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1(i) Let A be the ~~probability~~ event that the third barrel was chosen. Let B be the event that a ball is taken from the chosen barrel uniformly at random and the ball is red.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{6} + \frac{1}{6} + \frac{2}{5}}$$

$$= \frac{2}{11}$$

ii) $\frac{2}{11} \times \frac{2}{11} \times \frac{2}{11} = \frac{8}{1331}$

For three independent repeated trials, each from a sample space S, the overall sample space is $S \times S \times S$ and the probability function $\Pr((s_1, s_2, s_3))$ equals to $\Pr(s_1)\Pr(s_2)\Pr(s_3)$.

2i) 1

ii) $\frac{1}{4} \times (-1-1)^2 + \frac{1}{8} \times (-1)^2 + \frac{5}{8} \times (2-1)^2 = \frac{7}{4}$

iii) $-\frac{7}{3}$

iv)

z	-4	-1	0	2
$\Pr(Z=z)$	$\frac{5}{16}$	$\frac{1}{12}$	$\frac{13}{48}$	$\frac{1}{3}$

$$\Pr(XY=-4) = \Pr(X=2 \wedge Y=-2)$$
$$= \left(\frac{5}{8}\right)\left(\frac{1}{2}\right) = \frac{5}{16}$$

$$\Pr(XY=-1) = \Pr(X=-1 \wedge Y=1)$$
$$= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{12}$$

$$\Pr(XY=0) = \Pr(X=-1 \wedge Y=0) + \Pr(X=0 \wedge Y=-2) + \Pr(X=0 \wedge Y=0)$$
$$+ \Pr(X=0 \wedge Y=1) + \Pr(X=2 \wedge Y=0)$$
$$= \left(\frac{1}{4}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{6}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{3}\right) + \left(\frac{5}{8}\right)\left(\frac{1}{6}\right)$$
$$= \frac{13}{48}$$

$$\Pr(XY=2) = \Pr(X=-1 \wedge Y=-2) + \Pr(X=2 \wedge Y=1)$$
$$= \left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{8}\right)\left(\frac{1}{3}\right)$$
$$= \frac{1}{3}$$

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3i) $\Pr(X=0) = \frac{1}{6}$ $\Pr(Y=0) = \frac{1}{6}$ $\Pr(X=0 \cap Y=0) = \frac{1}{36}$

$\therefore \Pr(X=0 \cap Y=0) = \Pr(X=0)\Pr(Y=0)$, so the event "$X=0$" and "$Y=0$" are independent.

ii) X can take values in $\{-5, -4, -3, -2, -1, 0\}$ (each with probability $\frac{1}{6}$)

Probability distribution of Y is

y	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\Pr(Y=y)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$\Pr(X=-4 \wedge Y=4) = 0$

$\Pr(X=-4)\Pr(Y=4) = \frac{1}{108}$

So $\Pr(X=-4 \wedge Y=4) \neq \Pr(X=-4)\Pr(Y=4)$ and X and Y are not independent.

iii) $\Pr(X \leqslant -2 \mid Y \geqslant 1) = \frac{\Pr(X \leqslant -2 \wedge Y \geqslant 1)}{\Pr(Y \geqslant 1)} \ldots$

$$= \frac{\frac{6}{36}}{\frac{15}{36}} = \frac{2}{5}$$ #

Event $(X \leqslant -2 \wedge Y \geqslant 1)$ can take values in $\{(-4,5), (-3,4), (-3,5), (-2,3), (-2,4), (-2,5)\}$

Probability distribution of Y is

y	1	2	3	4	5
$\Pr(Y=y)$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$