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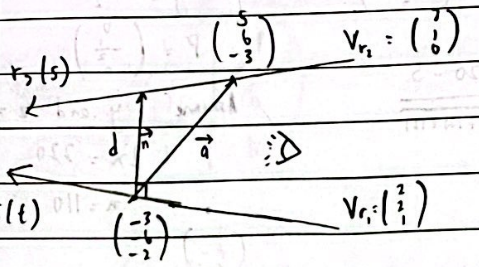
1a) To find the vector perpendicular to both lines, we need to find cross product of two line's direction vector.

$$\begin{pmatrix}2\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\2\\-2\end{pmatrix}$$

b)



Shortest distance between lines $= \dfrac{\vec{n}\cdot\vec{a}}{|\vec{n}|}$

$= \dfrac{-8+24+2}{\sqrt{1+4+4}}$

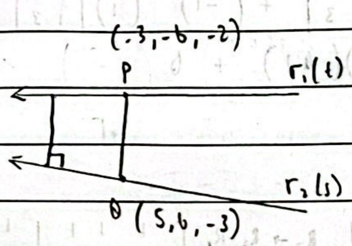
$= 6$ units

$\vec{a} = \begin{pmatrix}5\\6\\-3\end{pmatrix} - \begin{pmatrix}-3\\-6\\-2\end{pmatrix}$

$= \begin{pmatrix}8\\12\\-1\end{pmatrix}$

$\vec{n} = \begin{pmatrix}-1\\2\\-2\end{pmatrix}$ (same as 1a)

c)



Find $\vec{PQ} = \begin{pmatrix}5+2s\\6+s\\-3\end{pmatrix} - \begin{pmatrix}-3+2t\\-6+2t\\-2+t\end{pmatrix}$

$= \begin{pmatrix}2s-2t+8\\s-2t+12\\-1-t\end{pmatrix}$

$\vec{PQ}$ · direction vector of $r_1(t) = 0$

$\vec{PQ}$ · direction vector of $r_2(s) = 0$

$\begin{pmatrix}2s-2t+8\\s-2t+12\\-1-t\end{pmatrix}\cdot\begin{pmatrix}2\\1\\0\end{pmatrix} = 0$

$4s-4t+16+s-2t+12=0$

$5s-6t+28=0$ — ①

$\begin{pmatrix}2s-2t+8\\s-2t+12\\-1-t\end{pmatrix}\cdot\begin{pmatrix}2\\2\\1\end{pmatrix} = 0$

$39+6s-9t=0$

$13+2s-3t=0$ — ②

$5s-26-4s+28=0$ $13+2(-2)=3t$

$s=-2$ $t=3$

2) Normal vector of plane 1 : $2i-3j$

Normal vector of plane 2 : $3i+j+k$

$$\begin{pmatrix}2\\-3\\0\end{pmatrix} \times \begin{pmatrix}3\\1\\1\end{pmatrix} = \begin{pmatrix}-3\\-2\\11\end{pmatrix}$$

To find point on line of intersection, set $x = 0$

P1 : $-3y = 1$ P2 : $y+z=6$

P1 : $-18y = 6$ $z = \frac{19}{3}$

$-18y = 6$

$y = -\frac{1}{3}$

Point on line of intersection $= \begin{pmatrix}0\\-\frac{1}{3}\\\frac{19}{3}\end{pmatrix}$

$$r = \begin{pmatrix}0\\-\frac{1}{3}\\\frac{19}{3}\end{pmatrix} + t\begin{pmatrix}-3\\-2\\11\end{pmatrix}$$

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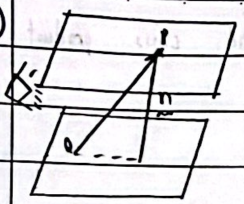
3) 

Two planes have normal $\vec{n} = \begin{pmatrix} 2 \\ 10 \\ -11 \end{pmatrix}$

choose P, Q from P1, P2

Assume x and $z = 0$

$10y = -5$

$y = -\frac{1}{2}$

$P = \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 0 \end{pmatrix}$

Assume y and $z = 0$

$2x = 220$

$x = 110$ $\quad Q = \begin{pmatrix} 110 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{QP} = \begin{pmatrix} -110 \\ -\frac{1}{2} \\ 0 \end{pmatrix}$

$$V_n = \frac{\overrightarrow{QP} \cdot \vec{n}}{|\vec{n}|}$$

$$= \frac{-220 - 5}{\sqrt{4+100+121}}$$

$= 15$ units

4a) fix row $i = 1$

column $j = 1, 2, 3$

$$\det(M) = \underbrace{(-1)^{1+1}(1)\begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix}}_{i=1,\ j=1} + \underbrace{(-1)^{1+2}(1)\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}}_{i=1,\ j=2} + \underbrace{(-1)^{1+3}(1)\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}}_{i=1,\ j=3}$$

$$= 2 + (-1)(1) + 0$$

$$= 1$$

4b)
$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\therefore M^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

c)
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$$

5a) fix row $i = 1$

column $j = 1, 2, 3$

$$\det(M) = (-1)^{1+1}(3)\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} + (-1)^{1+2}(1)\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + (-1)^{1+3}(1)\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix}$$

$$= (3)(-1) + (-1)(-1) + 2$$

$$= 0$$

5b) $\left(\begin{array}{ccc|c} 3 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & 1 & b \end{array}\right)$

Convert the augmented matrix to echelon form:

$$\left(\begin{array}{ccc|c} 3 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & 1 & b \end{array}\right) \xrightarrow{R_2 \to 3R_2 - R_1} \left(\begin{array}{ccc|c} 3 & 1 & 1 & 1 \\ 0 & -4 & 2 & -1 \\ 2 & 0 & 1 & b \end{array}\right) \xrightarrow{R_3 \to 3R_3 - 2R_1} \left(\begin{array}{ccc|c} 3 & 1 & 1 & 1 \\ 0 & -4 & 2 & -1 \\ 0 & -2 & 1 & 3b-2 \end{array}\right)$$

$$\xrightarrow{R_3 \to 2R_3 - R_2} \left(\begin{array}{ccc|c} 3 & 1 & 1 & 1 \\ 0 & -4 & 2 & -1 \\ 0 & 0 & 0 & 6b-3 \end{array}\right)$$

In order to have infinite solutions, $6b - 3$ must equals to 0.

$6b - 3 = 0$

$b = \frac{1}{2}$ ∗

c) Back substitution:

let $z = \alpha$

$R_2 \to -4y + 2\alpha = -1$

$y = \frac{2\alpha + 1}{4}$

$R_1 \to 3x + y + \alpha = 1$

$3x + \frac{2\alpha + 1}{4} + \alpha = 1$

$x = \frac{1 - 2\alpha}{4}$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$