$$e^{x} = 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + \dots + \frac{x^{n}}{n!} + x^{n} \varepsilon(x) = \sum_{k=0}^{n} \frac{x^{k}}{k!} + x^{n} \varepsilon(x)$$

$$\sin(x) = x - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + x^{2n+2} \varepsilon(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + x^{2n+2} \varepsilon(x)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \dots + (-1)^n \frac{x^{2n}}{2n!} + x^{2n+1} \varepsilon(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + x^{2n+1} \varepsilon(x)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + x^n \varepsilon(x) = \sum_{k=0}^{n} x^k + x^n \varepsilon(x)$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \frac{a(a-1)(a-2)}{3!}x^3 + \dots + \frac{a(a-1)\cdots(a-n+1)}{n!}x^n + x^n\varepsilon(x)$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + x^2\varepsilon(x)$$

$$\tan(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + x^6\varepsilon(x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1} \frac{x^n}{n} + x^n \varepsilon(x) = \sum_{k=1}^n (-1)^{k+1} \frac{x^k}{k} + x^n \varepsilon(x)$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + x^{2n+2} \varepsilon(x) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{2k+1} + x^{2n+2} \varepsilon(x)$$

$$\lim_{x \to +\infty} \frac{\ln(x)}{x} =$$

Autrement dit : 
$$\ln(x) = \underset{x \to \infty}{o}(x)$$

$$\forall (\alpha, \beta) \in (\mathbb{R}^{+*})^2, \lim_{x \to +\infty} \frac{(\ln(x))^{\alpha}}{x^{\beta}} =$$

Autrement dit : 
$$(\ln(x))^{\alpha} = o_{x \to \infty}(x^{\beta})$$

$$\forall (\alpha, \beta) \in (\mathbb{R}^{+*})^2, \quad \lim_{x \to 0^+} x^{\beta} \times |\ln(x)|^{\alpha} =$$

Autrement dit : 
$$|\ln(x)|^{\alpha} = o_{x\to 0^+} \left(\frac{1}{x^{\beta}}\right)$$

$$\forall a \in ]1, +\infty[, \ \forall \alpha \in \mathbb{R}, \ \lim_{x \to +\infty} \frac{a^x}{x^{\alpha}} =$$

Autrement dit : 
$$x^{\alpha} = \underset{x \to \infty}{o} (a^x)$$

$$\forall a \in ]1, +\infty[, \ \forall \alpha \in \mathbb{R}, \ \lim_{x \to -\infty} a^x |x|^{\alpha} =$$

Autrement dit : 
$$a^x = o_{x \to -\infty} \left(\frac{1}{|x|^{\alpha}}\right)$$

$$\sin(x) \underset{x \to 0}{\sim} x$$

$$\tan(x) \underset{x \to 0}{\sim} x$$

$$1 - \cos(x) \sim \frac{x^2}{2}$$

$$e^x - 1 \underset{x \to 0}{\sim} x$$

$$(1+x)^a - 1 \sim_{x\to 0} ax \text{ si } a \neq 0.$$

Pour 
$$n = \frac{1}{2}$$
, on obtient donc :  $\sqrt{1+x} - 1 \underset{x\to 0}{\sim} \frac{1}{2}x$ 

$$\ln(1+x) \underset{x\to 0}{\sim} x$$

ce qui s'écrit aussi : 
$$\ln(u) \underset{u \to 1}{\sim} u - 1$$