}$$

To calculate the downwash velocity at the plane of symmetry, we note that $y_1 = 0$ and:

$$w_{y_1} = \frac{1}{4\pi} \int_{-s}^{+s} \frac{\frac{d\Gamma}{dy}}{y - y_1} dy = \frac{1}{4\pi} \int_{-s}^{+s} \frac{1}{y} \frac{d\Gamma}{dy} dy$$

For the parabolic distribution: $\dfrac{d\Gamma}{dy} = \Gamma_{0,p}\left(-\dfrac{2y}{s^2}\right)$

Thus, $$w_{y_1=0} = \frac{1}{4\pi} \int_{-s}^{+s} \left(-\frac{2\Gamma_{0,p}}{s^2}\right) dy = -\frac{\Gamma_{0,p}}{\pi s}$$

7.2 Contd. Thus, the relation between the induced down-wash velocities at the plane of symmetry is:

$$\frac{(w_{y_1=0})_p}{(w_{y_1=0})_e} = \frac{-\dfrac{\Gamma_{0,p}}{\pi s}}{-\dfrac{\Gamma_{0,e}}{4s}} = \frac{4}{\pi}\frac{\Gamma_{0,p}}{\Gamma_{0,e}} = \frac{3}{2}$$

7.3

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi}\left\{\frac{1}{AR_2} - \frac{1}{AR_1}\right\}$$

If $AR_1 = \infty$ and $\alpha_1 = 4° = 0.0698$ radians ; $C_L = 1.0$.

The angle of attack required to generate the same lift coefficient when $AR_2 = 7.5$ is:

$$\alpha_2 = 0.0698 + \frac{1.0}{\pi}\left\{\frac{1}{7.5} - 0\right\} = 0.1123 \text{ rad}$$

$$\alpha_2 = 6.432°$$

The angle of attack required to generate the same lift coefficient if the aspect ratio is 6, i.e. $AR_2 = 6.0$

$$\alpha_2 = 0.0698 + \frac{1.0}{\pi}\left\{\frac{1}{6.0} - 0\right\} = 0.1229 \text{ rad}$$

$$\alpha_2 = 7.040°$$

7.4 For a planar wing, which has a NACA 0012 section, $\alpha_{twist} = 0.0°$, $\alpha_{0\ell} = 0.0°$ (across the wing), and $a_0 = 2\pi$. Thus, using a four term series, as given by equation (7.31), to represent the spanwise loading for a wing whose aspect ratio (AR) is 7.0 and whose taper ratio (λ) is 0.4, the four equations to be solved are:

7.4 Contd.

$$0.20109 A_1 + 0.74934 A_3 + 1.01319 A_5 + 0.52897 A_7 = 0.05465\alpha$$
$$0.63044 A_1 + 0.89132 A_3 - 1.15220 A_5 - 1.41308 A_7 = 0.13044\alpha$$
$$1.08160 A_1 - 0.63694 A_3 - 0.82586 A_5 + 2.4499 A_7 = 0.22805\alpha$$
$$1.32041 A_1 - 1.96122 A_3 + 2.60204 A_5 - 3.24286 A_7 = 0.32041\alpha$$

Solving:

$A_1 = 0.22147\alpha$; $A_3 = 8.0783 \times 10^{-4}\alpha$; $A_5 = 9.9666 \times 10^{-3}\alpha$;
$A_7 = -1.1200 \times 10^{-3}\alpha$

$$C_L = A_1 \pi AR = 4.8703\alpha$$

$$\delta = 1.0166 \times 10^{-6}$$

$$C_{D_v} = \frac{C_L^2}{\pi AR}(1+\delta) = 1.0896\alpha^2$$

Thus, for $\alpha = 6.0°$ and $\lambda = 0.4$: $C_L = 0.5100$; $C_{D_v} = 0.0011949$

The spanwise load distribution (for all angles of attack) is:

$\frac{2y}{b}$	0.92388	0.70711	0.38268	0.00000
$\frac{C_\ell}{C_L}$	0.85132	1.05490	1.04190	0.93264

The effect of the taper ratio is summarized below:

λ	C_L	δ	C_{D_v}
0.4	4.8703α	1.0166×10^{-2}	$1.0896\alpha^2$
0.5	4.8495α	1.2450×10^{-2}	$1.0827\alpha^2$
0.6	4.8226α	1.7934×10^{-2}	$1.0765\alpha^2$
1.0	4.6948α	5.5318×10^{-2}	$1.0577\alpha^2$

7.4 Contd. The spanwise load distributions (which are independent of α):

		$\frac{2y}{b}$			
	λ	0.92388	0.70711	0.38268	0.0000
$\frac{C_\ell}{C_L}$ for	0.4	0.85132	1.05490	1.04190	0.93264
	0.5	0.80059	1.03667	1.05319	0.96861
	0.6	0.76265	1.02240	1.06434	1.00384
	1.0	0.67621	0.98906	1.10492	1.13505

For $\alpha = 6°$:

λ	C_L	C_{D_V}
0.4	0.5100	1.1949×10^{-2}
0.5	0.5078	1.1873×10^{-2}
0.6	0.5050	1.1806×10^{-2}
1.0	0.4916	1.1599×10^{-2}

7.5 $AR = 7.52$; $\lambda = 0.69$; $\alpha_{twist} = -3.0°$

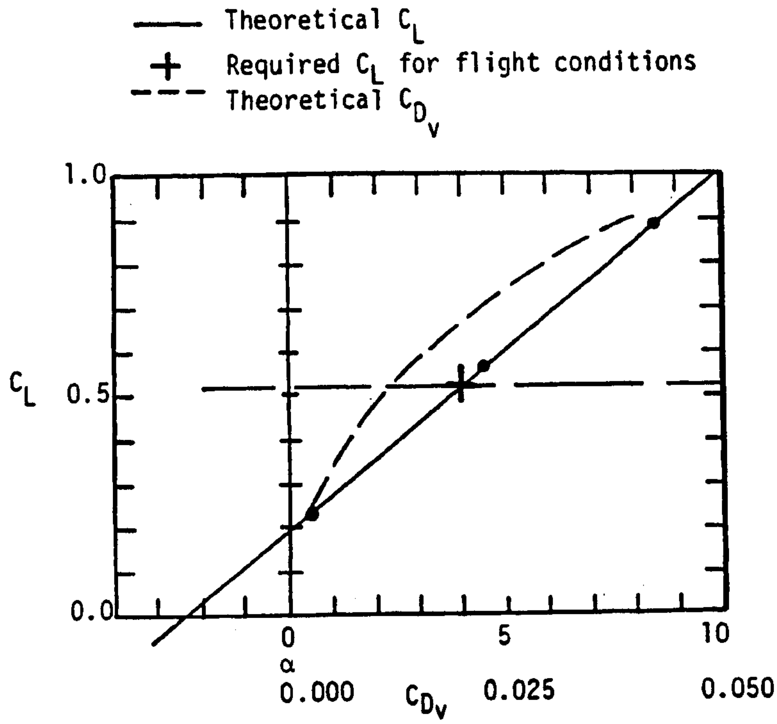
NACA 2412: $\alpha_{0\ell} = -2.095°$ (across the wing); $a_0 = 6.0$/radian

Solving the monoplane equation to calculate the aerodynamic coefficients for three angles of attack:

α	C_L	C_{D_V}
0.5°	0.2308	0.00246
4.5°	0.5588	0.01342
8.5°	0.8868	0.03736

The results are summarized in the sketch on the following page.

7.5 Contd.



The lift coefficient for the flight condition is:

$$C_L = \frac{L}{q_\infty S} = \frac{10,000}{(1200.5)(16.3)} = 0.5110$$

Thus, $\alpha = 4.0°$ and $C_{D_v} = 0.011$. The spanwise load distribution is:

	$\frac{2y}{b}$			
	0.92308	0.70711	0.38268	0.00000
$\frac{C_\ell}{C_L}$	0.6259	0.9213	1.0945	1.1618

7.6 $$\vec{V} = \frac{\Gamma_n}{4\pi}\left\{\frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|^2}\right\}\left\{\vec{r}_0 \cdot \left[\frac{\vec{r}_1}{r_1} - \frac{\vec{r}_2}{r_2}\right]\right\} \quad (7.37)$$

$$\vec{r}_0 = \overrightarrow{AB} = (x_{2n} - x_{1n})\hat{i} + (y_{2n} - y_{1n})\hat{j} + (z_{2n} - z_{1n})\hat{k}$$
$$\vec{r}_1 = (x - x_{1n})\hat{i} + (y - y_{1n})\hat{j} + (z - z_{1n})\hat{k}$$
$$\vec{r}_2 = (x - x_{2n})i + (y - y_{2n})\hat{j} + (z - z_{2n})\hat{k}$$

7.6 Contd.

$$\left\{ Fac\ 1_{AB} \right\} = \frac{\vec{r}_1 \times \vec{r}_2}{|\vec{r}_1 \times \vec{r}_2|^2} \quad \text{in equation (7.38a)}$$

$$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (x - x_{1n}) & (y - y_{1n}) & (z - z_{1n}) \\ (x - x_{2n}) & (y - y_{2n}) & (z - z_{2n}) \end{vmatrix}$$

$$= \hat{i}\left[(y - y_{1n})(z - z_{2n}) - (y - y_{2n})(z - z_{1n})\right]$$
$$+ \hat{j}\left[(x - x_{2n})(z - z_{1n}) - (x - x_{1n})(z - z_{2n})\right]$$
$$+ \hat{k}\left[(x - x_{1n})(y - y_{2n}) - (x - x_{2n})(y - y_{1n})\right]$$

$$|\vec{r}_1 \times \vec{r}_2|^2 = \Big\{ \left[(y - y_{1n})(z - z_{2n}) - (y - y_{2n})(z - z_{1n})\right]^2$$
$$+ \left[(x - x_{2n})(z - z_{1n}) - (x - x_{1n})(z - z_{2n})\right]^2$$
$$+ \left[(x - x_{1n})(y - y_{2n}) - (x - x_{2n})(y - y_{1n})\right]^2 \Big\}$$

$$\left\{ Fac\ 2_{AB} \right\} = \vec{r}_0 \cdot \frac{\vec{r}_1}{|\vec{r}_1|} - \vec{r}_0 \cdot \frac{\vec{r}_2}{|\vec{r}_2|} \quad \text{on page 296}$$

$$\vec{r}_0 \cdot \frac{\vec{r}_1}{|\vec{r}_1|} = \frac{(x_{2n} - x_{1n})(x - x_{1n}) + (y_{2n} - y_{1n})(y - y_{1n}) + (z_{2n} - z_{1n})(z - z_{1n})}{\sqrt{(x - x_{1n})^2 + (y - y_{1n})^2 + (z - z_{1n})^2}}$$

$$\vec{r}_0 \cdot \frac{\vec{r}_2}{|\vec{r}_2|} = \frac{(x_{2n} - x_{1n})(x - x_{2n}) + (y_{2n} - y_{1n})(y - y_{2n}) + (z_{2n} - z_{1n})(z - z_{2n})}{\sqrt{(x - x_{2n})^2 + (y - y_{2n})^2 + (z - z_{2n})^2}}$$

7.7 For a planar wing: $z_{1n} = z_{2n} = z_m = 0$

$$\vec{r}_0 = \vec{AB} = (x_{2n} - x_{1n})\hat{i} + (y_{2n} - y_{1n})\hat{j}$$

$$\vec{r}_1 = (x_m - x_{1n})\hat{i} + (y_m - y_{1n})\hat{j}$$

$$\vec{r}_2 = (x_m - x_{2n})\hat{i} + (y_m - y_{2n})\hat{j}$$

$$\vec{r}_1 \times \vec{r}_2 = \hat{k}\left[(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})\right]$$

$$\left|\vec{r}_1 \times \vec{r}_2\right|^2 = \left[(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})\right]^2$$

Thus,
$$\frac{\vec{r}_1 \times \vec{r}_2}{\left|\vec{r}_1 \times \vec{r}_2\right|^2} = \frac{\hat{k}}{\left[(x_m - x_{1n})(y_m - y_{2n}) + (x_m - x_{2n})(y_m - y_{1n})\right]}$$

$$\vec{r}_0 \cdot \frac{\vec{r}_1}{|\vec{r}_1|} = \frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}}$$

$$\vec{r}_0 \cdot \frac{\vec{r}_1}{|\vec{r}_1|} = \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}}$$

Thus,
$$\vec{V}_{AB} = \frac{\Gamma_n}{4\pi} \frac{\hat{k}}{\left[(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})\right]} \text{ times}$$

$$\left\{ \frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} - \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right\}$$

7.8] We are to calculate the downwash velocity at the control point (CP) of panel no. 4 induced by the horseshoe vortex of panel no.1 of the starboard wing.

$$x_m = 0.5875b; \quad y_m = 0.4375b$$

$$x_1 = 0.0500b; \quad y_1 = 0.0000b$$

$$x_2 = 0.1750b; \quad y_2 = 0.1250b$$

$$w_{4,1s} = \frac{\Gamma_1}{4\pi b}\left\{\frac{1.0}{(0.5375)(0.3125)-(0.4125)(0.4375)}\left[\frac{0.1250(0.5375)+0.1250(0.4375)}{\sqrt{(0.5375)^2+(0.4375)^2}}\right.\right.$$

$$\left. - \frac{0.1250(0.4125)+0.1250(0.3125)}{\sqrt{(0.4125)^2+(0.3125)^2}}\right]$$

$$+\frac{1.0}{(-0.4375)}\left[1.0+\frac{0.5375}{\sqrt{(0.5375)^2+(0.4375)^2}}\right]$$

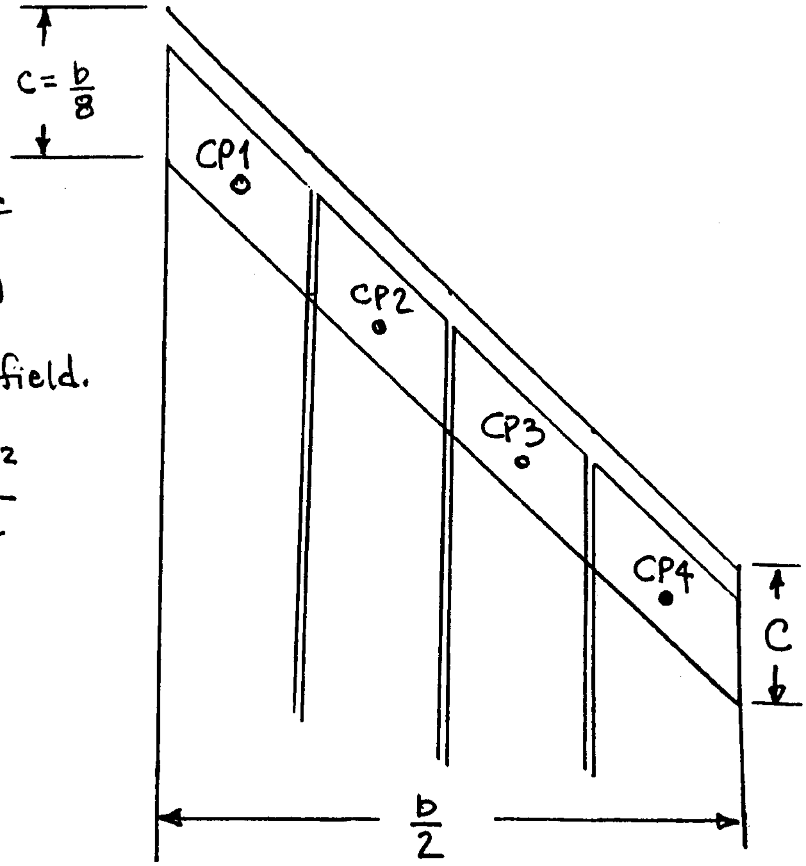
$$\left.-\frac{1.0}{(-0.3125)}\left[1.0+\frac{0.4125}{\sqrt{(0.4125)^2+(0.3125)^2}}\right]\right\}$$

$$w_{4,1s} = \frac{\Gamma_1}{4\pi b}\left[-0.0588 - 4.0584 + 5.7507\right]$$

$$w_{4,1s} = 1.6335\frac{\Gamma_1}{4\pi b}$$

7.9

And, of course, there is a port wing having mirror vortices that influence the flow field.



$$AR = 8 = \frac{b^2}{S} = \frac{b^2}{bc}$$

so that $b = 8c$

Table of the coordinates of the bound vortices and of the control points of the starboard (right) wing.

Panel	x_m	y_m	x_{1n}	y_{1n}	x_{2n}	y_{2n}
1	0.15625b	0.0625b	0.03125b	0.0000b	0.15625b	0.1250b
2	0.28125b	0.1875b	0.15625b	0.1250b	0.28125b	0.2500b
3	0.40625b	0.3125b	0.28125b	0.2500b	0.40625b	0.3750b
4	0.53125b	0.4375b	0.40625b	0.3750b	0.53125b	0.5000b

Let us use equation (7.44) to calculate the downwash velocity at the CP of panel 1 induced by the horseshoe vortex of panel 1 of the starboard wing.

$$w_{1,1s} = \frac{\Gamma_1}{4\pi}\left\{\frac{1.0}{(0.1250b)(-0.0625b) - (0b)(0.0625b)}\right.$$

7.9 Contd.

$$\left[\frac{(0.1250b)(0.1250b)+(0.1250b)(0.0625b)}{\sqrt{(0.1250b)^2+(0.0625b)^2}}\right.$$

$$\left.-\frac{(0.1250b)(0b)+(0.1250b)(-0.0625b)}{\sqrt{(0b)^2+(-0.0625b)^2}}\right]$$

$$+\frac{1.0}{-0.0625b}\left[1.0+\frac{(0.1250b)}{\sqrt{(0.1250b)^2+(0.0625b)^2}}\right]$$

$$\left.-\frac{1.0}{+0.0625b}\left[1.0+\frac{(0.0b)}{\sqrt{(0.0b)^2+(-0.0625b)^2}}\right]\right\}$$

$$w_{1,1s}=\frac{\Gamma_1}{4\pi b}\left\{-128.00\left[0.16771+0.12500\right]\right.$$

$$\left.-30.3108-16.0000\right\}=-83.7771\frac{\Gamma_1}{4\pi b}$$

Evaluating all of the various components (or influence coefficients), we find that at control point 1:

$$w_1=\frac{1}{4\pi b}\left[(0.24074\Gamma_4+0.51579\Gamma_3+1.72249\Gamma_2\right.$$

$$+17.25903\Gamma_1)_p+(-83.77708\Gamma_1+7.43706\Gamma_2$$

$$\left.+0.87294\Gamma_3+0.33104\Gamma_4)_s\right]$$

$$w_2=\frac{1}{4\pi b}\left[(0.23256\Gamma_4+0.45218\Gamma_3+1.09036\Gamma_2\right.$$

$$+3.48107\Gamma_1)_p+(19.77709\Gamma_1-83.77708\Gamma_2$$

$$+7.43706\Gamma_3+0.87294\Gamma_4)_s$$

7.9 Contd.

$$W_3 = \frac{1}{4\pi b}\left[\left(0.20666\,\Gamma_4 + 0.35871\,\Gamma_3 + 0.69318\,\Gamma_2 + 1.51108\,\Gamma_1\right)_p + \left(3.80720\,\Gamma_1 + 19.77709\,\Gamma_2 - 83.77708\,\Gamma_3 + 7.43706\,\Gamma_4\right)_s\right]$$

$$W_4 = \frac{1}{4\pi b}\left[\left(0.17665\,\Gamma_4 + 0.27956\,\Gamma_3 + 0.47020\,\Gamma_2 + 0.84579\,\Gamma_1\right)_p + \left(1.60961\,\Gamma_1 + 3.80720\,\Gamma_2 + 19.77709\,\Gamma_3 - 83.77708\,\Gamma_4\right)_s\right]$$

Since it is a planar wing with no dihedral, the "no flow" condition of equation (7.47) requires that

$$W_1 = W_2 = W_3 = W_4 = -U_\infty \alpha$$

$$-66.5181\,\Gamma_1 + 9.1596\,\Gamma_2 + 1.3887\,\Gamma_3 + 0.5718\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+23.2582\,\Gamma_1 - 82.6867\,\Gamma_2 + 7.8892\,\Gamma_3 + 1.1055\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+5.3183\,\Gamma_1 + 20.4702\,\Gamma_2 - 83.4184\,\Gamma_3 + 7.6437\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+2.4554\,\Gamma_1 + 4.2774\,\Gamma_2 + 20.0567\,\Gamma_3 - 83.6004\,\Gamma_4 = -4\pi b U_\infty \alpha$$

Solving for Γ_1, Γ_2, Γ_3, and Γ_4, we find that:

$$\Gamma_1 = 0.01810\,(4\pi b U_\infty \alpha)$$

$$\Gamma_2 = 0.01838\,(4\pi b U_\infty \alpha)$$

$$\Gamma_3 = 0.01859\,(4\pi b U_\infty \alpha)$$

$$\Gamma_4 = 0.01728\,(4\pi b U_\infty \alpha)$$

$$L = 2\rho_\infty U_\infty \sum_{n=1}^{4} \Gamma_n \Delta y_n$$

7.9 Contd.

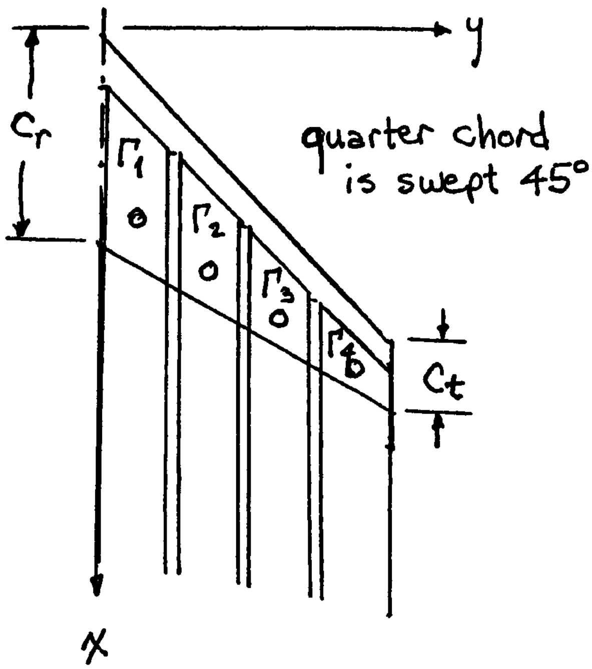
$$L = 2\rho_\infty U_\infty (4\pi b U_\infty \alpha)\left[0.01810 + 0.01838 + 0.01859 + 0.01728\right](0.1250b)$$

$$L = 0.07235\,\pi\,\rho_\infty U_\infty^2 b^2 \alpha$$

$$C_L = \frac{L}{q_\infty S} = \frac{0.07235\pi \rho_\infty U_\infty^2 b^2 \alpha}{\frac{1}{2}\rho_\infty U_\infty^2 \left[b\ \frac{b}{8}\right]}$$

$$C_L = 1.1576\,\pi\alpha$$

7.10



And, of course, there is a port wing having mirror vortices that influence the velocity field of the starboard wing.

$$c_t = 0.5c_r$$

$$S = 2\left[\left(\frac{b}{2}\right)\left(\frac{c_r + c_t}{2}\right)\right]$$

$$S = b\left(\frac{3}{4}c_r\right)$$

$$AR = 5 = \frac{b^2}{S} = \frac{b^2}{b\left(\frac{3}{4}c_r\right)} \Rightarrow c_r = \frac{4}{15}b$$

Panel	x_m	y_m	x_{1n}	y_{1n}	x_{2n}	y_{2n}
1	0.25417 b	0.0625b	0.0667 b	0.0000b	0.19167b	0.1250b
2	0.36250b	0.1875b	0.19167b	0.1250b	0.31667b	0.2500b
3	0.47083b	0.3125b	0.31667b	0.2500b	0.44167b	0.3750b
4	0.57916b	0.4375b	0.44167b	0.3750b	0.56667b	0.5000b

7.10 Contd.

$$w_1 = \frac{1}{4\pi b}\Big[(0.28397\,\Gamma_4 + 0.64163\,\Gamma_3 + 2.27049\,\Gamma_2$$

$$+ 19.09941\,\Gamma_1)_p + (-68.61193\,\Gamma_1 + 13.54764\,\Gamma_2$$

$$+ 1.23655\,\Gamma_3 + 0.41258\,\Gamma_4)_s\Big]$$

$$w_2 = \frac{1}{4\pi b}\Big[(0.25432\,\Gamma_4 + 0.49887\,\Gamma_3 + 1.19193\,\Gamma_2$$

$$+ 3.64019\,\Gamma_1)_p + (20.28998\,\Gamma_1 - 70.31355\,\Gamma_2$$

$$+ 12.0933\,\Gamma_3 + 1.12694\,\Gamma_4)_s\Big]$$

$$w_3 = \frac{1}{4\pi b}\Big[(0.21536\,\Gamma_4 + 0.37343\,\Gamma_3 + 0.71634\,\Gamma_2$$

$$+ 1.54024\,\Gamma_1)_p + (3.86458\,\Gamma_1 + 20.13701\,\Gamma_2$$

$$- 73.07623\,\Gamma_3 + 10.45307\,\Gamma_4)_s\Big]$$

$$w_4 = \frac{1}{4\pi b}\Big[(0.17897\,\Gamma_4 + 0.28293\,\Gamma_3 + 0.47471\,\Gamma_2$$

$$+ 0.85098\,\Gamma_1)_p + (1.61795\,\Gamma_1 + 3.83305\,\Gamma_2$$

$$+ 19.94846\,\Gamma_3 - 77.82260\,\Gamma_4)_s\Big]$$

$$w_1 = w_2 = w_3 = w_4 = -U_\infty \alpha$$

is the tangency condition for all four control points, since it is a planar wing with no dihedral. Thus,

$$-49.5125\,\Gamma_1 + 15.8181\,\Gamma_2 + 1.8782\,\Gamma_3 + 0.6966\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+23.9302\,\Gamma_1 - 69.1216\,\Gamma_2 + 12.5922\,\Gamma_3 + 1.3813\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+5.4048\,\Gamma_1 + 20.8534\,\Gamma_2 - 72.7028\,\Gamma_3 + 10.6684\,\Gamma_4 = -4\pi b U_\infty \alpha$$

$$+2.4689\,\Gamma_1 + 4.3078\,\Gamma_2 + 20.2314\,\Gamma_3 - 77.6436\,\Gamma_4 = -4\pi b U_\infty \alpha$$

Solving for Γ_1, Γ_2, Γ_3, and Γ_4, we find that:

$$\Gamma_1 = 0.03150\,(4\pi b U_\infty \alpha)$$

7.10 Contd.

$$L = 2\rho_\infty U_\infty \sum_{n=1}^{4} \Gamma_n \Delta y_n$$

$\Delta y_n = 0.1250b$ for each panel

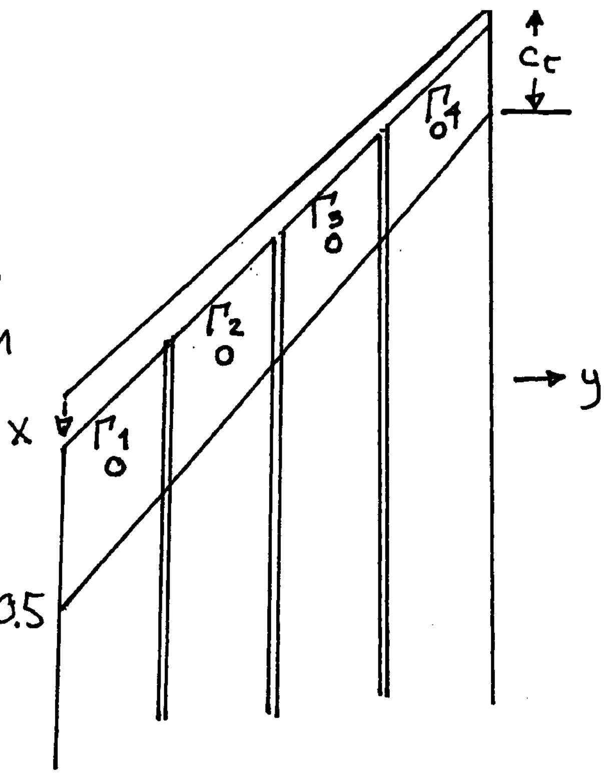
$$L = 2\rho_\infty U_\infty (4\pi b U_\infty \alpha)[0.03150 + 0.03100 + 0.02836 + 0.02299](0.1250b)$$

$$L = \rho_\infty U_\infty^2 b\,(0.11385)\,\alpha\pi$$

$$S = \frac{3}{4} b c_r \; ; \; c_r = \frac{4}{15} b \; ; \; S = \frac{1}{5} b^2$$

$$C_L = \frac{\rho_\infty U_\infty^2 b\,(0.11385)\,\alpha\pi}{\left(\frac{1}{2}\rho_\infty U_\infty^2\right)\left(\frac{1}{5}b^2\right)} = 1.1385\pi\alpha$$

7.11



Of course, there are mirror symmetric vortices on the port wing, which induce velocities at the control points on the starboard wing.

AR = 3.55; taper ratio = 0.5

$$S = 2\left(\frac{b}{2}\right)\left(\frac{c_r + c_t}{2}\right)$$

$$c_t = \frac{1}{2} c_r$$

$$S = 2\left(\frac{b}{2}\right)\frac{(c_r + \frac{1}{2}c_r)}{2} = \frac{3}{4} b c_r$$

$$AR = 3.55 = \frac{b^2}{S} = \frac{b^2}{0.75\,b\,c_r}$$

7.11 contd.] Thus,

$$C_r = \frac{4}{3}\frac{b}{3.55} = 0.3756b \;;\; C_t = \frac{1}{2}C_r = 0.18779b$$

Panel	x_m	y_m	x_{1n}	y_{1n}	x_{2n}	y_{2n}
1	0.20745b	0.0625b	0.09390b	0.0000b	−0.03110b	0.1250b
2	0.05898b	0.1875b	−0.03110b	0.1250b	−0.15610b	0.2500b
3	−0.08950b	0.3125b	−0.15610b	0.2500b	−0.28110b	0.3750b
4	−0.23797b	0.4375b	−0.28110b	0.3750b	−0.40610b	0.5000b

For the control points of our "flat plate" wing:

$$W_1 = W_2 = W_3 = W_4 = -U_\infty \alpha$$

$$W_1 = \frac{1}{4\pi b}\Big[\left(0.88435\Gamma_4 + 1.60158\Gamma_3 + 3.78165\Gamma_2 + 19.59125\Gamma_1\right)_p$$
$$+\left(-66.19173\Gamma_1 + 20.68658\Gamma_2 + 3.98390\Gamma_3 + 1.67207\Gamma_4\right)_s\Big]$$

$$W_2 = \frac{1}{4\pi b}\Big[\left(0.49752\Gamma_4 + 0.75184\Gamma_3 + 1.24515\Gamma_2 + 2.23909\Gamma_1\right)_p$$
$$+\left(+15.49708\Gamma_1 - 66.98395\Gamma_2 + 20.57927\Gamma_3 + 3.95631\Gamma_4\right)_s\Big]$$

$$W_3 = \frac{1}{4\pi b}\Big[\left(0.29461\Gamma_4 + 0.38599\Gamma_3 + 0.50414\Gamma_2 + 0.61055\Gamma_1\right)_p$$
$$+\left(1.26492\Gamma_1 + 13.87312\Gamma_2 - 66.29678\Gamma_3 + 20.44316\Gamma_4\right)_s\Big]$$

$$W_4 = \frac{1}{4\pi b}\Big[\left(0.18301\Gamma_4 + 0.21642\Gamma_3 + 0.24730\Gamma_2 + 0.26141\Gamma_1\right)_p$$
$$+\left(+0.38512\Gamma_1 + 1.11006\Gamma_2 + 11.83890\Gamma_3 - 70.67206\Gamma_4\right)_s\Big]$$

Applying the boundary conditions at the control point of each panel:

$$-46.6005\Gamma_1 + 24.4682\Gamma_2 + 5.5855\Gamma_3 + 2.5564\Gamma_4 = -4\pi b U_\infty \alpha$$
$$+17.7362\Gamma_1 - 65.7388\Gamma_2 + 21.3311\Gamma_3 + 4.4538\Gamma_4 = -4\pi b U_\infty \alpha$$
$$+1.8755\Gamma_1 + 14.3773\Gamma_2 - 67.9108\Gamma_3 + 20.7378\Gamma_4 = -4\pi b U_\infty \alpha$$
$$+0.6465\Gamma_1 + 1.3574\Gamma_2 + 12.0553\Gamma_3 - 70.4891\Gamma_4 = -4\pi b U_\infty \alpha$$

7.11 Contd. Solving for Γ_1, Γ_2, Γ_3, and Γ_4, we obtain:

$$\Gamma_1 = 0.04682\,(4\pi b U_\infty \alpha)$$

$$\Gamma_2 = 0.03917\ \ 4\pi b U_\infty \alpha)$$

$$\Gamma_3 = 0.03060\ \ (4\pi b U_\infty \alpha)$$

$$\Gamma_4 = 0.02060\ \ (4\pi b U_\infty \alpha)$$

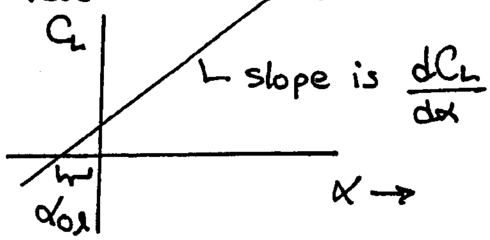
$$L = 2\rho_\infty U_\infty \left[0.04682 + 0.03917 + 0.03060 + 0.02060\right](4\pi b U_\infty \alpha)(0.1250b)$$

$$L = 0.13719\,\rho_\infty U_\infty^2\, b\, \pi\, \alpha$$

$$C_L = \frac{0.13719\,\rho_\infty U_\infty^2 b^2 \pi \alpha}{\frac{1}{2}\rho_\infty U_\infty^2 \left[\left(\frac{3}{4}b\right)(0.3756\,b)\right]} = 0.97401\pi\alpha$$

However, we have a wing with airfoil sections $\alpha_{0l} = -0.94°$. Thus, we could assume the linear variation between C_L and α:

$$C_L = \frac{dC_L}{d\alpha}(\alpha - \alpha_{0l})$$



Differentiating our expression for C_L with respect to α:

$$\frac{dC_L}{d\alpha} = 0.97401\pi$$

Thus,

$$C_L = 0.97401\pi(\alpha + 0.0164)$$

where α and α_{0l} are in radians

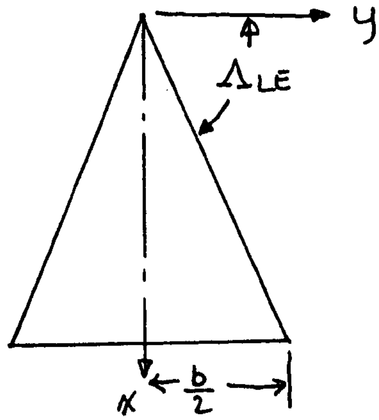
7.12

$$AR = \frac{b^2}{S} = \frac{b^2}{\frac{1}{2} b c_r} = 1.5$$

$$c_r = \frac{2b}{1.5} = \frac{4b}{3}$$

$$\tan \Lambda_{LE} = \frac{c_r}{b/2} = \frac{\frac{4b}{3}}{\frac{b}{2}} = \frac{8}{3}$$

$$\Lambda_{LE} = 69.444^\circ$$



The coordinates for the control points and for the corners of the bound vortices of the four panels of the starboard wing are:

Panel	x_m	y_m	x_{1n}	y_{1n}	x_{2n}	y_{2n}
1	1.04167b	0.0625b	0.33333b	0.0000b	0.58333b	0.1250b
2	1.12500b	0.1875b	0.58333b	0.1250b	0.83333b	0.2500b
3	1.20833b	0.3125b	0.83333b	0.2500b	1.08333b	0.3750b
4	1.29167b	0.4375b	1.08333b	0.3750b	1.33333b	0.5000b

Evaluating the various contributions to the velocities at the control points of the starboard wing:

$$w_1 = \frac{1}{4\pi b}\Big[(0.36165\,\Gamma_4 + 1.15109\,\Gamma_3 + 3.86042\,\Gamma_2 + 21.15021\,\Gamma_1)_p$$
$$+ (-64.19183\,\Gamma_1 + 20.79186\,\Gamma_2 + 2.96651\,\Gamma_3 + 0.58344\,\Gamma_4)_s\Big]$$

$$w_2 = \frac{1}{4\pi b}\Big[(0.28709\,\Gamma_4 + 0.67861\,\Gamma_3 + 1.59937\,\Gamma_2 + 4.13726\,\Gamma_1)_p$$
$$+ (21.19100\,\Gamma_1 - 64.39225\,\Gamma_2 + 19.66242\,\Gamma_3 + 1.70746\,\Gamma_4)_s\Big]$$

7.12 Contd.)

$$w_3 = \frac{1}{4\pi b}\Big[(0.22686\Gamma_4 + 0.44687\Gamma_3 + 0.86925\Gamma_2 + 1.73390\Gamma_1)_p$$

$$+ (4.16089\Gamma_1 + 21.07731\Gamma_2 - 65.27532\Gamma_3 + 14.80354\Gamma_4)_s\Big]$$

$$w_4 = \frac{1}{4\pi b}\Big[(0.18121\Gamma_4 + 0.31623\Gamma_3 + 0.54478\Gamma_2 + 0.94426\Gamma_1)_p$$

$$+ (1.74843\Gamma_1 + 4.09686\Gamma_2 + 20.73495\Gamma_3 - 88.17950\Gamma_4)_s\Big]$$

Since we have a planar wing with no dihedral:

$$w_1 = w_2 = w_3 = w_4 = -U_\infty \alpha$$

Equating the expressions for w_1, w_2, w_3, and w_4, we obtain:

$$-43.0416\Gamma_1 + 24.6523\Gamma_2 + 4.1176\Gamma_3 + 0.9451\Gamma_4 = -4\pi b U_\infty \alpha$$

$$25.3283\Gamma_1 - 62.7929\Gamma_2 + 20.3410\Gamma_3 + 1.9946\Gamma_4 = -4\pi b U_\infty \alpha$$

$$5.8948\Gamma_1 + 21.9465\Gamma_2 - 64.8285\Gamma_3 + 15.0304\Gamma_4 = -4\pi b U_\infty \alpha$$

$$2.8927\Gamma_1 + 4.6416\Gamma_2 + 21.0512\Gamma_3 - 87.9983\Gamma_4 = -4\pi b U_\infty \alpha$$

Solving for Γ_1, Γ_2, Γ_3, and Γ_4, we find that:

$$\Gamma_1 = 0.06073(4\pi b U_\infty \alpha)$$

$$\Gamma_2 = 0.05688(4\pi b U_\infty \alpha)$$

$$\Gamma_3 = 0.04504(4\pi b U_\infty \alpha)$$

$$\Gamma_4 = 0.02772(4\pi b U_\infty \alpha)$$

$$L = 2\rho_\infty U_\infty (4\pi b U_\infty \alpha)[0.06073 + 0.05688 + 0.04504$$

$$+ 0.02772](0.1250b)$$

$$L = \rho_\infty U_\infty^2 b^2 \alpha [0.19037]\pi$$

7.12 Contd.

$$C_L = \frac{\rho_\infty U_\infty^2 b^2 \alpha \left[0.19037\right]\pi}{\frac{1}{2}\rho_\infty U_\infty^2 \left[\left(\frac{1}{2}b\right)\left(\frac{4}{3}b\right)\right]} = 0.57111\pi\alpha = 1.7942\alpha$$

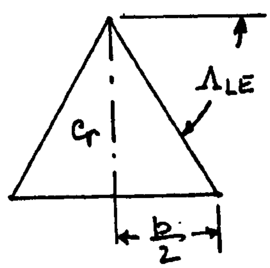
where α is in radians.

When $\alpha = 10°$, $\alpha = 0.1745$ radians and the calculated

$$C_L = 0.313$$

The experimental correlation of Fig. 7.44 indicates that the experimental value of $C_L \approx 0.35$ for an angle of attack of 10°

7.13 For a delta wing



$$S = \frac{c_r b}{2}$$

and $\tan\Lambda_{LE} = \dfrac{c_r}{b/2}$

Combining these two relations: $S = \frac{b^2}{4}\tan\Lambda_{LE}$; $\tan\Lambda_{LE} = \frac{4}{AR}$

Since the AR (aspect ratio) is 1.5, $\Lambda_{LE} = 69.45°$.

Using Fig. 7.42, $K_p = 1.76$ and using Fig. 7.43, $K_v = 3.18$ as the "constants" in equation (7.61)

$$C_L = K_p \sin\alpha\cos^2\alpha + K_v \sin^2\alpha\cos\alpha$$

we have:

α	0°	5°	10°	15°
C_L	0.000	0.176	0.391	0.631

These calculations reproduce those represented by the broken line of Fig. 7.44

7.14) Taking the coefficients found in the previous problem, i.e., $K_p = 1.76$ and $K_v = 3.18$, and substituting them into equations (7.61) and (7.62)

$$\Delta C_D = C_D - C_{D0} = C_L \tan\alpha$$

and using the results from the previous problem:

α	$0°$	$5°$	$10°$	$15°$
C_L	0.000	0.176	0.391	0.631
ΔC_D	0.000	0.015	0.069	0.169

7.15)

Given: The square-cube law and an airplane with the following characteristics:

$W = 92{,}000\ lbf$

$b = 75\ ft$

The square-cube law states that the wing area is proportional to the square of the span, and the wing volume (and thus wing weight) is proportional to the cube of the span. If we assume that the aircraft weight scales the same as the wing weight, then:

$$S \propto b^2 \text{ and } W \propto b^3$$

Wing loading is the aircraft weight divided by the wing area:

$$\frac{W}{S} \propto \frac{b^3}{b^2} = b$$

That is, as wing loading increases the span should also increase proportionally. This assumes that similar construction methods and materials are used. Typically, transport aircraft would be designed with fairly similar wing loadings (say 130 lbf/ft^2), but as the aircraft grows to very large sizes, a proportional growth in wing span would also be required in order to maintain aerodynamic efficiency due to induced drag effects. This would require higher and higher wing loadings, which may not be feasible due to structural or flight-related issues.

7.15) contd.

In order to estimate the new wing span for a 10% heavier airplane, use the weight proportionality relationship from above (subscript "1" is for the initial airplane and subscript "2" is for the larger airplane):

$$W \propto b^3 \qquad \text{or} \qquad \frac{W}{b^3} \propto 1 = C$$

$$\left(\frac{W}{b^3}\right)_1 = \left(\frac{W}{b^3}\right)_2 \qquad \left(\frac{92{,}000 lbf}{(75\,ft)^3}\right)_1 = \left(\frac{101{,}200 lbf}{b^3}\right)_2$$

$b_2 = 77.4\,ft$ which is only a 3.2% increase in wing span.

This analysis assumes that the wing weight would grow proportionally to the aircraft weight, which is not completely accurate. A more detailed analysis would require approximating the wing weight required to carry the additional load, as well as increased drag due to growth of the fuselage (if required). A more detailed analysis can be found in NASA Contractor Report 198351, "Advanced Configurations for Very Large Subsonic Transport Airplanes."

8.1 Assuming that the flow is one-dimensional, i.e., it does not vary across the cross section,

$$\dot{m} = \rho_\infty U_\infty A = \frac{p_\infty}{RT_\infty}\left[M_\infty\sqrt{\gamma R T_\infty}\right]A$$

$M_\infty = 3.5$; $T_\infty = 90.18\,\text{K}$; $p_\infty = 7.231 \times 10^3 \frac{\text{N}}{\text{m}^2}$

Thus,

$$\dot{m} = \left[\frac{7.231\times10^3\,\frac{\text{N}}{\text{m}^2}}{\left(287.05\,\frac{\text{N m}}{\text{kg K}}\right)(90.18\,\text{K})}\right]\left[(3.5)\sqrt{(1.4)\left(287.05\,\frac{\text{N m}}{\text{kg K}}\right)(90.18\,\text{K})}\right] \text{times}$$

$$\left[1.4884\,\text{m}^2\right] \qquad \text{(P8.1a)}$$

$$\dot{m} = 277.0\,\frac{\text{kg}}{\text{s}}$$

Since the density of the air at these conditions is $0.2793\,\frac{\text{kg}}{\text{m}^3}$ (this is the first bracketed factor in eq. (P8.1a), we can calculate the volumetric flow rate (Q):

$$Q = \frac{\dot{m}}{\rho} = 991.7\,\frac{\text{m}^3}{\text{s}}$$

8.2 Note that $T_t = 1000\,\text{K}$ and that the minimum allowable static temperature in the test section is 50K. Thus, the maximum allowable Mach number for this flow of perfect air can be found by solving eq. (8.34) for the Mach number.

Since $M_{ts}^2 = \frac{2}{\gamma - 1}\left[\frac{T_t}{T_{ts}} - 1\right]$; $M_{ts} = 9.747$

8.2 Contd.) We could also have used Table 8.1 to find the Mach number in the test section. Since we know that $(T_{ts}/T_t) = 0.05$, we could interpolate the temperature ratios on page 354 to find the corresponding Mach number. However, there are relatively few values presented in Table 8.1 in this Mach-number range. Although we can see that the test-section Mach number is between 9 and 10 (which is consistent with the value we calculated using eq. (8.34)), it is difficult to determine a precise value from Table 8.1. If one has access to NACA Report 1135 (Ames Research Center Staff, 1953), one could use the tables presented therein to determine the test-section Mach number with suitable accuracy.

8.3) The Reynolds number is:

$$Re_{\infty,L} = \frac{\rho_\infty U_\infty L}{\mu_\infty}, \text{ where } L = 0.75\,m$$

For a perfect gas, $\rho_\infty = \frac{p_\infty}{RT_\infty}$; $U_\infty = M_\infty\sqrt{\gamma R T_\infty}$;

$$\text{and } \mu_\infty = 1.458\times10^{-6}\,\frac{T_\infty^{1.5}}{T_\infty + 110.4} \quad (1.5a)$$

Since we know that the free-stream Mach number is 8, we could use Table 8.1 to find: $T_\infty = 0.07246\,T_t$ and $p_\infty = 1.02\times10^{-4}\,p_{t1}$. Since $T_t = 725K$, $T_\infty = 52.53K$, $U_\infty = 1162.4\,\frac{m}{s}$ and $\mu_\infty = 3.4072\times10^{-6}\,\frac{kg}{s\cdot m}$ (and are constants for this particular wind-tunnel application). The free-stream (or test-section) density, on the other

8.3 Contd. hand, is directly proportional to p_{t1}

$$\rho_\infty = \frac{1.02\times10^{-4}\,p_{t1}}{(287.05)(52.53)} \quad \begin{cases} = 4.6675\times10^{-3}\ \mathrm{kg/m^3} \\ \quad \text{for } p_{t1} = 6.90\times10^{5}\ \mathrm{N/m^2} \\ = 3.9911\times10^{-2}\ \mathrm{kg/m^3} \\ \quad \text{for } p_{t1} = 5.90\times10^{6}\ \mathrm{N/m^2} \end{cases}$$

$$= 6.76\times10^{-9}\,p_{t1}\ \frac{\mathrm{kg}}{\mathrm{m^3}}$$

p_{t1} should be $6.90\times10^{5}\ \mathrm{N/m^2}$, not $6.90\times10^{6}\ \mathrm{N/m^2}$. To compensate multiply the numbers on this page by 10.

As a result

$$Re_{\infty,L} = \frac{(6.76\times10^{-9}\,p_{t1})(1162.4\ \tfrac{m}{s})(0.75\,m)}{3.4072\times10^{-6}\ \frac{kg}{s\cdot m}}$$

$$Re_{\infty,L} = \begin{cases} 1.194\times10^{6} & \text{for } p_{t1} = 6.90\times10^{5}\ \mathrm{N/m^2} \\ & \text{and} \\ 1.021\times10^{7} & \text{for } p_{t1} = 5.90\times10^{6}\ \mathrm{N/m^2} \end{cases}$$

8.4 Recall that, for the wind-tunnel flow described in Problem 8.3, the air accelerates isentropically from a region where the air is "at rest" (the "stagnation chamber") and the stagnation temperature (T_t) is 725K and the stagnation pressure (p_{t1}) varies from $6.90\times10^{5}\ \mathrm{N/m^2}$ to $5.90\times10^{6}\ \mathrm{N/m^2}$ to a test section where the Mach number is 8. Because $M_\infty = 8$, we can use Table 8.1 to find the temperature and the pressure in the test section.

8.4 Contd. $p_\infty = 1.02\times10^{-4}\, p_{t_1}$ and $T_\infty = 0.07246\, T_t$

(a) Since $T_\infty = 0.07246\, T_t = 0.07246(725K) = 52.53K$

Referring to the standard atmosphere presented in Table 1.2, we see that the lowest temperature presented is 216.65K. Thus, the static temperature in the test section does not simulate the temperature altitude at any altitude of 30km, or less. In fact, if the temperature were any lower, oxygen would begin to condense in the test section. Nevertheless, the gas properties calculated using the perfect-gas relations are, in fact, in reasonable agreement with the actual properties.

(b) The perfect-gas equation for the velocity in the test section is:

$$U_\infty = M_\infty\sqrt{\gamma R T_\infty} = (8.0)\sqrt{(1.4)(287.05)(52.53)} = 1162.4\ \frac{m}{s}$$

For a perfect-gas flow, the value of the velocity in the test section is independent of the static pressure.

(c) The static pressure in the test section (p_∞) is $1.02\times10^{-4}\, p_{t1}$. Thus, the maximum value of p_∞ (that correponding to $p_{t1} = 5.90\times10^6\ N/m^2$) is approximately $604.2\ N/m^2$. Referring to Table 1.2, it is clear that this pressure corresponds to an altitude in excess of 30km. In fact, it corresponds to the static pressure at an altitude of approximately 35km.

8.5 (a) If the flow accelerates isentropically from the reservoir to the test section where the Mach number is 0.8, the static pressure in the test section is

$$p_{ts} = 0.65602\, p_{t1}$$

(see Table 8.1 or eq. (8.36)). Furthermore, since the nozzle exhaust is subsonic and the streamlines are straight, the static pressure in the exhaust flow is equal to the static pressure in the tank. Thus, $p_{ts} = p_{atm}$ $= 1.01325 \times 10^5\ N/m^2$. Combining the two expressions:

$$p_{t1} = \frac{1.01325 \times 10^5}{0.65602} = 1.5445 \times 10^5 \frac{N}{m^2}$$

(b) $T_t = 40°C = 313K$

For $M_\infty = 0.80$, $T_\infty = 0.88652\, T_t$ (see Table 8.1 or eq.(8.34))

Thus, $T_\infty = 277.5\ K$

Note that when using the tabulated ratios to evaluate the fluid properties, you must make use of the parameters in absolute units, e.g., K or °R

(c) To evaluate the Reynolds number for this flow, where;

$M_\infty = 0.8$; $T_\infty = 277.K$; and $p_\infty = 1.01325 \times 10^5 \frac{N}{m^2}$

$$\mu_\infty = 1.458 \times 10^{-6} \frac{T_\infty^{1.5}}{T_\infty + 110.4} = 1.7374 \times 10^{-4} \frac{kg}{m^3}$$

$$U_\infty = M_\infty \sqrt{\gamma R T_\infty} = 267.15 \frac{m}{s}$$

Thus, $$Re_c = \frac{\rho_\infty U_\infty c}{\mu_\infty}$$

8.5 Contd.] $Re_c = \frac{(1.2721)(267.15)(0.30)}{1.7374 \times 10^{-5}} = 5.868 \times 10^6$

(d) Since the flow is subsonic, the pressure at the stagnation point of the airfoil is equal to the stagnation pressure in the tunnel reservoir, i.e., $p_t = p_{t1}$. Recall, as shown in Example 8.3, that

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2$$

Thus, $C_{p,t} = \frac{p_t - p_\infty}{q_\infty} = \left(\frac{p_t}{p_\infty} - 1\right)\frac{2}{\gamma M_\infty^2} = \left(\frac{p_{t1}}{p_\infty} - 1\right)\frac{2}{\gamma M_\infty^2}$

$$C_{p,t} = \left(\frac{1}{0.65602} - 1\right)\frac{2}{(1.4)(0.8)^2} = 1.1704.$$

Note that, even though the flow is subsonic, the pressure coefficient at the stagnation point is not one. Recall that, when we applied Bernoulli's equations, the flow was steady, inviscid, irrotational, incompressible with negligible body forces and $C_{p,t} = 1.0$. For the present flow, the flow is steady, inviscid, irrotational, COMPRESSIBLE with negligible body forces and $C_{p,t} > 1$. As will be seen when discussing the hypersonic flow of Problem 8.20, $C_{p,t}$ approaches 2 as M_∞ becomes very large.

8.6)

Given: A convergent nozzle as shown in Fig. P8.6 with various back pressures depending on altitude.

Solution results are given in the book on pp. 438-439. John Bertin told me that the solution should remain in the book and serve as a guide for students to use while learning to perform convergent nozzle calculation.

8.6) contd.

Using Table 1.2A, and assuming that the pressure inside the cabin is the total pressure and the pressure outside is the back pressure:

h (km)	p_b / p_{SL}	p_b (N/m^2)	p_b / p_c
6	0.46600	0.47218×10^5	0.94436
8	0.35185	0.35651×10^5	0.71302
10	0.26153	0.26500×10^5	0.53000
12	0.19145	0.19399×10^5	0.38798

The Mach number for isentropic acceleration to the back pressure may be solved for from Eqn. 8.36:

$$M_b = \left\{ \frac{2}{\gamma - 1} \left[\left(\frac{p_c}{p_b} \right)^{(\gamma-1)/\gamma} - 1 \right] \right\}^{0.5}$$

which yields the following Mach numbers: 0.2872 at 6 km, 0.7122 at 8 km, 0.9972 at 10 km, and 1.2462 at 12 km. These values can be verified by using Table 8.1 and interpolating.

For conditions (a) through (c) the back pressure is such that the flow would only accelerate to subsonic speeds. When flow exhausts subsonically from a duct, the static pressure in the exit plane is equal to the static pressure of the environment into which it exhausts. Thus, for conditions (a) through (c), $p_{ne} = p_b$, which is illustrated in the pressure distributions in Fig, P8.6a, and in the correlations for the nozzle exit pressure as a function of back pressure in Fig. P8.6b.

An isentropic expansion to the back pressure of condition (d) would produce supersonic flow. However, from the discussion of one-dimensional flow, we know that the maximum speed that can be achieved in a convergent-only nozzle is the sonic value. Furthermore, that would occur at the throat (i.e., the nozzle exit plane). The existence of sonic conditions in the exit plane prevents changes in the conditions outside from propagating upstream. As a result, we conclude that the flow expands to sonic conditions in the nozzle exit plane for condition (d). Further acceleration (with the corresponding decrease in pressure) takes place outside the nozzle. Any back pressure below that required for sonic flow would produce a similar "choking" of the nozzle to sonic conditions, with subsequent expansion taking place outside the airplane.

8.7)

Given: The conditions of Prob. 8.6 with an exit diameter of 0.75 cm and a temperature in the cabin of 22°C.

First note that $T_t = T_c = 295.15$ K since the flow is adiabatic. For altitude of 6 km, the flow is not choked, so the mass flow rate is given by:

$$\dot{m} = \dot{m}_{ne} = \rho_{ne} U_{ne} A_{ne}$$

where the nozzle exit area is $A_{ne} = \pi d^2/4 = \pi(7.5\times10^{-3})^2/4 = 4.4179\times10^{-5}\ \text{m}^2$. Since the pressure at the nozzle exit is $0.47218\times10^5\ \text{N/m}^2$, and from Eqn. 8.37:

$$\rho_{ne} = \frac{\rho_t}{\left(1+\dfrac{\gamma-1}{2}M_{ne}^2\right)^{1/(\gamma-1)}} = 0.56652\ \text{kg/m}^3$$

The temperature and the speed of sound at the nozzle exit are:

$$T_{ne} = \frac{p_{ne}}{R\rho_{ne}} = \frac{0.47218\times10^5}{(287.05)(0.56652)} = 290.36\ \text{K}$$

$$a_{ne} = \sqrt{\gamma R T_{ne}} = \sqrt{(1.4)(287.05)(290.36)} = 341.59\ \text{m/s}$$

The velocity at the nozzle exit is:

$$U_{ne} = M_{ne} a_{ne} = (0.2872)(341.59) = 98.107\ \text{m/s}$$

The mass flow rate is given by:

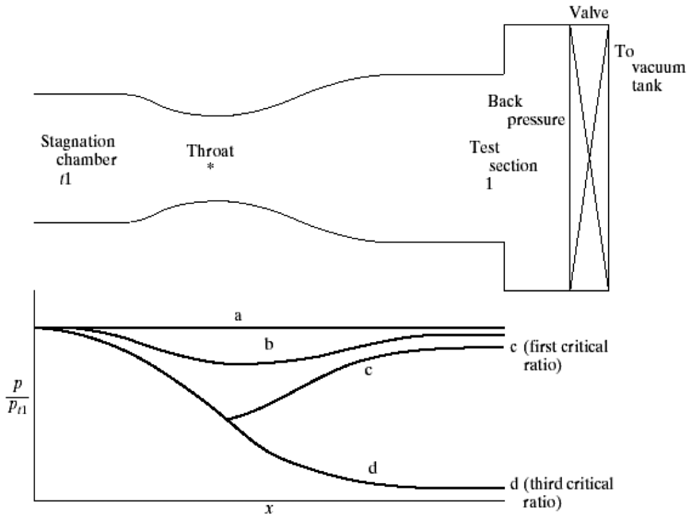
$$\dot{m} = \dot{m}_{ne} = \rho_{ne} U_{ne} A_{ne} = (0.56652)(98.107)(4.4179\times10^{-5}) = 2.4555\times10^{-3}\ \text{kg/s}$$

For an altitude of 12 km, we know that the flow in the nozzle is choked, so we can use Eqn. 8.45:

$$\dot{m}_{ne} = \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}\frac{p_{t_1} A^*}{\sqrt{T_t}} = 5.1965\times10^{-3}\ \text{kg/s}$$

8.8)

Given: Flow in a convergent-divergent nozzle as shown below.



The total temperature is given by $T_t = 200^\circ F = 659.6^\circ R$. Since the back pressure is equal to the total pressure for condition (a), there is no flow. Thus for condition (a):

$$p_1 = 100.000 \text{ psia} \qquad T_t = 659.6^\circ R \qquad M_1 = 0.0 \qquad U_1 = 0.0$$

Before proceeding with the calculations, first find the values of the various parameters that correspond to $A/A^* = 2.035$. From Table 8.1 the Mach number can be either 0.30 or 2.23, with corresponding values for the pressure ratio, p/p_{t_1}, of 0.93947 or 0.09117, respectively. The first values correspond to the situation where the flow accelerates in the convergent section, reaches sonic speed at the throat, and then, because of relatively high back pressure, decelerates in the divergent section. This is known as the first critical ratio. Thus, for condition (c):

$$p_1 = 93.947 \text{ psia} \qquad M_1 = 0.30$$

Using Table 8.1:

$$T_1 = \frac{T}{T_t} T_t = 0.98232(659.6) = 647.94^\circ R$$

and the velocity is:

$$U_1 = M_1 a_1 = 0.30\sqrt{(1.4)(1716.16)(647.94)} = 374.3 \text{ ft/s}$$

8.8) contd.

The second set of values from Table 8.1 corresponds to the situation where the flow accelerates in the convergent section, reaches sonic speed at the throat, and then, because of relatively low back pressure, continues to accelerate isentropically in the divergent section. This is known as the third critical ratio. The test section Mach number is 2.23 for condition (d) and:

$$p_1 = 9.117 \text{ psia} \qquad T_t = 332.69^\circ R \qquad M_1 = 2.23 \qquad U_1 = 1993.88 \text{ ft/s}$$

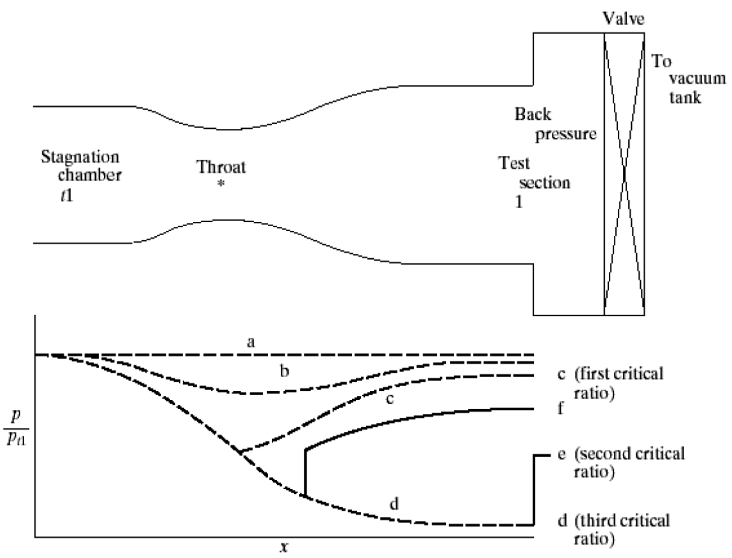
For condition (b) the flow is subsonic throughout the nozzle. Thus using Table 8.1:

$$p_1 = 97.250 \text{ psia} \qquad T_t = 654.36^\circ R \qquad M_1 = 0.20 \qquad U_1 = 250.79 \text{ ft/s}$$

What happens to the flow when the back pressure is between 9.117 psia and 93.947 psia? To be able to define the flow field for pressures between the third critical pressure ratio and the first critical pressure ratio, we need to be able to understand shock waves.

8.9)

Given: Flow in a convergent-divergent nozzle as initially described in Prob. 8.8 and shown in Fig. P8.9 (reproduced below).



Calculate the static pressure, static temperature, Mach number, and velocity in the test section for the following back pressures: (e) $p_b = 51.38$ psia and (f) $p_b = 75.86$ psia.

8.9) contd.

Recall that, if the flow accelerates isentropically to the supersonic Mach number that corresponds to the area ratio of the test section (2.035), the Mach number there is 2.23. That corresponds to the third critical pressure ratio (i.e., condition (d) of Fig. P8.8). Assume that a normal shock wave occurs in the test section at this condition We can use Table 8.3 to calculate the conditions downstream of the shock wave. For $M_1 = 2.23$, the pressure ratio is:

$$\frac{p_2}{p_{t_1}} = \left(\frac{p_2}{p_1}\right)\left(\frac{p_1}{p_{t_1}}\right) = (5.6358)(0.09117) = 0.5138$$

Thus, $p_2 = 51.38$ psia. This corresponds to the value of the back pressure for condition (e). Since the thickness of the shock wave is infinitesimal, there are actually two sets of conditions in the test section for condition (e): one set upstream of the shock wave (corresponding to the condition designated "1" in the shock wave nomenclature) and a second set downstream of the shock wave (corresponding to the condition designated "2" in the shock wave nomenclature). The former set of conditions is identical to those of condition (d) in Prob. 8.8. The latter set of conditions (i.e., those downstream of a normal shock wave) are those of condition (e).Continuing with the parameters of Table 8.3:

$$M_2 = 0.5432$$

$$T_2 = \left(\frac{T_2}{T_1}\right)T_1 = (1.8836)(332.69) = 626.65^\circ R$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{(1.4)(1716.16)(626.65)} = 1227.03 \text{ ft/s}$$

$$U_2 = M_2 a_2 = (0.5432)(1227.03) = 666.52 \text{ m/s}$$

The pressure downstream of a normal shock wave in the test section when written in dimensionless form as p_2/p_{t_1} is known as the second critical pressure ratio.

For condition (f), a normal shock wave occurs in the divergent section of the nozzle between the throat and the test section, as shown in Fig. P8.9. Upstream of this shock wave, the flow accelerates isentropically to the supersonic speed corresponding to the area ratio at the location where the shock is located. Downstream of the shock wave, the flow is subsonic. Since subsonic flow in a divergent section decelerates, the velocity decreases as the flow proceeds in the divergent section. The static pressure increases in the streamwise direction, matching the back pressure at the downsream boundary condition.

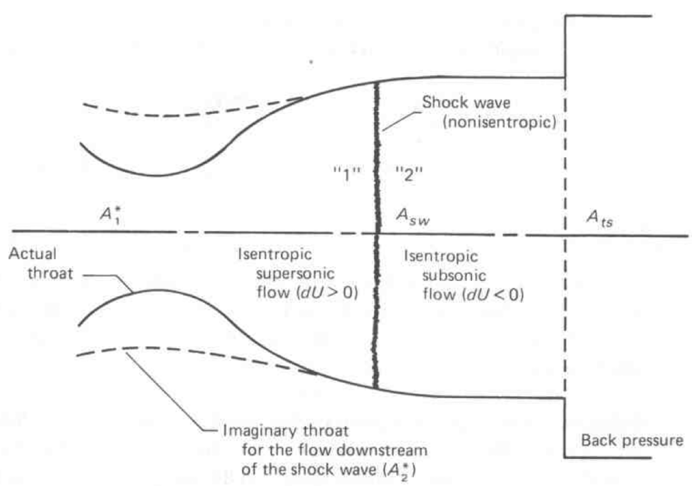
8.9) contd.

We will use an iterative procedure to determine where the shock wave occurs such that the pressure of the subsonic flow in the test section equals the back pressure. For the first iteration, assume that a normal shock wave occurs in the divergent section where the area ratio, A_{sw}/A^*, is 1.4952. A sketch of the flow and the relevant nomenclature is presented below. Using Table 8.1, we see that the conditions ahead of the shock wave are $M_1 = 1.85$ and $p_1/p_{t_1} = 0.16120$. We can use Table 8.3 to determine the conditions downstream of the shock wave:

$$M_2 = 0.6057$$

$$p_{t_2}/p_{t_1} = 0.79021$$

Having determined the downstream Mach number, M_2, we can calculate the area ratio for the second, "imaginary" nozzle (see the broken line in the figure below) that would produce this flow by an isentropic process.



From Table 8.1, the area ratio for the imaginary nozzle is $A_{sw}/A_2^* = 1.1819$. Now determine the conditions in the test section by modeling a subsonic deceleration for the test-section area ratio for the second nozzle. The corresponding area ratio is:

$$\frac{A_{ts}}{A_2^*} = \frac{A_{ts}}{A_1^*}\frac{A_{sw}/A_2^*}{A_{sw}/A_1^*} = 2.035\frac{1.1819}{1.4952} = 1.6086$$

The subsonic Mach number corresponding to this area ratio is 0.3944. For this Mach number, the pressure ratio in Table 8.1 is 0.8984. Although this ratio is p_1/p_{t_1} in the table, it is actually

8.9) contd.

p_{ts}/p_{t_2}, since we are considering the flow downstream of the shock wave, and since the area ratio is that of the imaginary (second) nozzle. The resultant pressure ratio is:

$$\frac{p_{ts}}{p_{t_1}} = \left(\frac{p_{ts}}{p_{t_2}}\right)\left(\frac{p_{t_2}}{p_{t_1}}\right) = (0.8984)(0.79021) = 0.7099$$

The pressure from this iteration is less than that specified for condition (f). Now repeat the iterative procedure until obtaining the proper value for the pressure ratio in the test section. The appropriate values for the flow upstream of the shock wave are:

$$\frac{A_{sw}}{A_1^*} = 1.3865 \qquad M_1 = 1.75 \qquad \frac{p_1}{p_{t_1}} = 0.7594$$

The conditions immediately downstream of the shock wave are:

$$\frac{A_{sw}}{A_2^*} = 1.1571 \qquad M_2 = 0.62809 \qquad \frac{p_{t_2}}{p_{t_1}} = 0.83456$$

For the test section:

$$\frac{A_{ts}}{A_2^*} = 1.6983 \qquad M_{ts} = 0.3694 \qquad \frac{p_{t_2}}{p_{t_1}} = 0.7954$$

Note that for this pressure ratio the pressure in the test section is essentially that prescribed for condition (f). Since the flow across a shock wave is adiabatic, $T_{t_1} = T_{t_2}$. Thus the static temperature in the test section for this condition is determined by using Table 8.1 for $M_{ts} = 0.3694$.

$$T_{ts} = \left(\frac{T_{ts}}{T_{t_2}}\right) T_{t_2} = (0.9733)(659.6) = 642.0^\circ R$$

$$U_{ts} = M_{ts} a_{ts} = 0.3694\sqrt{(1.4)(1716.16)(642.0)} = 458.82 \text{ ft/s}$$

8.10] We know nothing of the geometry between the two stations nor the direction of the flow. We do know that the flow is one-dimensional and steady. Furthermore, $A_1 = A_2 = 5.0 \text{ ft}^2$. Furthermore, the streamtube is insulated. Thus, $H_{t_1} = H_{t_2}$. So that, for a perfect-gas, $T_{t_1} = T_{t_2}$, 8.10 Contd. Let us first examine the flow at station 2.

$$\frac{p_2}{p_{t2}} = \frac{2101}{2116} = 0.9929. \text{ Thus, } M_2 = 0.10$$

As a result, $\frac{T_2}{T_{t2}} = 0.9980$; so that $T_2 = 499\,°R$

Now we have enough information to calculate U_2.

$$U_2 = M_2 a_2 = M_2 \sqrt{\gamma R T_2} = 109.54 \frac{ft}{s}$$

$$\rho_2 = \frac{p_2}{R T_2} = \frac{2101}{(1716)(499)} = 0.00246 \frac{slugs}{ft^3}$$

$$\dot{m}_2 = \rho_2 U_2 A_2 = (0.00246)(109.54)(5) = 1.344 \frac{slugs}{s}$$

Let us now examine the flow at station 1. Of course, $\dot{m}_1 = \dot{m}_2$. Solving equation (8.44) for p_{t1}:

$$p_{t1} = \frac{\dot{m}}{A}\sqrt{\frac{RT_t}{\gamma}} \; \frac{\left[\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\gamma+1/(2(\gamma-1))}}{M}$$

$$p_{t1} = \frac{1.344}{5}\sqrt{\frac{(1716)(500)}{1.4}} \; \frac{21.92}{3} = 1543.0 \frac{lbf}{ft^2}$$

Let us calculate the entropy change from 1 to 2. We know that

$$S_2 - S_1 = C_p \ln \frac{T_{t2}}{T_{t1}} - R \ln \frac{p_{t2}}{p_{t1}}$$

$T_{t2} = T_{t1}$. $p_{t2} = 2116$. $p_{t1} = 1543.0 \frac{lbf}{ft^2}$

Thus, $\frac{S_2 - S_1}{R} = -\ln \frac{p_{t2}}{p_{t1}} < 0$. But the entropy of a system undergoing an adiabatic process cannot decrease, Thus, the flow must be from 2 to 1!

8.11) Note that the shock wave immediately ahead of the face of the probe is normal to the approach flow. Thus, the entropy of the flow at the stagnation point ($s_{t2} = s_2$) equals the entropy downstream of a normal shock wave. From the stagnation point, the flow is assumed to expand isentropically to the static pressure p_2. Thus,

$$\frac{p_{t2}}{p_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}} \qquad (8.36)$$

Solving for M_2^2: $\quad M_2^2 = \frac{2}{\gamma-1}\left[\left(\frac{p_{t2}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad$ (a)

However, using the normal-shock relations, eq. (8.75)

$$M_2^2 = \frac{(\gamma-1)M_\infty^2 + 2}{2\gamma M_\infty^2 - (\gamma-1)}$$

Let us solve this expression for M_∞^2 in terms of M_2^2:

$$M_2^2\left(2\gamma M_\infty^2\right) - M_2^2(\gamma-1) = (\gamma-1)M_\infty^2 + 2$$

$$M_\infty^2\left[2\gamma M_\infty^2 - (\gamma-1)\right] = M_2^2(\gamma-1) + 2$$

Thus: $\quad M_\infty^2 = \frac{M_2^2(\gamma-1)+2}{\left[2\gamma M_\infty^2 - (\gamma-1)\right]} \qquad$ (b)

Substituting the pressure ratios of (a) for M_2^2 in (b)

$$M_\infty^2 = \frac{2\left[\left(\frac{p_{t2}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + 2}{\frac{4\gamma}{\gamma-1}\left[\left(\frac{p_{t2}}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] - (\gamma-1)} = \frac{2\left(\frac{p_{t2}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}}{\frac{4\gamma}{\gamma-1}\left[\left(\frac{p_{t2}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}\right] - \frac{(\gamma+1)^2}{(\gamma-1)}}$$

For perfect air, $\gamma = 1.4$ and $\frac{\gamma-1}{\gamma} = \frac{2}{7} = a$;

$$M_\infty^2 = \left(\frac{p_{t2}}{p_2}\right)^a \Big/ \left[7\left(\frac{p_{t2}}{p_2}\right)^a - 7.2\right]$$

8.12) Recall that:

$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}\ \frac{p_{t1}}{\sqrt{T_t}} \qquad (8.45)$$

Rearranging and adding the appropriate subscripts to the parameters to designate their relation to the flow upstream of the shock wave:

$$p_{t1} A_1^* = \frac{\dot{m}_1 \sqrt{T_{t1}}}{\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}}$$

By continuity, the mass flow rate is constant through the shock wave, i.e., $\dot{m}_1 = \dot{m}_2$ and the stagnation temperature is constant across a shock wave for a perfect gas, i.e., $T_{t1} = T_{t2} = T_t$. Thus,

$$p_{t1} A_1^* = p_{t2} A_2^*$$

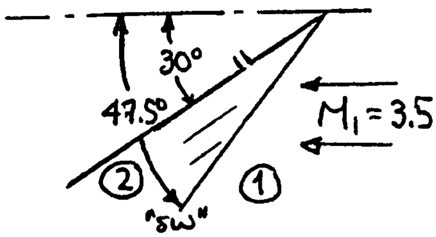
(a) $$\frac{A_2^*}{A_1^*} = \frac{p_{t1}}{p_{t2}}$$, which is given in terms of the free-stream Mach number in eq. (8.76)

(b) as $M_1 \to 1$, $p_{t2} \approx p_{t1}$ and $\frac{A_2^*}{A_1^*} \approx 1$

as $M_1 \to \infty$, $p_{t2} \to 0$ and $\frac{A_2^*}{A_1^*} \to \infty$

(c) A_1^* is the area that the streamtube would be if the supersonic flow corresponding to M_1 were decelerated isentropically to sonic velocity. A_2^* is the area that the streamtube would be if the supersonic flow corresponding to M_1 passed through a normal shock wave and THEN accelerated isentropically to the sonic velocity.

8.13) When a sharp wedge with a deflection angle of 30° is placed in an airstream of Mach 3.5, a straight (linear) oblique shock wave attached to the leading edge is formed. As shown in the sketch, the shock wave makes an angle of 47.5° with respect to the free stream. The shock wave turns the streamlines downstream



of the shock wave parallel to the surface of the wedge and parallel to the other streamlines. The pressure coefficient in the shock layer is 0.775. Let us use the subscript "1" to designate the flow upstream upstream of the shock wave, i.e., the free-stream conditions. Let us use the subscript "2" to designate the flow downstream of the shock wave, i.e., the flow in the shock layer between the surface and the shock layer.

For $M_\infty = M_1 = 3.5$:

$$p_\infty = p_1 = \frac{p_1}{p_{t1}} p_{t1} = (0.01311)(6.0 \times 10^5 \frac{N}{m^2}) = 7.866 \times 10^3 \frac{N}{m^2}$$

$$C_p = 0.775 = \frac{p_2 - p_1}{q_1} = \left(\frac{p_2}{p_1} - 1\right)\frac{2}{\gamma M_1^2}$$

$$p_2 = \frac{p_2}{p_1} p_1 = \left(1 + \frac{\gamma M_1^2}{2} C_p\right) p_1 = (7.646)(7.866 \times 10^3 \frac{N}{m^2})$$

(a) $p_2 = 5.2274 \times 10^4 \frac{N}{m^2}$ is the static pressure on the wedge.

(b) The U-tube manometer senses the difference between p_2 and p_1. $p_2 - p_1 = 4.4408 \times 10^4 \frac{N}{m^2}$

8.13 Contd. However, using the manometry equation:

$$p_2 = p_1 + \rho g \Delta h$$

where the density of mercury (ρ) is $13.6 \times 10^3 \frac{kg}{m^3}$.

Thus,

$$\Delta h = \frac{p_2 - p_1}{\rho g} = \frac{4.4408 \times 10^4 \frac{kg}{m\, s^2}}{\left(13.6 \times 10^3 \frac{kg}{m^3}\right)\left(9.8066 \frac{m}{s^2}\right)} = 0.3330 m$$
$$= 33.30 cm$$

(c) $q_1 = \frac{\gamma}{2} p_1 M_1^2 = 0.7\left(7.866 \times 10^3 \frac{N}{m^2}\right)(3.5)^2 = 6.745 \times 10^4 \frac{N}{m^2}$

The barometric pressure is not required in this problem, since the stagnation pressure is given in absolute units and since the difference between the wall orifice and the surface orifice is measured directly by the U-tube measurement.

8.14 (a) A uniform (irrotational) stream of air where the Mach number is 3.0 is to be turned compressively by 10°

(i) For a simple, single ramp which turns the flow 10° in a single deflection, we will use the charts of Fig. 8.13. For $M_1 = 3.0$; $\delta = 10°$: $\theta = 27.5°$ (Fig. 8.13a); $1 - \frac{1}{M_2} = 0.6$ (Fig. 8.13c). Thus, $M_2 = 2.5$.

The component of the free-stream Mach number normal to the shock wave is:

$$M_{n1} = M_1 \sin\theta = 1.385$$

Referring to Table 8.3 for $M_{n1} = 1.385$: $\frac{p_{t2}}{p_{t1}} = 0.9619$

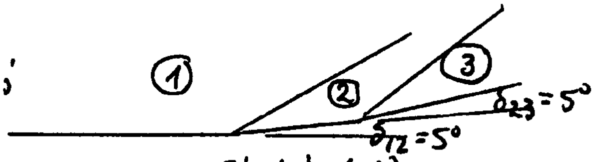
sketch i

Therefore, $\frac{\Delta s}{R} = -\ln \frac{p_{t2}}{p_{t1}} = 0.0388$

8.14 Contd.] For two successive 5° sharp turns, we will use the oblique shock-wave relations to accomplish the 10° turn in two 5° turns. Referring to sketch (ii): For $M_1 = 3.0$;

$\delta_{12} = 5°$: $\theta = 23.2°$ (Fig. 8.13a);

$1 - \frac{1}{M_2} = 0.635$ (Fig. 8.13c).



sketch (ii)

Thus, $M_2 = 2.740$.

The component of the free-stream Mach number normal to the shock wave: $M_{n1} = M_1 \sin\theta = 1.182$. Referring to Table 8.3 for $M_{n1} = 1.182$: $\frac{p_{t2}}{p_{t1}} = 0.9944$

$$\frac{\Delta s_{21}}{R} = 0.0056$$

To get to state 3 (the final state), the flow in region 2 is turned 5° by a second weak shock wave (see sketch (ii)). For this second turn, the flow in region 2 is the flow upstream of the shock wave (designated 1 in the graphs) and the flow in region 3 is the flow downstream of the shock waves (designated 2 in the graphs)

$$M_2 = 2.74 \text{ and } \delta_{23} = 5°: \; 1 - \frac{1}{M_3} = 0.603 \; ; \theta_3 = 25.3°$$

Thus, $M_3 = 2.519$. Furthermore, the component of the free-stream Mach number normal to the shock wave is:

$$M_{n2} = M_2 \sin\theta_3 = 1.171$$

Referring to Table 8.3, $\frac{\Delta s}{R} = \ln \frac{p_{t3}}{p_{t2}} = \ln(0.9952)$

$$\frac{\Delta s_{32}}{R} = 0.0048 \; ; \text{ Thus, } \frac{\Delta s_{31}}{R} = \frac{\Delta s_{32} + \Delta s_{21}}{R} = 0.0104$$

8.14 Contd. An infinite number of infinitesimal turns may be assumed to be an isentropic process. Thus, we can use the Prandtl-Meyer relations to calculate the change as we cross the left-running Mach waves on right-running characteristics

$$d\nu = \nu_f - \nu_1 = -d\theta = -(\theta_f - \theta_1) = -10°$$

Since $M_1 = 3.0$, $\nu_1 = 49.757°$. Therefore, $\nu_f = 39.757°$

and $M_f = 2.527$

There is no change in entropy for an isentropic process.

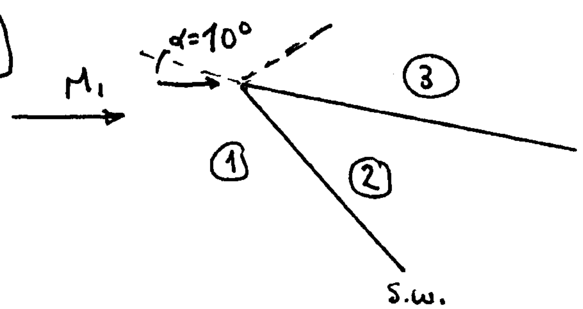
(b) To summarize the results from part (a)

	M_{final}	$\frac{\Delta s}{R}$	$\frac{p_{tf}}{p_{t1}}$
(i) Single 10° turn	2.50	0.0388	0.9619
(ii) Two 5° turns	2.519	0.0104	0.9896
(iii) Infinite infinitesimal turns	2.527	0.0000	1.0000

Thus, the more gradual the process by which a total deflection is accomplished, the more nearly the process is isentropic. As a result, the final Mach number and the final stagnation pressure are greatest, the more gradual the turn.

(c) The results apply to compressive turns only, not to expansive turns.

8.15



Since the angle of attack of the airfoil is 10°, the flow undergoes a compressive turn in going from region 1

8.15 Contd. (the free-stream) to region 2 (the lower surface of the airfoil). Let us assume that this compressive flow is turned by a single, attached, oblique shock wave. The flow along the entire length of the lower surface (downstream of the shock wave) is uniform (i.e., the pressure is constant; the Mach number is constant, etc.) Using the charts of Fig. 8.13 for $M_1 = 2.0$ and $\delta = 10°$:

$$C_{p_2} = \frac{p_2 - p_1}{q_1} = 0.25; \quad \frac{p_2}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_{p_2} = 1 + (0.7)(4)(0.25)$$

$$p_2 = 1.70\, p_1$$

The flow undergoes an isentropic expansion from the free-stream conditions in region 1 to the local flow on the upper surface in region 3. The flow is uniform along the entire length of the upper surface. To calculate the flow properties in region 3, note that the flow crosses the left-running Mach waves emanating from the leading edge.

$$\nu_3 - \nu_1 = -d\theta = -(\theta_3 - \theta_1) = -[-10° - 0°] = +10°$$

Using Table 8.2: $\nu_3 = \nu_1 + 10° = 26.38° + 10° = 36.38°$

Thus, $M_3 = 2.385$. Since the flow undergoes an isentropic acceleration from region 1 to region 3, $p_{t1} = p_{t3}$. Thus, using Table 8.1

$$M_1 = 2.00: \quad \frac{p_1}{p_{t1}} = 0.1278$$

$$M_3 = 2.385: \quad \frac{p_3}{p_{t3}} = 0.07003$$

$$\frac{p_3}{p_1} = \frac{p_3/p_{t3}}{p_1/p_{t1}} = \frac{0.07003}{0.1278} = 0.5479$$

Since the pressures in region 2 and in region 3 are constant

8.15 Contd. over the surface, the resultant force can be considered to act at the midchord location, normal to the chord line. The net force is

$$f_{net} = (1.700 p_1 - 0.5479 p_1) c = 1.1521 p_1 c$$

where c, the chord length of the airfoil, is the wing area per unit span. Since this is a two-dimensional airfoil

$$c_\ell = \frac{\ell}{q_1 c} = \frac{f_{net} \cos\alpha}{\frac{\gamma}{2} p_1 M_1^2 \, c}$$

$$c_\ell = \frac{(1.1521 p_1 c)(\cos 10°)}{\frac{\gamma}{2} p_1 (4) c} = 0.4052$$

Similarly,

$$c_d = \frac{d}{q_1 c} = \frac{f_{net} \sin\alpha}{\frac{\gamma}{2} p_1 M_1^2 \, c}$$

$$c_d = \frac{(1.1521 p_1 c)(\sin 10°)}{(0.7) p_1 (4) c} = 0.0715$$

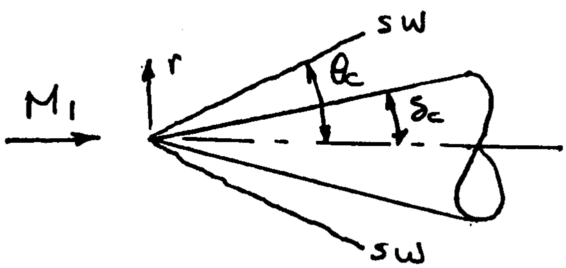
Note that $\frac{\ell}{d} = \frac{c_\ell}{c_d} = \frac{\cos\alpha}{\sin\alpha} = \cot\alpha = 5.67$

Since the net force acts at the midchord of the airfoil,

$$c_{M_{0.5c}} = 0.0$$

It is clear that we did not need to know p_1 (or, equivalently, the altitude) or the dimension (scale) of the airfoil to generate the desired coefficients

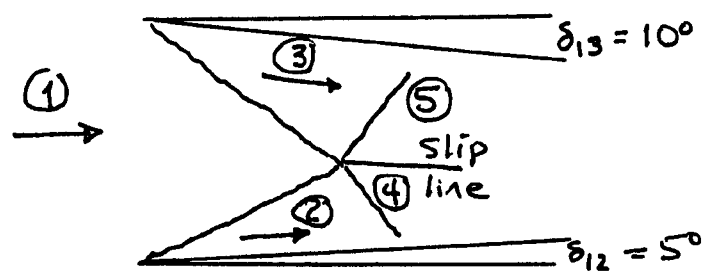
8.16



If $M_1 = 2.80$ and $\delta_c = 8°$, then $\theta_c = 22°$ (Fig. 8.16a)

$$l = \frac{r}{\tan\theta_c} = \frac{1.0}{\tan 22°} = 2.475\,m$$

8.17



Note the slip line which divides the flow in region 4 from that in region 5 is a fluid/fluid interface. Thus, the flow direction in region 4 is the same as the flow direction in region 5, i.e., $\theta_4 = \theta_5$. Furthermore, the static pressure in region 4 is the same as the static pressure in region 5, i.e., $p_4 = p_5$. However, since the flow in region 4 has passed through different strength shockwaves than the flow in region 5, $s_4 \neq s_5$ (the entropies are different). Similarly, $M_4 \neq M_5$, etc.

Let us use Fig. 8.13 to calculate directly the flows in regions 2 and 3

Region 2: $M_1 = 2.5$, $\delta_{12} = 5°$ (the flow deflection in going from region 1 to region 2). Using Fig. 8.13c,

$1 - \frac{1}{M_2} = 0.56$, $M_2 = 2.272$. We know $\theta_2 = +5°$.

However, with $\delta_{12} = 5°$, it is difficult to use Chart 8.13b

8.17 Contd. to determine a realistic value of C_{p_2}. Thus, let us use Fig. 8.13a to find $\theta_w = 27.5°$. Thus, we can calculate the normal component of the Mach number $M_{n1} = M_1 \sin\theta_w = 1.154$. Therefore, $\frac{p_2}{p_1} = 1.387$ ($p_2 = 6.395 \times 10^3\ N/m^2$),

Region ③: $M_1 = 2.5$, $\delta_{13} = 10°$. $M_{n1} = M_1 \sin\theta_w$. Since $\theta_w = 31.9°$, $M_{n1} = 1.321$. Therefore, $\frac{p_3}{p_1} = 1.869$ ($p_3 = 9.345 \times 10^3\ N/m^2$). Using Fig. 8.13c, $1 - \frac{1}{M_3} = 0.518$.

Therefore, $M_3 = 2.075$. Since the flow in region ③ is parallel to the wall, $\theta_3 = -10°$

To calculate the flow in regions ④ and ⑤, we use an iterative procedure until $p_4 = p_5$ AND $\theta_4 = \theta_5$.

Iteration No. 1: Assume $\delta_{35} = 4°$. Thus, $\theta_5 = -6°$. If $\theta_5 = -6°$, θ_4 must also equal $-6°$. Thus, $\delta_{24} = 11°$.

For $M_2 = 2.272$ and $\delta_{24} = 11°$:

$\theta_w = 35.8°$; $M_{2n} = M_2 \sin\theta_w = 1.329$; $\frac{p_4}{p_2} = 1.893$; and $p_4 = 2.626\, p_1$

For $M_3 = 2.075$ and $\delta_{35} = 4°$:

$\theta_w = 32.1°$; $M_{3n} = M_3 \sin\theta_w = 1.103$; $\frac{p_5}{p_3} = 1.250$; and $p_5 = 2.336\, p_1$

Thus, for our assumed deflection $p_4 > p_5$.

Iteration No. 2: Apparently, a better assumption would have $\delta_{35} = 5.5°$. Thus, $\theta_5 = -4.5°$. If $\theta_5 = -4.5°$, θ_4 must equal $-4.5°$. Thus, $\delta_{24} = 9.5°$

For $M_2 = 2.272$ and $\delta_{24} = 9.5°$:

$\theta_w = 34.2°$; $M_{2n} = M_2 \sin\theta_w = 1.277$; $\frac{p_4}{p_2} = 1.736$; and $p_4 = 2.408\, p_1$

8.17 Contd. For $M_3 = 2.075$ and $\delta_{35} = 5.5°$:

$\theta_w = 34.0°$; $M_{3n} = M_3 \sin\theta_w = 1.160$; $\frac{p_5}{p_3} = 1.403$; and $p_5 = 2.622 p_1$

We still need better agreement.

Iteration No. 3: Let us try $\delta_{35} = 4.7°$, so that $\theta_5 = \theta_4 = -5.3°$

For $M_2 = 2.272$ and $\delta_{24} = 10.3°$:

$\theta_w = 35.1°$; $M_{2n} = M_2 \sin\theta_w = 1.306$; $\frac{p_4}{p_2} = 1.823$; and $p_4 = 2.529 p_1$

For $M_3 = 2.075$ and $\delta_{35} = 4.7°$:

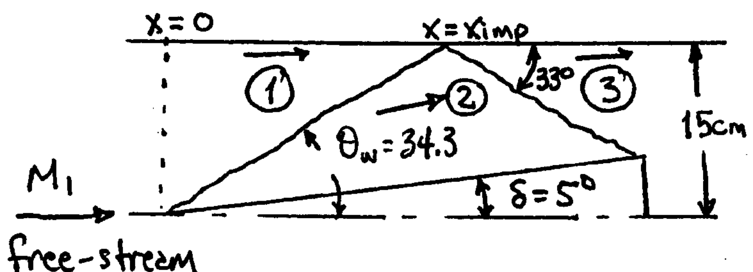
$\theta_w = 33.0°$; $M_{3n} = M_3 \sin\theta_w = 1.130$; $\frac{p_5}{p_3} = 1.323$; and $p_5 = 2.473 p_1$

We are close enough that we can say:

Iteration No. 4: $p_4 = p_5 \approx 2.50 p_1$; $\theta_4 = \theta_5 \approx -5.2°$

Thus, $\delta_{35} = 4.8°$ and $\delta_{24} = 10.2°$. As a result, $1 - \frac{1}{M_5} = 0.47$; $M_5 = 1.886$. $1 - \frac{1}{M_4} = 0.47$, $M_4 = 1.886$, within our accuracy: $M_4 = M_5$. This results because the turning angles are small, so the small deflection approximations are reasonably valid. (In a general case, $M_4 \neq M_5$ and $p_{t4} \neq p_{t5}$!)

8.18



Note that the flows in regions 1 and 3 are parallel to the wind tunnel wall ($\theta_1 = \theta_3 = 0°$) and the flow in region 2 is parallel to the airfoil surface ($\theta_2 = +5°$).

Since $M_1 = 2$ and $\delta_{12} = \delta = 5°$, we can use Fig. 8.13a:

8.18 Contd. $\theta_w = 34.3°$, as shown in the sketch. If we say that the leading edge of our airfoil corresponds to the origin of our streamwise coordinate (i.e., $x=0$), then the shock wave impinges on the wall at:

$$x_{imp} = \frac{15\text{ cm}}{\tan 34.3°} = 21.9892\text{ cm}$$

For the flow in the region between the leading-edge shock wave and the airfoil surface, i.e., in region (2), $M_2 = 1.818$. The flow in region (1) is inclined 5° and, therefore, must turn 5° in going from region (2) to region (3).

$M_2 = 1.818$ and $\delta_{23} = 5°$: $\theta_w = 38° \Rightarrow$ Thus, the reflected shock wave makes an angle of 38° to the flow direction in region (2) or 33° relative to the wall.

The equation for the reflected shock wave is

$$\frac{y-15}{x-x_{imp}} = \tan(-33°)$$

$$y = -0.6494x + 29.2789$$

The equation for the upper surface of the airfoil is:

$$\frac{y}{x} = \tan 5°$$

$$y = 0.08749x$$

Equating the two expressions for y and solving for x:

$$x = 39.734\text{ cm}$$

This is the maximum chord length of an airfoil before the reflected wave affects its flowfield. It should be noted that this analysis neglects shock/boundary-layer interactions.

8.19 If the static temperature of the air is $-75°F$, $T_1 = 384.6°R$. The speed of sound, therefore, is:

$$a_1 = 49.02\sqrt{T_1} = 49.02\sqrt{384.6} = 961.34\ \frac{ft}{s}$$

and the Mach number is:

$$M_1 = \frac{U_1}{a_1} = \frac{880\ \frac{ft}{s}}{961.34\ \frac{ft}{s}} = 0.915$$

Using Table 8.1: for $M_1 = 0.915$: $\frac{T_1}{T_t} = 0.8565$ and $\frac{p_1}{p_{t1}} = 0.5817$

Thus, $$T_t = \frac{T_1}{T_1/T_t} = \frac{384.6}{0.8565} = 449.0°R$$

and $$p_{t1} = \frac{p_1}{p_1/p_{t1}} = \frac{474\ lbf/ft^2}{0.5817} = 814.85\ \frac{lbf}{ft^2} = 0.385\ p_{SL}$$

8.20 We are to consider the case where the Space Shuttle is flying at 7.62 Km/s at an altitude of 75 Km. At this altitude the free-stream temperature is 200.15 K and the free-stream pressure is $2.50\ \frac{N}{m^2}$. Referring to Chapter 1, the free-stream speed of sound is

$$a_1 = \sqrt{\gamma RT} = 20.047\sqrt{T(K)} = 283.61\ \frac{m}{s}$$

Thus, the free-stream Mach number is:

$$M_1 = \frac{U_1}{a_1} = 26.868.$$

To calculate the flow just downstream of the normal (portion) of the shock wave for the perfect-gas assumptions, we will use equations (8.72) with $\sin\theta = 1$.

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} = 842.01;\ p_2 = 2105\ \frac{N}{m^2} = 0.02077\ p_{SL}$$

8.20 Contd.

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma-1)][(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2 M_1^2} = 141.31$$

$T_2 = 28,282.5\,K$ (Note how high the temperature downstream of the normal portion of the shock wave is for the perfect-gas model.

$$M_2^2 = \frac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)} \quad ; \quad M_2 = 0.3793$$

Since the flow decelerates isentropically from a point just downstream of the shock (designated by the symbol 2 in the first part of this problem) to the stagnation point (outside of the boundary layer and at the same entropy as point 2; we will designate these conditions t2). Starting with a Mach number at point 2 of 0.3793 and decelerating the flow to t2, we can use Table 8.1 to find:

$\frac{p_2}{p_{t2}}$ (which corresponds to $\frac{p}{p_{t1}}$ of Table 8.1) $= 0.9055$

Thus, $$p_{t2} = \frac{p_2}{p_2/p_{t2}} = \frac{2105.0}{0.9055} = 2324.7\,\frac{N}{m^2}$$

$\frac{T_2}{T_{t2}}$ (which corresponds to $\frac{T}{T_t}$ of Table 8.1) $= 0.9720$

Thus, $$T_{t2} = \frac{T_2}{T_2/T_{t2}} = \frac{28282.5}{0.9720} = 29,097.2\,K$$

$$C_{p,t2} = \frac{p_{t2} - p_1}{q_1} = \left(\frac{p_{t2}}{p_1} - 1\right)\frac{2}{\gamma M_1^2}$$

8.21) $M_\infty = M_1 = 11.5$; $T_t = 1970K$; $p_\infty = p_1 = 1070\ \frac{N}{m^2}$

If $\delta_c = 12°$, we can use Fig. 8.16 to determine the parameters at the surface of the cone (neglecting the effects of the boundary layer)

$$C_{p_c} = 0.092 \quad \text{and} \quad 1 - \frac{1}{M_c} = 0.855$$

$$\frac{p_c}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_{p_c} = 9.52\ ;\ p_c = 10186\ \frac{N}{m^2} = 0.1005\ p_{SL}$$

$$M_c = 6.9$$

Thus, since $M_c = 6.9$: $\frac{p_c}{p_{tc}} \left(= \frac{p}{p_{t1}}\right) = 0.2646 \times 10^{-3}$

and $p_{tc} = 3.85 \times 10^7\ \frac{N}{m^2} = 379.97\ p_{SL}$

Furthermore, $T_c = \left(\frac{T_c}{T_t}\right) T_t = (0.9504 \times 10^{-1})\, 1970K = 187.2K$

To calculate the Reynolds number, we use equation (1.10) to calculate the density:

$$\rho_c = \frac{p_c}{R T_c} = \frac{10186\ \frac{N}{m^2}}{\left(287.05\ \frac{Nm}{kg\,K}\right)(187.2K)} = 0.1895\ \frac{kg}{m^3}$$

and we use equation (1.12a) to calculate the viscosity:

$$\mu_c = 1.458 \times 10^{-6}\ \frac{T_c^{1.5}}{T_c + 110.4} = 1.255 \times 10^{-5}\ \frac{kg}{s \cdot m}$$

and we use equation (1.14a) to calculate the speed of sound:

$$a_c = 20.047 \sqrt{T_c(K)} = 274.3\ m/s$$

$$U_c = M_c a_c = 1893\ \frac{m}{s} \quad ; \quad x = 0.1m$$

Thus, $Re_x = \frac{\rho_c U_c x}{\mu_c} = 2.858 \times 10^6$

8.18) Again, $M_\infty = M_1 = 11.5$; $T_t = 1970K$; $p_\infty = p_1 = 1070 \frac{N}{m^2}$.
But this time the 12° deflection of the flow is made by a wedge, or two-dimensional configuration. Thus, we use Fig. 8.13 to find the solution for a two-dimensional flowfield.

$$C_{p_2} = 0.115 \text{ and } 1 - \frac{1}{M_2} = 0.845$$

$$\frac{p_2}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_{p_2} = 11.65 ; \; p_2 = 12461 \frac{N}{m^2} = 0.1230 \, p_{SL}$$

$M_2 = 6.45$. Since $M_2 = 6.45$,

$$\frac{p_2}{p_{t2}} \left(\text{which corresponds to } \frac{p}{p_{t1}} \text{ in Table 8.1}\right) = 0.4045 \times 10^{-3}$$

Thus $$p_{t2} = \frac{p_2}{p_2/p_{t2}} = 3.080 \times 10^7 \frac{N}{m^2} = 303.97 \, p_{SL}$$

Furthermore, $$T_2 = \left(\frac{T_2}{T_{t2}}\right) T_{t2} = (0.1073)(1970) = 211.38K$$

(noting that $T_{t2} = T_{t1} = T_t$)

To calculate the Reynolds number, we again must first calculate ρ_2, μ_2, and U_2

$$\rho_2 = 0.2054 \frac{kg}{m^3} ; \; \mu_2 = 1.392 \times 10^{-5} \frac{kg}{s \cdot m}$$

$$U_2 = M_2 a_2 = (6.45)(291.5 \tfrac{m}{s})$$

Thus, $$Re_x = \frac{\rho_2 U_2 x}{\mu_2} = 2.773 \times 10^6$$

The free-stream flows of Problem 8.21 (the previous problem) and of this problem were identical. Both free-stream flows were turned 12°, one by a cone

8.22 Cont) (an axisymmetric configuration), the other by a wedge (a two-dimensional configuration). Let us compare the parameters at the surface of these two configurations:

Property at the "edge of the boundary layer"	Cone	Wedge
M	6.9	6.45
C_p	0.09	0.115
$p\ (N/m^2)$	1.02×10^4	1.25×10^4
$p_{t''e''}\ (N/m^2)$	3.85×10^7	3.08×10^7
$T\ (K)$	187.2	211.4
$T_t\ (K)$	1970	1970

Note that p (the static pressure), T (the static temperature), and C_p (the pressure coefficient are greater for the wedge; while M (the local Mach number) and $p_{t''e''}$ (the total pressure downstream of the shock wave are less. Although both shock waves are "weak", i.e., straight shock waves attached at the apex of the configuration, the shock wave for the wedge is stronger than that for the cone.

8.17) $M_\infty = M_1 = 11.5$; $T_t = 1970K$; $p_\infty = p_1 = 1070 \frac{N}{m^2}$

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Thus, since $M_c = 6.9$: $\frac{p_c}{p_{tc}} \left(= \frac{p}{p_{t1}} \right) = 0.2646 \times 10^{-3}$

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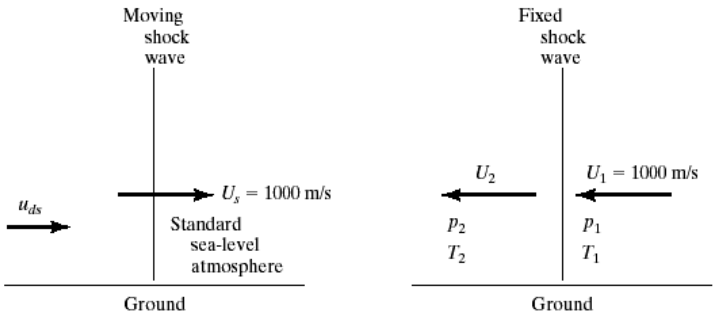
Problem 8.23 Solution

Given: A blast wave traveling at $U_s = 1000$ m/s at standard sea-level conditions.

As shown in Fig. P8.23 (and reproduced below), the moving blast wave can be transformed into an equivalent steady flow with $U_1 = 1000$ m/s.



For standard sea-level conditions from Table 1.2A:

$$p_{SL} = p_1 = 1.01325 \times 10^5 \text{ N/m}^2$$

$$T_{SL} = T_1 = 288.150 \text{ K}$$

$$a_{SL} = a_1 = 340.29 \text{ m/s}$$

The Mach number of the flow is:

$$M_1 = \frac{U_1}{a_1} = \frac{1000}{340.29} = 2.939$$

For flow through a normal shock wave at this Mach number:

$$M_2 = 0.47886$$

$$p_2 = (9.911)(1.01325 \times 10^5) = 1.00423 \times 10^6 \text{ N/m}^2$$

$$T_2 = (2.608)(288.150) = 751.50 \text{ K}$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{(1.4)(287.05)(751.50)} = 549.55 \text{ m/s}$$

$$U_2 = M_2 a_2 = (0.47886)(549.55) = 263.16 \text{ m/s}$$

However, this is the velocity of the shock in the shock-wave-fixed coordinate system. The actual velocity of the air behind the shock the shock wave is determined by returning to the ground-based coordinate system:

$$u_{ds} = 1000 - U_2 = 1000 - 263.16 = 736.84 \text{ m/s}$$

and the Mach number of the shock-induced motion is:

$$M_{ds} = \frac{u_{ds}}{a_2} = \frac{736.84}{549.55} = 1.341$$

9.1] Starting with the integral form of the energy equation for a steady, one-dimensional adiabatic flow:

$$H_t = h_1 + \frac{U_1^2}{2} = h_2 + \frac{U_2^2}{2} \qquad (8.29)$$

For a perfect gas, this equation can be written:

$$c_p T_1 + \frac{U_1^2}{2} = c_p T_2 + \frac{U_2^2}{2} \qquad (8.30)$$

Furthermore, for a perfect gas

$$c_p T = \frac{\gamma R}{\gamma - 1} T = \frac{a^2}{\gamma - 1}$$

Let the "1" in equation (8.30) represent the free-stream (∞) conditions and the "2" represent the local conditions. Thus, $U_1^2 = U_\infty^2$ and $U_2^2 = u^2 + v^2 + w^2$. Making these substitutions, equation (8.30) becomes:

$$a_\infty^2 + \frac{\gamma-1}{2} U_\infty^2 = a^2 + \frac{\gamma-1}{2}(u^2+v^2+w^2) \qquad (P.9.1a)$$

Noting that: $u^2 = [U_\infty + u']^2 = U_\infty^2 + 2u'U_\infty + (u')^2$,

equation (P.9.1a) becomes:

$$a^2 = a_\infty^2 + \frac{\gamma-1}{2}\left[U_\infty^2 - U_\infty^2 - 2u'U_\infty - (u')^2 - (v')^2 - (w')^2\right]$$

Combining terms, dividing through by a_∞^2, and rearranging terms, this equation becomes:

$$\frac{a^2}{a_\infty^2} = 1 - \frac{\gamma-1}{2}\frac{U_\infty^2}{a_\infty^2}\left[\frac{2u'U_\infty + (u')^2 + (v')^2 + (w')^2}{U_\infty^2}\right]$$

Since only small perturbations are considered, the sum of the terms in the bracket is small and we can use the expression for a binomial series to invert the left-hand side:

$$\frac{a_\infty^2}{a} = 1 + \frac{\gamma-1}{2} M_\infty^2 \left[\frac{2u'}{U_\infty} + \frac{(u')^2 + (v')^2 + (w')^2}{U_\infty^2}\right] \qquad (9.7)$$

9.2] A rectangular wing having an aspect ratio of 3.5 is flown at $M_\infty = 0.85$ at 12 km. Introducing the affine transformation:

$$x' = \frac{x}{\sqrt{1 - M_\infty^2}} \; ; \; y' = y \; ; \text{ and } z' = z$$

where the ' quantities denote the values for the equivalent incompressible, subsonic flow. Thus, the chord length for the equivalent incompressible, subsonic flow would be:

$$c' = \frac{c}{\sqrt{1 - M_\infty^2}} = 1.8983c$$

However, the span would be unchanged. The aspect ratio of the equivalent wing in an incompressible flow is:

$$AR' = \frac{b'}{c'} = \frac{b}{1.8983c} = 0.5268\, AR = 1.8438$$

Since the z coordinate is unchanged by the transformation, the maximum thickness remains 0.06 of the original chord length. Since the chord length of the equivalent wing in an incompressible flow is 1.8983c, the thickness ratio is 0.0316. Thus, the airfoil section is approximately that for an NACA 0003 section.

9.3] We are to consider steady flow between two streamlines. By continuity:

$$\rho_e U_e \, dy_e = \rho_\infty U_\infty \, dy_\infty$$

For an isentropic flow of a perfect gas:

9.3 Contd.

$$\frac{\rho_\infty}{\rho_e} = \frac{\rho_\infty}{\rho_{t1}} \frac{\rho_{t1}}{\rho_{te}} \frac{\rho_{te}}{\rho_e}$$

Since the flow is isentropic: $\rho_{t1} = \rho_{te}$. For $M_\infty = 1.20$:

$\frac{\rho_e}{\rho_{te}} = 0.53114$. For $M_\infty = 0.85$: $\frac{\rho_\infty}{\rho_{t1}} = 0.71361$ (see Table 8.1) Thus,

$$\frac{\rho_\infty}{\rho_e} = [0.71361]\left[\frac{1}{0.53114}\right] = 1.3435$$

From the definition of the Mach number:

$$\frac{U_\infty}{U_e} = \frac{M_\infty\sqrt{T_\infty}}{M_e\sqrt{T_e}} = \frac{M_\infty\sqrt{T_\infty/T_t}}{M_e\sqrt{T_e/T_t}}$$

$$\frac{U_\infty}{U_e} = \frac{0.85}{1.20}\frac{\sqrt{0.87374}}{\sqrt{0.77640}} = 0.75142$$

Thus, $$dy_e = \frac{\rho_\infty U_\infty}{\rho_e U_e} dy_\infty = 1.0096\, dy_\infty$$

As was noted, in those regions where the local flow has expanded considerably relative to the free-stream conditions, the streamlines have diverged. The application of the area rule is designed to compensate for this streamtube divergence.

10.1 For linear theory:

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

independent of the cross section. In addition:

$$C_{m_{x_0}} = \frac{-4\alpha}{\sqrt{M_\infty^2 - 1}}\left(\frac{1}{2} - \frac{x_0}{c}\right) + \frac{4}{\sqrt{M_\infty^2 - 1}}\int_0^1 \frac{dz_c}{dx}\,\frac{x - x_0}{c}\,d\left(\frac{x}{c}\right)$$

But for a symmetric section, $z_c \equiv 0$. Furthermore, since we are to calculate the pitching moment about the midchord location, $x_0 = \frac{c}{2}$. Thus

$$C_{m_{0.5c}} = 0.0$$

To calculate the section drag coefficient, note that

$$\overline{\sigma_u^2} = \overline{\sigma_l^2}$$

Applying this fact to equation (10.16) gives:

$$C_d = \frac{4}{\sqrt{M_\infty^2 - 1}}\left[\alpha^2 + \overline{\sigma_u^2}\right]$$

Therefore, we need to evaluate $\overline{\sigma_u^2}$. Referring to the sketch in the problem statement:

$$\frac{dz_u}{dx} = \tan\delta_1 = \frac{(t/2)}{a_1 c} \quad \text{for} \quad 0 \le \frac{x}{c} \le a_1$$

and $$\frac{dz_u}{dx} = \tan\delta_2 = \frac{(-t/2)}{a_2 c} \quad \text{for} \quad (1 - a_2) \le \frac{x}{c} \le 1$$

Thus, $$\overline{\sigma_u^2} = \int_0^{a_1}\left(\frac{t}{2a_1 c}\right)^2 d\left(\frac{x}{c}\right) + \int_{1-a_2}^{1}\left(\frac{-t}{2a_2 c}\right)^2 d\left(\frac{x}{c}\right)$$

$$\overline{\sigma_u^2} = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[\frac{1}{a_1} + \frac{1}{a_2}\right] = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[\frac{a_1 + a_2}{a_1 a_2}\right]$$

10.1 Contd.] Note that if $a_2 = 1 - a_1$

$$\overline{\sigma_u^2} = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[\frac{1}{a_1} + \frac{1}{1-a_1}\right] = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[\frac{1}{a_1(1-a_1)}\right]$$

$$\frac{d\overline{\sigma_u^2}}{da_1} = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[-\frac{1}{a_1^2} + \frac{1}{(1-a_1)^2}\right] = \frac{1}{4}\left(\frac{t}{c}\right)^2\left[\frac{2a_1 - 1}{a_1^2(1-a_1)^2}\right]$$

and, therefore,

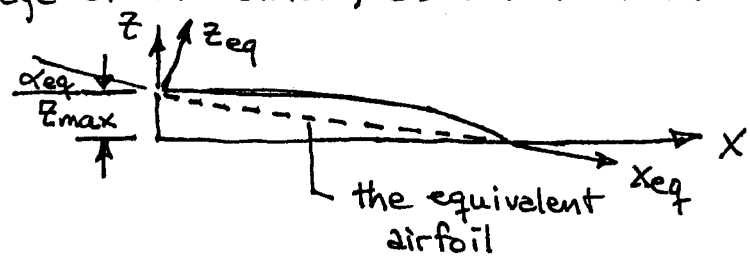
$$\frac{d\overline{\sigma_u^2}}{da_1} = 0 \quad \text{when} \quad a_1 = \frac{1}{2} \text{ (and, therefore, } a_2 = \frac{1}{2}\text{)}$$

It can also be shown that the second derivative is negative when $a_1 = \frac{1}{2}$. I.e.,

$$\frac{d^2\overline{\sigma_u^2}}{da_1^2} < 0 \quad , \text{when } a_1 = \frac{1}{2}$$

10.2] Parts (a) and (c)

For these two parts of the problem, let us replace the given airfoil by an equivalent airfoil whose chord line is the straight line joining the leading edge and the trailing edge of the airfoil, as shown in the sketch.



For the equivalent airfoil, we can define the coordinates by the approximations:

$$x_{eq} = x \quad ; \quad z_{eq} = z - z_{max}\left(1 - \frac{x}{c}\right)$$

Thus, $\quad z = z_{eq} + z_{max} - z_{max}\frac{x}{c}$

As a result, $\quad x^2 = -\frac{c^2}{z_{max}}(z - z_{max})$ becomes

10.2 Contd. $x_{eq}^2 = \frac{-c^2}{z_{max}}\left(z_{eq} - z_{max}\frac{x_{eq}}{c}\right).$

Taking the derivative with respect to x_{eq}:

$$2x_{eq} = \frac{-c^2}{z_{max}}\left(\frac{dz_{eq}}{dx_{eq}} - \frac{z_{max}}{c}\right)$$

$$\frac{dz_{eq}}{dx_{eq}} = -2\left(\frac{x_{eq}}{c}\right)\left(\frac{z_{max}}{c}\right) + \frac{z_{max}}{c}$$

This equivalent airfoil operates at an angle of attack, α_{eq}:

$$\alpha_{eq} = \tan^{-1}\frac{z_{max}}{c} \approx \frac{z_{max}}{c}$$

Thus, $C_\ell = \frac{4\alpha_{eq}}{\sqrt{M_\infty^2 - 1}} = \frac{4\frac{z_{max}}{c}}{\sqrt{M_\infty^2-1}}$. If $\frac{z_{max}}{c} = 0.1$,

and $M_\infty = 2.059$, $C_\ell = 0.2222$, which compares favorably with the value of 0.2286 obtained in Example 8.4.

To calculate the section drag coefficient

$$\overline{\sigma_u^2} = \int_0^1 \left(\frac{dz_u}{dx}\right)^2 d\left(\frac{x}{c}\right)$$

$$\overline{\sigma_u^2} = \int_0^1 \left[+4\left(\frac{x_{eq}}{c}\right)^2\left(\frac{z_{max}}{c}\right)^2 - 4\left(\frac{x_{eq}}{c}\right)\left(\frac{z_{max}}{c}\right)^2 + \left(\frac{z_{max}}{c}\right)^2\right] d\left(\frac{x_{eq}}{c}\right)$$

$$\overline{\sigma_u^2} = \frac{1}{3}\left(\frac{z_{max}}{c}\right)^2$$

Thus, $C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2-1}} + \frac{2\left(\frac{1}{3}\frac{z_{max}^2}{c^2}\right)}{\sqrt{M_\infty^2-1}}$

which is equal to 0.0259 for our assumed approximate (equivalent) airfoil. This value is within 20% of the value calculated in Example 8.3. Also, $\frac{C_\ell}{C_d} = 8.5792$

10.2 Contd. To calculate the pitching moment about the leading edge (i.e., $x = x_{eq} = 0$), we use the expression for $z_{eq}(x_{eq})$ to represent the mean camber line in equation (10.22). Thus,

$$C_{m_0} = -\frac{2\alpha}{\sqrt{M_\infty^2-1}} + \frac{4}{\sqrt{M_\infty^2-1}} \int_0^1 \left[-2\left(\frac{x_{eq}}{c}\right)\left(\frac{z_{max}}{c}\right) + \frac{z_{max}}{c}\right]\left(\frac{x_{eq}}{c}\right) d\left(\frac{x_{eq}}{c}\right)$$

$$C_{m_0} = -\frac{2\alpha}{\sqrt{M_\infty^2-1}} + \frac{4}{\sqrt{M_\infty^2-1}} \left[-\frac{2}{3}\left(\frac{z_{max}}{c}\right) + \frac{1}{2}\left(\frac{z_{max}}{c}\right)\right]$$

$$C_{m_0} = \frac{1}{\sqrt{M_\infty^2-1}} \left[-2\alpha - \frac{2}{3}\left(\frac{z_{max}}{c}\right)\right] = -0.1482$$

(b) To calculate the pressure distribution, let us return to the original x, z coordinate system. Note that $\theta = 0$ for the free-stream flow. Thus, $\frac{dz}{dx} = -2\left(\frac{x}{c}\right)\left(\frac{z_{max}}{c}\right) \approx \theta$ for use in the equation for the pressure coefficient, equation (10.1c):

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2-1}} = 1.1112\theta \approx -0.2222\frac{x}{c}$$

For the graph:

C_p \ x/c	0.0	0.1	0.3	0.5	0.7	0.9	1.0
upper surface	0.0	−0.0222	−0.0667	−0.1111	−0.1556	−0.2000	−0.2222
lower surface	0.0	+0.0222	+0.0667	+0.1111	+0.1556	+0.2000	+0.2222

Using the information presented in the table on page 364 to obtain the corresponding values from Example 8.3:

C_p \ x/c	0.1	0.3	0.5	0.7	0.9
upper surface	−0.0206	−0.0640	−0.0977	−0.1381	−0.1587
lower surface	+0.0236	+0.0762	+0.1280	+0.1884	+0.2558

10.3) If $\frac{t}{c} = 0.04$, the angle between the chord line and a facet of the double-wedge airfoil is:

$$\delta_w = \tan^{-1}\left[\frac{t/2}{c/2}\right] = \tan^{-1} 0.04 = 2.29°$$

$\frac{dz_c}{dx} = 0$ at all points, since this is a symmetric airfoil.

Furthermore, $\overline{\sigma_l^2} = \overline{\sigma_u^2} = \delta_w^2$. Since we are to neglect the effects of the boundary layer when calculating the force and the moment characteristics, the free-stream conditions (and, hence, the altitude) do not affect the dimensionless aerodynamic coefficients.

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \;;\; C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{4\delta_w^2}{\sqrt{M_\infty^2 - 1}} \;; \text{and}$$

$$C_{m_0} = \frac{-2\alpha}{\sqrt{M_\infty^2 - 1}}$$

Thus,

α	C_l	C_d	C_l/C_d	C_{m_0}
2.29°	0.09230	0.00738	12.51	−0.04615
5.00°	0.20153	0.02128	9.47	−0.10077

	C_p			
α	Region 2	Region 3	Region 4	Region 5
2.29°	+0.0923	0.0	0.0	−0.0923
5.00°	+0.1469	−0.0546	+0.0546	−0.1469

10.4) Using Table 10.1 to find the values of the coefficients C_1 and C_2, when $M_1 = M_\infty = 2.0$, $C_1 = 1.155$ and $C_2 = 1.467$. Thus, since

$$C_p = C_1\theta + C_2\theta^2$$

10.4 Contd.

α	C_p Region 2	Region 3	Region 4	Region 5
2.29°	+0.1017	0.0	0.0	−0.0829
5.00°	+0.1706	−0.0513	+0.0579	−0.1232

Let us use equation (10.24b) to calculate the section lift coefficient and equation (10.25b) to calculate the section drag coefficient.

For $\alpha = 2.29°$:

$$C_\ell = \frac{1}{2\cos\delta_w}\left[C_{p2}(\cos(4.58)) - C_{p5}(\cos(4.58))\right] = 0.09208$$

$$C_d = \frac{1}{2\cos\delta_w}\left[C_{p2}(\sin(4.58)) - C_{p5}(\sin(4.58))\right] = 0.00738$$

$$C_\ell / C_d = 12.48$$

We will need to develop our own equation to calculate the pitching moment about the leading edge. Note that a nose-up pitching moment is positive. The chordwise contribution of the pressure acting on a facet is $p\left(\frac{c}{2}\tan\delta_w\right)\left(\frac{c}{4}\tan\delta_w\right)$, since $p\left(\frac{c}{2}\tan\delta_w\right)$ is the chordwise force per unit span acting on a facet and $\left(\frac{c}{4}\tan\delta_w\right)$ is the distance from the chord line to the center of the facet, i.e., the moment arm.

Thus,

$$C_{M_0} = -\frac{1}{8}C_{p2} + \frac{3}{8}C_{p5} + \left(-C_{p2} + C_{p4} + C_{p3} - C_{p5}\right)\frac{1}{8}\tan^2\delta_w$$

For $\alpha = 2.29°$, $C_{M_0} = -0.04380$

Note that the contribution due to the chordwise components, i.e., the third (and last) term in the above equation, is -3.8×10^{-6}. It could easily have been neglected for this

10.4 Contd.] thin airfoil section.

For $\alpha = 5.00°$

$$C_l = \frac{1}{2\cos\delta_w}\Big[C_{p_2}\cos(7.29) + C_{p_4}\cos(2.71) - C_{p_3}\cos(2.71)$$
$$- C_{p5}\cos(7.29)\Big] = 0.20041$$

$$C_d = \frac{1}{2\cos\delta_w}\Big[C_{p_2}\sin(7.29) + C_{p4}\sin(2.71) - C_{p_3}\sin(2.71)$$
$$- C_{p5}\sin(7.29)\Big] = 0.02124$$

$$C_l / C_d = 9.44$$

$$C_{m_0} = -\frac{1}{8}C_{p_2} - \frac{3}{8}C_{p4} + \frac{1}{8}C_{p_3} + \frac{3}{8}C_{p5} + \Big(-C_{p_2} + C_{p4}$$
$$+ C_{p_3} - C_{p5}\Big)\frac{1}{8}\tan^2\delta_w = -0.09565$$

10.5] For $M_1 = 2.0$, $\nu_1 = 26.38°$ and $\frac{p_1}{p_{t1}} = 0.1278$

For $\alpha = 2.29°$:

Flow from 1 to 2 passes through a weak shock wave, which turns the flow 4.58°. Using Fig. 8.13:

$$C_{p_2} = \frac{p_2 - p_1}{\frac{\gamma}{2} p_1 M_1^2} = 0.10 \; ; \; 1 - \frac{1}{M_2} = 0.455$$

Thus, $\frac{p_2}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_p = 1.28$; $M_2 = 1.835$

Since we now know M_2, we find that:

$$\nu_2 = 21.73° \quad \text{and} \quad \frac{p_2}{p_{t2}} = 0.1650$$

In going from region 2 to region 4, we cross right-running Mach waves on a left-running characteristic. Thus, $d\nu = d\theta = +4.58°$; and $\nu_4 = 26.31°$. Using this value

10.5 Contd.] of ν_4, we find that $M_4 = 1.998$ and $\frac{p_4}{p_{t4}} = 0.1282$.
(Note that since the flow from region 1 to region 2 passes through a weak shock wave, the expansion to region 4 results in a Mach numbes (1.998) slightly below the free-stream value (2.000). Compare this with the characteristics approach, which would yield a value of $M_4 = 2.00$). Furthermore, since the acceleration from region 2 to region 4 is an isentropic process, $p_{t4} = p_{t2}$. Therefore,

$$\frac{p_4}{p_2} = \left(\frac{p_4}{p_{t4}}\right)\left(\frac{p_{t4}}{p_{t2}}\right)\left(\frac{p_{t2}}{p_2}\right) = 0.777$$

and

$$\frac{p_4}{p_1} = \left(\frac{p_4}{p_2}\right)\left(\frac{p_2}{p_1}\right) = 0.9945$$

$$C_{p_4} = \left(\frac{p_4}{p_1} - 1\right)\frac{2}{\gamma M_1^2} = -0.0020$$

Note that the accuracy inherent in using these tables and charts for these small angles introduce errors approvimately equal in magnitude to the pressure coefficient in region 4 itself.

Since the surface of region 3 is parallel to the free-stream, the flow is unchanged in direction in going from region 1 to region 3. Thus, $\nu_3 = 26.38°$; $p_3 = p_1$; and $C_{p_3} = 0$.
Going from region 3 to region 5, we move on a right-running characteristic. Since the flow over the upper surface is isentropic, $p_{t5} = p_{t3} = p_{t1}$. Thus, $d\nu = -d\theta = -(-4.58°)$; $\nu_5 = 30.96°$; $M_5 = 2.17$; $\frac{p_5}{p_{t5}} = 0.0980$; $\frac{p_5}{p_1} = 0.766$, $C_{p_5} = -0.0836$

10.5 Contd. $\alpha = 5.00°$

Flow from region 1 to region 2 turns 7.29° through a weak, oblique shock wave. Thus, $p_{t2} < p_{t1}$. $C_{p_2} = 0.170$;

$\frac{p_2}{p_1} = 1.476$; $1 - \frac{1}{M_2} = 0.423$; $M_2 = 1.733$; $\frac{p_2}{p_{t2}} = 0.1927$;

$\nu_2 = 18.78°$.

Going from region 2 to region 4 (with $p_{t2} = p_{t4}$)

$d\nu = d\theta = +4.58°$; $\nu_4 = 23.36°$; $M_4 = 1.892$;

$\frac{p_4}{p_{t4}} = 0.1511$; $\frac{p_4}{p_2} = 0.7814$; $\frac{p_4}{p_1} = 1.157$; $C_{p_4} = 0.0561$

From region 1 to region 3 (with $p_{t1} = p_{t3}$)

$d\nu = -d\theta = -(-2.71°)$; $\nu_3 = 29.09°$; $M_3 = 2.10$;

$\frac{p_3}{p_{t3}} = 0.1094$; $\frac{p_3}{p_1} = 0.8560$; $C_{p_3} = -0.0514$

From region 3 to region 5 (with $p_{t3} = p_{t5}$)

$d\nu = -d\theta = -(-4.58°)$; $\nu_5 = 33.67°$; $M_5 = 2.275$;

$\frac{p_5}{p_{t5}} = 0.0830$; $\frac{p_5}{p_1} = 0.6495$; $C_{p_5} = -0.1252$

Let us summarize the pressure coefficients calculated using the shock/expansion technique.

	C_p			
α	Region 2	Region 3	Region 4	Region 5
2.29°	0.100	0.0	−0.0020	−0.0836
5.00°	0.170	−0.0514	+0.0561	−0.1252

We can see that these pressure coefficients are in excellent agreement with those obtained in Problem 10.4 for the second-order theory and in reasonable agreement

10.5 Contd.] with those obtained in Problem 10.3 for linear theory. This correlation is not unexpected since this is indeed a thin airfoil section operating at a relatively small angle of attack.

To calculate the force coefficients, let us use the relations developed in Problem 10.4 (noting that $C_{p_4} \neq 0$ for this problem).

$\alpha = 2.29°$

$C_\ell = 0.09058$; $C_d = 0.00734$; $C_\ell / C_d = 12.35$; $C_{M_0} = -0.04385$

$\alpha = 5.00°$

$C_\ell = 0.20026$; $C_d = 0.02129$; $C_\ell / C_d = 9.41$; $C_{M_0} = -0.09566$

10.6) Let us calculate the corresponding coefficients assuming that the flow is inviscid and incompressible subsonic. Since the airfoil is symmetric, $z_c \equiv 0$. Further, recall that there is no wave drag for an incompressible flow.

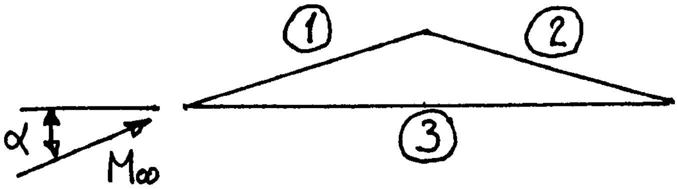
For an inviscid, incompressible flow past any (two-dimensional) airfoil, $C_d = 0$

For $\alpha = 2.29°$: $C_\ell = 2\pi\alpha = 0.2511$; $C_{M_0} = -\frac{C_\ell}{4} = -0.0628$

For $\alpha = 5.00°$: $C_\ell = 2\pi\alpha = 0.5483$; $C_{M_0} = -\frac{C_\ell}{4} = -0.1371$

10.7) We are to calculate the aerodynamic coefficients for the single-wedge airfoil for Fig. 10.6. The thickness ratio, $\tau = t_{max}/c$, is 0.063 and M_∞ is 2.13. To compare with the results presented in Fig. 10.6, let us calculate the coefficients for $\alpha = 0°$ and for $\alpha = 8°$ using second-order theory. Thus, $C_p = C_1\theta + C_2\theta^2$, where $C_1 = \frac{2}{\sqrt{M_\infty^2 - 1}} = 1.0635$

10.7 Contd. and $C_2 = \left[\dfrac{(\gamma+1)M_\infty^4 - M_\infty^2 + 4}{2(M_\infty^2 - 1)^2}\right] = 1.4090$



$$\delta_w = \tan^{-1}\frac{t_{max}}{c/2} = \tan^{-1} 0.126 = 7.181^\circ$$

$\alpha = 0.0^\circ$

$\theta_1 = 0.1253$ radians ; $\theta_2 = -0.1253$ radians ; $\theta_3 = 0.0$ radians

$C_{p_1} = 0.15542$, $C_{p_2} = -0.11115$; $C_{p_3} = 0.0$

$$d = p_1 \frac{0.5c}{\cos\delta_w}\sin\delta_w - p_2\frac{0.5c}{\cos\delta_w}\sin\delta_w$$

$$C_d = \frac{1}{2}\left[C_{p_1} - C_{p_2}\right]\tan\delta_w = 0.0168$$

$$\ell = p_3 c - p_1\frac{0.5c}{\cos\delta_w}\cos\delta_w - p_2\frac{0.5c}{\cos\delta_w}\cos\delta_w$$

$$C_\ell = C_{p_3} - \frac{1}{2}\left[C_{p_1} + C_{p_2}\right] = -0.0221$$

$$m_{0.5c} = -p_1\frac{c}{2}\frac{c}{4} + p_1\frac{c}{2}\tan\delta_w\frac{c}{4}\tan\delta_w + p_2\frac{c}{2}\frac{c}{4}$$
$$- p_2\frac{c}{2}\tan\delta_w\frac{c}{4}\tan\delta_w$$

$$C_{m_{0.5c}} = \left[-C_{p_1} + C_{p_2}\right]\frac{1}{8} + \left[C_{p_1} - C_{p_2}\right]\frac{1}{8}\tan^2\delta_w = -0.0328$$

$\alpha = 8.0^\circ$

$\theta_1 = -0.0143$ rad.; $\theta_2 = -0.2650$ rad; $\theta_3 = +0.1396$ rad

10.7 Contd.

$C_{p_1} = -0.01491$; $C_{p_2} = -0.18285$; $C_{p_3} = 0.17596$

$$C_d = C_{p_3}\sin\alpha - \frac{1}{2\cos\delta_w}\left[C_{p_1}\sin(\alpha-\delta_w) + C_{p_2}\sin(\alpha+\delta_w)\right]$$

$$C_d = 0.0487$$

$$C_l = C_{p_3}\cos\alpha - \frac{1}{2\cos\delta_w}\left[C_{p_1}\cos(\alpha-\delta_w) + C_{p_2}\cos(\alpha+\delta_w)\right]$$

$$C_l = 0.2707$$

$$C_{m_{0.5c}} = \left[-C_{p_1} + C_{p_2}\right]\frac{1}{8} + \left[C_{p_1} - C_{p_2}\right]\frac{1}{8}\tan^2\delta_w = -0.0206$$

Note that these calculated values are in excellent agreement with the results presented in Fig. 10.6.

10.8 Since both configurations are symmetric, $\overline{\sigma_u^2} = \overline{\sigma_l^2}$.
For a double-wedge airfoil section whose thickness ratio is τ:

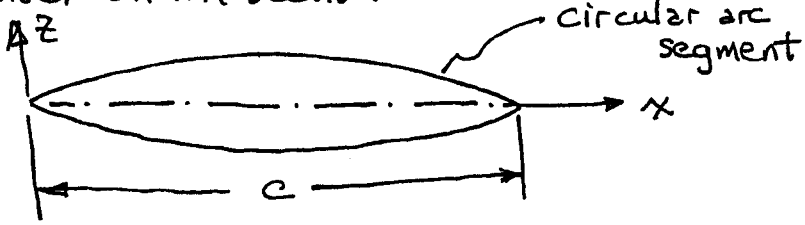
$$\frac{dz_u}{dx} = \tan\delta_w = \frac{t/2}{c/2} = \tau \quad \text{for } 0 \le x \le \frac{c}{2}.$$

$$\frac{dz_u}{dx} = -\tau \quad \text{for } \frac{c}{2} \le x \le c. \text{ Thus,}$$

$$\overline{\sigma_u^2} = \int_0^1 \tau^2\, d\left(\frac{x}{c}\right) = \tau^2. \text{ As a result,}$$

$$C_{d,\text{thickness}} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left[\overline{\sigma_u^2} + \overline{\sigma_l^2}\right] = \frac{4\tau^2}{\sqrt{M_\infty^2 - 1}}$$

For a biconvex airfoil section:



10.8 Contd. $\left(x - \frac{c}{2}\right)^2 + (z - z_0)^2 = R^2 = \left(\frac{t}{2} - z_0\right)^2$

To solve for z_0, we note that $z=0$ when $x=0$. Thus,

$$\frac{c^2}{4} + z_0^2 = \frac{t^2}{4} - z_0 t + z_0^2$$

and

$$z_0 = \frac{t}{4} - \frac{c^2}{4t}$$

Thus, the equation for the upper surface of the section is:

$$\left(x - \frac{c}{2}\right)^2 + \left(z_u - \frac{t}{4} + \frac{c^2}{4t}\right)^2 = \left(\frac{t}{4} + \frac{c^2}{4t}\right)^2$$

Differentiating with respect to x:

$$2\left(x - \frac{c}{2}\right) + 2\left(z_u - \frac{t}{4} + \frac{c^2}{4t}\right)\frac{dz_u}{dx} = 0$$

Solving for $\frac{dz_u}{dx}$:

$$\frac{dz_u}{dx} = \frac{1 - 2\left(\frac{x}{c}\right)}{2\frac{z_u}{c} - \frac{t}{2c} + \frac{c}{2t}}$$

Let us examine the relative magnitude of the terms in the denominator. For a thin airfoil, $\tau = \frac{t}{c}$, which is much less than 1. Thus, $\frac{c}{2t} >> \frac{2z_u}{c} - \frac{t}{2c}$. Thus, we can approximate the expression for dz_u/dx as:

$$\frac{dz_u}{dx} \approx \frac{2t}{c}\left[1 - 2\left(\frac{x}{c}\right)\right] = 2\tau\left[1 - 2\left(\frac{x}{c}\right)\right]$$

As a result: $\overline{\sigma_u^2} = \int_0^1 \left(\frac{dz_u}{dx}\right)^2 d\left(\frac{x}{c}\right) = 4\tau^2 \int_0^1 \left[1 - 4\left(\frac{x}{c}\right) + 4\left(\frac{x}{c}\right)^2\right] d\left(\frac{x}{c}\right)$

$$\overline{\sigma_u^2} = 4\tau^2\left[\left(\frac{x}{c}\right) - 2\left(\frac{x}{c}\right)^2 + \frac{4}{3}\left(\frac{x}{c}\right)^3\right]_0^1 = \frac{4}{3}\tau^2$$

Thus, for a biconvex airfoil section:

$$C_{d,\text{thickness}} = \frac{4\overline{\sigma_u^2}}{\sqrt{M_\infty^2 - 1}} = \frac{5.33\tau^2}{\sqrt{M_\infty^2 - 1}}$$

10.9 First we calculate δ_w

$$\tan \delta_w = \frac{t/2}{c/2} = \frac{t}{c} = 0.07; \quad \text{Thus, } \delta_w \approx 4^\circ$$

a) Given that $\delta_w = 4^\circ$ and $\alpha = 6^\circ$, the flow from region 1 to region 2 undergoes an expansion which turns the flow 2°. Thus, $\Delta\nu = \Delta\theta = +2^\circ$. For $M_1 = 2.2$, $\nu_1 = 31.732^\circ$ from Table 8.2. Thus, $\nu_2 = 33.732^\circ$ and $M_2 = 2.28$ (also from Table 8.2). Since the flow is isentropic in going from Region 1 to Region 2, $p_{t1} = p_{t2} = 125$ psia. Since $M_2 = 2.28$, we can use Table 8.1 to find that $\frac{p_2}{p_{t2}} = 0.08251$.

$$\text{Thus, } p_2 = \left(\frac{p_2}{p_{t2}}\right) p_{t2} = (0.08251)(125) = 10.31 \text{ psia}$$

The flow from region 2 to region 4 undergoes an expansion which turns the flow through an angle $2\delta_w = 8^\circ$. As a result, $\nu_4 = \nu_2 + 8^\circ = 41.732^\circ$, giving a value of $M_4 = 2.61$. From Table 8.1, $\frac{p_4}{p_{t4}} = 0.04935$. Since the expansion process is isentropic, $p_{t4} = p_{t2} = p_{t1}$. Thus,

$$p_4 = \left(\frac{p_4}{p_{t4}}\right) p_{t4} = (0.04935)(125) = 6.17 \text{ psia}$$

The flow from Region 1 to Region 3 undergoes a compressive turning of 10°. This turning of the flow is accomplished by a weak, linear shock wave attached to the leading edge. From Fig. 8.13b:

$$C_{p_3} = 0.19 = \frac{p_3 - p_1}{q_1} = \left(\frac{p_3}{p_1} - 1\right)\frac{2}{\gamma M_1^2}$$

10.9 Contd. Thus, $p_3 = \frac{p_1}{p_{t1}}\left[\frac{\gamma M_1^2 C_{p_3}}{2} + 1\right] p_{t1}$

But $M_1 = 2.2$, so $\frac{p_1}{p_{t1}} = 0.09352$. As a result:

$$p_3 = 19.2 \text{ psia}$$

From Fig. 8.13c, $M_3 = 1.75$. Thus, $\nu_3 = 19.273°$.

The flow from region 3 to region 5 undergoes an isentropic expansion of 8°. Thus, $\nu_5 = \nu_3 + 8°$. Therefore, $\nu_5 = 27.273°$, which means that $M_5 = 2.03$. For $M_5 = 2.03$,

$$\frac{p_5}{p_{t5}} = 0.122$$

Since the expansion is isentropic, $p_{t5} = p_{t3}$. Since $M_3 = 1.75$, $\frac{p_3}{p_{t3}} = 0.1878$. Since $p_3 = 19.2$, $p_{t3} = 102.2$ psia.

Thus, $p_5 = \left(\frac{p_5}{p_{t5}}\right) p_{t5} = (0.122)(102.2) = 12.47$ psia

b) Linear theory relates the local pressure to the local flow direction relative to the free-stream flow direction through equation 10.1c. For each region, we have:

$$\theta_2 = -2° = -0.035 \text{ radians}$$

$$\theta_3 = 10° = 0.174 \text{ radians}$$

$$\theta_4 = -10° = -0.174 \text{ radians}$$

$$\theta_5 = -2° = -0.035 \text{ radians}$$

Also for $M_1 = 2.20$, $\sqrt{M_1^2 - 1} = 1.96$

Equation 10.1c gives: $C_{p_2} = -0.036$; $C_{p_3} = 0.178$;

$C_{p_4} = -0.178$; and $C_{p_5} = -0.036$

10.9 Contd. Since $C_{p_i} = \dfrac{p_i - p_1}{q_1}$,

$$p_i = C_{p_i} q_1 + p_1$$

For $M_1 = 2.20$, $\dfrac{p_1}{p_{t1}} = 0.09352$ and $p_1 = 11.69$ psia

Furthermore, because $q_1 = \frac{\gamma}{2} p_1 M_1^2 = 39.6$ psia

Therefore,

$$p_2 = 10.26 \text{ psia}$$
$$p_3 = 18.69 \text{ psia}$$
$$p_4 = 4.59 \text{ psia}$$
$$p_5 = 10.26 \text{ psia}$$

Comparing the results from part (a) with those from part (b)

	p_i (psia)			
	p_2	p_3	p_4	p_5
shock/expansion	10.31	19.2	6.17	12.47
linear	10.26	18.69	4.59	10.26

(c) The axial force coefficient is defined by: $C_A \equiv \dfrac{A}{q_1 c}$

where A is the axial force. The area per unit span of each facet of the airfoil is given by $\dfrac{c/2}{\cos \delta_w}$. Selecting the direction from the leading edge to the trailing edge as positive, we have:

$$A = (p_2 + p_3 - p_4 - p_5)\left(\frac{c/2}{\cos \delta_w}\right) \sin \delta_w$$

Thus, $C_A = 0.0096$

Similarly, the normal force coefficient is:

10.9 Contd.

$$C_N \equiv \frac{N}{q_1 c}$$

where $N = (-p_2 + p_3 - p_4 + p_5)\left(\frac{c/2}{\cos\delta_w}\right)(\cos\delta_w)$

Thus, $C_N = 0.192$

From geometry: $C_\ell = -C_A \sin\alpha + C_N \cos\alpha$
$C_d = C_A \cos\alpha + C_N \sin\alpha$

Thus, $C_\ell = 0.19$ and $C_d = 0.03$

To determine the moment coefficient about the midchord, we note that the moment about the midchord due to the constant pressure force acting on facet 2 is given by

$$m_2\ c/2 = \frac{p_2 c^2}{8}(\tan^2\delta_w - 1)$$

Similarly:

$$m_3\ c/2 = -\frac{p_3 c^2}{8}(\tan^2\delta_w - 1)$$

$$m_4\ c/2 = -\frac{p_4 c^2}{8}(\tan^2\delta_w - 1)$$

$$m_5\ c/2 = \frac{p_5 c^2}{8}(\tan^2\delta_w - 1)$$

Note that since the pressure is constant (uniform) over each facet, the pressure force acts at the center of that facet. The total pitching moment is thus:

$$m_{c/2} = \frac{c^2}{8}\left[-p_2 + p_3 + p_4 - p_5\right](1 - \tan^2\delta_w)$$

Since $C_{m_{c/2}} = \frac{m_{c/2}}{q_1\ c\ c}$ where one c represents the

10.9 Contd. reference area per unit span of the two-dimensional airfoil and the second c represents the characteristic dimension used to nondimensionalize the moment arm. Therefore,

$$C_{m\,c/2} = \frac{M_{c/2}}{\frac{\gamma}{2} p_1 M_1^2 c c}$$

$$= \frac{1}{8}\left[\frac{-p_2 + p_\infty + p_3 - p_\infty + p_4 - p_\infty - p_5 + p_\infty}{\frac{\gamma}{2} p_1 M_1^2}\right]\left(1 - \tan^2 \delta_w\right)$$

Substituting the pressure coefficients calculated previously, we have:

$$C_{m\,c/2} = 0.008$$

Note that the values for pressure that are required to calculate C_A, C_N, C_l, C_d, and $C_{m\,c/2}$ are those obtained in part (a).

10.10 Let us first calculate δ_4 and δ_5.

We know $t = 0.07c$; $\delta_2 = 8°$; $\delta_3 = 2°$. Thus,

$$\sin \delta_2 = \frac{t_2}{0.4c} \quad . \text{ Hence, } t_2 = 0.057c$$

$$\sin \delta_3 = \frac{t_3}{0.4c} \quad . \text{ Hence, } t_3 = 0.014c$$

Where t_2 is the distance from the upper shoulder of the airfoil to the chord line and t_3 is the distance from the lower shoulder to the chord line. Thus,

$$\sin \delta_4 = \frac{t_2}{0.6c} \quad . \text{ Hence, } \delta_4 = 5.32°$$

$$\sin \delta_5 = \frac{t_3}{0.6c} \quad . \text{ Hence, } \delta_5 = 1.32°$$

(a) The solution proceeds in a manner identical to that of

10.10 Contd.] Problem 10.9a. The flow from region 1 to region 2 undergoes an isentropic expansion which turns the flow through 2° since the angle of attack is 10°. Thus, $\Delta\nu = +2°$.

Since $M_1 = 2.00$, $\nu_1 = 26.5°$ (from Table 8.2)

Therefore, $\nu_2 = \nu_1 + \Delta\nu = 28.5°$ and $M_2 = 2.078$

Since the expansion is isentropic:

$$p_{t1} = p_{t2} = 125 \text{ psia}$$

With $M_2 = 2.078$, Table 8.1 or equation (8.36) can be used to determine that $\frac{p_2}{p_{t2}} = 0.1135$.

Thus,
$$p_2 = \left(\frac{p_2}{p_{t2}}\right) p_{t2} = (0.1135)(125) = 14.16 \text{ psia}$$

The flow also undergoes an isentropic expansion in going from region 2 to region 4. The change in flow direction is $\delta_2 + \delta_4 = 13.32°$. Thus, $\nu_4 = \nu_2 + 13.32° = 41.82°$. Using Table 8.2, $M_4 = 2.618$. Again, since the expansion is isentropic, $p_{t4} = p_{t2} = p_{t1} = 125$ psia. With $M_4 = 2.618$, Table 8.1 or equation (8.36) can be use to determine that

$$\frac{p_4}{p_{t4}} = 0.0488$$

Therefore,
$$p_4 = \left(\frac{p_4}{p_{t4}}\right) p_{t4} = (0.0488)(125) = 6.097 \text{ psia}$$

In going from region 1 to region 3, the flow undergoes a compressive turn of $\delta_3 + \alpha = 12°$. Using Fig. 8.13b

$$C_{p3} = 0.32 = \frac{p_3 - p_1}{q_1} = \frac{p_3 - p_1}{\frac{\gamma}{2} p_1 M_1^2}$$

10.10 Contd. Since $M_1 = 2.00$, $\frac{p_1}{p_{t1}} = 0.1278$ (from Table 8.1)

Thus,
$$p_1 = \left(\frac{p_1}{p_{t1}}\right) p_{t1} = (0.1278)(125) = 15.975 \text{ psia}$$

Therefore,
$$p_3 = p_1 + C_{p_3} \frac{\gamma}{2} p_1 M_1^2 = 30.289 \text{ psia}$$

Note that the pressure is nearly doubled (p_3/p_1) as the flow passes through the oblique shock wave.

Also, from Fig. 8.13c, for $M_1 = 2.00$ and $\delta = 12°$:

$$1 - \frac{1}{M_3} = 0.354 \quad \text{so that} \quad M_3 = 1.548 \text{ and } \nu_3 = 13.324°$$

Using Table 8.1 or equation (8.36): $\frac{p_3}{p_{t3}} = 0.2540$. Hence,

$$p_{t3} = \frac{p_3}{(p_3/p_{t3})} = \frac{30.289}{0.2540} = 119.24 \text{ psia}$$

In going from region 3 to region 5, the flow undergoes an isentropic expansion while turning through an angle of $\delta_3 + \delta_5 = 3.32°$. Hence, $\nu_5 = \nu_3 + 3.32° = 16.644°$.

When $\nu_5 = 16.644°$, Table 8.2 can be used to find $M_5 = 1.66$. For $M_5 = 1.66$, $\frac{p_5}{p_{t5}} = 0.21523$ (using equation (8.36)). Since the expansion from region 3 to region 5 is isentropic, $p_{t5} = p_{t3} = 119.24$ psia.

b) As was done in Problem 10.9b, we can determine the flow (the pressure and the Mach number) in each region if we now the flow direction in that region relative to

10.10 Contd. the free-stream flow direction ($\theta_1 = 0$)

$$\theta_2 = \delta_2 - \alpha = -2° = -0.0349 \text{ radians}$$

$$\theta_3 = \delta_3 + \alpha = 12° = +0.2094 \text{ radians}$$

$$\theta_4 = \theta_2 - \delta_2 - \delta_4 = -15.32° = -0.2674 \text{ radians}$$

$$\theta_5 = \theta_3 - \delta_3 - \delta_5 = 8.68° = 0.1515 \text{ radians}$$

Also, for $M_1 = 2.00$; $\sqrt{M_1^2 - 1} = \sqrt{3} = 1.732$

Using equation (10.1c):

$$C_{p_2} = -0.0403$$

$$C_{p_3} = 0.2418$$

$$C_{p_4} = -0.3088$$

$$C_{p_5} = 0.1749$$

Since $C_{p_i} = \dfrac{p_i - p_1}{q_1}$; $p_i = q_1 C_{p_i} + p_1$

Since $q_1 = \frac{\gamma}{2} p_1 M_1^2$; $p_i = p_1\left(\frac{\gamma}{2} M_1^2 C_{p_i} + 1\right)$. With

$p_1 = 15.975$ psia

$$p_2 = 14.172 \text{ psia}$$

$$p_3 = 26.79 \text{ psia}$$

$$p_4 = 2.162 \text{ psia}$$

$$p_5 = 23.80 \text{ psia}$$

Summarizing our results from parts (a) and (b), we have:

	p_i (psia)			
	p_2	p_3	p_4	p_5
Shock/Exp	14.16	30.289	6.097	25.66
Linear	14.172	26.79	2.162	23.80

10.10 Contd. To find the axial force coefficient, we follow the procedure used in Problem 10.9, but we note that we must account for the fact that each facet of the airfoil makes a different angle with respect to the chord line.

$$A = p_2(0.4c)\tan\delta_2 + p_3(0.4c)\tan\delta_3 - p_4(0.6c)\tan\delta_4 - p_5(0.6c)\tan\delta_5$$

Since $C_A \equiv \dfrac{A}{q_1 c}$, we have, using the pressures calculated above in part (a):

$$C_A = 0.0117$$

Similarly, the normal force coefficient is:

$$C_N \equiv \frac{N}{q_1 c}$$

where $N = -p_2(0.4c) + p_3(0.4c) - p_4(0.6c) + p_5(0.6c)$

Hence, $C_N = 0.4066$

From geometry: $c_l = -C_A \sin\alpha + C_N \cos\alpha$

$c_d = C_A \cos\alpha + C_N \sin\alpha$

Hence, $c_l = 0.398$ and $c_d = 0.082$

To calculate the pitching moment, a reasonable approximation for thin airfoils is to neglect the component of the moment arms normal to the chord line when we calculate the pitching moment coefficient about the midchord. This leaves the pressure forces acting on the

10.10 Contd. facets normal to the chord line as the main contributor to the pitching moment. Since the pressure is constant (or uniform) over each facet, the pitching moment may be calculated by considering the pressure force acts at the center of the facet. Using nose up as a positive pitching moment:

$$m_{0.5c} = -p_2(0.4c)(0.3c) + p_3(0.4c)(0.3c)$$
$$+ p_4(0.6c)(0.2c) - p_5(0.6c)(0.2c) = -0.4126c^2$$

The pitching moment coefficient is:

$$C_{m_{0.5c}} = \frac{m_{0.5c}}{q_1 c c} = -0.009$$

Had we done an exact calculation, accounting for the fact that the pressure forces on a given facet are not quite normal to the chord line, we would have still obtained a non-zero value for the pitching moment coefficient about the midchord. The fact that we get a non-zero result here, as well as in Problem 10.9 reflects the fact that the aerodynamic center of an airfoil in a supersonic stream is not exactly at the midchord, as predicted by linear theory. Recall also that the aerodynamic center for an airfoil in an incompressible flow is near, but not exactly at the quarter-chord point as predicted by linear theory in that flow regime.

10.11)

Given: A biconvex airfoil in a supersonic wind tunnel.

Problems 10.11, 10.12, and 10.13 all require the geometry for a bi-convex airfoil, which is given in the problem statements as:

$$R(x) = 2t\frac{x}{L}\left(1 - \frac{x}{L}\right)$$

In order to use the theories in these problems, the local slope of the airfoil surface is required, which is given by:

$$\frac{dR}{dx} = \frac{2t}{L}\left(1 - \frac{2x}{L}\right)$$

Shock-expansion theory requires knowing the slope of the upper and lower surfaces, which can be found using Eqns. 10.2. In general, for any position on the airfoil, the local slope is given by (assuming small angles, i.e. $\tan\theta \approx \theta$):

$$\delta_u = \frac{dR}{dx} - \alpha = \frac{2t}{L}\left(1 - \frac{2x}{L}\right) - \alpha$$

$$\delta_l = \frac{dR}{dx} + \alpha = \frac{2t}{L}\left(1 - \frac{2x}{L}\right) + \alpha$$

The leading-edge slope of the airfoil will be required for the initial shock calculation, which is different between the upper surface and the lower surface due to the angle of attack of $\alpha = 6^\circ$. Since $dR/dx = \tan\delta$, the local slope at $x = 0$ is:

$$\delta = \tan^{-1}(2t/L) = \tan^{-1}(0.14) = 7.97^\circ \approx 8^\circ$$

$$\delta_u = \delta - \alpha = 2^\circ$$

$$\delta_l = \delta + \alpha = 14^\circ$$

For $M_1 = 2.2$ and using the oblique shock tables (Fig. 8.12) and Table 8.2, the shock angle, pressure coefficient, downstream Mach number, and the Prandtl-Meyer function for the upper surface are:

$$\theta_u = 28.6^\circ \qquad M_{2_u} = 2.09 \qquad C_{p_u} = 0.038 \qquad \nu_u \approx 29^\circ$$

and for the lower surface:

$$\theta_l = 37.2^\circ \qquad M_{2_l} = 1.67 \qquad C_{p_l} = 0.35 \qquad \nu_l \approx 17^\circ$$

Now that the initial condition for each surface has been found, an analysis of the expansion may be conducted. Since the local slope for each surface decreases continuously from the leading edge to the trailing edge, the pressure will increase continuously. In this case, there are an infinite number of isentropic expansion waves along the airfoil surfaces. All that is required is to

10.11) contd.

find the total turning angle of the flow through the continuous isentropic expansion and utilize the relationship from p. 408:

$$\Delta \nu = \pm \Delta \theta$$

Following the procedure of Example 8.3, the airfoil is broken up into five segments over the upper and lower surfaces. For the upper surface ($\Delta \nu = -\Delta \theta$):

Segment	$\Delta\theta$ (deg)	ν_u (deg)	M_u	p_u/p_{t_a}	p_u/p_1	C_{p_u}	p_u (lbf/in^2)
a	0	29	2.09	0.11113	1.12870	0.03798	13.195
b	-4	33	2.25	0.08648	0.87834	-0.0359	10.268
c	-8	37	2.41	0.06737	0.68425	-0.0932	7.999
d	-12	41	2.58	0.05173	0.52540	-0.1401	6.141
e	-16	45	2.76	0.03919	0.39804	-0.1777	4.652

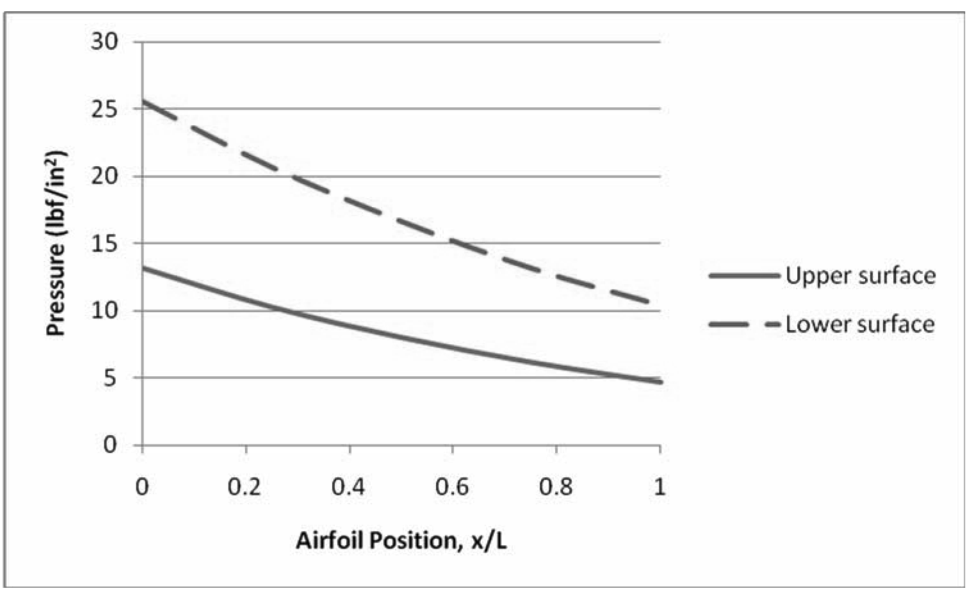
and for the lower surface ($\Delta \nu = +\Delta \theta$):

Segment	$\Delta\theta$ (deg)	ν_l (deg)	M_l	p_l/p_{t_a}	p_l/p_1	C_{p_l}	p_l (lbf/in^2)
a	0	17	1.67	0.21207	2.1858	0.35000	25.552
b	+4	21	1.81	0.17147	1.7673	0.22648	20.661
c	+8	25	1.95	0.13813	1.4237	0.12505	16.645
d	+12	29	2.10	0.10935	1.1271	0.03751	13.175
e	+16	33	2.25	0.08648	0.8913	-0.0321	10.419

Note for finding the local pressure coefficient that values needed to be referred back to freestream values by using the following relationships:

$$p_u = \frac{p_u}{p_{t_a}} \frac{p_{t_a}}{p_a} \frac{p_a}{p_1} \frac{p_1}{p_{t_1}} p_{t_1} \qquad C_p = \frac{2}{\gamma M_1^2}\left(\frac{p}{p_1} - 1\right)$$

The results are included in the following graph:



10.12)

Given: The conditions of Prob. 10.11 but using linear theory.

For linear theory, no estimation of the position of shocks or expansion waves is required, merely knowing the local slope of each surface relative to the freestream flow. This was obtained in Prob. 10.11 as:

$$\delta_u = \frac{dR}{dx} - \alpha = \frac{2t}{L}\left(1 - \frac{2x}{L}\right) - \alpha$$

$$\delta_l = \frac{dR}{dx} + \alpha = \frac{2t}{L}\left(1 - \frac{2x}{L}\right) + \alpha$$

The local pressure coefficient is given by Eqn. 10.1c as:

$$C_p = \frac{2\delta}{\sqrt{M_\infty^2 - 1}}$$

So the upper and lower pressure coefficient distributions are defined by:

$$C_{p_u} = \frac{2\delta_u}{\sqrt{M_\infty^2 - 1}} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left[\frac{2t}{L}\left(1 - \frac{2x}{L}\right) - \alpha\right]$$

$$C_{p_l} = \frac{2\delta_l}{\sqrt{M_\infty^2 - 1}} = \frac{2}{\sqrt{M_\infty^2 - 1}}\left[\frac{2t}{L}\left(1 - \frac{2x}{L}\right) + \alpha\right]$$

For the supersonic tunnel conditions described in the problem statement:

$$M_\infty = 2.2 \qquad t = 0.07L \qquad \alpha = 6^\circ$$

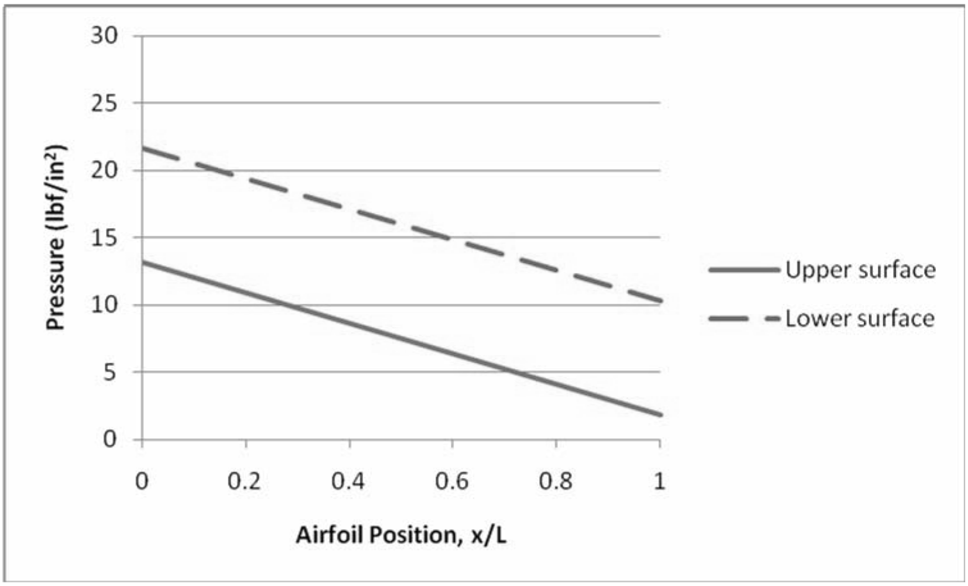
and the resulting pressure coefficient variations are (assuming the airfoil has a length of $L = 1$):

$$C_{p_u} = 1.021[0.14(1 - 2x) - 0.1047]$$

$$C_{p_l} = 1.021[0.14(1 - 2x) + 0.1047]$$

The resulting pressure coefficient variation is shown below. Notice that the variation of pressure is linear between the leading edge and trailing edge. Also notice that the lower surface has positive pressure coefficients (pressure greater than free stream pressure) over most of its length, and the upper surface has negative pressure coefficients (pressure less than free stream pressure) over most of its length.

10.12) contd.



The linear theory results are similar to the shock/expansion results from Prob. 10.11, although some obvious differences are evident. The upper surface results at the leading edge are quite similar for the two approaches, but near the trailing edge difference arise due to the higher angles (and the lack of using "small" angles in the linear theory). Notice that the upper surface pressure near the trailing edge is approaching zero, which is an error that linear theory can cause when the expansion surfaces are at high enough angles. The lower surface results are also quite different near the leading edge. At this point the local angles are not "small" and the basic limitations of linear theory become more obvious. The obvious advantage of linear theory, however, is the ease of computation, with the shock/expansion results taking a great deal more time. Therefore, as long as the small angle assumptions are met, linear theory is a good approach for estimating supersonic airfoil aerodynamics.

10.13)

Given: The conditions of Prob. 10.12.

Note: The problem statement provides some conflicting requests, namely in one sentence the student is asked to do the problem with linear theory and in another sentence they are asked to use shock/expansion theory. Students should be informed to do the entire problem using linear theory only.

The problem asks to compare the drag coefficient variation with angle of attack for a diamond wedge airfoil and a bi-convex airfoil. According to Eqn. 10.14 the drag coefficient as a function of thickness and angle of attack is given by:

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{2}{\sqrt{M_\infty^2 - 1}} \int_0^1 \left[\left(\frac{dz_u}{dx} \right)^2 + \left(\frac{dz_l}{dx} \right)^2 \right] d\left(\frac{x}{c} \right) = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + C_{d,thickness}$$

10.13) contd.

The linear theory drag coefficient due to thickness for the two airfoils was done in Prob. 10.8. The diamond wedge airfoil zero-lift drag coefficient is:

$$C_{d,thickness} = \frac{4(t/L)^2}{\sqrt{M_\infty^2 - 1}}$$

and the zero-lift drag coefficient for a bi-convex airfoil is:

$$C_{d,thickness} = \frac{16(t/L)^2}{3\sqrt{M_\infty^2 - 1}}$$

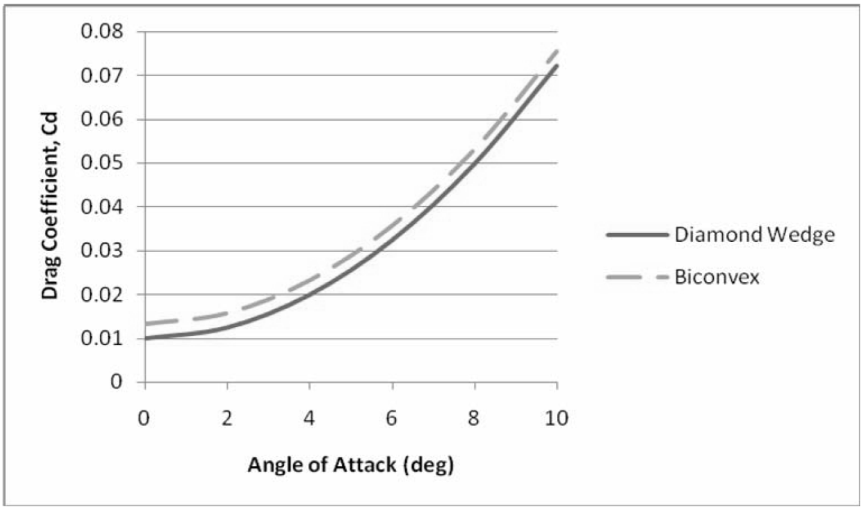
So the drag coefficient variation with angle of attack for a diamond wedge airfoil is:

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{4(t/L)^2}{\sqrt{M_\infty^2 - 1}} = \frac{4\left[\alpha^2 + (t/L)^2\right]}{\sqrt{M_\infty^2 - 1}}$$

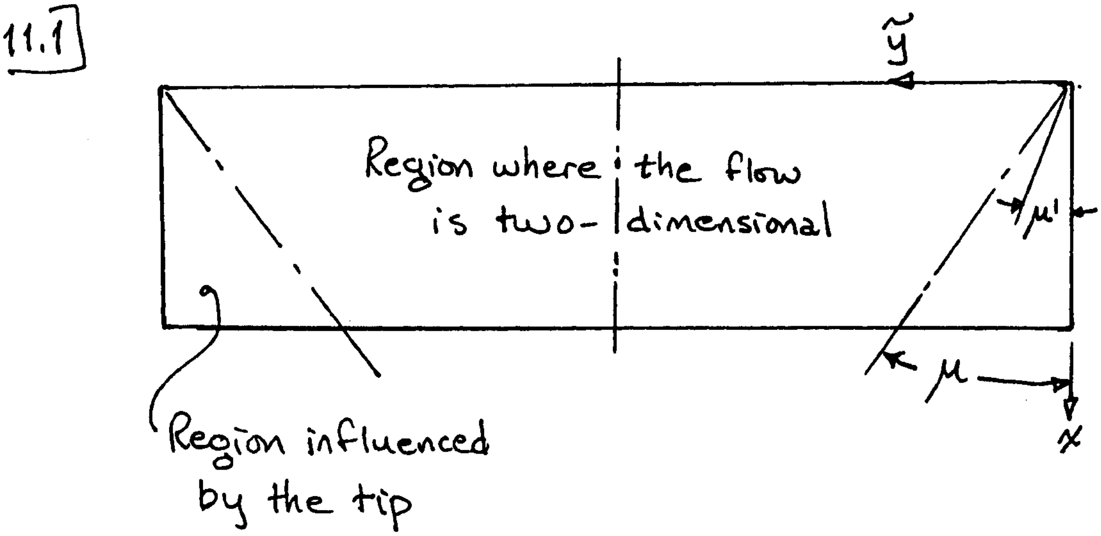
and the drag coefficient variation with angle of attack for a bi-convex airfoil is:

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{16(t/L)^2}{3\sqrt{M_\infty^2 - 1}} = \frac{4\left[\alpha^2 + \frac{4}{3}(t/L)^2\right]}{\sqrt{M_\infty^2 - 1}}$$

The following graph shows the comparison of the drag coefficients for the two airfoils between 1 and 10 degrees angle of attack. Note that the bi-convex airfoil has 33% more drag at 0 degrees, but the difference between the two airfoils becomes less noticeable as the angle of attack is increased since the alpha variation dominates the results. Also note that the drag coefficient does not depend on the total properties within the wind tunnel since the coefficient is non-dimensionalized by the freestream conditions.



11.1



We are to consider supersonic flow past an unswept, rectangular flat-plate wing at a small angle of attack. Flow is such that $\beta\, AR > 2$ so that the Mach cones emanating from the tips do not overlap.

In the region where the flow is two-dimensional, we can use linear theory to determine $C_{p,2d}$:

$$C_{p,2d} = \frac{2\theta}{\sqrt{M_\infty^2 - 1}} = \frac{2\theta}{\beta} \qquad (10.1c)$$

To determine the net force in the direction normal to the plate surface due to the pressure, we note that

$$C_{p,2d} = \frac{2\alpha}{\beta} \text{ on the windward (lower) surface}$$

and $C_{p,2d} = -\dfrac{2\alpha}{\beta}$ on the leeward (upper) surface.

Thus, the net lifting $\Delta C_{p,2d}$ acting on an elemental area is $\frac{4\alpha}{\beta}$. The pressure differential acts over the

11.1 Contd. region between the two Mach waves emanating from the wing-tip leading edge.

In the region influenced by the wing tip:

$$\frac{C_p}{C_{p,2d}} = \frac{2}{\pi} \sin^{-1}\sqrt{\frac{\tan\mu'}{\tan\mu}}$$

where: $\tan\mu = \frac{1}{\sqrt{M_\infty^2 - 1}} = \frac{1}{\beta}$ and $\tan\mu' = \frac{\tilde{y}}{x}$

where $\tilde{y}$ is measured from the wing tip, as shown in the sketch. Thus,

$$\frac{C_p}{C_{p,2d}} = \frac{2}{\pi} \sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}$$

To determine the normal force due to the pressure differential acting on the flat-plate wing, we integrate the pressures over the semi-span area:

$$0 \le x \le c \quad \text{and} \quad 0 \le \tilde{y} \le \frac{b}{2}$$

and multiply by 2 to obtain the desired force.

$$N = 2q_\infty \int_0^c dx \left\{ \int_0^{\frac{x}{\beta}} \Delta C_p \, d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} \Delta C_{p,2d} \, d\tilde{y} \right\}$$

$$N = 2\Delta C_{p,2d}\, q_\infty \int_0^c dx \left\{ \frac{2}{\pi} \int_0^{\frac{x}{\beta}} \sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}\, d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} d\tilde{y} \right\}$$

To evaluate the first integral, let us replace $\sqrt{\tilde{y}}$ by ϕ. For this transformation $d\tilde{y} = 2\phi\, d\phi$ and the upper limit of the integral is $\sqrt{x/\beta}$. Thus, the first integral

11.1 Contd. becomes:

$$\frac{2}{\pi}\int_0^{\frac{x}{\beta}} \sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}\, d\tilde{y} = \frac{4}{\pi}\int_0^{\sqrt{\frac{x}{\beta}}} \phi\left[\sin^{-1}\sqrt{\frac{\beta}{x}}\,\phi\right]d\phi$$

$$= \frac{4}{\pi}\left\{\frac{\phi^2}{2}\left[\sin^{-1}\sqrt{\frac{\beta}{x}}\,\phi\right] - \frac{x}{4\beta}\left[\sin^{-1}\sqrt{\frac{\beta}{x}}\,\phi\right] + \frac{\phi}{4}\sqrt{\frac{x}{\beta}}\sqrt{1-\frac{\beta}{x}\phi^2}\right\}\Bigg|_0^{\sqrt{\frac{x}{\beta}}}$$

$$= \frac{4}{\pi}\left\{\frac{x}{2\beta}\sin^{-1}1 - \frac{x}{4\beta}\sin^{-1}1\right\} = \frac{x}{2\beta}$$

Thus, we can integrate the expression on the bottom of page 11.2 and obtain:

$$N = 2\,\Delta C_{p,2d}\, q_\infty \int_0^c dx\left[\frac{x}{2\beta} + \frac{b}{2} - \frac{x}{\beta}\right] \quad (P.11.1a)$$

so that

$$N = 2\,\Delta C_{p,2d}\, q_\infty \left[\frac{cb}{2} - \frac{c^2}{4\beta}\right]$$

Recall that the net lifting $\Delta C_{p,2d}$ is $\frac{4\alpha}{\beta}$. Thus,

$$N = 2\left\{\frac{4\alpha}{\beta}\right\} q_\infty\, cb\left[\frac{1}{2} - \frac{c}{b}\left(\frac{1}{4\beta}\right)\right]$$

$$C_L = \frac{N\cos\alpha}{q_\infty cb} = \frac{4\alpha}{\beta}\left[1 - \frac{1}{2\,AR\,\beta}\right] \quad (P.11.1b)$$

and:

$$C_D = \frac{N\sin\alpha}{q_\infty cb} \approx \frac{4\alpha^2}{\beta}\left[1 - \frac{1}{2\,AR\,\beta}\right] \quad (P.11.1c)$$

11.1 Contd. In deriving equations (P.11.1b) and (P.11.1c), we have made use of the approximations that, for small angles of attack, $\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$.

Comparing the lift coefficient for an (infinite aspect ratio) airfoil, equation (10.8), with that for a finite-span wing, equation (P.11.1b), we see that the lift is reduced by the second factor in equation (P.11.1b). Similarly, comparing the drag coefficient for an airfoil, equation (10.6), with that for a finite-span wing, equation (P.11.1c), we see that the drag is reduced by the second factor of equation (P.11.1c).

To calculate the pitching moment about the leading edge, we need only to go to equation (P.11.1a) and note that the moment is the incremental pressure force times the moment arm x. Thus,

$$M_0 = -2\,\Delta C_{p,2d}\, q_\infty \int_0^c x\,dx \left[\frac{b}{2} - \frac{x}{2\beta}\right]$$

where the negative sign indicates that the pressure force produces a nose down pitching moment about the leading edge. Integrating:

$$M_0 = -2\,\Delta C_{p,2d}\, q_\infty \left[\frac{bc^2}{4} - \frac{c^3}{6\beta}\right]$$

$$M_0 = -\frac{2\alpha}{\beta} q_\infty (bc) c \left[1 - \frac{2}{3\,AR\,\beta}\right]$$

$$C_{M_0} = \frac{M_0}{q_\infty (bc) c} = -\frac{2\alpha}{\beta}\left[1 - \frac{2}{3\,AR\,\beta}\right]$$

11.1 Contd. To calculate the center of pressure:

$-N x_{cp} = M_o$ (since we have a nose down pitching moment about the leading edge due to N)

$$x_{cp} = \frac{-\frac{2\alpha}{\beta} q_\infty (bc)\, c \left[1 - \frac{2}{3\,AR\,\beta}\right]}{-\frac{4\alpha}{\beta} q_\infty (bc) \left[1 - \frac{1}{2\,AR\,\beta}\right]}$$

$$x_{cp} = \frac{c}{2} \frac{\left[1 - \frac{2}{3AR\beta}\right]}{\left[1 - \frac{1}{2AR\beta}\right]}$$

For an infinite aspect ratio (AR), we see that $x_{cp} = \frac{c}{2}$, the result for an airfoil.

11.2 Since

$$A' = \frac{\text{airfoil cross-sectional area}}{c^2}$$

we can find the cross-sectional area of a double-wedge airfoil to be:

$$A_{cs} = \left[\frac{1}{2}\left(\frac{t}{2}\right)(c)\right]2 = \frac{1}{2} tc$$

Thus, $A' = \dfrac{\frac{1}{2}tc}{c^2} = \dfrac{1}{2}\dfrac{t}{c} = \dfrac{\tau}{2}$ Q.E.D.

11.3 The airfoil section in Fig. 10.5 is a double-wedge airfoil, $\delta_w = 10°$, in a Mach 2.0 stream at an α of 10°.

Thus,

11.3 Contd.] $\tau = \frac{t}{c} = \frac{t/2}{c/2} = \tan\delta_w = 0.1763$

$\alpha = 10° = 0.1745$ radians ; $\beta = \sqrt{M_\infty^2 - 1} = 1.7321$

$$C_3 = \frac{\gamma M_\infty^4 + (M_\infty^2 - 2)^2}{2(M_\infty^2 - 1)^{3/2}} = 2.5403 \; ; \; A' = 0.0882$$

Using the results from Table 11.1

$$C_L = \frac{4\alpha}{\beta}\left[1 - \frac{1}{2AR\beta}(1 - C_3 A')\right]$$

$$C_L = (0.4030)\left[1 - 0.0722(1 - 0.2241)\right] = 0.3804$$

$$C_D = \frac{4\alpha^2}{\beta}\left[1 - \frac{1}{2AR\beta}(1 - C_3 A')\right] + \frac{K_1 \tau^2}{\beta} + C_{D,\text{friction}}$$

$$C_D = (0.0703)\left[1 - 0.0722(1 - 0.2241)\right] + 0.0718 + C_{D,\text{friction}}$$

$$C_D = 0.1382 + C_{D,\text{friction}}$$

Recall that this is a very thick airfoil section (which was chosen primarily because it could be easily used to demonstrate the techniques of Chapter 10).

11.4] The data of Figs. 9.11 and 9.12 are for a rectangular wing with an aspect ratio of 2.75 and with a NACA 65A005 airfoil section. We are told to assume that the airfoil section is biconvex with a thickness ratio of 0.05. From Table 11.1:

$$C_L = \frac{4\alpha}{\beta}\left[1 - \frac{1}{2AR\beta}(1 - C_3 A')\right]$$

11.4 Contd. For the biconvex section:

$$A' = \frac{2}{3}\tau = 0.0333$$

For $M_\infty = 1.50$: $\beta = \sqrt{M_\infty^2 - 1} = 1.1180$

$$C_3 = \frac{\gamma M_\infty^4 + (M_\infty^2 - 2)^2}{2(M_\infty^2 - 1)^{1.5}} = 2.5581$$

Thus, the lift coefficient is 3.0455α (where α is in radians) or 0.05315α (where α is in °). Let us compare this result with the data presented in Fig. 9.11. For $M_\infty =$ 1.5, the experimentally-determined values of the lift coefficient vary linearly with α up to an angle of attack of 16°. Thus, we will compare the theoretical and the experimental values of C_L at $\alpha = 12°$. The (interpolated) experimental value from Fig. 9.11 is 0.66. Using the theoretical relation:

$$C_L = 0.05315\alpha = 0.638 \text{ (when } \alpha = 12°)$$

To calculate the drag coefficient:

$$C_D = C_{D,\text{ due to lift}} + C_{D,\text{thickness}} + C_{D,\text{friction}}$$

Using the relation in Table 11.1

$$C_{D,\text{thickness}} = \frac{K_1 \tau^2}{\beta} = \frac{(5.33)(0.0333)^2}{1.1180} = 0.00529$$

Thus,

$$C_D = C_L\alpha + 0.00529 + C_{D,\text{friction}} \qquad \text{(P.11.4a)}$$

Using the data from Fig. 9.12, $C_D = 0.027$ when $\alpha = 0°$ for $M_\infty = 1.5$. Thus, solving equation P.11.4a for $C_{D,\text{friction}}$ using the experimental value that $C_D = 0.027$ when $\alpha = 0°$, 11.4 Contd.] we obtain an empirical value for $C_{D,friction}$ of 0.0217. Using this value of $C_{D,friction}$ in equation (P. 11.4a)

$$C_D = C_L \alpha + 0.00529 + 0.0217 = C_L \alpha + 0.0270$$

From the first part of this problem, $C_L = 0.638$ when $\alpha = 12° = 0.2094$ radians. Thus, the theoretical value for the drag coefficient for $\alpha = 12°$ is $C_D = 0.1606$. The corresponding experimental value of C_D (as taken from Fig. 9.12) is 0.1607, which is in exceptionally good agreement with the theoretical value. This good agreement is due (in part) to the use of the experimental value of C_D at zero angle of attack to estimate the value of $C_{D,friction}$.

11.5] For the Northrop F5E, the quarter chord is swept 24°, i.e., $\Lambda_{c/4} = 24°$ (as given in Table 5.1). Thus, it makes an angle of 66° with the vehicle axis.

When $M_\infty = 1.23$, the Mach angle (μ) is 54.39°. Thus, the quarter-chord line is supersonic.

For the quarter-chord line to be sonic:

$$\mu = 90° - \Lambda_{c/4} = 66.0°$$

$$M_1 = \frac{1}{\sin 66°} = 1.095$$

11.6] Referring to Fig. 11.15 and using equation (11.10), we see that:

$$M_{\infty e} = M_\infty \left(1 - \sin^2 \Lambda \cos^2 \alpha\right)^{0.5} \qquad (11.10)$$

11.6 Contd. For small angles of attack, $\cos\alpha \approx 1$. Thus,

$$M_{\infty e} \approx M_\infty (1 - \sin^2\Lambda)^{0.5}$$

If the leading edge is to be sonic, $M_{\infty,e} = 1$. Thus,

$$\sin\Lambda = \left[1 - \frac{1}{M_\infty^2}\right]^{0.5} = \frac{(M_\infty^2 - 1)^{0.5}}{M_\infty} = \frac{\beta}{M_\infty}$$

As a result, $\Lambda = \sin^{-1}\dfrac{\beta}{M_\infty}$

The following calculations can be used to prepare the graph:

Λ	0°	10°	20°	30°	40°	60°	70°	80°
M_∞	1.000	1.015	1.064	1.155	1.414	2.000	2.924	5.759

11.7 From equation (11.15):

$$C_l = C_{le}\left(\frac{M_{\infty e}}{M_\infty}\right)^2 = C_{le}(1 - \sin^2\Lambda\cos^2\alpha) \qquad \text{(P.11.7a)}$$

For small angles of attack, $\cos\alpha \approx 1$. Further, from equation (11.17)

$$C_{le} = \frac{4\alpha_e}{\sqrt{M_{\infty e}^2 - 1}} \qquad \text{(P.11.7b)}$$

Using equation (11.10): $\alpha_e = \tan^{-1}\left(\dfrac{\tan\alpha}{\cos\Lambda}\right) \approx \dfrac{\alpha}{\cos\Lambda}$ (P.11.7c)

Using equation (11.9):

$$M_{\infty e}^2 = M_\infty^2(1 - \sin^2\Lambda\cos^2\alpha) \approx M_\infty^2\cos^2\Lambda \qquad \text{(P.11.7d)}$$

Combining equations (P.11.7a), (P.11.7b), (P.11.7c), and (P.11.7d), we obtain:

$$C_l = 4\left[\frac{\alpha}{\cos\Lambda}\right]\frac{\cos^2\Lambda}{\sqrt{M_\infty^2\cos^2\Lambda - 1}} = \frac{4\cos\Lambda}{\sqrt{M_\infty^2\cos^2\Lambda - 1}}\alpha$$

11.8 The solution is not valid when the denominator approaches zero, i.e., when $M_\infty^2 \cos^2 \Lambda \to 1$. Thus, as $\Lambda \to \cos^{-1} \frac{1}{M_\infty}$, the validity of the relation derived in Problem 11.7 breaks down. Note that the condition that $\Lambda = \cos^{-1} \frac{1}{M_\infty}$ corresponds to the situation where the leading edge is sonic.

Note also that, in developing equation (P.11.7c), we assumed that:

$$\tan^{-1}\left(\frac{\tan\alpha}{\cos\Lambda}\right) \approx \frac{\tan\alpha}{\cos\Lambda} \approx \frac{\alpha}{\cos\Lambda}$$

which is valid providing $\frac{\alpha}{\cos\Lambda}$ is small. This is not true for large sweep back angles. Furthermore, it is valid only for small angles of attack.

11.9

$$\left(\frac{\phi_z}{U_\infty + \phi_x}\right)_{\text{surface}} = \frac{dz_s}{dx} \qquad (11.6)$$

Using a Taylor's series expansion:

$$\left(\frac{\phi_z}{U_\infty + \phi_x}\right)_{\text{surface}} = \left(\frac{\phi_z}{U_\infty + \phi_x}\right)_{z=0}$$

$$+ z_s \left[\frac{\partial}{\partial z}\left(\frac{\phi_z}{U_\infty + \phi_x}\right)\right]_{z=0}$$

$$+ \frac{z_s^2}{2!}\left[\frac{\partial^2}{\partial z^2}\left(\frac{\phi_z}{U_\infty + \phi_x}\right)\right]_{z=0} + \text{h.o.t.}$$

11.9 Contd. Neglecting all but the first term:

$$\left(\frac{\phi_z}{U_\infty+\phi_x}\right)_{surface} \approx \left(\frac{\phi_z}{U_\infty+\phi_x}\right)_{z=0} = \frac{(\phi_z)_{z=0}}{U_\infty}\left[\frac{1}{1+\frac{(\phi_x)_{z=0}}{U_\infty}}\right]$$

But $$\frac{(\phi_x)_{z=0}}{U_\infty} = \frac{(u')_{z=0}}{U_\infty} \ll 1$$

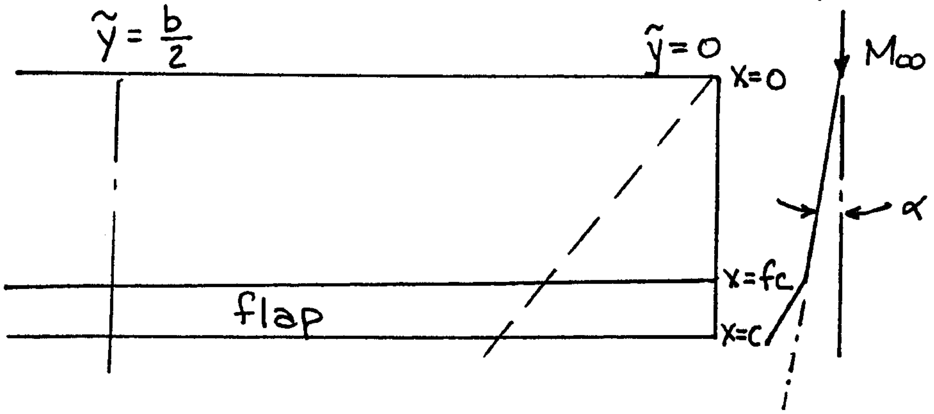
Therefore, $$\left[1+\frac{(\phi_x)_{z=0}}{U_\infty}\right]^{-1} = 1-\frac{(\phi_x)_{z=0}}{U_\infty}+\cdots$$

which is approximately equal to one. Thus, neglecting the higher order terms and combining the above expressions:

$$\left(\frac{\phi_z}{U_\infty+\phi_x}\right)_{surface} \approx \frac{(\phi_z)_{z=0}}{U_\infty}$$

So that: $$(\phi_z)_{z=0} \approx U_\infty \frac{dz_s}{dx}$$ Q.E.D.

11.10 The flat-plate rectangular wing is in a supersonic stream such that $\beta \cdot AR > 2$, i.e., the Mach cones emanating from the wing tips do not overlap.



Using linear theory and the procedures used in the

11.10 Contd. Solution of Problem 11.1, we find that:

$$\Delta C_{p,2d} = \frac{4\alpha}{\beta}$$

for the region between the leading edge ($x=0$) and the flap hinge line ($x=fc$) and that:

$$\Delta C_{p,2d} = \frac{4(\alpha+\delta)}{\beta}$$

for the region between the flap hinge line ($x=fc$) and the trailing edge ($x=c$). Continuing with the procedure, we must take into account the effect of the wing tip on the pressure distribution. Thus, in the tip region, i.e., $0 \leq \tilde{y} \leq \frac{x}{\beta}$,

$$\Delta C_p = \Delta C_{p,2d}\,\frac{2}{\pi}\left[\sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}\right]$$

For small angles of attack:

$$L = N_a \cos\alpha + N_b \cos(\alpha+\delta) \approx N_a + N_b$$

and the lift for the wing is:

$$L = 2q_\infty\left\{\int_0^{fc} dx\left[\int_0^{\frac{x}{\beta}} \Delta C_p\, d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} \Delta C_{p,2d}\, d\tilde{y}\right]\right.$$

$$\left. + \int_{fc}^{c} dx\left[\int_0^{\frac{x}{\beta}} \Delta C_p\, d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} \Delta C_{p,2d}\, d\tilde{y}\right]\right\}$$

11.10 Contd.

$$\frac{L}{2q_\infty} = \frac{4\alpha}{\beta}\int_0^{fc} dx\left[\frac{2}{\pi}\int_0^{\frac{x}{\beta}}\left(\sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}\right)d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} d\tilde{y}\right]$$

$$+ \frac{4(\alpha+\delta)}{\beta}\int_{fc}^{c} dx\left[\frac{2}{\pi}\int_0^{\frac{x}{\beta}}\left(\sin^{-1}\sqrt{\frac{\beta\tilde{y}}{x}}\right)d\tilde{y} + \int_{\frac{x}{\beta}}^{\frac{b}{2}} d\tilde{y}\right]$$

Integrating with respect to $\tilde{y}$

$$\frac{L}{2q_\infty} = \frac{4\alpha}{\beta}\left[\frac{bx}{2} - \frac{x^2}{4\beta}\right]\Bigg|_0^{fc} + \frac{4(\alpha+\delta)}{\beta}\left[\frac{bx}{2} - \frac{x^2}{4\beta}\right]\Bigg|_{fc}^{c}$$

$$C_L = \frac{L}{q_\infty c b} = \frac{4\alpha}{\beta}\left[f - \frac{f^2}{2AR\beta}\right]$$

$$+ \frac{4(\alpha+\delta)}{\beta}\left[(1-f) - \frac{(1-f^2)}{2AR\beta}\right]$$

or equivalently:

$$C_L = \frac{4\alpha f}{\beta}\left[1 - \frac{f}{2AR\beta}\right] + \frac{4(\alpha+\delta)(1-f)}{\beta}\left[1 - \frac{(1+f)}{2AR\beta}\right]$$

Note that, if $f=1$ (i.e., there is no flap),

$$C_L = \frac{4\alpha}{\beta}\left[1 - \frac{1}{2AR\beta}\right]$$

as would be expected from Table 11.1. Also, if $f=0$ (i.e., the entire wing is at the incidence angle of the flaps, which is $(\alpha+\delta)$)

$$C_L = \frac{4(\alpha+\delta)}{\beta}\left[1 - \frac{1}{2AR\beta}\right]$$ as expected.

11.11 Recall that for a delta wing:

$$S = \frac{bc_r}{2} \quad \text{and} \quad \tan\Lambda_{LE} = \frac{c_r}{(b/2)}$$

Combining these two relations:

$$\tan\Lambda_{LE} = \frac{4}{AR}$$

If the leading edge is to be sonic, $\mu = 90° - \Lambda_{LE}$

$$\tan\mu = \tan(90° - \Lambda_{LE}) = \operatorname{ctn}\Lambda_{LE}$$

Thus, $$\frac{AR}{4} = \operatorname{ctn}\Lambda_{LE} = \tan\mu$$

so that $$AR = 4\tan\mu = \frac{4}{\sqrt{M_\infty^2 - 1}}$$

11.12 The average value of R in an element (which we shall designate as $\bar{R}$)

$$\bar{R} = \frac{1}{\Delta x\, \Delta\beta y} \int_{\text{Element}} R\, dx\, d\beta y$$

where Δx and $\Delta\beta y$ are the dimensions of the element. Since R is assumed to have a negligible dependence on x over an element (an assumption which is justified by the numerical integrations):

$$\bar{R} = \frac{1}{\Delta\beta y} \int R\, d\beta y$$

But $\Delta\beta y = 1$ for an element. Thus,

$$\bar{R} = \int R\, d\beta y$$

11.12 Contd. Using the definition for R on page 539,

$$\bar{R}(x-x_1, y-y_1) = \int_{\beta y_A}^{\beta y_B} \frac{(x-x_1)\, d\beta y}{\beta^2 (y-y_1)^2 \left[(x-x_1)^2 - \beta^2 (y-y_1)^2\right]^{0.5}}$$

$$= \left. \frac{(x-x_1)\left[(x-x_1)^2 - \beta^2 (y-y_1)^2\right]^{0.5}}{(x-x_1)^2 \beta (y-y_1)} \right|_{\beta y_A}^{\beta y_B}$$

where βy_A and βy_B are the spanwise coordinates of an element. But in our numerical nomenclature:

$$\beta y = N^* ; \ \beta y_A = N - 0.5 ; \ \beta y_B = N + 0.5$$

Also, evaluating $x - x_1$ at the midchord of an element:

$$x - x_1 = L^* - L + 0.5$$

Thus,

$$\bar{R}(L^*-L, N^*-N) = \frac{\left[(L^*-L+0.5)^2 - (N^*-N-0.5)^2\right]^{0.5}}{(L^*-L+0.5)(N^*-N-0.5)}$$

$$- \frac{\left[(L^*-L+0.5)^2 - (N^*-N+0.5)^2\right]^{0.5}}{(L^*-L+0.5)(N^*-N+0.5)}$$

11.13 The slope of BD is λ_1 ; the slope of DE is $\lambda_1 - (\lambda_1 + \lambda_2) = -\lambda_2$. The slope of BD' remains $-\lambda_1$, but the slope of D'E will be $-\lambda_1 - (-\lambda_1 - \lambda_2)$, which is λ_2. Thus, in region BCDC', the source strength will be:

$$C_1(x, y) = \lambda_1 \frac{U_\infty}{\pi}$$

while in the region DCC', the source strength will be:

11.13 Contd.

$$C(x,y) = -\lambda_2 \frac{U_\infty}{\pi} = C_1 + C_2$$

Thus,

$$C_2(x,y) = -(\lambda_1 + \lambda_2)\frac{U_\infty}{\pi}$$

11.14 & 11.15 We can summarize the results obtained as part of Example 11.3:

Element (L^*, N^*)	$\Delta C_{pa}(L^*, N^*)$	$\Delta C_{pb}(L^*, N^*)$	$\Delta C_p(L^*, N^*)$
1, 0	$4.0000\frac{\alpha}{\beta}$	$-0.8016\frac{\alpha}{\beta}$	$2.7996\frac{\alpha}{\beta}$
2, 0	$0.6393\frac{\alpha}{\beta}$	$+2.0128\frac{\alpha}{\beta}$	$0.9827\frac{\alpha}{\beta}$
2, 1	$5.6804\frac{\alpha}{\beta}$	$+3.3820\frac{\alpha}{\beta}$	$3.9730\frac{\alpha}{\beta}$
2, -1	$5.6804\frac{\alpha}{\beta}$	$+3.3820\frac{\alpha}{\beta}$	$3.9730\frac{\alpha}{\beta}$

To calculate $\Delta C_{pa}(3,0)$, i.e., $L^* = 3$, $N^* = 0$, we must know the pressures at $(1,0)$, $(2,0)$, $(2,+1)$, and $(2,-1)$ — which have been found earlier in the chapter and are tabulated above.

$$\Delta C_{pa}(3,0) = \frac{4\alpha}{\beta}$$

$$+ \frac{1}{\pi}\left\{\frac{[(2.5)^2 - (-0.5)^2]^{0.5}}{(2.5)(-0.5)} - \frac{[(2.5)^2 - (+0.5)^2]^{0.5}}{(2.5)(+0.5)}\right\} 2.7996\frac{\alpha}{\beta}$$

$$+ \frac{1}{\pi}\left\{\frac{[(1.5)^2 - (-0.5)^2]^{0.5}}{(1.5)(-0.5)} - \frac{[(1.5)^2 - (+0.5)^2]^{0.5}}{(1.5)(+0.5)}\right\} 0.9827\frac{\alpha}{\beta}$$

$$+ \frac{1}{\pi}\left\{\frac{[(1.5)^2 - (-1.5)^2]^{0.5}}{(1.5)(-1.5)} - \frac{[(1.5)^2 - (-0.5)^2]^{0.5}}{(1.5)(-0.5)}\right\} \frac{1}{3}\left(3.9730\frac{\alpha}{\beta}\right)$$

$$+ \frac{1}{\pi}\left\{\frac{[(1.5)^2 - (0.5)^2]^{0.5}}{(1.5)(+0.5)} - \frac{[(1.5)^2 - (1.5)^2]^{0.5}}{(1.5)(+1.5)}\right\} \frac{1}{3}\left(3.9730\frac{\alpha}{\beta}\right)$$

[11.14 and 11.15 Contd.] The last two lines handle individually the influence of $\Delta C_p(2,1)$ and of $\Delta C_p(2,-1)$, respectively. Because these two elements have the same influence (by symmetry) on the pressure at (3,0), we could have multiplied the first line by two.

Carrying out the indicated operations;

$$\Delta C_{pa}(3,0) = 0.9176\frac{\alpha}{\beta}$$

$$\Delta C_{pa}(3,1) = \frac{4\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(2.5)^2-(0.5)^2]^{0.5}}{(2.5)(0.5)} - \frac{[(2.5)^2-(1.5)^2]^{0.5}}{(2.5)(1.5)}\right\}2.7996\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(1.5)^2-(0.5)^2]^{0.5}}{(1.5)(0.5)} - \frac{[(1.5)^2-(1.5)^2]^{0.5}}{(1.5)(1.5}\right\}0.9827\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(1.5)^2-(-0.5)^2]^{0.5}}{(1.5)(-0.5)} - \frac{[(1.5)^2-(0.5)^2]^{0.5}}{(1.5)(0.5)}\right\}\frac{1}{3}(3.9730\frac{\alpha}{\beta}$$

For this element, i.e., $L^*=3, N^*=1$, the elements that contribute to the pressure differential are those (1,0), (2,0), and (2,+1). Thus, carrying out the indicated operations:

$$\Delta C_{pa}(3,1) = 4.2711\frac{\alpha}{\beta}$$

To calculate $\Delta C_p(3,0)$, w must calculate a preliminary value of $\Delta C_{p,b}(3,0)$. Thi invdves consideration of the element at (4,0). We note that we have the final values of ΔC_p at the elements (1,0), (2,0), (2,+1), and

$\boxed{\text{11.14 and 11.15 Contd.}}$ $(2,-1)$.

$$\Delta C_p(1,0) = 2.7996\,\frac{\alpha}{\beta}$$

$$\Delta C_p(2,0) = 0.9827\,\frac{\alpha}{\beta}$$

$$\Delta C_p(2,1) = 3.9730\,\frac{\alpha}{\beta} = \Delta C_p(2,-1)$$

We have "preliminary" values for the ΔC_p's of the influencing elements in the same "L" row as the field-point under consideration. For these, we will use:

$$\Delta C_{p_a}(3,0) = 0.9176\,\frac{\alpha}{\beta}$$

$$\Delta C_{p_a}(3,1) = 4.2711\,\frac{\alpha}{\beta} = \Delta C_{p_a}(3,-1)$$

In the calculation of $\Delta C_{p_b}(3,0)$, we will make use of symmetry with respect to points $(2,\pm 1)$ and points $(3,\pm 1)$. Recall that $\Delta C_{p_b}(3,0)$ is calculated with $L^* = 4$, $N^* = 0$.

$$\Delta C_{p_b}(3,0) = \frac{4\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(3.5)^2-(-0.5)^2]^{0.5}}{(3.5)(-0.5)} - \frac{[(3.5)^2-(0.5)^2]^{0.5}}{(3.5)(0.5)}\right\}2.7996\,\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(2.5)^2-(-0.5)^2]^{0.5}}{(2.5)(-0.5)} - \frac{[(2.5)^2-(0.5)^2]^{0.5}}{(2.5)(0.5)}\right\}0.9827\,\frac{\alpha}{\beta}$$

$$+\frac{2}{\pi}\left\{\frac{[(2.5)^2-(-1.5)^2]^{0.5}}{(2.5)(-1.5)} - \frac{[(2.5)^2-(-0.5)^2]^{0.5}}{(2.5)(-0.5)}\right\}\left(\frac{1}{3}\right)(3.9730)\,\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(1.5)^2-(-0.5)^2]^{0.5}}{(1.5)(-0.5)} - \frac{[(1.5)^2-(0.5)^2]^{0.5}}{(1.5)(0.5)}\right\}0.9176\,\frac{\alpha}{\beta}$$

$$+\frac{2}{\pi}\left\{\frac{[(1.5)^2-(-1.5)^2]^{0.5}}{(1.5)(-1.5)} - \frac{[(1.5)^2-(-0.5)^2]^{0.5}}{(1.5)(-0.5)}\right\}4.2711\,\frac{\alpha}{\beta}$$

[11.14 and 11.15 Contd.] $\Delta C_{p_b}(3,0) = 4.4742\,\frac{\alpha}{\beta}$

so that

$$\Delta C_p(3,0) = \frac{3}{4}\,C_{p_a}(3,0) + \frac{1}{4}\,C_{p_b}(3,0) = 1.8067\,\frac{\alpha}{\beta}$$

To calculate $\Delta C_{p_b}(3,1)$ involves consideration of the element at $L^* = 4$, $N^* = 1$. For this we need to evaluate the contributions of $\Delta C_p(1,0)$, $\Delta C_p(2,0)$, $\Delta C_p(2,1)$, $\Delta C_p(2,-1)$, $\Delta C_{p_a}(3,0)$, and $\Delta C_{p_a}(3,1)$.

$$\Delta C_{p_b}(3,1) = \frac{4\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(3.5)^2-(0.5)^2]^{0.5}}{(3.5)(0.5)} - \frac{[(3.5)^2-(1.5)^2]^{0.5}}{(3.5)(1.5)}\right\}2.7996\,\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(2.5)^2-(0.5)^2]^{0.5}}{(2.5)(0.5)} - \frac{[(2.5)^2-(1.5)^2]^{0.5}}{(2.5)(1.5)}\right\}0.9827\,\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(2.5)^2-(-0.5)^2]^{0.5}}{(2.5)(-0.5)} - \frac{[(2.5)^2-(0.5)^2]^{0.5}}{(2.5)(0.5)}\right\}\left(\frac{1}{3}\right)(3.9730)\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(2.5)^2-(1.5)^2]^{0.5}}{(2.5)(1.5)} - \frac{[(2.5)^2-(2.5)^2]^{0.5}}{(2.5)(2.5)}\right\}\left(\frac{1}{3}\right)(3.9730)\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(1.5)^2-(0.5)^2]^{0.5}}{(1.5)(0.5)} - \frac{[(1.5)^2-(1.5)^2]^{0.5}}{(1.5)(1.5)}\right\}0.9176\,\frac{\alpha}{\beta}$$

$$+\frac{1}{\pi}\left\{\frac{[(1.5)^2-(-0.5)^2]^{0.5}}{(1.5)(-0.5)} - \frac{[(1.5)^2-(0.5)^2]^{0.5}}{(1.5)(0.5)}\right\}4.2711\,\frac{\alpha}{\beta}$$

$$\Delta C_{p_b}(3,1) = -0.3303\,\frac{\alpha}{\beta}$$

Thus $\Delta C_p(3,1) = \frac{3}{4}\,\Delta C_{p_a}(3,1) + \frac{1}{4}\,\Delta C_{p_b}(3,1) = 3.1208\,\frac{\alpha}{\beta}$

11.16) Since $M_\infty = 1.5$,

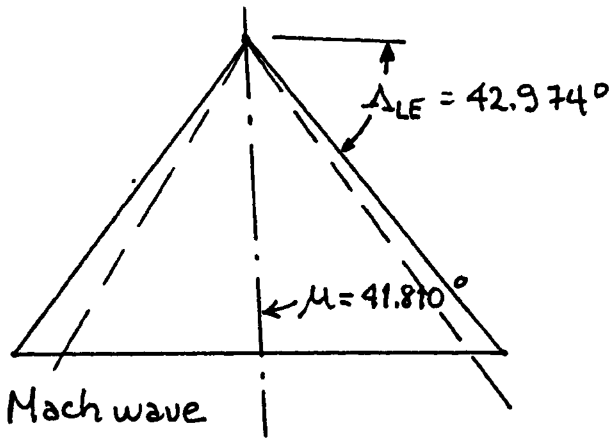
$$\mu = \sin^{-1}\frac{1}{M_\infty} = 41.810°$$

Thus, the leading edge is supersonic.

$$\beta = \sqrt{M_\infty^2 - 1} = 1.118$$

and $\beta \cot \Lambda_{LE} = 1.200$

which is the same as that shown in Fig. 11.30



11.17) and 11.18) should follow the process outlined in 11.16)

11.19)

Given: An axisymmetric slender body.

Problems 11.19 and 11.20 require the geometry for a bi-convex shape, which is given in the problem statements as:

$$R(x) = 2t\frac{x}{L}\left(1-\frac{x}{L}\right)$$

In order to use the theories required for these problems, the local slope of the airfoil surface is required, which is given by:

$$\frac{dR}{dx} = \frac{2t}{L}\left(1-\frac{2x}{L}\right)$$

In order to evaluate the pressure coefficient for this axisymmetric body using Eqn. 11.44, the cross-sectional area distribution and its derivatives are required.

$$S(x) = \pi R(x)^2 = 4\pi t^2\left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)^2$$

$$S'(x) = \frac{8\pi t^2}{L}\left(\frac{x}{L}\right)\left[\left(1-\frac{x}{L}\right)^2 - \frac{x}{L}\left(1-\frac{x}{L}\right)\right]$$

$$S''(x) = \frac{8\pi t^2}{L}\left(\frac{x}{L}\right)\left[\frac{1}{x}\left(1-\frac{x}{L}\right)^2 - 4\frac{x}{L}\left(1-\frac{x}{L}\right) + \frac{1}{L}\left(\frac{x}{L}\right)\right]$$

The pressure coefficient is given in Eqn. 11.44 as:

$$C_p(x) = \frac{S''(x)}{\pi}\ln\left(\frac{2}{R(x)\sqrt{1-M_\infty^2}}\right) + \frac{1}{\pi}\frac{d}{dx}\int_0^x S''(\xi)\ln(x-\xi)d\xi - \left(\frac{dR(x)}{dx}\right)^2$$

The first term in the pressure coefficient relation becomes:

$$\frac{S''(x)}{\pi}\ln\left(\frac{2}{R(x)\sqrt{1-M_\infty^2}}\right)$$

$$= \frac{8\pi t^2}{L}\left(\frac{x}{L}\right)\left[\frac{1}{x}\left(1-\frac{x}{L}\right)^2 - 4\frac{x}{L}\left(1-\frac{x}{L}\right) + \frac{1}{L}\left(\frac{x}{L}\right)\right]\ln\left(\frac{2}{\left(2t\frac{x}{L}\left(1-\frac{x}{L}\right)\right)\sqrt{1-M_\infty^2}}\right)$$

11.19) contd.

The third term becomes:

$$\left(\frac{dR(x)}{dx}\right)^2 = \left(\frac{2t}{L}\left(1-\frac{2x}{L}\right)\right)^2$$

The second term requires taking derivatives and integrals. Evaluating the integral requires finding integrals of the form:

$$\int \ln(ax+b)dx = \ln(ax+b) - x$$

$$\int x^m \ln(ax+b)dx = \frac{1}{m+1}\left[x^{m+1} - (-b/a)^{m+1}\right]\ln(ax+b) - \frac{1}{m+1}\left(-\frac{b}{a}\right)^{m+1}\sum_{r=1}^{m+1}\frac{1}{r}\left(-\frac{ax}{b}\right)^r$$

Evaluating the limits of integration can also be tricky, since they often are undefined. They take the form:

$$\lim(x \to 0)\left[x\ln(x)\right]$$

To perform these, let $f(x) = \ln x$ and $g(x) = 1/x$ and each limit is undefined, allowing the use of L'Hopital's rule.

$$\lim(x \to 0)\left[\frac{f(x)}{g(x)}\right] = \lim(x \to 0)\left[\frac{f'(x)}{g'(x)}\right] \qquad f'(x) = \frac{1}{x} \qquad g'(x) = -\frac{1}{x^2}$$

$$\lim(x \to 0)\frac{f'(x)}{g'(x)} = \lim(x \to 0)(-x) = 0$$

The resulting pressure coefficient is given by:

$$C_p = 8\left(\frac{t}{L}\right)^2\left\{6(x/L) - (3x/L)^2 + \left(1 - 6(x/L) + 6(x/L)^2\right)\ln\left(\frac{L}{t(1-(x/L))\sqrt{M_\infty^2 - 1}}\right)\right\}$$

11.20)

Given: The conditions of Prob. 11.19 but using linear theory.

If the same radius distribution is used on a two-dimensional airfoil and evaluated with linear theory:

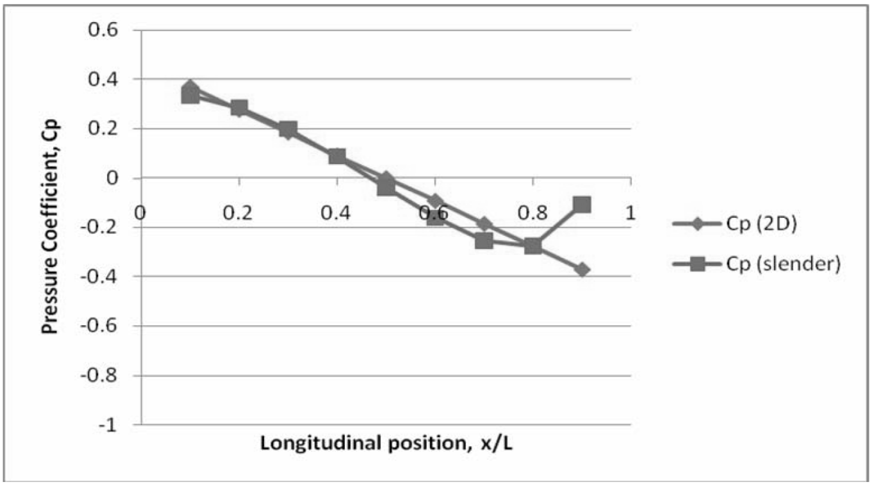
$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

where $\tan\theta = dR/dx$ from above. This yields a pressure coefficient distribution of:

$$C_p = 4\left(\frac{t}{L}\right)\frac{1-2(x/L)}{\sqrt{M_\infty^2 - 1}}$$

Evaluating the two pressure coefficients for $t = 0.2$, $L = 1.0$, and $M = 2.0$ (any reasonable values could be chosen):

x/L	C_p (2D)	C_p (slender)
0.1	0.370	0.335
0.2	0.277	0.285
0.3	0.185	0.199
0.4	0.092	0.086
0.5	0.0	-0.041
0.6	-0.092	-0.163
0.7	-0.185	-0.256
0.8	-0.277	-0.278
0.9	-0.370	-0.110



11.21)

Given: An indraft supersonic wind tunnel with a wing being tested.

Note: The problem statement is missing some details to make it appropriate for inclusion in Chapter 11. Instructors should give students the following additional information:

Calculate the *L/D* ratio for an aspect ratio 5 wing that uses the two-dimensional airfoil section given in Problem 11.20 as a function of angle of attack from 0° to 10°.

Using the results of Table 11.1:

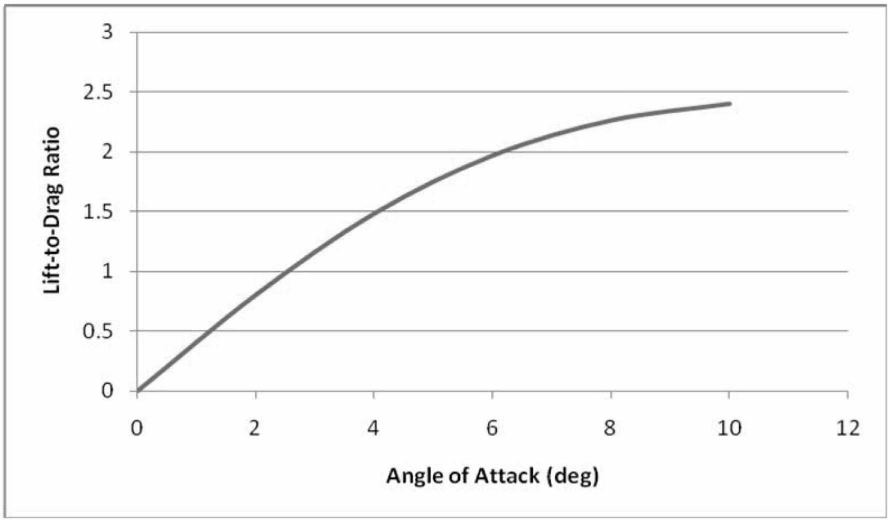
$$C_L = \frac{4\alpha}{\beta}\left[1 - \frac{1}{2AR \cdot \beta}(1 - C_3 A')\right]$$

$$C_D = \frac{K_1 \tau^2}{\beta} + C_{D,friction} + \frac{4\alpha^2}{\beta}\left[1 - \frac{1}{2AR \cdot \beta}(1 - C_3 A')\right]$$

$\beta = \sqrt{M_\infty^2 - 1} = 4.899$ $\quad AR = 5$ $\quad A' = \frac{2}{3}\tau = 0.133$ $\quad K_1 = 5.33$

$$C_3 = \frac{\gamma M_\infty^4 + \left(M_\infty^2 - 2\right)^2}{2\left(M_\infty^2 - 1\right)^{3/2}} = 5.966$$

From Fig. 10.6 a reasonable approximation for the zero-lift friction can be found as $C_{D,friction} \approx 0.015$. This yields a lift-to-drag ratio variation as shown below,



which matches the results from Fig. 11.12.

12.1) At 200,000 ft: $p_1 = 0.4715 \frac{lbf}{ft^2}$; $T_1 = 449°R$;

$\rho_1 = 6.119 \times 10^{-7} \frac{slugs}{ft^3}$; and $h_1 = 2.703 \times 10^6 \frac{ft\,lbf}{slug}$

To determine the flow across a normal shock wave, we will use the continuity equation, (12.7), the momentum equation, (12.8), and the energy equation, (12.9). To account for the departure from a thermally perfect gas, we will use the parameter z (the molecular weight ratio) in the equation of state:

$$\rho_2 = \frac{p_2}{z_2 R T_2}$$

To solve for the properties downstream of a normal shock wave, we will use the following iterative scheme:

(a) $p_2 = (p_1 + \rho_1 U_1^2) - \rho_2 U_2^2 = PM\O M - \rho_2 U_2^2$

(b) $h_2 = (h_1 + \frac{1}{2} U_1^2) - \frac{1}{2} U_2^2 = HT\O T - \frac{1}{2} U_2^2$

We will then use the thermodynamic properties presented in Fig. 12.5 to find:

(c) $T_2(p_2, h_2)$ and $z_2(p_2, h_2)$

Next we will calculate ρ_2 using the relation:

(d) $$\rho_2 = \frac{p_2}{z_2 R T_2}$$

Solving the continuity equation for U_2

(e) $$U_2 = \frac{\rho_1 U_1}{\rho_2} = \frac{MFLUX}{\rho_2}$$

The values from (d) and (e) are substituted into (a)

12.1 Contd. and (b) to obtain new iterates for p_2 and h_2. Then use Fig 12.5 to find T_2 and Z_2, which are the used to update ρ_2. If there is essentially no change in the density value from one iteration to the next, the iteration process can be terminated. If not, the new values from (d) and (e) are substituted into (a) and (b) and the iteration process continues.

First, let us calculate PMØM, HTØT, and MFLUX:

$$PM\emptyset M = p_1 + \rho_1 U_1^2 = 0.4715 \frac{lbf}{ft^2} + \left(6.119\times10^{-7} \frac{lbf\,s^2}{ft^4}\right)\left(10{,}000 \frac{ft}{s}\right)^2$$

$$= 0.4715 + 61.19 = 61.6615 \frac{lbf}{ft^2}$$

$$HT\emptyset T = h_1 + \frac{U_1^2}{2} = 2.703\times10^6 \frac{ft\,lbf}{\left(\frac{lbf\,s^2}{ft}\right)} + \frac{\left(10{,}000 \frac{ft}{s}\right)^2}{2}$$

$$= 0.2703\times10^7 + 5.000\times10^7 = 5.2703\times10^7 \frac{ft^2}{s^2}$$

$$MFLUX = \rho_1 U_1 = 0.006119 \frac{lbf\,s}{ft^3}$$

For the first iteration, we will assume: $U_2 \approx 0$. Thus, using equations (a) and (b):

$$p_2 = 61.6615 \frac{lbf}{ft^2} \frac{atm}{2116.22 \frac{lbf}{ft^2}} = 0.0291 \text{ atm}$$

$$h_2 = 5.2703\times10^7 \frac{ft^2}{s^2} \underbrace{\left[\frac{Btu}{778.2\ ft\,lbf}\right]\left[\frac{\frac{lbf\cdot s^2}{ft}}{32.174\ lbm}\right]}_{25{,}037.8\ \frac{Btu/lbm}{ft^2/s^2}}$$

$$= 2105 \frac{Btu}{lbm}$$

Having determined p_2 and h_2, we use Fig. 12.5 to determine $T_2 = 5200\,°R$ and $Z_2 = 1.078$. Then using equation (d)

12.1 Contd.

$$\rho_2 = \frac{61.6615\ \text{lbf/ft}^2}{(1.078)(1716.16\ \frac{\text{ft}^2}{\text{s}^2\,{}^\circ\text{R}})(5200\ {}^\circ\text{R})} = 6.41\times10^{-6}\ \frac{\text{lbf s}^2}{\text{ft}^4}$$

Using equation (e)

$$U_2 = \frac{MFLUX}{\rho_2} = 954.65\ \frac{\text{ft}}{\text{s}}$$

Next iteration is to obtain new values for p_2 and h_2:

$h_2 = 5.2247\times10^7\ \frac{\text{ft}^2}{\text{s}^2} = 2087\ \frac{\text{Btu}}{\text{lbm}}$

$p_2 = 55.820\ \frac{\text{lbf}}{\text{ft}^2} = 0.0264$ atm

Returning to Fig. 12.5, we find that:

$T_2 = 5160\ {}^\circ\text{R}$ and $z_2 = 1.077$

Therefore, for this iteration:

$$\rho_2 = \frac{55.82}{(1.077)(1716.16)(5160)} = 5.85\times10^{-6}\ \frac{\text{lbf s}^2}{\text{ft}^4}$$

Using equation (e)

$$U_2 = \frac{MFLUX}{\rho_2} = 1045.5\ \frac{\text{ft}}{\text{s}}$$

$h_2 = 5.2156\times10^7\ \frac{\text{ft}^2}{\text{s}^2} = 2083\ \frac{\text{Btu}}{\text{lbm}}$

$p_2 = 55.264\ \frac{\text{lbf}}{\text{ft}^2} = 0.0261$ atm

Using these values for h_2 and p_2, there are essentially no changes in T_2, z_2, and (as a result) ρ_2. Thus, the iteration is complete. Note, in working the problem, I used the charts in the original NACA TN 4265. This allowed me to get better resolution for the iter-

12.1 Contd.] ation process.

For a perfect gas:

$$M_1 = \frac{U_1}{49.02\sqrt{T_1}} = 9.627$$

$$\frac{p_2}{p_1} = 108 \Rightarrow p_2 = 50.92 \ \frac{\text{lbf}}{\text{ft}^2}$$

$$\frac{\rho_2}{\rho_1} = 5.693 \Rightarrow \rho_2 = 3.48 \times 10^{-6} \ \frac{\text{slugs}}{\text{ft}^3}$$

$$\frac{T_2}{T_1} = 18.98 \Rightarrow T_2 = 8522\,°R$$

$$h_2 = \frac{h_2}{h_1} h_1 = \frac{T_2}{T_1} h_1 = 51.303 \times 10^6 \ \frac{\text{ft lbf}}{\text{slug}}$$

12.2] Let us follow the procedures for Problem 12.1:

$U_1 = 20{,}000$ ft/s ; alt = 200,000 ft

$$PM\varnothing M = 0.4715 + (6.9117 \times 10^{-7})(20{,}000)^2$$
$$= 0.4715 + 244.760 = 245.232 \ \frac{\text{lbf}}{\text{ft}^2} = 0.1159 \text{ atm}$$

$$HT\varnothing T = 2.703 \times 10^6 + \frac{1}{2}(20{,}000)^2$$
$$= 0.2703 \times 10^7 + 20.0000 \times 10^7 = 20.270 \times 10^7 \ \frac{\text{ft}^2}{\text{s}^2}$$

$$MFLUX = 0.01224 \ \frac{\text{lbf} \cdot \text{s}}{\text{ft}^3}$$

For the first iteration, we will assume that $U_2 \approx 0$.

Thus, $p_2 = PM\varnothing M = 0.1159$ atm

and $h_2 = HT\varnothing T = 8096 \ \frac{\text{Btu}}{\text{lbm}}$

For these values of p_2 and h_2, we can use the thermodynamic charts of Moeckel and Weston to find that, 12.2 Contd.] for equilibrium air,

$T_2 = 10{,}450\ °R,\ z_2 = 1.436$

Thus, we can calculate the density downstream of the normal shock wave:

$$\rho_2 = \frac{p_2}{z_2 R T_2} = 9.52 \times 10^{-6}\ \frac{\text{lbf s}^2}{\text{ft}^4}$$

The corresponding value of U_2 is:

$$U_2 = \frac{MFLUX}{\rho_2} = 1285.2\ \frac{\text{ft}}{\text{s}}$$

Completing the first iterative cycle.

The second iteration begins with this value of U_2

$$p_2 = PM\emptyset M - \rho_2 U_2^2$$

$$p_2 = 245.232 - (9.52 \times 10^{-6})(1285.2)^2 = 229.504\ \frac{\text{lbf}}{\text{ft}^2}$$

$$h_2 = HT\emptyset T - \frac{1}{2} U_2^2$$

$$h_2 = 20.270 \times 10^7 - 0.082 \times 10^7 = 20.188 \times 10^7\ \frac{\text{ft}^2}{\text{s}^2}$$

Thus, for the second iteration:

$p_2 = 0.1084$ atm and $h_2 = 8063\ \frac{\text{Btu}}{\text{lbm}}$

From the thermodynamic charts of Moeckel and Weston (1958)

$T_2 = 10{,}430\ °R,\ z_2 = 1.435$

Thus, the density downstream of the normal shock wave is:

$$\rho_2 = \frac{p_2}{z_2 R T_2} = \frac{229.504\ \frac{\text{lbf}}{\text{ft}^2}}{(1.435)(1716.16\ \frac{\text{ft}^2}{\text{s}^2\ °R})(10{,}430\ °R)} = 8.94 \times 10^{-6}\ \text{slugs/ft}^3$$

The corresponding value of U_2 is:

$$U_2 = \frac{MFLUX}{\rho_2} = 1369.7\ \frac{\text{ft}}{\text{s}}$$

12.2 Contd. The third iteration begins with this value of U_2:

$$p_2 = PH\phi M - \rho_2 U_2^2 = 228.470 \ \frac{lbf}{ft^2} = 0.1080 \ atm$$

and

$$h_2 = HT\phi T - \frac{1}{2} U_2^2 = 20.177 \times 10^7 \ \frac{ft^2}{s^2} = 8058 \ \frac{Btu}{lbm}$$

Since these values are essentially unchanged from the previous iteration, we could not discern changes in z_2 and T_2 in the thermodynamic charts (even though we are using the original charts, as presented in NACA TN 4265).

If the gas is assumed to behave as a perfect gas:

$$M_\infty = \frac{20{,}000}{(49.02)(449)^{0.5}} = 19.255$$

$$\frac{p_2}{p_1} = 432.38 \ ; \quad p_2 = 203.867 \ \frac{lbf}{ft^2}$$

$$\frac{\rho_2}{\rho_1} = 5.920 \ ; \quad \rho_2 = 3.62 \times 10^{-6} \ \frac{lbf \ s^2}{ft^4}$$

$$\frac{T_2}{T_1} = 73.03 \ ; \quad T_2 = 32{,}790 \ °R$$

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} = 73.03 \ ; \quad h_2 = 7884.1 \ \frac{Btu}{lbm}$$

Note that the perfect-gas value of T_2 (32,790 °R) is over 3 times greater than the equilibrium value of T_2 (10,430 °R). However, the enthalpy values are relatively close:

$$h_{2, \text{perfect gas}} = 7884.1 \ \frac{Btu}{lbm}$$

$$h_{2, \text{equilibrium}} = 8058 \ \frac{Btu}{lbm}$$

12.2 Contd. The difference in the values of the static enthalpy (h_2) is due to the difference in U_2, since the total enthalpy ($H_t = h_1 + \frac{U_1^2}{2} = h_2 + \frac{U_2^2}{2}$) is unchanged across the shock wave independent of the flow model. The static temperature (T_2), however, is very sensitive to the flow model. Since the composition of "perfect air" is "frozen", the the temperature increases as the flow is slowed across the shock wave. For the assumed model that the flow is in equilibrium across the shock wave, a considerable fraction of the available energy is absorbed by the dissociation of the molecules.

12.3 Let us again follow the procedure used in Problem 12.1.

$$PM\phi M = 0.4715 + (6.119 \times 10^{-7})(26{,}400)^2$$

$$= 0.4715 + 426.470 = 426.941 \frac{lbf}{ft^2} = 0.2017 \text{ atm}$$

$$HT\phi T = 2.703 \times 10^6 + \frac{1}{2}(26{,}400)^2$$

$$= 0.2703 \times 10^7 + 34.848 \times 10^7 = 35.118 \times 10^7 \frac{ft^2}{s^2}$$

$$MFLUX = 0.01615 \frac{lbf\ s}{ft^3}$$

Again, we will start the first iteration with the assumption that $U_2 \approx 0$:

$p_2 = PM\phi M = 0.2017$ atm and $h_2 = HT\phi T = 14{,}026 \frac{Btu}{lbm}$

Using the Mollier chart:

$T_2 = 12{,}350\,°R$; $z_2 = 1.787$

$$\rho_2 = \frac{p_2}{z_2 R T_2} = \frac{426.941}{(1.787)(1716.16)(12{,}350)} = 1.127 \times 10^{-5} \frac{slugs}{ft^3}$$

$$U_2 = \frac{MFLUX}{\rho_2} = 1432.7 \frac{ft}{s}$$

12.3 Contd.

$$p_2 = PM\phi M - \rho_2 U_2^2 = 426.941 - 23.133 = 403.808 \ \frac{lbf}{ft^2}$$

$$h_2 = HT\phi T - \frac{1}{2} U_2^2 = 35.118 \times 10^7 - 0.103 \times 10^7 = 35.016 \times 10^7$$

So that $p_2 = 0.1909$ atm and $h_2 = 13,985 \ \frac{Btu}{lbm}$

Using the Mollier chart:

$$T_2 = 12,300 \ °R; \ z_2 = 1.786$$

$$\rho_2 = \frac{p_2}{z_2 R T_2} = \frac{403.808}{(1.786)(1716.16)(12,300)} = 1.071 \times 10^{-5} \ \frac{sl}{ft^3}$$

$$U_2 = \frac{MFLUX}{\rho_2} = 1508.2 \ \frac{ft}{s}$$

$$p_2 = PM\phi M - \rho_2 U_2^2 = 426.941 - 24.363 = 402.578 \ \frac{lbf}{ft^2}$$

$$h_2 = HT\phi T - \frac{1}{2} U_2^2 = 35.118 \times 10^7 - 0.114 \times 10^7 = 35.004 \times 10^7$$

So that $p_2 = 0.1902$ atm; $h_2 = 13,981 \ \frac{Btu}{lbm}$

Thus, the iteration cycle is complete.

The equilibrium values for the flow properties downstream of the normal shock wave are:

$$p_2 = 402.578 \ \frac{lbf}{ft^2} = 0.1902 \text{ atm}$$

$$h_2 = 35.004 \times 10^7 \ \frac{ft \ lbf}{slug} = 13,981 \ \frac{Btu}{lbm}$$

$$\rho_2 = 1.071 \times 10^{-5} \ \frac{slugs}{ft^3}$$

$$T_2 = 12,300 \ °R$$

12.3 Contd.] If the air behaves as a perfect gas:

$$M_\infty (\equiv M_1) = \frac{26,400}{(49.02)(449)^{0.5}} = 25.416$$

$$\frac{\rho_2}{\rho_1} = 5.957 \; ; \; \rho_2 = 0.365 \times 10^{-5} \; \frac{\text{slugs}}{\text{ft}^3}$$

$$\frac{T_2}{T_1} = 136.7 \; ; \; T_2 = 61,378 \; ^\circ R$$

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} \; ; \quad h_2 = 36.950 \times 10^7 \; \frac{\text{ft lbf}}{\text{slug}} = 14,758 \; \frac{\text{Btu}}{\text{lbm}}$$

$$\frac{p_2}{p_1} = 815.6 \; ; \; p_2 = 384.555 \; \frac{\text{lbf}}{\text{ft}^2}$$

12.4] If $M_1 = 6$ and $\delta = 30°$, the wedge value of $C_p = 0.666$

For the Newtonian flow model: $C_p = 2 \sin^2 \delta = 0.500$

For the modified Newtonian flow model:

$$C_p = C_{p,t2} \sin^2 \delta$$

$$C_{p,t2} = \left(\frac{p_{t2}}{p_1} - 1 \right) \frac{2}{\gamma M_1^2} = (46.816 - 1) \frac{2}{(1.4)(36)} = 1.8181$$

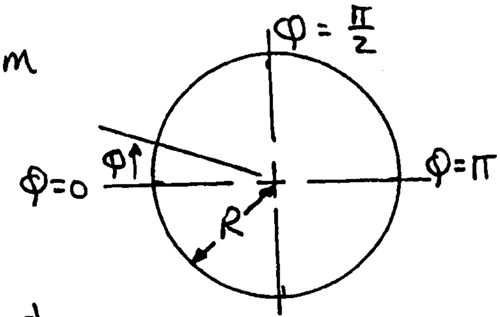
$$C_p = 0.4545$$

12.5] If $M_1 = 6$ and $\delta = 30° = \theta_c$ (the cone semi-vertex angle), the conical value of $C_p = 0.535$

The values (and procedures) for the Newtonian flow model and for the modified Newtonian flow are as in Problem 12.4. For Newtonian theory: $C_p = 0.500$

For the modified Newtonian flow model: $C_p = 0.4545$

12.6 Let's formulate this problem in the coordinate system shown at the left.



The drag per unit span is:

$$d = \oint p \, ds \cos\phi = \oint p \, R \, d\phi \cos\phi$$

From this expression, we can subtract $\oint p_\infty R \, d\phi \cos\phi$ since the net drag due to a uniform pressure acting over a closed surface is zero. Combining the two integrals:

$$d = q_\infty \oint \frac{(p - p_\infty)}{q_\infty} R \, d\phi \cos\phi = q_\infty \oint C_p R \, d\phi \cos\phi$$

In this coordinate system $\phi = \frac{\pi}{2} - \theta$, so that

$$C_p = C_{p,t2} \cos^2\phi \quad \text{from } -\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

and $C_p = 0$ from $\frac{\pi}{2} \leq \phi \leq \frac{3\pi}{2}$ (since that is a shaded region). Thus,

$$d = 2q_\infty \left[\int_0^{\pi/2} C_{p,t2} \cos^2\phi \, R \, d\phi \cos\phi + \int_{\frac{\pi}{2}}^{\pi} 0 \; R \, d\phi \cos\phi \right]$$

$$d = 2 R q_\infty C_{p,t2} \int_0^{\pi/2} \cos^3\phi \, d\phi$$

$$d = 2 R q_\infty C_{p,t2} \left[\sin\phi - \frac{1}{3}\sin^3\phi \right|_0^{\pi/2}$$

$$d = 2 R q_\infty C_{p,t2} \left[1 - \frac{1}{3} \right] = 2 R q_\infty C_{p,t2} \frac{2}{3}$$

$$C_d = \frac{d}{q_\infty [S/\text{span}]} = \frac{d}{q_\infty 2R} = \frac{2}{3} C_{p,t2}$$!! Q.E.D.

12.7] 12.8] 12.9] 12.10] The equations derived in Example 12.2 to calculate the aerodynamic coefficients for Newtonian flow past a sharp cone where $-\theta_c \leq \alpha \leq +\theta_c$, were rearranged to give:

$$\frac{C_A}{C_{p,t2}} = \sin^2\theta_c + 0.5\sin^2\alpha\,(1 - 3\sin^2\theta_c) \qquad (12.34)'$$

$$\frac{C_N}{C_{p,t2}} = 0.5\sin 2\alpha\,\cos^2\theta_c \qquad (12.36)'$$

$$\frac{C_L}{C_{p,t2}} = \sin\alpha\left[\cos^2\alpha\cos^2\theta_c - \sin^2\theta_c - 0.5\sin^2\alpha\,(1 - 3\sin^2\theta_c)\right] \qquad (12.37)'$$

$$\frac{C_D}{C_{p,t2}} = \cos\alpha\left[\sin^2\alpha\cos^2\theta_c + \sin^2\theta_c + 0.5\sin^2\alpha\,(1 - 3\sin^2\theta_c)\right] \qquad (12.38)'$$

$$\frac{C_{M_0}}{C_{p,t2}} = \frac{-\sin 2\alpha}{3\tan\theta_c} \qquad (12.44)'$$

The aerodynamic coefficients have been divided by $C_{p,t2}$ so that the right-hand sides of these equations are functions of θ_c and of α only. As such, the expressions are valid for the Newtonian flow model and for the modified Newtonian flow model and are independent of the Mach number.

These primed equations have been evaluated using a spreadsheet routine. The results are shown in the attached table.

12.7 through 12.10 continued

thetc(deg)	sinthtc	sinthtc^2	costhtc	costhtc^2	tanthtc			
10	0.173648	0.030154	0.984808	0.969846	0.176327			
	alpha	CA/CPT2	CN/CPT2	CL/CPT2	CD/CPT2	L/D	CM0/CPT2	YCP/XL
	(deg)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
	-10	0.04387	-0.16585	-0.15572	0.07200	-2.16272	0.64656	-0.0784
	-9	0.04128	-0.14985	-0.14155	0.06422	-2.20422	0.58417	-0.0752
	-8	0.03896	-0.13366	-0.12694	0.05719	-2.21980	0.52107	-0.0711
	-7	0.03691	-0.11731	-0.11194	0.05093	-2.19795	0.45734	-0.0659
	-6	0.03512	-0.10082	-0.09660	0.04547	-2.12448	0.39304	-0.0595
	-5	0.03361	-0.08421	-0.08096	0.04082	-1.98329	0.32827	-0.0519
	-4	0.03237	-0.06749	-0.06507	0.03700	-1.75876	0.26310	-0.0432
	-3	0.03140	-0.05069	-0.04898	0.03401	-1.44007	0.19760	-0.0335
	-2	0.03071	-0.03383	-0.03273	0.03187	-1.02714	0.13187	-0.0228
	-1	0.03029	-0.01692	-0.01639	0.03058	-0.53600	0.06597	-0.0116
	0	0.03015	0.00000	0.00000	0.03015	0.00000	0.00000	0.0000
	1	0.03029	0.01692	0.01639	0.03058	0.53600	-0.06597	0.0116
	2	0.03071	0.03383	0.03273	0.03187	1.02714	-0.13187	0.0228
	3	0.03140	0.05069	0.04898	0.03401	1.44007	-0.19760	0.0335
	4	0.03237	0.06749	0.06507	0.03700	1.75876	-0.26310	0.0432
	5	0.03361	0.08421	0.08096	0.04082	1.98329	-0.32827	0.0519
	6	0.03512	0.10082	0.09660	0.04547	2.12448	-0.39304	0.0595
	7	0.03691	0.11731	0.11194	0.05093	2.19795	-0.45734	0.0659
	8	0.03896	0.13366	0.12694	0.05719	2.21980	-0.52107	0.0711
	9	0.04128	0.14985	0.14155	0.06422	2.20422	-0.58417	0.0752
	10	0.04387	0.16585	0.15572	0.07200	2.16272	-0.64656	0.0784

For Newtonian flow: $C_{p,t2} = 2$

For the modified Newtonian flow model:

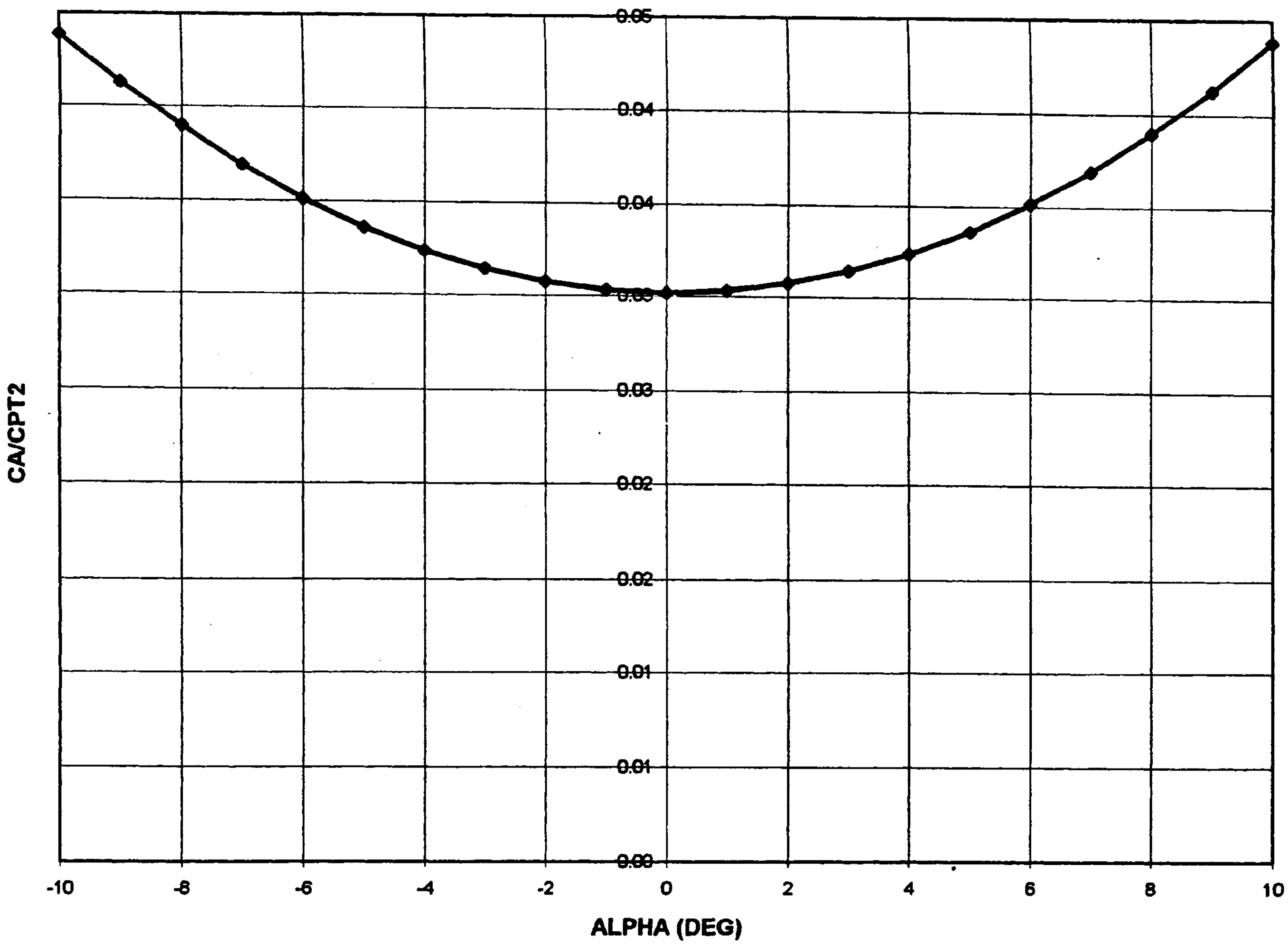
For $M_1 = 10$

$$C_{p,t2} = \left(\frac{p_{t2}}{p_1} - 1\right)\frac{2}{\gamma M_1^2} = (129.2 - 1)\frac{1}{70} = 1.8316$$

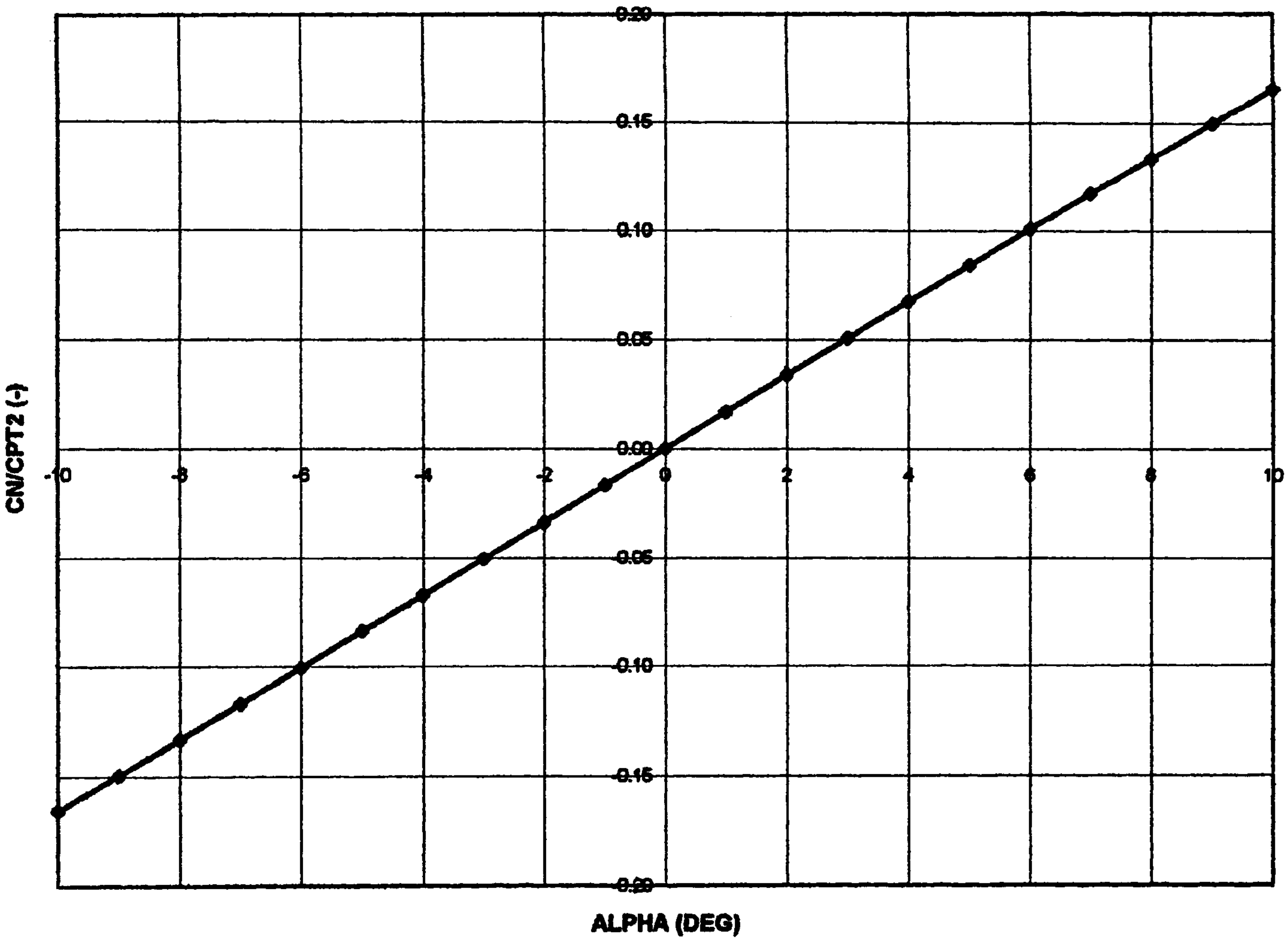
Thus, when $\alpha = 8°$, $C_D = 0.1047$ for the modified Newtonian flow model and $C_D = 0.1144$ for the Newtonian flow model.

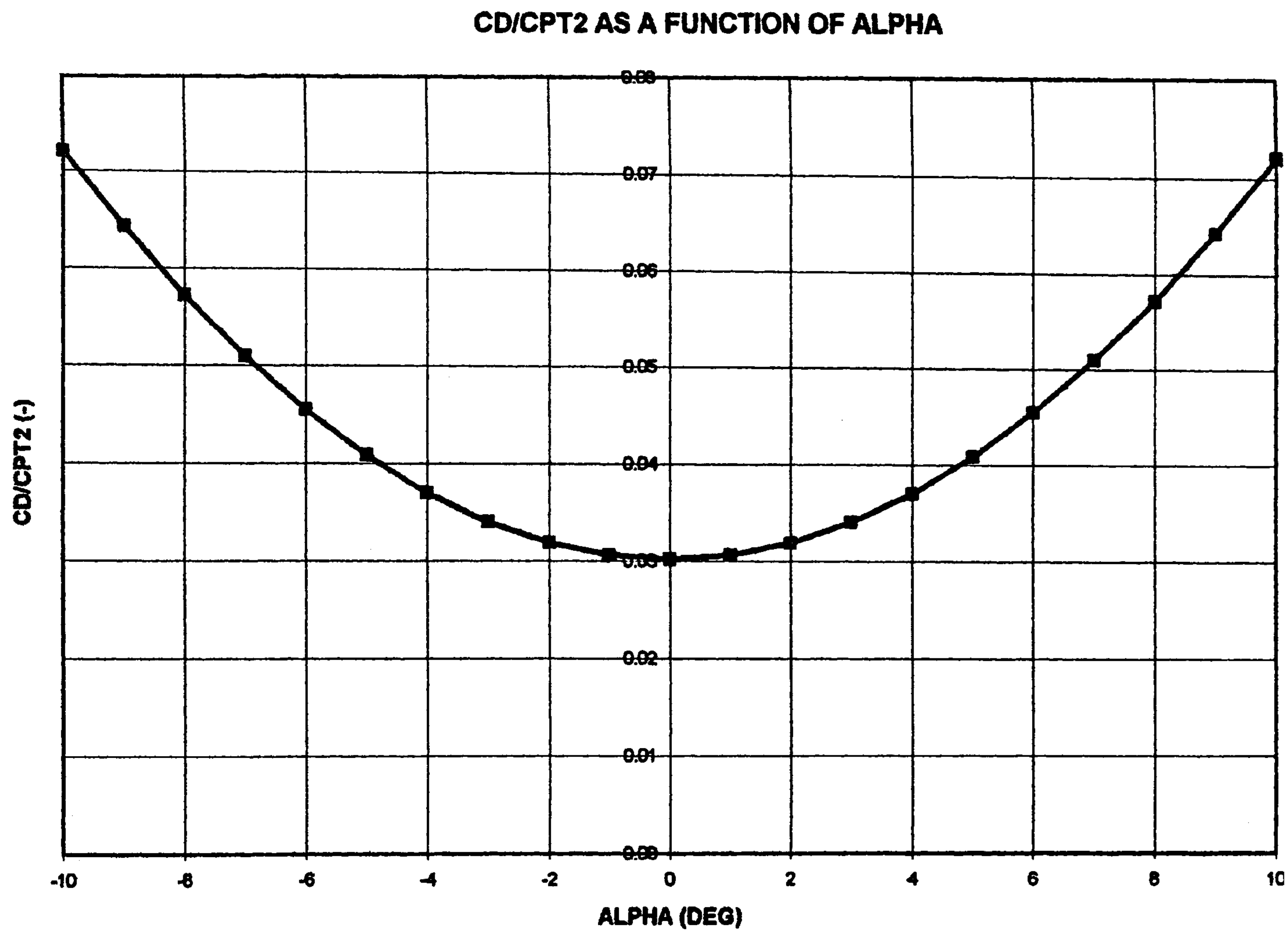
The required graphs for Problems 12.7 through 12.10 with the coefficients divided by the $C_{p,t2}$ are presented on the following pages.

CA/CPT2 AS A FUNCTION OF ALPHA



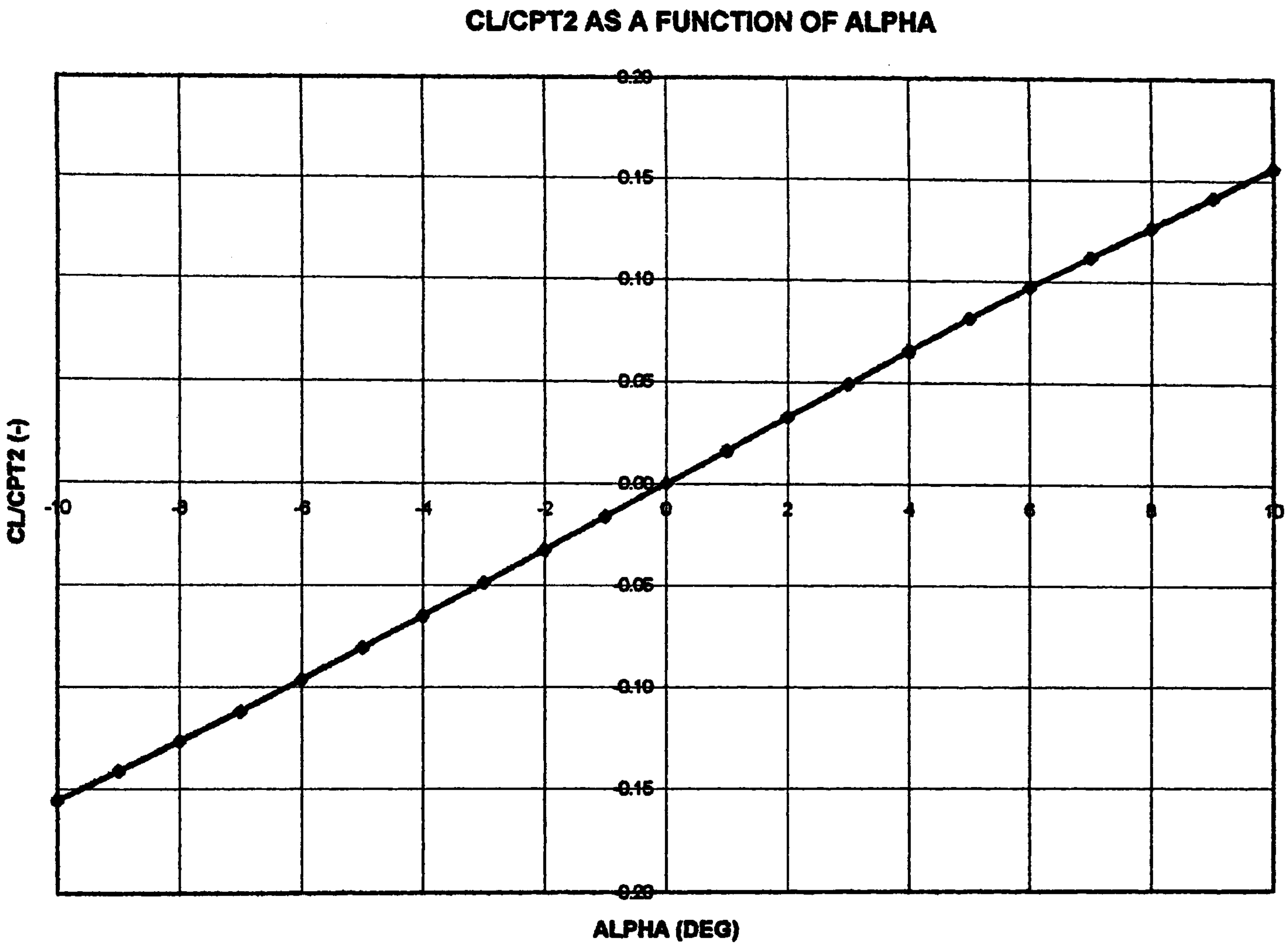
CN/CPT2 AS A FUNCTION OF ALPHA



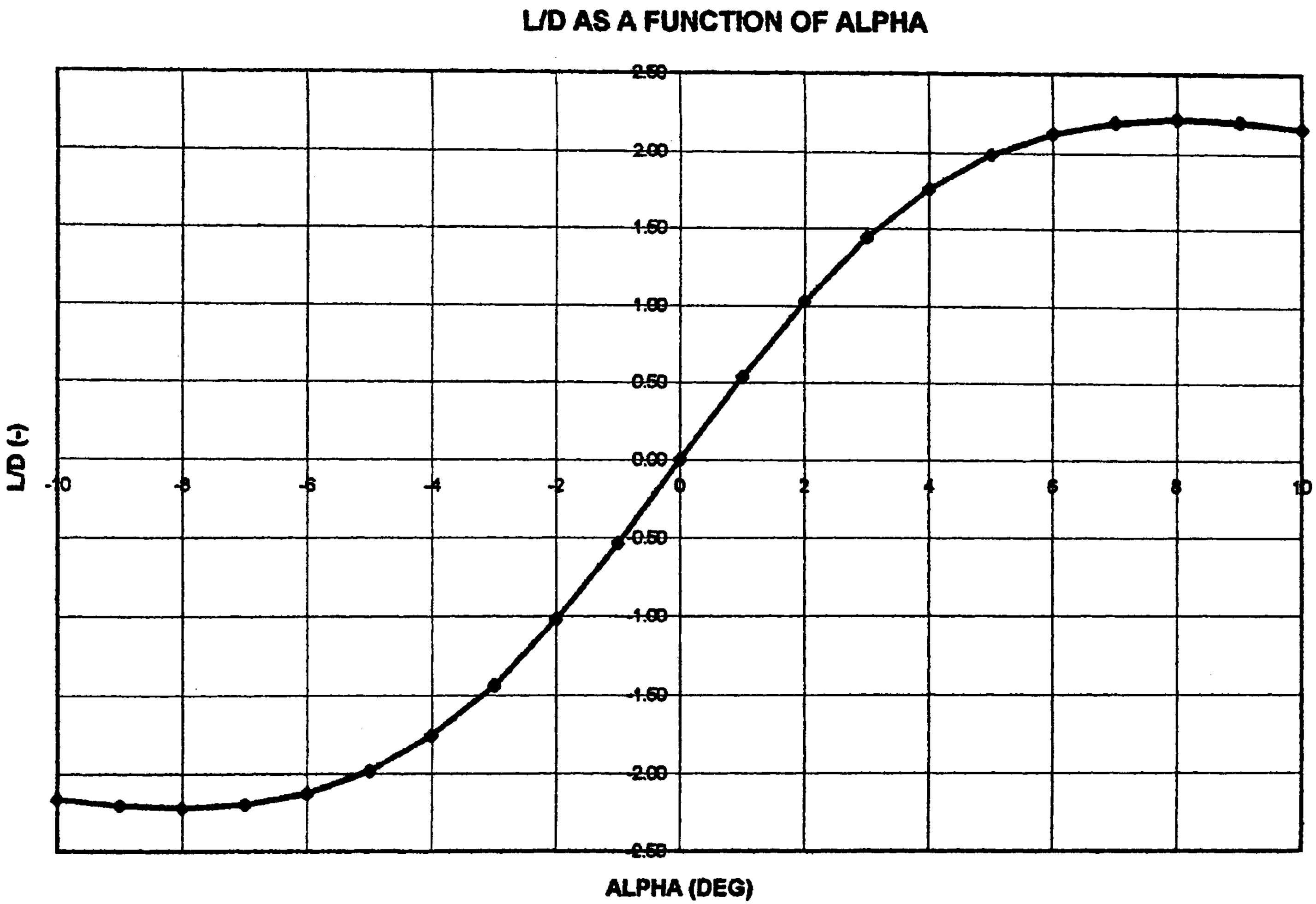


CD/CPT2 AS A FUNCTION OF ALPHA
CD/CPT2 (-)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
-10
-8
-6
-4
-2
0
2
4
6
8
10
ALPHA (DEG)

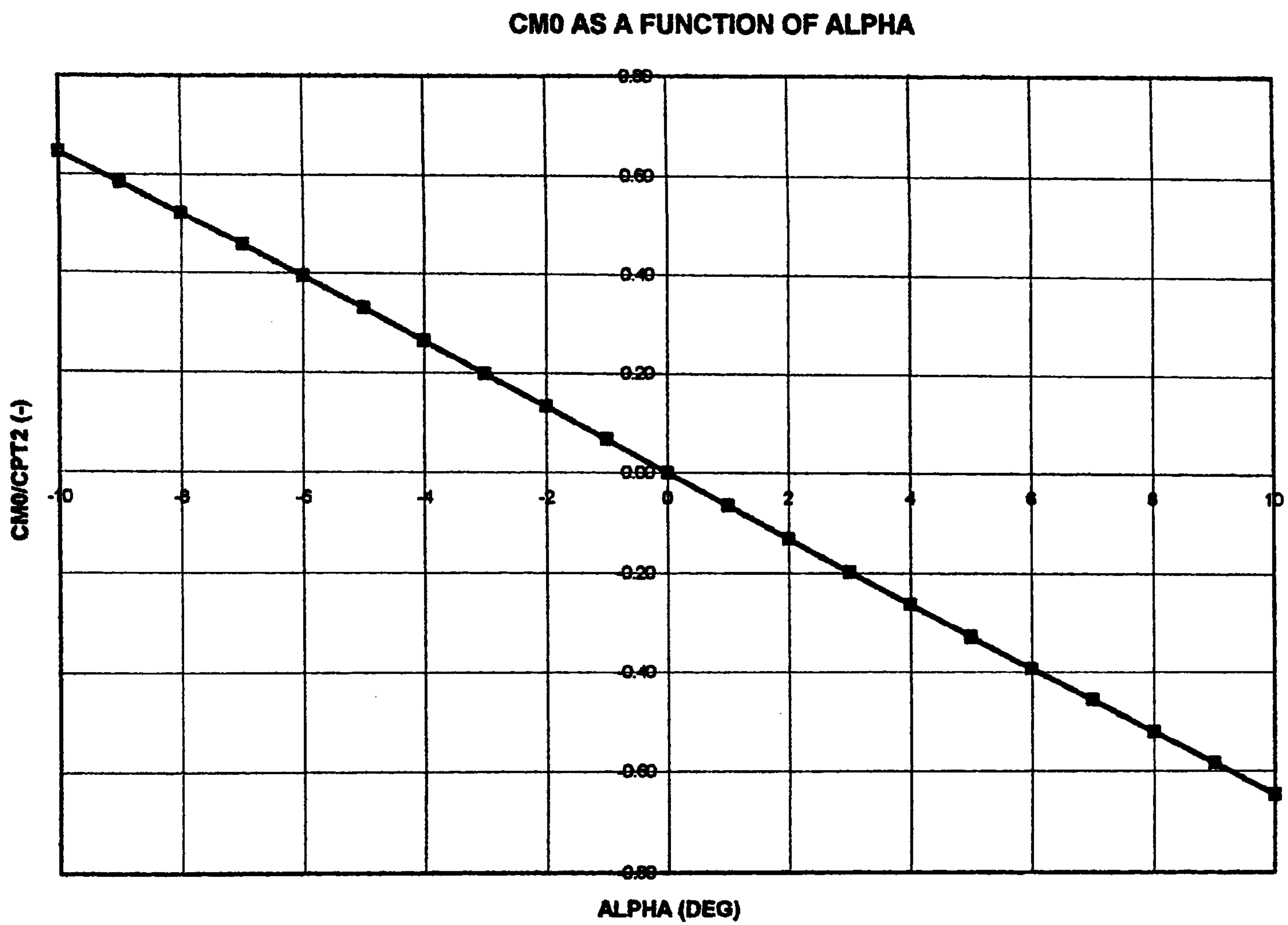
PROBLEM 12.7


CL/CPT2 AS A FUNCTION OF ALPHA
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-10
-8
-6
-4
-2
0
2
4
6
8
10
CL/CPT2 (-)
ALPHA (DEG)

PROBLEM 12.8


L/D AS A FUNCTION OF ALPHA
L/D (-)
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
-10
-8
-6
-4
-2
0
2
4
6
8
10
ALPHA (DEG)

PROBLEM 12.9



PROBLEM 12.10

12.11

$$C_A = \frac{2C_{p,t2}}{\pi R_b^2} \int_0^{x_L} \left\{ \int_0^{\beta_u} (\cos^2\alpha \sin^2\theta_c \right.$$

$$+ 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\cos\beta$$

$$\left. + \sin^2\alpha\cos^2\theta_c\cos^2\beta)(x\tan\theta_c)\,d\beta\left(\frac{dx}{\cos\theta_c}\right)\sin\theta_c\right.$$

Integrating first with respect to β (we'll leave the upper limit general for now as β_u – but $\beta_u = \frac{\pi}{2}$ for our half cone):

$$C_A = \frac{2C_{p,t2}}{\pi R_b^2} \int_0^{x_L} \left\{ \left[\cos^2\alpha \sin^2\theta_c \beta_u \right.\right.$$

$$+ 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\sin\beta_u$$

$$\left.\left. + \sin^2\alpha\cos^2\theta_c\left(\frac{\beta_u}{2} + \frac{1}{4}\sin 2\beta_u\right)\right]\tan^2\theta_c\right\} x\,dx$$

Integrating over x from $x=0$ to $x=x_L$ and noting that:

$$\tan\theta_c = \frac{R_b}{x_L},$$

$$C_A = \frac{C_{p,t2}}{\pi}\left[\cos^2\alpha\sin^2\theta_c\,\beta_u + 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\sin\beta_u\right.$$

$$\left. + \sin^2\alpha\cos^2\theta_c\left(\frac{\beta_u}{2} + \frac{1}{4}\sin 2\beta_u\right)\right]$$

To calculate C_N,

$$C_N = \frac{2C_{p,t2}}{\pi R_b^2} \int_0^{x_L} \left\{ \int_0^{\beta_u} (\cos^2\alpha\sin^2\theta_c \right.$$

$$+ 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\cos\beta$$

$$+ \sin^2\alpha\cos^2\theta_c\cos^2\beta)(x\tan\theta_c)\,d\beta\,\frac{dx}{\cos\theta_c}\cos\theta_c\cos\beta$$

12.11 Contd.] Integrating with respect to β:

$$C_N = \frac{2\,C_{p,t2}}{\pi R_b^2}\int_0^{x_L}\Big\{\Big[\cos^2\alpha\,\sin^2\theta_c\,\sin\beta_u + 2\sin\alpha\cos\alpha\,\sin\theta_c\cos\theta_c\left(\frac{\beta_u}{2}+\frac{1}{4}\sin 2\beta_u\right) + \sin^2\alpha\,\cos^2\theta_c\,\frac{1}{3}\sin\beta_u\left(\cos^2\beta_u+2\right)\Big]\tan\theta_c\Big\}\,x\,dx$$

Again, everything within the brackets $\{\,\}$ is constant for a sharp cone, whose half angle is θ_c, so that:

$$C_N = \frac{2\,C_{p,t2}}{\pi R_b^2}\Big[\cos^2\alpha\,\sin^2\theta_c\,\sin\beta_u + 2\sin\alpha\cos\alpha\,\sin\theta_c\cos\theta_c\left(\frac{\beta_u}{2}+\frac{1}{4}\sin 2\beta_u\right) + \sin^2\alpha\,\cos^2\theta_c\,\frac{1}{3}\sin\beta_u\left(\cos^2\beta_u+2\right)\Big](\tan\theta_c)\,\frac{x_L^2}{2}$$

For the half-cone model, $\beta_u = \frac{\pi}{2}$ and for $0 \le \alpha \le \theta_c$:

$$C_A = \frac{C_{p,t2}}{\pi}\Big\{\cos^2\alpha\,\sin^2\theta_c\,\frac{\pi}{2} + \sin 2\alpha\,\sin\theta_c\cos\theta_c + \sin^2\alpha\,\cos^2\theta_c\,\frac{\pi}{4}\Big\}$$

$$C_N = \frac{C_{p,t2}}{\pi\tan\theta_c}\Big\{\cos^2\alpha\,\sin^2\theta_c + \sin 2\alpha\,\sin\theta_c\cos\theta_c\,\frac{\pi}{4} + \sin^2\alpha\,\cos^2\theta_c\,\frac{2}{3}\Big\}$$

$$C_L = C_N\cos\alpha - C_A\sin\alpha$$

$$C_D = C_N\sin\alpha + C_A\cos\alpha$$

So that:

12.11 Contd.

$$C_L = \frac{C_{p,t2}}{\pi}\left\{\left[\cos^2\alpha \sin^2\theta_c + \sin 2\alpha \sin\theta_c \cos\theta_c \frac{\pi}{4} + \sin^2\alpha \cos^2\theta_c \frac{2}{3}\right]\frac{\cos\alpha}{\tan\theta_c} - \left[\cos^2\alpha \sin^2\theta_c \frac{\pi}{2} + \sin 2\alpha \sin\theta_c \cos\theta_c + \sin^2\alpha \cos^2\theta_c \frac{\pi}{4}\right]\sin\alpha\right\}$$

$$C_D = \frac{C_{p,t2}}{\pi}\left\{\left[\cos^2\alpha \sin^2\theta_c + \sin 2\alpha \sin\theta_c \cos\theta_c \frac{\pi}{4} + \sin^2\alpha \cos^2\theta_c \frac{2}{3}\right]\frac{\sin\alpha}{\tan\theta_c} + \left[\cos^2\alpha \sin^2\theta_c \frac{\pi}{2} + \sin 2\alpha \sin\theta_c \cos\theta_c + \sin^2\alpha \cos^2\theta_c \frac{\pi}{4}\right]\cos\alpha\right\}$$

$$\frac{L}{D} = \frac{C_L}{C_D}$$

Note that, if $\alpha = 0°$,

$$C_L = \frac{C_{p,t2}}{\pi}\,\frac{\sin^2\theta_c}{\tan\theta_c} = \frac{C_p}{\pi \tan\theta_c}$$

$$C_D = \frac{C_{p,t2}}{\pi}\sin^2\theta\,\frac{\pi}{2} = \frac{1}{2}C_p$$

Thus, $C_D = 0.5$ times the pressure coefficient on the windward surface. Verify to yourself that

$$C_D = \frac{1}{2}C_p$$

is a logical result for this inviscid flow model.

12.12 Following the example in the book,

$$C_{M_0} = -\frac{2\tan\theta_c}{\pi R_b^2 R_b}\left\{\int_0^{x_L}\left[\int_0^{\beta_u} C_{p,t2}\left(\cos^2\alpha\sin^2\theta_c\right.\right.\right.$$

$$+ 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\cos\beta$$

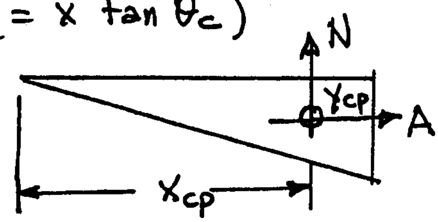
$$\left.\left.\left. + \sin^2\alpha\cos^2\theta_c\cos^2\beta\right)d\beta\cos\beta\right]x^2\,dx\right\}$$

$$+ \frac{2\tan^3\theta_c}{\pi R_b^2 R_b}\left\{\int_0^{x_L}\left[\int_0^{\beta_u} C_{p,t2}\left(\cos^2\alpha\sin^2\theta_c\right.\right.\right.$$

$$+ 2\sin\alpha\cos\alpha\sin\theta_c\cos\theta_c\cos\beta$$

$$\left.\left.\left. + \sin^2\alpha\cos^2\theta_c\cos^2\beta\right)d\beta\cos\beta\right]x^2\,dx\right\}$$

The first term is the contribution of the normal force acting with a moment arm of x. The second term is the contribution of the axial force acting with a moment arm of $r(= x\tan\theta_c)$



Thus,

$$M_0 = -N x_{cp} - A y_{cp}$$

or

$$C_{M_0} q_\infty \pi R_b^2 R_b = -C_N q_\infty \pi R_b^2 x_{cp} - C_A q_\infty \pi R_b^2 y_{cp}$$

$$C_{M_0} = +C_{M_{0,N}} + C_{M_{0,A}}$$

$$C_{M_0} = -C_N \frac{x_{cp}}{R_b} - C_A \frac{y_{cp}}{R_b}$$

Thus, $C_{M_{0,N}} = -C_N \dfrac{x_{cp}}{R_b}$

12.12 Contd. So that:

$$-\frac{2C_{p,t2}\tan\theta_c}{\pi X_L^3\tan^3\theta_c}\left[\cos^2\alpha\sin^2\theta_c+\sin 2\alpha\sin\theta_c\cos\theta_c\frac{\pi}{4}\right.$$
$$\left.+\sin^2\alpha\cos^2\theta_c\frac{2}{3}\right]\frac{X_L^3}{3}=-\frac{C_{p,t2}}{\pi\tan\theta_c}\left[\cos^2\alpha\sin^2\theta_c\right.$$
$$\left.+\sin 2\alpha\sin\theta_c\cos\theta_c\frac{\pi}{4}+\sin^2\alpha\cos^2\theta_c\frac{2}{3}\right]\frac{X_{cp}}{R_b}$$

the term in bracket is common to both sides, as is $C_{p,t2}$, π, and $\tan\theta_c$. Simplifying,

$$X_{cp}=\frac{2}{3}\frac{R_b}{\tan\theta_c}=\frac{2}{3}X_L$$

Similarly, $C_{M,0}=-C_A\dfrac{Y_{cp}}{R_b}$

Using the second term from the moment equation of problem 12.11 and the expression for C_A in problem 12.11,

$$-\frac{2C_{p,t2}\tan^3\theta_c}{\pi R_b^2 R_b}\left[\cos^2\alpha\sin^2\theta_c+\sin 2\alpha\sin\theta_c\cos\theta_c\frac{\pi}{4}\right.$$
$$\left.+\sin^2\alpha\cos^2\theta_c\frac{2}{3}\right]\frac{X_L^3}{3}=-\frac{C_{p,t2}}{\pi}\left[\cos^2\alpha\sin^2\theta_c\frac{\pi}{2}\right.$$
$$\left.+\sin 2\alpha\sin\theta_c\cos\theta_c+\sin^2\alpha\cos^2\theta_c\frac{\pi}{4}\right]\frac{Y_{cp}}{R_b}$$

$$\frac{Y_{cp}}{R_b}=\frac{2}{3}\frac{\left[\cos^2\alpha\sin^2\theta_c+\sin 2\alpha\sin\theta_c\cos\theta_c\frac{\pi}{4}+\sin^2\alpha\cos^2\theta_c\frac{2}{3}\right]}{\left[\cos^2\alpha\sin^2\theta_c\frac{\pi}{2}+\sin 2\alpha\sin\theta_c\cos\theta_c+\sin^2\alpha\cos^2\theta_c\frac{\pi}{4}\right]}$$

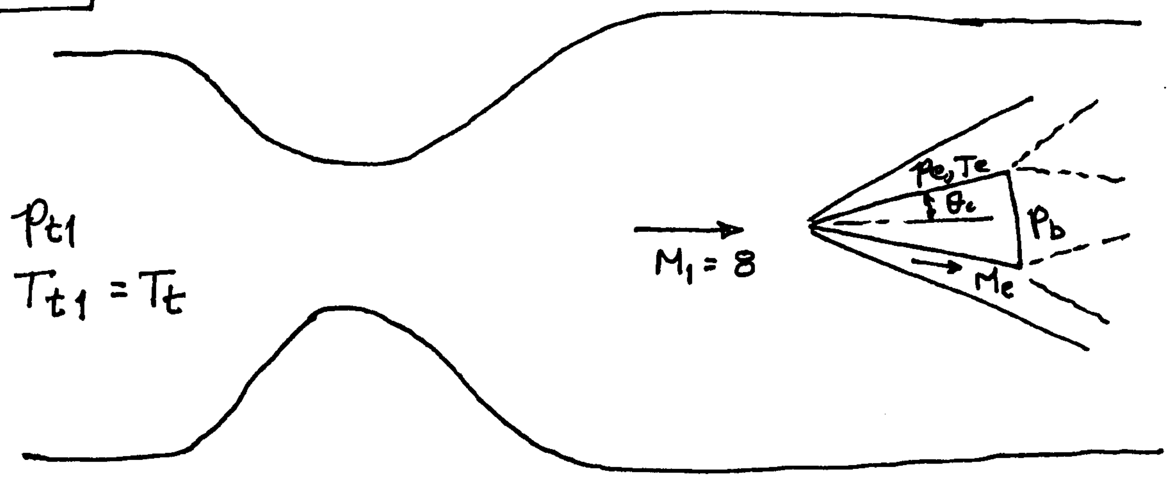
12.13 This problem has been solved using the spreadsheet described in problems (12.7) through (12.10). Thus, the tabulated for $\frac{C_D}{C_{p,t2}}$ as a function of α are valid here.

For the Newtonian flow model, $C_{p,t2} = 2$.

For the modified Newtonian flow model, when $M_1 = 8$

$$C_{p,t2} = \left(\frac{p_{t2}}{p_1} - 1\right)\frac{2}{\gamma M_1^2} = (82.85 - 1)\frac{1}{44.8} = 1.8270$$

12.14



$\theta_c = 20°$ (the cone half angle); $M_1 = 8$ (the test-section Mach number); $p_{t1} = 850$ psia and $T_t = 1350°R$ (the conditions in the tunnel reservoir).

Using the charts of Fig. 8.16:

(a) $\theta_{sw} = 23.3°$; (b) $C_p = 0.254 = \dfrac{p_e - p_1}{\frac{\gamma}{2} p_1 M_1^2}$;

(c) $1 - \frac{1}{M_e} = 0.764$ and, therefore, $M_e = 4.237$

Using the pressure coefficient,

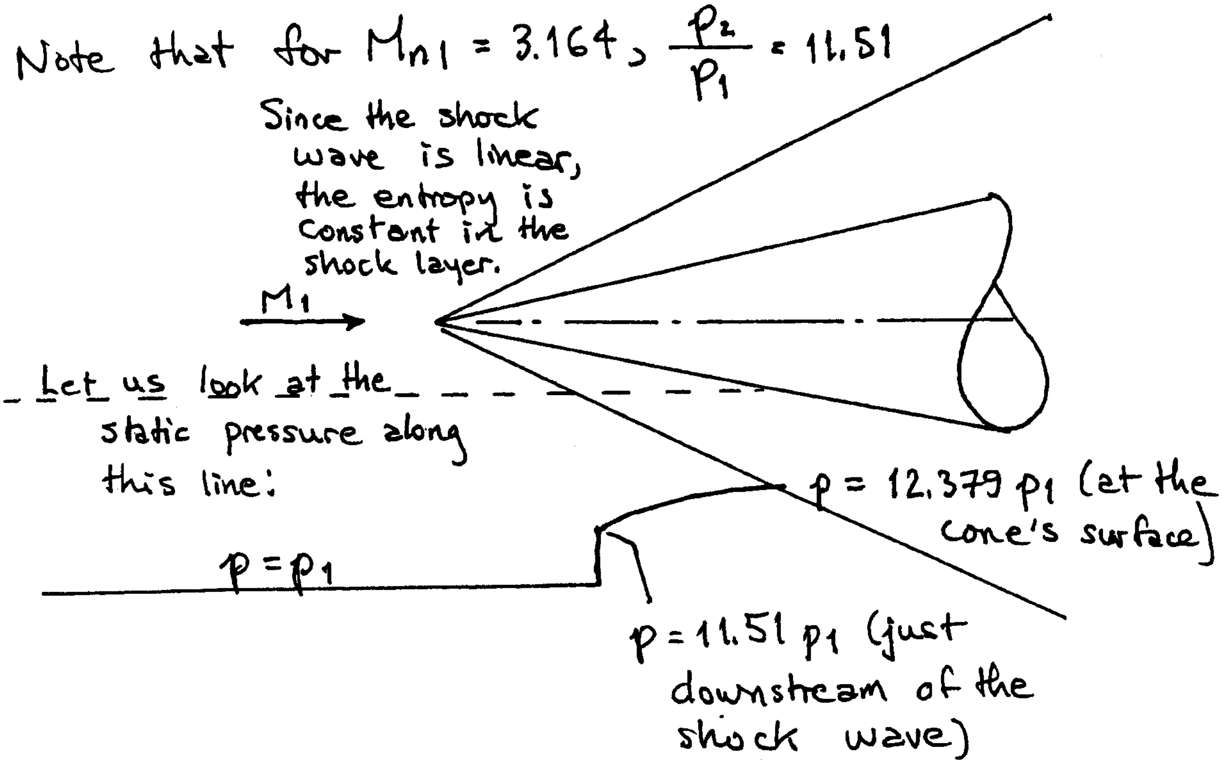
$$\frac{p_e}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_p = 12.379$$

12.14 Contd.] (Note that the subscript "1" designates the free-stream conditions and the subscript "e" designates the conditions at the edge of the boundary layer. Note further, that for a cone the conditions at the edge of the boundary layer, i.e., at the "surface" of the cone for the inviscid flow model are different than those immediately downstream of the shock wave. This differs from a wedge flow.

Let us calculate the flow downstream of the shock wave!

$$M_{n1} = M_1 \sin \theta_{sw} = 8 [\sin 23.3] = 3.164$$

Note that for $M_{n1} = 3.164$, $\frac{p_2}{p_1} = 11.51$



In the inviscid portion of the shock layer, flow properties are constant along a conical generator.

Is the boundary layer laminar or turbulent? Let us

12.14 contd] calculate the Reynolds number at the edge of the boundary layer.

$$\frac{T_e}{T_t} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-1} = 0.2261 \; ; \quad T_e = 305.2\,°R$$

$$p_e = \left(\frac{p_e}{p_1}\right)\left(\frac{p_1}{p_{t1}}\right) p_{t1} = [12.379][0.1024 \times 10^{-3}][850]$$

$$p_e = 1.0775 \text{ psia}$$

$$U_e = M_e \sqrt{\gamma R T_e} = 3628.5 \text{ ft/s}$$

$$\rho_e = \frac{p_e}{R T_e} = \frac{(1.0775)(144)}{(1716.16)(305.2)} = 0.0002962 \; \frac{\text{lbf s}^2}{\text{ft}^4}$$

$$\mu_e = 2.27 \times 10^{-8} \frac{T_e^{1.5}}{T_e + 198.6} = 2.4 \times 10^{-7} \; \frac{\text{lbf s}}{\text{ft}^2}$$

The wetted length for the conical generator is:

$$L = \frac{3.0}{\sin 20°} = 8.771 \text{ in} = 0.731 \text{ ft}$$

Thus, the Reynolds number based on the flow properties at the edge of the boundary layer and the wetted length of a conical generator is:

$$Re_L = \frac{\left(0.0002962 \frac{\text{lbf s}^2}{\text{ft}^4}\right)\left(3628.5 \frac{\text{ft}}{\text{s}}\right)(0.731 \text{ ft})}{2.4 \times 10^{-7} \frac{\text{lbf s}}{\text{ft}^2}}$$

So that $Re_L = 3{,}270{,}632$. For this hypersonic flow, our experience suggests that the boundary layer goes transitional near the end of the cone. The shear layer of the separated flow is probably turbulent. But the

12.14 Contd. boundary layer on the cone is laminar for the purposes of calculating the skin friction.

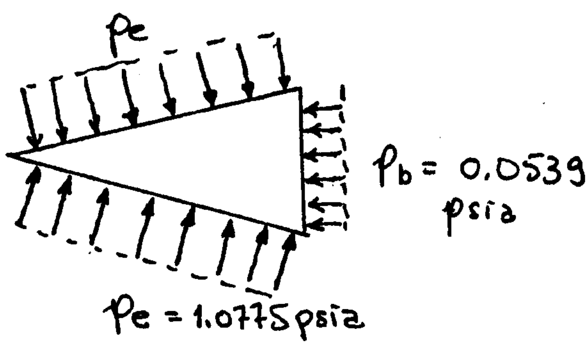
Since we have assumed that the separated flow is turbulent, we can use Fig. 12.19 to calculate the base pressure.

$$\frac{p_b}{p_e} = 0.05$$

Thus, $$\frac{p_b}{p_1} = \frac{p_b}{p_e}\frac{p_e}{p_1} = 0.6190$$

and $$p_b = \frac{p_b}{p_1}\frac{p_1}{p_{t1}} p_{t1} = (0.6190)(0.1024\times10^{-3})(850)$$

$$p_b = 0.0539 \text{ psi}$$



The Newtonian assumption implies that $C_{p,b} = 0$ (i.e., $p_b = p_1$). Using Fig. 12.19,

$$C_{p,b} = \frac{p_b - p_1}{q_1} = \left[\frac{p_b}{p_e}\frac{p_e}{p_1} - 1\right]\frac{2}{\gamma M_1^2} = -0.0085$$

To calculate the skin friction drag, let us use Eckert's reference temperature and Reynolds analogy. Since the boundary layer is laminar:

$$r = \sqrt{Pr} = \sqrt{0.70} = 0.8367$$

$$T^* = 0.5(T_e + T_w) + 0.22r(T_{te} - T_e)$$

$$T^* = 0.5(305.2 + 600) + 0.22(0.8367)(1350 - 305.2)$$

$$T^* = 644.94\,°R$$

12.14 Contd.

$$Re_l^* = \left[\frac{\rho_e}{RT^*}\right]\left[U_e\right]\left[l\right] \Big/ \left[\frac{2.27\times10^{-8}\,T^{*\,1.5}}{T^* + 198.6}\right]$$

$$Re_l^* = \frac{[0.0001402][3628.5][l]}{4.4\times10^{-7}} = 1.154\times10^{6}\,l$$

For a sharp cone: $St = C_h = \dfrac{0.575}{(Re_x)^{0.5}}$

Using Reynolds analogy:

$$C_f = 2C_h = \frac{1.150}{(Re_l^*)^{0.5}} = \frac{0.00107}{l^{0.5}}$$

where l is the wetted distance from the apex in feet.

$$\tau = C_f\, q_e = C_f\left[\frac{\gamma}{2} p_e M_e^2\right] = \frac{2.08656}{l^{0.5}}\ \frac{lbf}{ft^2}$$

$$D = \int p_e\, 2\pi r\, dl \sin\theta_c + \int \tau\, 2\pi r\, dl \cos\theta_c - p_b \pi R_b^2$$

$$r = l \sin\theta_c \quad \text{and} \quad dr = dl \sin\theta_c$$

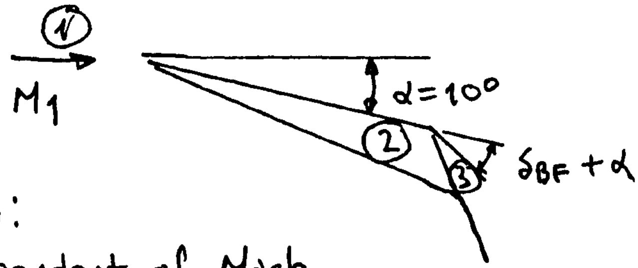
$$D = p_e \pi R_b^2 + (2.08656)(2\pi)(\cos\theta_c)(\sin\theta_c)\int_0^L l^{0.5}\,dl - p_b \pi R_b^2$$

$$D = (1.0775)(9\pi) + 4.21355\left[\tfrac{2}{3}L^{1.5}\right] - (0.0539)(9\pi)$$

$$D = 30.4656 + 1.7556 - 1.5240$$

$$D = 30.6972 \text{ lbs}$$

12.15



(a) For Newtonian flow:

$C_p = 2\sin^2\theta_b$ independent of Mach number. Thus,

$C_{p_2} = 0.06031$ and $C_{p_3} = 0.23396$

(b) Let us calculate the flows in regions 2 and 3 using the wedge relations of Figs. 8.13:

Going from region 1, the freestream, to region 2

M_1	M_2	C_{p_2}	$\frac{p_2}{p_1} = 1 + \frac{\gamma}{2} M_1^2 C_{p_2}$
2	1.639	0.25	1.700
4	3.268	0.13	2.456
6	4.608	0.106	3.671
8	5.814	0.09	5.032
10	6.667	0.085	6.950
20	9.091	0.073	21.440

to go from region 2 to region 3, we use the correlations of Figs. 8.13 treating region 2 as the "free stream" and region 3 as the "downstream flow"

M_1	M_2	M_3	$C_{p\text{"}3\text{"}}$	$\frac{p_3}{p_2} = 1 + \frac{\gamma}{2} M_2^2 C_{p\text{"}3\text{"}}$	C_{p_3}
2	1.639	1.282	0.34	1.639	0.6382
4	3.268	2.597	0.155	2.159	0.3841
6	4.608	3.704	0.122	2.813	0.3702
8	5.814	4.484	0.11	3.603	0.3824
10	6.667	5.000	0.097	4.018	0.3846
20	9.091	6.250	0.087	6.033	0.4584

13.1) If the airplane is cruising at constant altitude and at constant airspeed, the product $q_\infty S$ is constant. Since the lift balances the weight:

$$\frac{W}{q_\infty S} = \frac{L}{q_\infty S} = C_L = C_{L\alpha}\left(\alpha - \alpha_{OL}\right)$$

Thus, the angle of attack is given by:

$$\alpha = \alpha_{OL} + \frac{W}{q_\infty S C_{L,\alpha}}$$

Note that the weight of the aircraft is the only variable quantity in the right-hand side of this equation. It is clear that α decreases as fuel is consumed during the flight, since the weight of the aircraft decreases as the fuel is consumed.

13.2] The maximum lift-to-drag ratio occurs when the induced drag equals the parasite drag.

$$C_{Di} = k C_L^2 = C_{Do}$$

Thus, the total drag is $2C_{Do}$ and the L/D ratio is given by:

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{\left[C_{Do}/K\right]^{0.5}}{2C_{Do}} = \frac{1}{2\sqrt{k C_{Do}}}$$

To increase $(L/D)_{max}$, we can either decrease C_{Do} or k.

13.2 Contd. C_{D_0} includes those contributions to the drag which exist when the configuration generates zero lift. Thus, C_{D_0} includes such non-lift-related drag components, as: (1) the profile drag (i.e., that both due to skin friction and to the pressure distribution) of the wing, fuselage, tail, nacelles, etc.; (2) losses of momentum of the airstream due to power plant cooling, leakage through the skin, etc.; and (3) the drag which results when the flow fields of the various components interact. An example of this latter component (i.e., the drag due to the interactions) occurs at the wing/fuselage juncture, where the presence of the already-developed boundary layer on the fuselage reduces the local velocities of the air molecules as they flow over the wing root. Because the air molecules in the fuselage boundary layer have already been slowed, they are more likely to be separated in the presence of an adverse pressure gradient created as the flow in the boundary layer approaches the wing root. Proper design of a wing-root fillet can minimize these effects. Reductions in skin-friction drag can be accomplished by cleaning up (or smoothing) the surface of the vehicle. Thus, use is made of non-protuding fasteners (flush rivets), care is taken that the panel joints are smooth.

Referring to Chapter 5,

$$C_{Di} = kC_L^2 = \frac{1}{\pi\, e\, AR}\, C_L^2$$

13.2 Contd. Thus, we can reduce the induced drag (or the drag-due-to-lift) by increasing the aspect ratio or by increasing e (which is referred to as Oswald's efficiency factor). The airplane efficiency factor includes the parasite drag-due-to-lift. Recall that, as the angle of attack (and, therefore, the lift) increases, the pressure distribution changes and there are regions where the adverse pressure gradient is relatively large. Thus, separation may occur and the form drag would increase. The wing planform, the airfoil section, and twist may be used to minimize the effect.

We can increase e by using devices which produce equivalent increases in the aspect ratio, such as by adding winglets at the wing tips, which is discussed in this chapter.

13.3 A laminar flow airfoil section would be desireable for a transport aircraft that operates for long periods of time in cruise flight, since the reduction of friction drag would be an important consideration. Operating at low angles of attack, flow separation and, therefore, form drag would not be as important.

The maneuvers of a stunt airplane would be at large angles of attack, with large pressure variations in the flow field, with regions of flow separation and (as a result) with relatively high form drag. Since boundary-layer separation (or stall) is more likely to occur if the boundary layer is laminar, laminar boundary layers would be undesireable.

It should be recalled that it is difficult to maintain a laminar boundary layer at the high Reynolds numbers characteristic of flight.

Problem 13.4 Solution

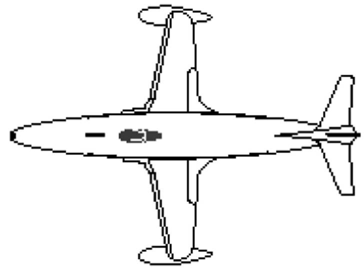
Winglets would be desirable for a transport aircraft that operates for long periods of time in cruise flight, since the reduction of induced drag would be an important consideration.

The maneuvers of a stunt airplane would be at large angles of attack. In addition, a stunt airplane typically only flies for fairly short periods of time, making induced drag reduction only marginally important for performance and fuel efficiency.

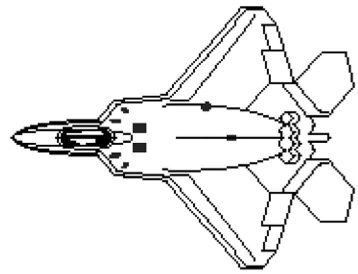
13.5)

The results for Mach number and aspect ratio come from Jane's All The World's Aircraft, while the lift-to-drag ratios are estimated from p. 244.

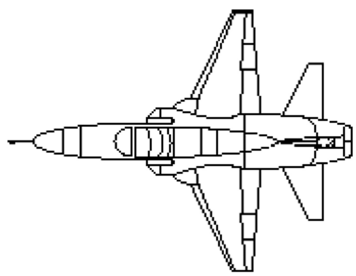
M_∞:0.78 at 25 kft
AR: 6.38
$(L/D)_{\max}$: 12
Name: P-80 (F-80) Shooting Star
Original company: Lockheed



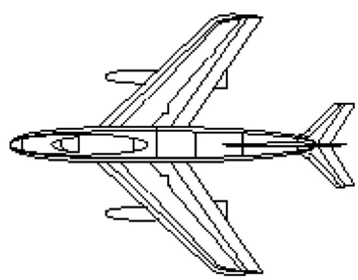
M_∞: 1.58 at 30 kft (w/o afterburner)
AR: 2.4
$(L/D)_{\max}$: 8
Name: F-22 Raptor
Original company: Lockheed Martin



M_∞: 1.4 at 36 kft
AR: 3.75
$(L/D)_{\max}$: 10
Name: F-5 Freedom Fighter
Original company: Northrop

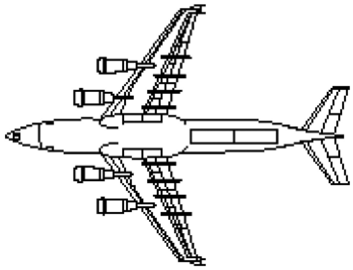


M_∞: 0.88
AR: 5.02
$(L/D)_{\max}$: 12
Name: F-86 Sabre
Original company: North American

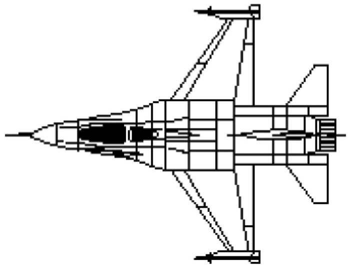


13.5) contd.

M_∞: 0.75 at 28 kft
AR: 7.2
$(L/D)_{max}$: 18
Name: C-17 Globemaster III
Original company: McDonnell-Douglas



M_∞: 2.04 at 40 kft
AR: 3.2
$(L/D)_{max}$: 12
Name: F-16 Fighting Falcon
Original company: General Dynamics



13.6)

Given:

T-41 with the following characteristics:

$$U_\infty = 125kts = 74.0\,ft/s$$
$$h = 10{,}000\,ft$$
$$b = 35\,ft$$
$$c = 7\,ft$$

rectangular wing planform

The boundary layer transition location for subsonic flow is assumed to occur for:

$$\mathrm{Re}_{x_{tr}} = 500{,}000 = \frac{\rho_\infty U_\infty x_{tr}}{\mu_\infty}$$

so the transition location is found by:

$$x_{tr} = \frac{500{,}000\mu_\infty}{\rho_\infty U_\infty}$$

13.6) contd.

For a standard day at 10,000 ft:

$$\rho_\infty = (0.73859)(0.002377) = 0.001756 \text{ slug/ft}^3$$

$$\mu_\infty = (0.94569)(3.740\times10^{-7}) = 3.537\times10^{-7} \text{ lb-s/ft}^2$$

Yielding a transition location of:

$$x_{tr} = \frac{500{,}000(3.537\times10^{-7})}{(0.001756)(74.0)} = 1.36 \text{ ft}$$

The boundary layer does not remain laminar for the entire airfoil section.
Since the wing has a chord of 7 ft, transition occurs at 19.4% of the wing chord, which means the laminar boundary layer is not having a large impact on the wing.