Simulated Design of an MPC Controller for Rocket Vertical Landing

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The objective of this study is to develop a controller for thrust vectoring of a bipropellant liquid rocket engine. This is meant to allow for landing of the UF Liquid Propulsion Development Team flight vehicle. To this end, the controller can command the elongation of two linear actuators to control thrust vector direction within ±10° and can command a throttle valve to control the thrust magnitude down to 40% of the maximum. The controller is designed to maintain a zero-degree pitch angle and to command the rocket to follow a prescribed landing trajectory. The simulated performance of the controller is evaluated based on landing success rate, overshoot, rise time. The controller designed herein achieved a landing success rate of 20% in simulations with Monte Carlo perturbation.

I. Nomenclature

COM	=	center of mass
COP	=	center of pressure
MPC	=	model predictive controller
TVC	=	thrust vector control

II. Introduction

The aim of this paper is the development of a model predictive controller (MPC) controller for thrust vectoring of a liquid rocket engine. The controller is designed to allow the rocket to hover and land at low velocities. Hence, several effects such as drag and aerodynamic forces on the fins are neglected for simplification of the model. The paper is divided into two main sections: one to derive the nonlinear dynamics of the system and one for simulation of the controller.

The MPC controller was chosen due to its ability to account for natural constraints in the physical system. These include the limits of engine gimballing, maximum engine thrust, as well as safe acceleration and landing rates. The constraints are assumed to be given based on the hopper vehicle of the UF Liquid Propulsion Development Team. The pertinent specifications include the maximum thrust of 500 lbf, the engine gimbal angle of ±10°, and the throttle range of 40% maximum thrust. The controller must maintain a zero-degree pitch and yaw angle while commanding the rocket to follow the prescribed hover and landing trajectories.

III. 6-DOF Mathematical Model

Figure 1 shows the ground-fixed ($\mathcal{G}$) and rocket-fixed ($\mathcal{R}$) coordinate systems. The roll rate ($\dot{\theta}_1$), pitch rate ($\dot{\theta}_2$), and yaw rate ($\dot{\theta}_3$) describe the angular velocity of the rocket in $\mathcal{R}$. The origin, O, of the $\mathcal{R}$ frame lies along the center axis of the rocket and is located at the center of mass (COM). The $\mathcal{G}$ frame is defined by $\mathcal{R}$ at t=0.

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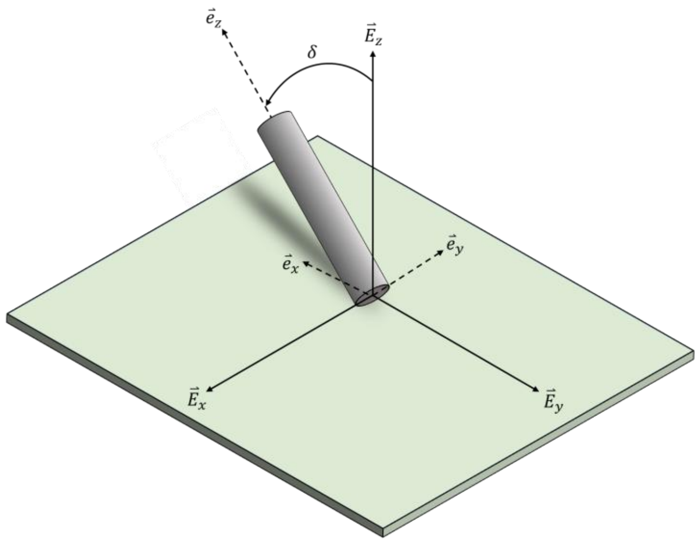

Fig. 1 Ground-fixed frame is $\{\vec{\boldsymbol{E}}_x, \vec{\boldsymbol{E}}_y, \vec{\boldsymbol{E}}_z\}$ and rocket-fixed frame $\{\vec{\boldsymbol{e}}_x, \vec{\boldsymbol{e}}_y, \vec{\boldsymbol{e}}_z\}$.

A. Forces

This model considers three forces on the rocket: thrust, aerodynamic drag, and gravity. Thrust is given by the rocket thrust equation, where $\dot{m}$ is the mass propellant mass flow rate, v_e is the exhaust velocity, P_e the exit pressure, P_a the ambient pressure, and A_e the exit area.

$$T = \dot{m}v_e + (P_e - P_a)A_e \tag{1}$$

At the design condition, $P_e = P_a$; thus, the reference thrust can be defined by Eq. (2).

$$T_{ref} = \dot{m}v_e \tag{2}$$
$$T = T_{ref} + (P_e - P_a)A_e \tag{3}$$

This reference thrust is the measured thrust from static tests of the rocket engine. The ambient pressure is defined by the US Standard Atmosphere [1]. However, for initial low-altitude tests, the pressure term of Eq. (3) is not significant.

Since the pitch angle is small, aerodynamic drag is assumed to only act in the $\boldsymbol{e_z}$ direction. This assumption is valid for the short hops that the first version of the rocket will perform. For higher altitude launches and landings, lateral aerodynamic drag must also be accounted for, as well as the effects of fins. Under these assumptions, drag is given by (4).

$$\boldsymbol{D} = \frac{1}{2}\rho({}^{\boldsymbol{G}}\boldsymbol{v} \cdot {}^{\boldsymbol{G}}\boldsymbol{v})C_d S\, \boldsymbol{e_z} \tag{4}$$

Here ρ is the air density (defined in [1]), ${}^{\boldsymbol{G}}\boldsymbol{v}$ is the rocket velocity, C_d is the aerodynamic drag coefficient, and S is the cross-sectional area of the rocket body.

Gravity acts purely in the $\boldsymbol{E_z}$ direction and is given by Eq. (5).

$$\boldsymbol{F_g} = -mg\, \boldsymbol{E_z} \tag{5}$$

Mass, m, and gravitational acceleration, g, are given by the following.

$$m = m_0 - \int_0^T \dot{m}\, dt \tag{6}$$

$$g = g_0 \left(\frac{R_e}{R_e + h}\right)^2 \tag{7}$$

Mass decreases from the initial mass, m_0, until all propellants are exhausted. The gravitational parameter is a function of altitude; however, for small hops it can be assumed to be the ground-level constant, g_0.

B. Rocket Orientation

The rocket orientation is parameterized in terms of the 3-2-1 Euler angles θ_1, θ_2, and θ_3, corresponding to roll, pitch, and yaw respectively. With this parametrization, the rotation matrix between $\mathcal{R}$ and $\mathcal{G}$ and be defined as follows.

$$\boldsymbol{R_{GR}} = \begin{bmatrix} \cos\theta_1\cos\theta_3 - \sin\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2 & \cos\theta_1\sin\theta_3 + \sin\theta_1\sin\theta_2\cos\theta_3 \\ \sin\theta_1\cos\theta_3 + \cos\theta_1\sin\theta_2\sin\theta_3 & \cos\theta_1\cos\theta_2 & \sin\theta_1\sin\theta_3 - \cos\theta_1\sin\theta_2\cos\theta_3 \\ -\cos\theta_2\sin\theta_3 & \sin\theta_2 & \cos\theta_2\cos\theta_3 \end{bmatrix} \tag{8}$$

Furthermore, the angular velocity of $\mathcal{R}$ in $\mathcal{G}$ can be written in both the body-fixed and inertial coordinates as follows.

$$\begin{aligned} {}^{G}\boldsymbol{\omega}^{R} &= \Omega_x\boldsymbol{E_x} + \Omega_y\boldsymbol{E_y} + \Omega_z\boldsymbol{E_z} \\ &= \left(\dot\theta_2\cos\theta_1 - \dot\theta_3\sin\theta_1\cos\theta_2\right)\boldsymbol{E_x} + \left(\dot\theta_2\sin\theta_1 + \dot\theta_3\cos\theta_1\cos\theta_2\right)\boldsymbol{E_y} + \left(\dot\theta_1 + \dot\theta_3\sin\theta_2\right)\boldsymbol{E_z} \end{aligned} \tag{9}$$

$$\begin{aligned} {}^{G}\boldsymbol{\omega}^{R} &= \omega_x\boldsymbol{e_x} + \omega_y\boldsymbol{e_y} + \omega_z\boldsymbol{e_z} \\ &= \left(\dot\theta_2\cos\theta_3 - \dot\theta_1\cos\theta_2\sin\theta_3\right)\boldsymbol{e_x} + \left(\dot\theta_3 + \dot\theta_1\sin\theta_2\right)\boldsymbol{e_y} + \left(\dot\theta_2\sin\theta_3 + \dot\theta_1\cos\theta_2\cos\theta_3\right)\boldsymbol{e_z} \end{aligned} \tag{10}$$

C. Gimbal Coordinate Transformation

We must determine the direction of the thrust vector in $\mathcal{R}$. This representation can then later be transformed into the $\mathcal{G}$ frame. The thrust vectoring mechanism is a two-axis gimbal controlled by two linear actuators. These are aligned with the $\boldsymbol{e_x}$ and $\boldsymbol{e_y}$ directions. We will arbitrarily choose the actuator aligned with the $\boldsymbol{e_y}$ direction as parent of the actuator aligned with the $\boldsymbol{e_x}$ direction. Thus, rotation about the $\boldsymbol{e_x}$ axis causes the actuator aligned with the $\boldsymbol{e_x}$ direction to rotate as well[2].

The thrust vector is aligned with the axis of the engine and points upwards (i.e. initially, it is aligned with $\boldsymbol{e_z}$). Thus, using the above parenting structure, the two rotations of the thrust vector are depicted in Fig. 2. First, the $\boldsymbol{e_y}$-aligned actuator rotates the engine so that the thrust vector points in the $\boldsymbol{e_z}'$ direction. The $\boldsymbol{e_x}$-aligned actuator then rotates the engine so that the thrust vector points in the $\boldsymbol{e_z}''$ direction. Thus, the final thrust vector direction can be expressed as follows.

$$\boldsymbol{e_z}' = \sin\gamma_y\,\boldsymbol{e_y} + \cos\gamma_y\,\boldsymbol{e_z} \tag{11}$$

$$\boldsymbol{e_z}'' = \sin\gamma_x\,\boldsymbol{e_x} + \cos\gamma_x\,\boldsymbol{e_z}' = \sin\gamma_x\,\boldsymbol{e_x} + \cos\gamma_x\sin\gamma_y\,\boldsymbol{e_y} + \cos\gamma_x\cos\gamma_y\,\boldsymbol{e_z} \tag{12}$$

D. System Kinematics

The kinematics of the rocket is described by the following. The position, $\boldsymbol{r}$, velocity, ${}^{G}\boldsymbol{v}$, and acceleration, ${}^{G}\boldsymbol{a}$, are of the origin of $\mathcal{R}$, which is the center of mass. Although the center of mass is not constant, we can assume that for small time increments, the system behaves as if it is.

$$\boldsymbol{r} = x\boldsymbol{E_x} + y\boldsymbol{E_y} + z\boldsymbol{E_z} \tag{13}$$

$${}^{G}\boldsymbol{v} = \dot{x}\boldsymbol{E_x} + \dot{y}\boldsymbol{E_y} + \dot{z}\boldsymbol{E_z} \tag{14}$$

$${}^{G}\boldsymbol{a} = \ddot{x}\boldsymbol{E_x} + \ddot{y}\boldsymbol{E_y} + \ddot{z}\boldsymbol{E_z} \tag{15}$$

[2] Note that rotation about the $\boldsymbol{e_y}$ axis DOES NOT cause the actuator aligned with the $\boldsymbol{e_y}$ direction to rotate. This is again due to the parenting structure.

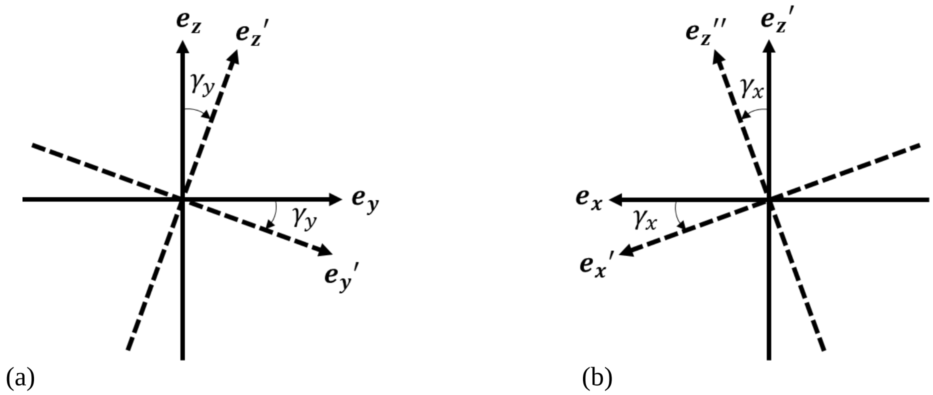

Fig. 2 Engine coordinate rotation. (a) The e_y-aligned actuator rotation. (b) The e_x-aligned actuator rotation.

E. System Dynamics

The dynamics of the system are governed by Newton's 2nd law and an alternate form of Euler's 2nd law, which are given in [2]. Note that the net aerodynamic force acts on the center of pressure (COP) of the rocket. Thus, if the shape of the rocket remains fixed, then the COP is fixed in the rocket frame.

$$\boldsymbol{F} = m^{\mathcal{G}}\overline{\boldsymbol{a}} \tag{16}$$

$$\boldsymbol{M_O} - \overline{\boldsymbol{r}}_{\boldsymbol{O}} \times m(-{}^{\mathcal{G}}\overline{\boldsymbol{a}}_{\boldsymbol{O}}) = \frac{{}^{G}d}{dt}({}^{\mathcal{G}}\boldsymbol{H_O}) \tag{17}$$

The moment applied to $\mathcal{R}$ relative to O is defined as

$$\boldsymbol{M_O} = \sum_{i=1}^{n} \boldsymbol{r_i} \times \boldsymbol{F_i}\,. \tag{18}$$

The following is the moment of inertia tensor, $\boldsymbol{I_O^{\mathcal{R}}}$, of the rocket relative to O and expressed in $\mathcal{R}$. The exact coefficients will be computed in a later section.

$$\boldsymbol{I_O^{\mathcal{R}}} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}_{\mathcal{R}} \tag{19}$$

F. Governing Differential Equations

1. Newton's Second Law

First, we find the net force acting the rocket and express all terms in $\mathcal{G}$.

$$\boldsymbol{F} = \boldsymbol{T} + \boldsymbol{D} + \boldsymbol{F_g} = \left(t_x\boldsymbol{e_x} + t_y\boldsymbol{e_y} + t_z\boldsymbol{e_z}\right) - D\boldsymbol{e_z} - mg\boldsymbol{E_z} \tag{20}$$

$$[\boldsymbol{T} + \boldsymbol{D}]_G = \boldsymbol{R_{GR}}[\boldsymbol{T} + \boldsymbol{D}]_R \tag{21}$$

Substituting the results into (16), we get the following system of differential equations.

$$\begin{aligned} m\ddot{x} &= c_{11}t_x + c_{12}t_y + c_{13}(t_z - D) \\ m\ddot{y} &= c_{21}t_x + c_{22}t_y + c_{23}(t_z - D) \\ m\ddot{z} &= c_{31}t_x + c_{32}t_y + c_{33}(t_z - D) - mg \end{aligned} \tag{22}$$

Here the coefficients c_{nm} represent the coefficients of the rotation matrix in Eqn. (8).

2. Euler's Second Law

We assume that the principal axis basis of the rocket is aligned with $\mathcal{R}$, thus allowing the use of the simplified Euler equations for rigid body motion. With this assumption, the non-diagonal terms of Eqn. (19) are zero and the remaining terms at labelled $I_1, I_2,$ and I_3 for the first, second, and third rows respectively.

$$\begin{aligned} M_1 &= I_1\dot{\omega}_1 + (I_3 - I_2)\omega_3\omega_2 \\ M_2 &= I_1\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 \\ M_3 &= I_1\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 \end{aligned} \tag{23}$$

Next, we compute the moment relative to *O*. The drag force axis intersects *O* and thus it has a moment of zero. Similarly, the gravitational force acts at *O* and produces zero moment. Thus, only the thrust force produces a moment about *O*.

$$\boldsymbol{M_O} = \sum_{i=1}^{n} \boldsymbol{r_i} \times \boldsymbol{F_i} = \boldsymbol{r_{T/O}} \times \boldsymbol{T} = -\ell \mathrm{e}_\mathrm{z} \times \left(t_x\boldsymbol{e_x} + t_y\boldsymbol{e_y} + t_z\boldsymbol{e_z}\right) = \ell t_y\boldsymbol{e_x} - \ell t_x\boldsymbol{e_y} \tag{24}$$

Assuming that the angular momentum of the rocket is small, we can substitute the results from Eq. (24) into (23), to get the following system of differential equations.

$$\begin{aligned} \dot{\omega}_1 &= \frac{\ell t_y}{I_3} \\ \dot{\omega}_2 &= \frac{-\ell t_x}{I_2} \\ \dot{\omega}_3 &= 0 \end{aligned} \tag{25}$$

IV. Results and Discussion

The MPC controller was simulated using the nonlinear equations of motion as the system plant. From the results in Fig. 3 and Fig. 4, it is clear that controller was able to successfully react to small perturbations in the rocket orientation. However, for larger perturbations, the controller performed much worse (Fig. 5), with the rocket reaching the ground with significant pitch and yaw angle. This was likely due to the effects of the engine gimbal rate. With the linear actuators used on the hopper vehicle, the lower actuation speed results in more phase lag in the system response. Simulation of faster actuator response, results in similar performance to Fig. 3. The control of rocket landing velocity is less effected by the gimbal rate of the engine. The throttle rate of the engine also seemed to be sufficient for both large and small perturbation tests.

While the MPC controller was able to perform well for most control scenarios, it suffered in terms of controller run time. The loop time of the code would not be acceptable on lower-power hardware and hence future work is needed to improve the execution speed of the model and reduce the number of computations. Further work is also needed to characterize the effect of the gimbal speed on the system response. This can be done by modelling the gimbal dynamics and including the effects in the MPC model.

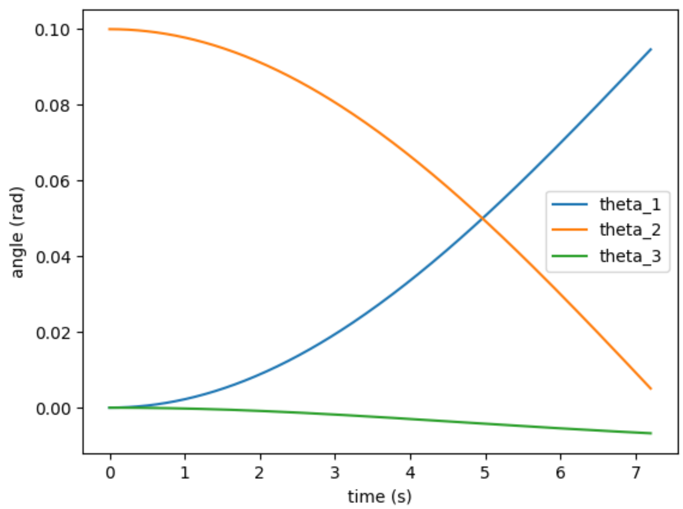

Fig. 3 Orientation of the rocket after small perturbation.

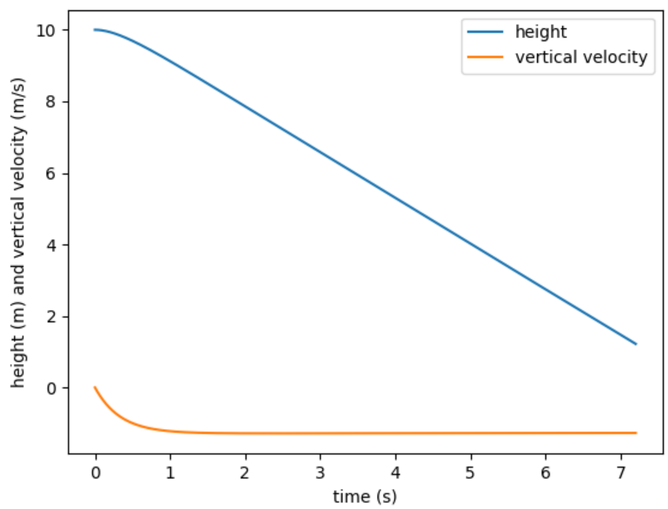

Fig. 4 Translation of the rocket.

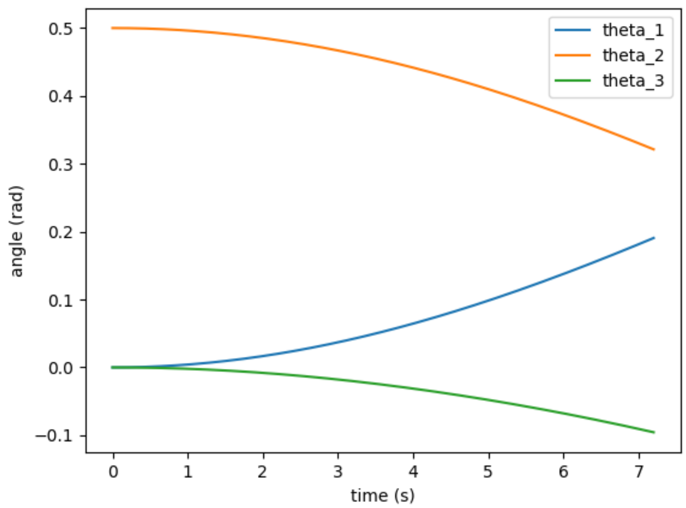

Fig. 5 Orientation of the rocket after large perturbation.

References

[1] National Oceanic and Atmospheric Administration, "U.S. Standard Atmosphere," National Oceanic and Atmospheric Administration, 1976.

[2] A. V. Rao, Dynamics of Particles and Rigid Bodies: A Systematic Approach, Gainesville: Cambridge Unversity Press, 2006.