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1) Let $P(n)$ be the statement $4 + 9 + 14 + 19 + \ldots + (5n-1) = \frac{n(5n+3)}{2}$ for each integer $n \geq 1$.

Base Step: when $n = 1$, LHS, $5(1) - 1 = 4$ and indeed LHS = RHS.

RHS, $\frac{1(5(1)+3)}{2} = 4$

$\Rightarrow P(1) \equiv T$

Inductive Step: Assume $P(n)$ is true when $n = k$, we have to prove $P(k+1)$ is true.

when $n = k+1$, $4 + 9 + 14 + 19 + \ldots + (5k-1) + (5k+4)$

$= \frac{k(5k+3)}{2} + 5k + 4$

$= \frac{k(5k+3)}{2} + \frac{2(5k+4)}{2}$

$= \frac{5k^2 + 13k + 8}{2}$

$= \frac{(k+1)(5k+8)}{2}$

Thus $P(k+1)$ is true.

Conclusion: $\therefore P(1) \equiv T$

$\therefore P(k) \rightarrow P(k+1)$

So we have proved by weak induction that $4 + 9 + 14 + 19 + \ldots + (5n-1) = \frac{n(5n+3)}{2}$

2) Let $P(n)$ be the statement 2^n divides a_n for all integers $n \geq 1$.

Base Step: when $n = 1$, $a_1 = 4$ (can be divided by 2) $\Rightarrow P(1) \equiv T$

when $n = 2$, $a_2 = 12$ (can be divided by 4) $\Rightarrow P(2) \equiv T$

when $n = 3$, $a_3 = 8$ (can be divided by 8) $\Rightarrow P(3) \equiv T$

Inductive Step: Assume that $P(n)$ is true for all $n \leq k$, we have to prove $P(k+1)$ is true.

when $n = k+1$, $a_{k+1} = 10a_k - 24a_{k-2}$.

Assumption states that $P(k)$ and $P(k-2)$ are true, so $a_k = (y)(2^k)$ and $a_{k-2} = (x)(2^{k-2})$ for some integer x and y.

$a_{k+1} = 10a_k - 24a_{k-2}$

$= 10(y2^k) - 24(x2^{k-2})$

$= 5 \times 2(y2^k) - 8 \times 3(x2^{k-2})$

$= 5y(2^k)(2) - 3x(2^{k-2})(2^3)$

$= (5y)(2^{k+1}) - (3x)(2^{k+1})$

$= 2^{k+1}(5y - 3x)$

Thus $P(k+1)$ is true.

Conclusion:

$\therefore P(1) \equiv T$

$\therefore P(1) \wedge P(2) \equiv T$

$\therefore P(1) \wedge P(2) \wedge P(3) \equiv T$

$\therefore P(1) \wedge P(2) \wedge P(3) \wedge \ldots \wedge P(k)$

$\rightarrow P(k+1)$

So we have proved by strong induction that $P(n)$ is true for all integers $n \geq 1$.

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3i) 4

ii) $\{-3, -2, 3\}$

iii) $\{\{\}, \{1,2\}\}$

iv) $\{(-2,1), (-2,2), (-3,1), (-3,2)\}$

v) 48

4) "$X \subseteq Y$ or $Y \subseteq X$"