Simulated Area Observation Mission along the Mississippi River via Low Earth Orbit

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With the recent droughts in the Mississippi river disrupting water-based industry, there has been a need to understand the behavior of one of the largest water systems in our country. The purpose of this study is to demonstrate the viability of the CubeSat platform for monitoring the Area along the Mississippi river. A simulated CubeSat mission was created to monitor the Mississippi river using the Systems tool kit. Using perturbations and constraints, this study aims to accurately simulate the mission's lifespan. This paper presents physical modeling as well as the creation and verification of a Reaction Wheel algorithm to be used for spacecraft attitude control.

I. Nomenclature

STK	=	Systems Tool Kit
e	=	Eccentricity
a	=	Semimajor axis
i	=	inclination
Ω	=	Right Angle Ascending Node
ω	=	Argument of Periapsis
$\boldsymbol{\omega}$	=	Angular Velocity
G	=	Gravity of earth
m	=	Mass
$\ddot{\vec{r}}$	=	Relative Acceleration
$\ddot{\vec{r}}_{ad}$	=	Relative Acceleration due to Atmospheric Drag
$\ddot{\vec{r}}_{sp}$	=	Relative Acceleration due to Solar Pressure
$\vec{r}$	=	Relative Velocity
μ	=	Gravity Constant
ρ	=	Atmospheric Density
L_s	=	Luminosity
C_d	=	Coefficient of Drag
L	=	Angular Momentum
I	=	Mass Moment of Inertia
MMOI	=	Mass Moment of Inertia
TLE	=	Two-Line Element Sets
HPOP	=	High-Precision Orbit Propagator
RWA	=	Reaction Wheel Assembly

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II. Introduction

The fields such as potamology and hydrogeology, have been helping us humans understand the importance of freshwater sources to our everyday lives. From drinking water to transportation, it would be no exaggeration to say that our lives depend on them. Therefore, it is always important to deepen the well of knowledge on the environmental exchange of groundwater. In October of 2022, following a record hot summer, news channels reported on record low water levels in the Mississippi river. Although these low levels lead to the discovery of many lost boats. This also had the effect of stopping water-based industry on the Mississippi river for the better parts of last winter. The Mississippi is the largest river in the U.S. This body of water is crucial to transport of goods, power generation, and agriculture along its 2,340-mile body. With global temperatures getting higher and drought seasons becoming more frequent it is important to have a means of fast monitoring of the river's watershed. This monitoring can serve as an early warning system for cities and businesses along the river. To achieve this, we want to simulate the use of the CubeSat platform to monitor the watershed region. This paper will focus on the design and application of such a CubeSat mission. Although some physical properties of the sensors will be used analytically, some knowledge about the operation of the radiometer and optics will be outside of the scope of this paper. Additional inspiration was taken from missions like RACE (Radiometer Atmospheric CubeSat Experiment).

III. Modeling and Material Properties

A. 3D model overview

For this project a 3D model is necessary for determining a number of aspects about the satellite. The mass properties such as total mass, center of gravity, and moment of inertia are incredibly important for the attitude determination system. In addition to this, the 3D model of the system is imperative to make sure all the components are compatible and fit with the other components in the satellite. The parts have been modeled using the software SOLIDWORKS, a 3D CAD software provided by Mississippi State University to students [12]. Many of the parts are intentionally generic and are created to just get an idea for the mass as they are outside the scope of this project.

B. 3D model overview

The system is made in the fashion of a 3-u satellite, with a total mass of 2.41 kg and includes systems such as the optical payload, radiometer payload, reaction wheel assembly, central computing unit, power bank, star tracker, magnetometer, antennas, and solar panels modeled as well as all associated connecting materials and hardware. Overall, the system is able to effectively fit the basic mass and size requirements of the 3u CubeSat model. The total mass of the satellite fits well under the 4 kg limit for a 3u satellite and measures in at 298.5 x 98 x 98 mm, which fits snugly below the 300 x 100 x 100 mm size maximum 3u satellites are allowed to take up. Pictured in Fig 1 is the model of the satellite, the satellite with the exterior panels removed, and a top-down view of the satellite.

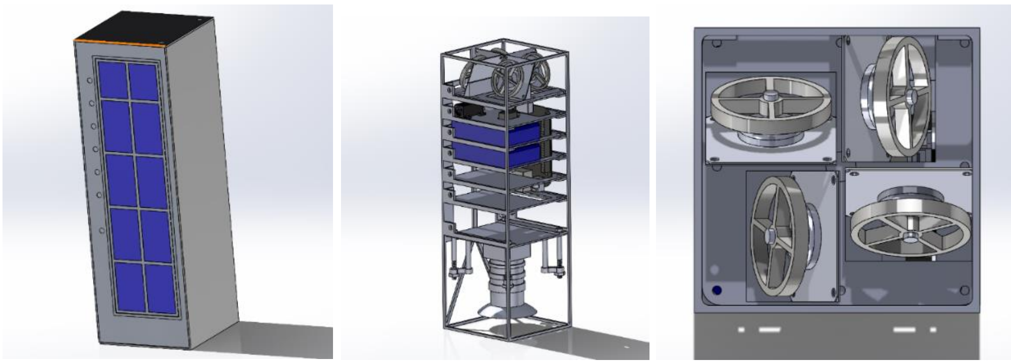

Figure 1. 3D model of the CubeSat system.

C. Derivable Properties

The properties pulled from the CubeSat 3D model are useful to the attitude determination system as well as deriving the fit of all parts into the system. The first and most obvious attribute gathered from the model is total system mass. For the satellite created, a total mass of 2.41 kg is estimated, which is below the conventional limit of 4 kg. The other important properties are the mass moments of inertia (MMOI) of the satellite and its components. The components MMOI that are considered useful for the Reaction Wheel simulation are the satellite as a whole, the Reaction Wheel Assembly (RWA), and the flywheel. These MMOIs are all incorporated into the Simulink model. Additionally important is the center of gravity of the satellite.

D. Future Improvements

The model here fits well for the purposes of this design but has a few areas that can be improved upon. The first and most obvious is a lack of wiring and harnessing, this is rather negligible for the purpose of this project, but it does have effects that can change the outcome of the simulation. Another area of potential improvement is changing the location of different sensors and components. It is likely to be beneficial to change the placement of the star tracker, RWA, or power supply depending on how those effect total system of inertia for the system, or potentially for aspects outside the scope of this study like thermal management or power efficiency.

IV. System Design

The main benefit of CubeSats is their relatively low price point compared to larger Satellite options. With their lightweight and compact designs, a single launch vehicle could deliver a number of these small satellites to orbit at once. The CubeSats, as the name implies, is a cubic satellite measured in units with a unit being roughly 10cm x 10cm x10cm. For this design a 3U system was modeled. This has dimensions of 10cm x 10cm x 30cm. In designing this system, the authors focused on low weight and low cost.

A. Payload

The main payload taken on the CubeSats are the optical sensors. One CubeSat is equipped with a multispectral camera. This allows for imaging in the Near IR as well as visible light spectrum. This allows the system to monitor things like changes in the temperature surrounding the river and activates around it. Another CubeSat will house the water Vapor radiometer.

V. Orbital Design

For the validation of our constellation, we used systems tool kit (STK). STK is software that allows you to bring in and observe different objects pertaining to a desired orbit. This software allows the orbit designer to not only simulate the behavior of orbital dynamics, but also atmospheric models. With the right constraints and parameters for sensors, the objective is to simulate the access behaviors between the CubeSats and the target. The simulation starts with the satellites starting a set position around Mississippi State University on February 14, 2023. The CubeSats will start at an altitude of 500 km. This height is based on the swath of some of our optical payloads. With this model the research wants to determine the frequency of access as well as which parts of the river get the most coverage. To model a river STK's line target was utilized. A line target is a set of points that define a location to be focused on. Each satellite is equipped with a sensor that represents the view of its payload.

A. Constellation

For this mission to take advantage of the CubeSat platform and to be able to observe the target area in the desired capacity, a 3 CubeSat trailing formation in STK was designed. The Mississippi River stretches nearly North to South from Louisiana to Minnesota. Because of this, it is ideal to get a near polar orbit with an inclination of 92 degrees. The satellites are brought under the High Precision Orbit Propagator, HPOP. This propagator includes perturbation from atmospheric drag, solar pressure, and earth's gravity.

Table 1. initial orbital elements

Orbital Elements	Satellite 1	Satellite 2	Satellite 3
e	0	0	0
a	6878.1 km	687.1 km	6878.1 km
i	92 deg	92 deg	92 deg
Ω	328 deg	328 deg	328 deg
ω	39 deg	36 deg	

B. Perturbation

To simulate the lifetime more accurately, the authors applied perturbations onto our orbit. To do so we used HPOP propagator. HPOP allows you to account for multiple Perturbation models. Originally the relative motion of the satellite could be assumed to be constant and described in Equ(1) and Equ(4).

$$\ddot{r} = -\frac{Gm_e}{r^2} * \frac{r}{|r|} - \frac{Gm_{sat}}{r^2} * \frac{r}{|r|} \qquad (1)$$

$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\frac{\vec{r}}{|r|} \qquad (2)$$

Were μ is the is the gravitational constant. From this ideal equation, we can introduce the perturbation from the earth's gravity, atmosphere. We also may choose to not include any third body influence from the sun or moon because we are in such a low orbit. We also choose to disable the tidal forces as well. With these assumptions the relative motion can be expressed as a new form of acceleration. This is shown in Equ(3) and (4)

$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\frac{\vec{r}}{r} + \ddot{\vec{r}}_{ad} + \ddot{\vec{r}}_{sp} \qquad (3)$$

$$\ddot{\vec{r}} = -\frac{\mu}{r^2}\frac{\vec{r}}{r} - \frac{1}{2}C_d * \frac{A_d}{M} * \frac{\rho * \vec{r}^2}{2} - k * Cr * \frac{A_r}{M}\frac{L_s}{4\pi * cr^2} \qquad (4)\text{Si}$$

the terms r_{ad} and r_{sp} correspond to the atmospheric and solar acceleration acting perpendicular to the direction velocity. These forces act to retard the spacecraft's initial movements. The Cd corresponds to the coefficient of drag.

In our equation we take Cd to be 2.0 for spherical bodies. Rho (ρ) is representative of the atmospheric density. Luminosity (L_s) represents the luminosity of the sun. This value is given as 3.826e26 Watts. Cr is the coefficient of solar radiation and taken as 1.0. For this model to be valid a few assumptions are made. First it is assumed the ideal gas for density value. For the solar pressure assume that there is no absorption by the body and the solar pressure is fully reflective across the exposed surface. The decay of the orbits can be visualized by STK's built in lifetime graphs. Each graph represents how eccentricity apogee and perigee drop over the months. The system tells that the mission has a lifespan of 2.4 years before these drops are experienced experience these drops.

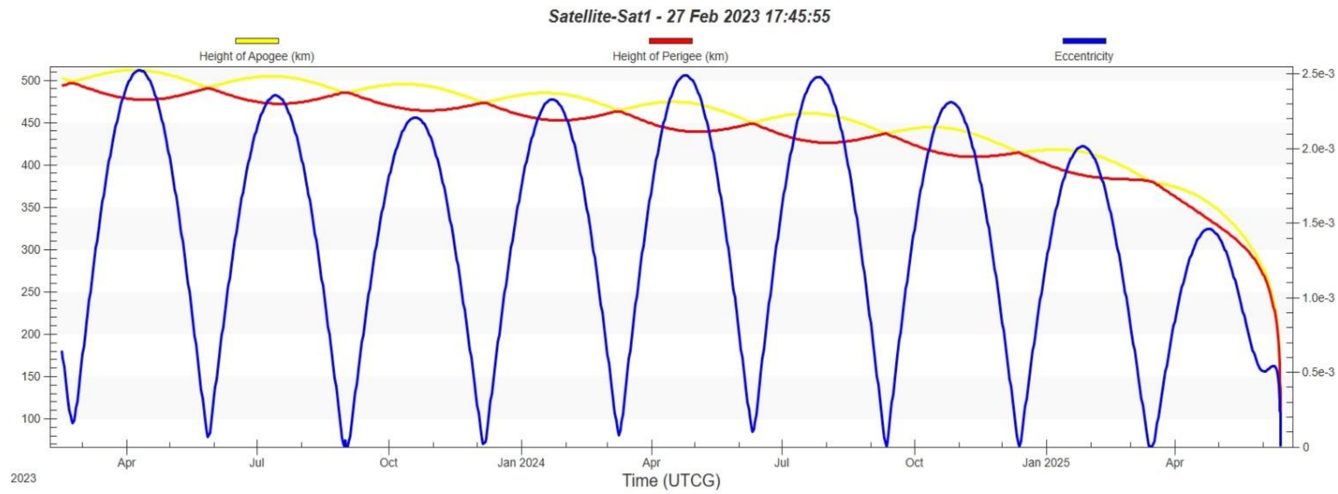

Figure 2. Lifetime of Satellite 1

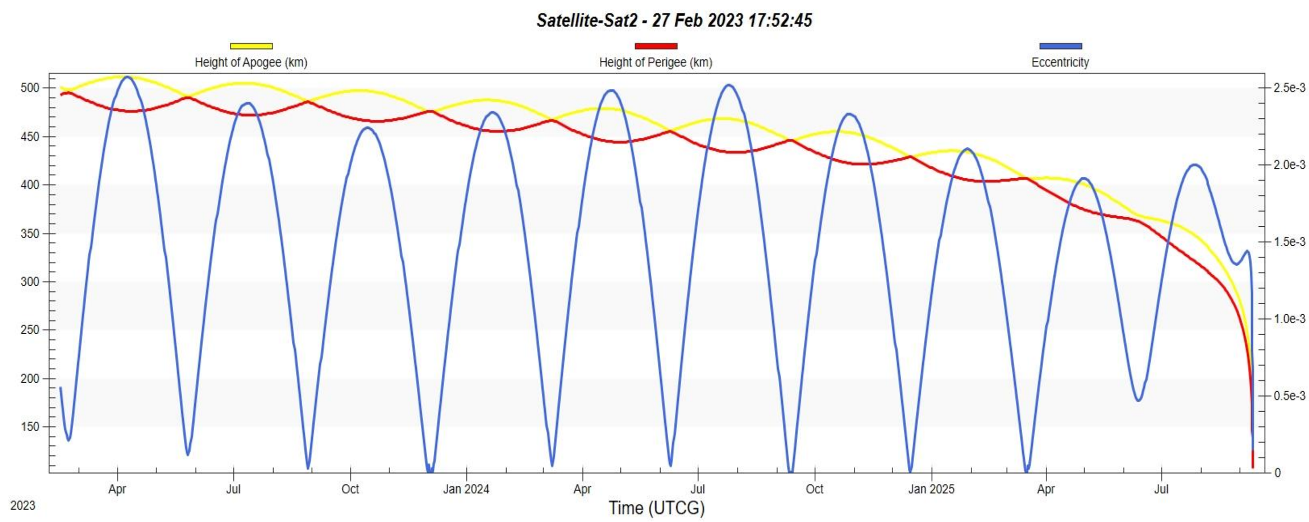

Figure 3. Lifetime of Satellite 2

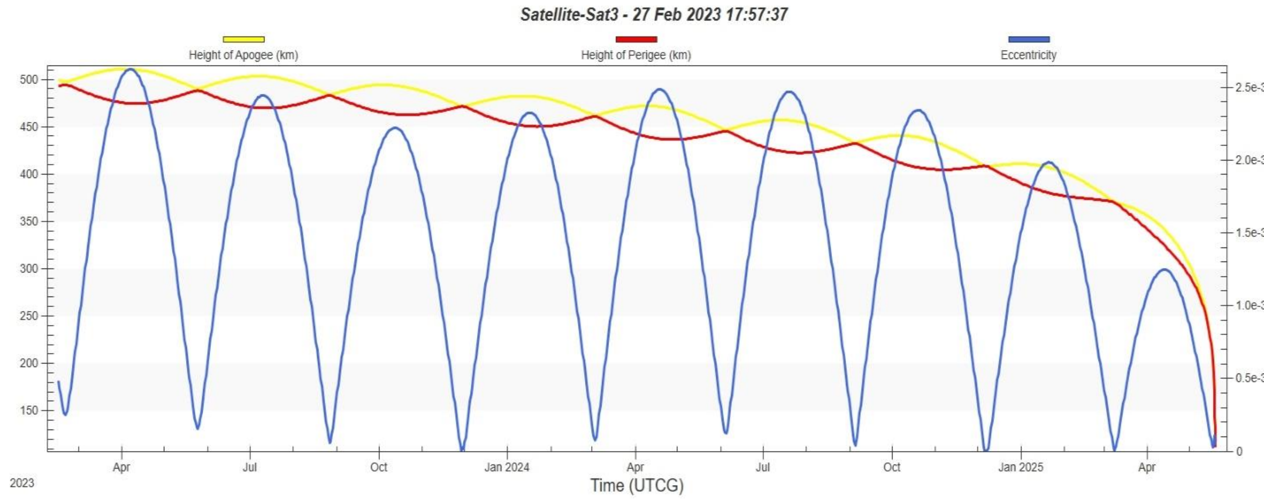

Figure 4. Lifetime of Satellite 3

B. Constraints

When designing the orbit, the satellites, the top level requirement was the ability to take data. After picking sensor that could take the data, the authors need to ensure that t satellites have coverage. Realistically Optical payloads like cameras will not be able to continuously observe a target. For this there are limitations like light and distance. For the scenario, our observations are limited to direct sunlight and penumbra. Another constraint set is the field of view of our sensors. All optical payloads have a basic observation behavior to simplify the orbit. Based on the Swath data from the sensors we set our vison type to conic and set the half angle to 7.125 deg. This should represent an optical sensor that has a swath of 125km at a 500km altitude. The half angle required is represented by equation(5).

$$\alpha = tan^{-1}(\frac{125km}{2*500km}) \qquad (5)$$

C. Access Results

The way that STK evaluates a connection between two bodies is through a built-in access function. Access is taken when constraints are met, and the two bodies have line of sight with each other. Because the orbit of our satellite group travels the length of the river, each instance of access is taken to be one pass along the whole river. The report that is generated through STK shows us just how often in this two-year period we can monitor the river area and for how long. Fig (5) through Fig (7) represents the use of a fixed sensor object on satellites. This did not produce a lot of access time. It only averaged 3.5 hours for the mission duration. Fig (8) is a representation of a facility placed along the river area that was set to target the passing satellites. This produced more access times and for a longer duration. From the data we see that our satellites are observed by the station an average of 335 times with a total of 6.6 hours of observation time. This difference in observation time shows that a targeting system works much better than the fixed sensors. Ultimately more research must be done.

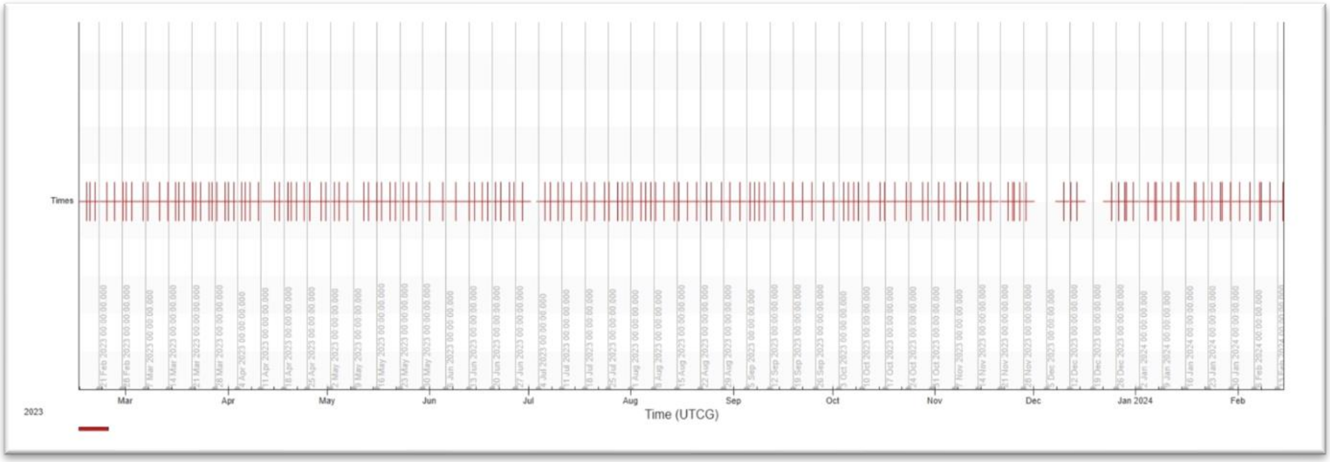

Figure 5. Access times of Satellite 1

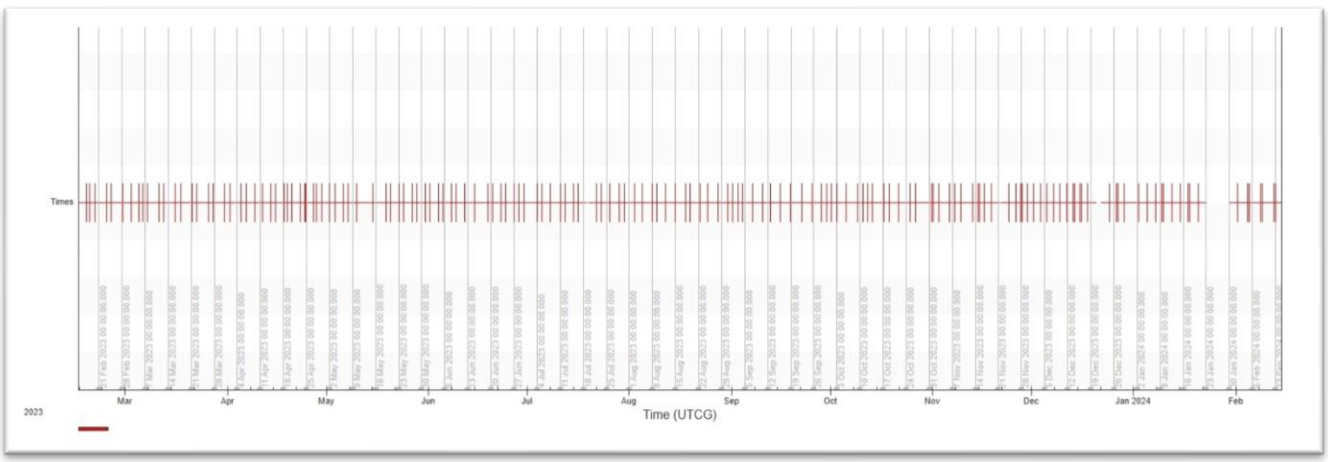

Figure 6. Access times of Satellite 2

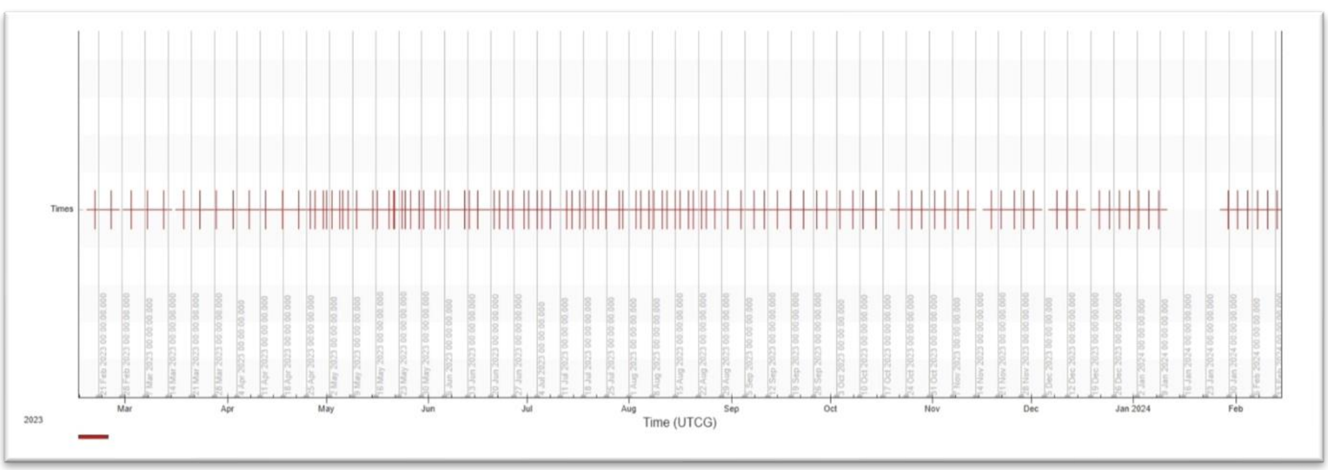

Figure 7. Access times of Satellite 3

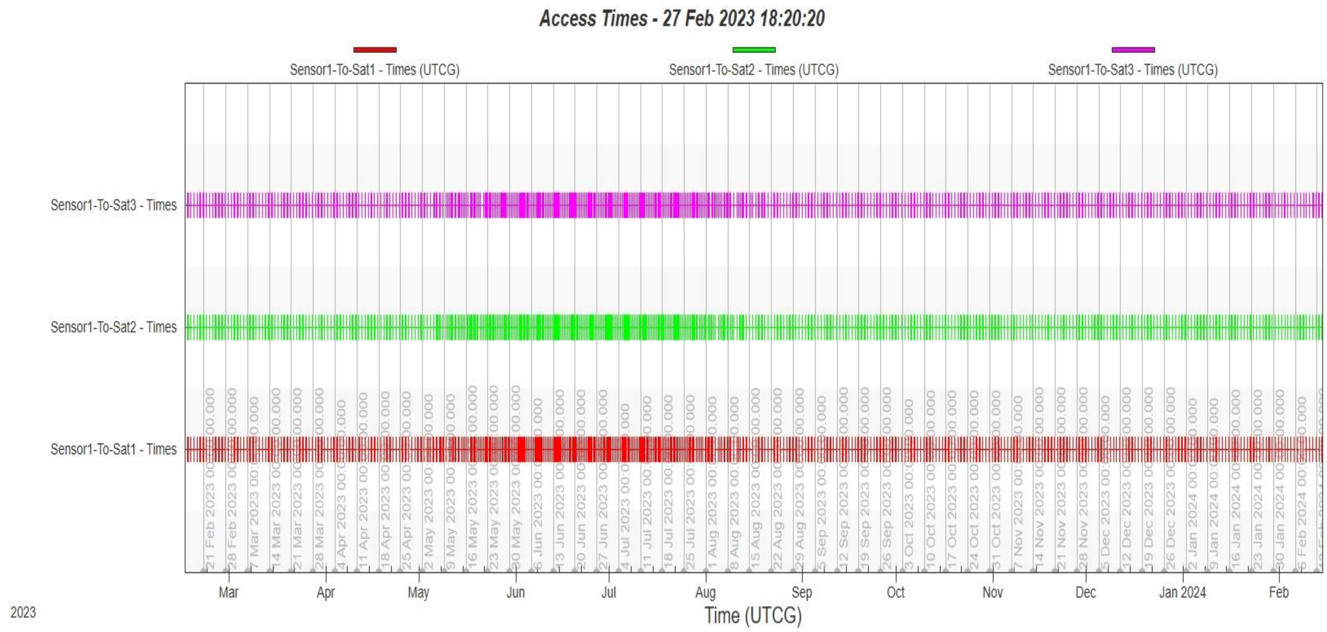

Figure 8. Access from Facility to Satellites

D. Future Improvements

The access that is provided is far from refined. We notice that even though the mission scenario is set for two years, the total access time is low. Parameters that provide a slower orbital velocity, or a system that can focus on a target can remedy this. Further research is needed to improve our results. The next step towards streamlining our analysis is STK's MATLAB scripting capabilities. The scenario will be able to accept our given initial orbital elements and be able to present the decay against time.

VI. Reaction Wheel Assembly

A. Overview

A particular focus of this project is on the attitude control system laid out by a series of four reaction wheels. Reaction wheels are an instrument that utilize the conservation of angular momentum to point the satellite. The wheels work by continuously spinning a fly wheel – one per each motor – wherein each one can be adjusted to spin faster or slower. The resulting change in angular velocity, $d\boldsymbol{\omega}$, which governs a change in the attitude of the spacecraft. The equation to determine resultant change in spacecraft angular velocity is determined in the set of Eq 5-7:

$$L_{Wheel} = -L_{Sat} \tag{5}$$

$$I_{Wheel} * \boldsymbol{\omega}_{Wheel} = I_{Sat} * \boldsymbol{\omega}_{Sat} \tag{6}$$

$$\boldsymbol{\omega}_{Sat} = -\frac{L_{Wheel}}{I_{Sat}} \tag{7}$$

Where $\boldsymbol{\omega}_{Sat}$ is the angular velocity of the entire satellite, L_{Wheel} is the angular momentum of the flywheel, and I_{Sat} is the mass moment of inertia for the entire satellite system, which is the structure and all the attached instrumentation[1]. The mass properties of the satellite are covered in the previous section.

B. Simulink Model

In Simulink, a simulation was created to determine the change in angular velocity due to the transfer of momentum from the flywheel to the satellite proper with the only input value being the voltage input to the 4 motors in the system. The system works with a voltage signal builder block with 4 signals, 1 signal for the voltage being fed into each motor.

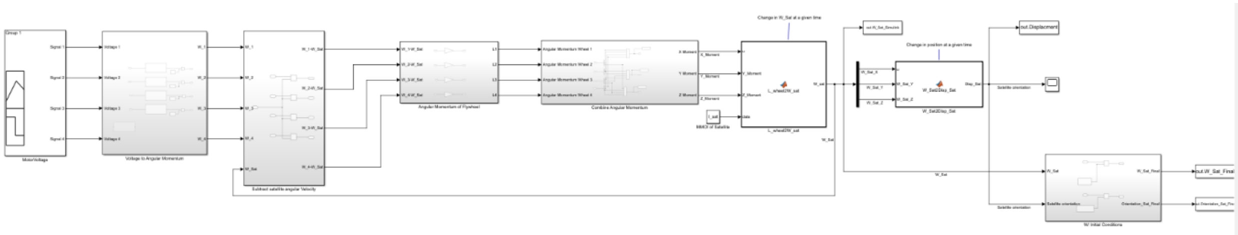

Figure 9. Overview of Simulink model.

The Simulink model expands the principal of the previously set equations, and accounts for a few other factors. The model is set over a period of 10 seconds with a fixed timestep. The model starts with a voltage input to one of four motors, where each voltage is a signal and a motor transforms voltage into an angular speed[2]. After angular speed, denoted $\boldsymbol{\omega}_{Wheel}$ is denoted, it must be combined with the angular speed of the satellite, $\boldsymbol{\omega}_{Sat}$, to achieve an overall speed of the wheel. Once the overall speed is found, it is multiplied by the MMOI of the flywheel, denoted I_{Wheel}, which determines the total angular momentum of the flywheel.

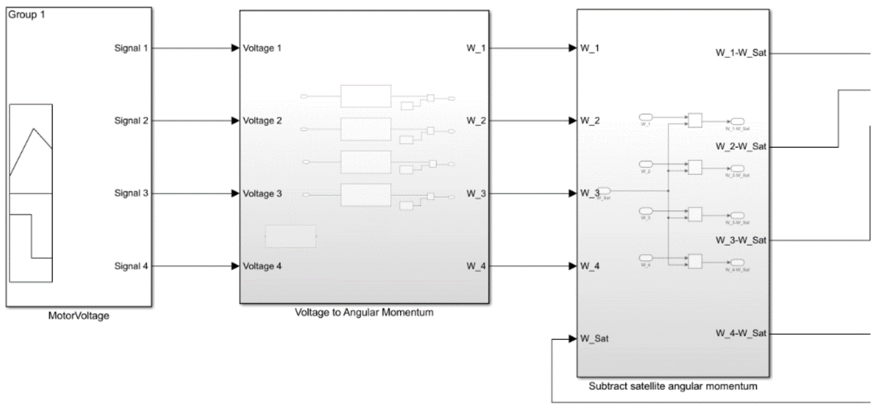

Figure 10. Simulink Model Overview

Once the overall speed is found, it is multiplied by the MMOI of the flywheel, denoted I_{Wheel}, which determines the total angular momentum of the flywheel.

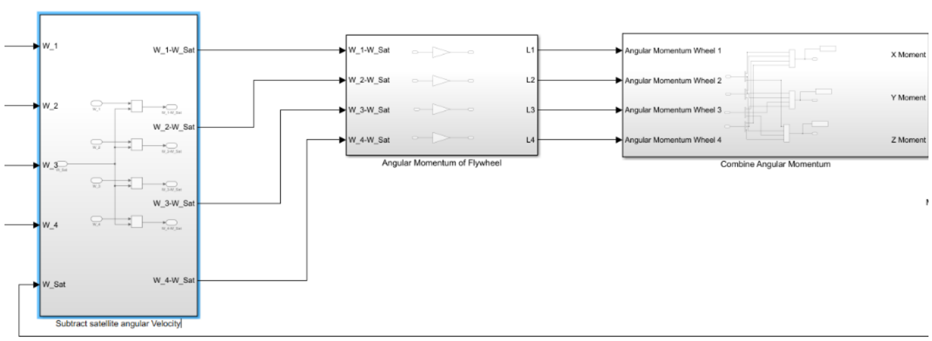

Figure 11. Finding angular momentum of the flywheel.

Once the angular momentum is found, it can be transferred to the angular velocity of the satellite by dividing the negative angular momentum of the flywheel by the MMOI of the satellite as shown in the equation below. Where L_{Wheel} is a 1x3 vector and I_{Sat} is a 3x3 matrix. The output is the angular velocity of the satellite with 3 component vectors about the X, Y, and Z axis respectively.

$$\boldsymbol{\omega}_{Sat} = -\frac{L_{Wheel}}{I_{Sat}} \tag{8}$$

After $\boldsymbol{\omega}_{Sat}$ is found, it is returned to the function that determines the angular displacement, θ_{Sat}, by multiplying $\boldsymbol{\omega}_{Sat}$ by our constant function timestep, Δt.

$$\theta_{Sat} = \boldsymbol{\omega}_{Sat} * \Delta t \tag{9}$$

Where θ_{Sat} is the angular displacement of the satellite and Δt is the change in time over the system. For the x-y-z directions. The process is shown in image 5 where the component vectors of $\boldsymbol{\omega_{Sat}}$ are input to find the angular displacement of the satellite. Using the above listed equation. The simulation uses a timestep of 0.05 seconds for Δt.

The final step of this is to combine the changes found in the simulation with a set of initial satellite conditions that show the real change in the system orientation. The system takes the initial condition and combines them into an output that can be graphically displayed.

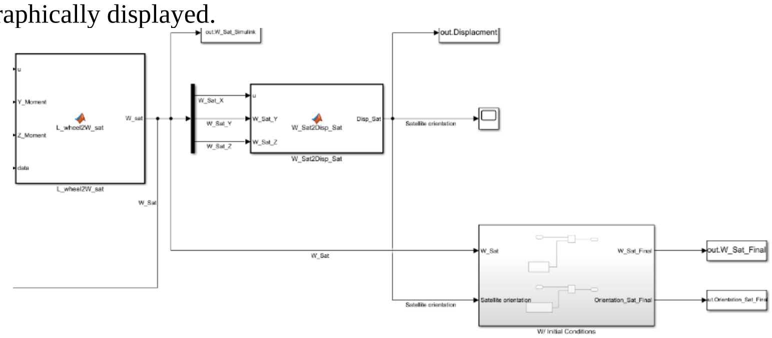

Figure 13. Using angular velocity to find angular displacement.

C. Simulation Results

The results of the simulation yield a solid display of change in velocity and angle. With a final angular displacement and velocity as shown below in Table 1. These results are after a 10 second duration. After a slightly longer duration a much larger offset can be attained.

Table 2. list of final displacement and velocity.

The total angular displacement about the X axis	0.45819097 Degrees
The total angular displacement about the Y axis	-7.96523290 Degrees
The total angular displacement about the Z axis	4.32397288 Degrees
Total change in angular velocity about the X axis	0.11454774 Degrees /sec
Total change in angular velocity about the Y axis	-1.99130822 Degrees/sec
Total change in angular velocity about the Z axis	1.08099322 Degrees/sec

In figure 14 respectively, they give a graphical representation of the resultant change in displacement and angular velocity of the satellite.

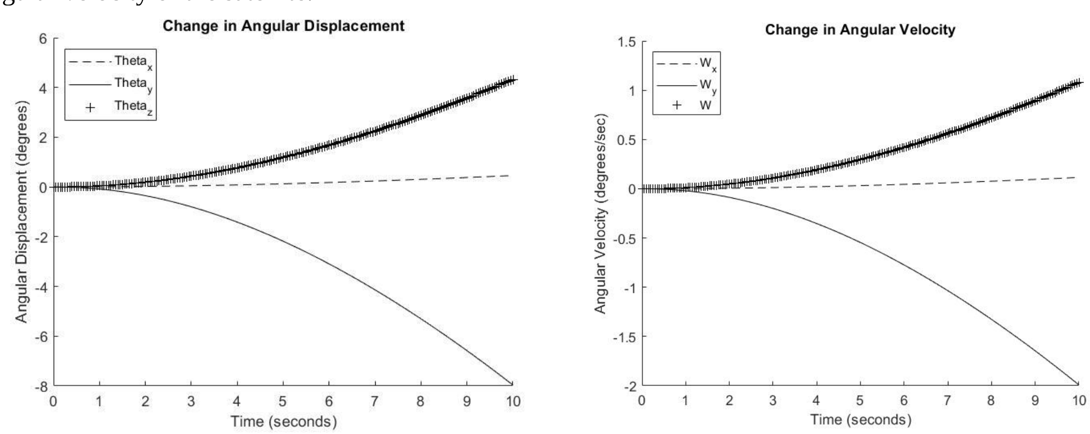

Figure 14. Change in angular displacement and change in angular velocity.

D. Future Improvements

In terms of future improvements, more spacecraft dynamics features would be ideal to add into the simulation for a capability review and as a useful learning opportunity, including a full kinematic model of the satellite. In the future, processes that add in the orbital elements to simulate the attitude over the cycle of an orbit would be a very useful improvement to the program. In addition to this, playing around with the parameters of certain elements like motor properties, mass properties, etc. could yield interesting changes and make the system more accurate. In the end, it would be best to be able to tune the input voltage to change the outcome in the loop. The team's advisor has recommended a change to use quaternion kinematics to determine propagation. At the moment, the simulation shows that the simulation can be changed, and shows a capability of the ability to control, but not much more than that. The team will implement systems to measure the satellite's ability to point relative to the earth. There is much improvement to be done.

VII. Conclusion

This CubeSat mission simulation can be a great consideration for a mission designed around monitoring the Mississippi river's watershed region. 3 key parts of the mission were modeled: physical design, orbital simulation and attitude determination. The designers continue to improve and add on to this project throughout the year. The next step in our project is to really improve our overall mission design. For the RWA we want to verify the robustness by testing against different mission scenarios and seeing if we can control the movement of the Satellites. It is also desired to test the design with the different orbital elements for our simulated mission and verify if we can control our CubeSats pointing capabilities. For the CubeSat model we would like to research more into the electronic controls and power management to further validate our choice of components. The information given from the scenario can also be improved to better represent the actual access of the desired region. We will research ways that a link budget between our satellites and a ground station can be modeled.

Acknowledgments

We would like to acknowledge the advice and guidance of Dr. Yang Cheng. The Authors would also like to thank their mothers.

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