Study Notes

An Introduction of Probability Theory and Its Applications

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1 The Sample Space

1.1 The Empirical Background

Events Events are the results of experiments or obervations. The events should be distinguished between *compound(decomposable)* and *simple(indecomposable) events*. A compound event is an aggregate of certain simple events.

Sample points Sample points are just the simple events. Every indecomposable result of the (idealized) experiment is represented by one, and only one, sample point.

Sample Space Sample space is the aggregate of all sample points.

1.2 Examples

Notes

1. All intuitive background sampling problems are abstractly equivalent to the scheme of **placing r balls into n cells**, in the sense that the outcomes differ only in their verbal description.
   (There are some examples listed on P10.)

2. Here comes a question that how to calculate the size of sampling space (how many sampling points in the sampling space) when placing r balls into n cells? This question should be answered under different assumption.

   - Placing r **distinguishable** balls into n **distinguishable** cells.

$$N = n^r$$

- Placing r **indistinguishable** balls into n **distinguishable** cells.

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- Placing r **distinguishable** balls into n **indistinguishable** cells.

?

- Placing r **indistinguishable** balls into n **indistinguishable** cells.

?

When we facing some particular problems, the model of distinguishable or indistinguishable balls is purely a matter of purpose and convenience.

1.3 The Sample Space and Events

Sample space is the universe of the sample points. All sample points are indecomposable and cover all the outcomes of an idealized experiment. The event is an aggregate of some sample points (one or the aggregate of all sample points is also an event).

New events can be define in terms of two or more given events. Here comes the notation of the formal *algebra of events* (algebra of point sets).

1.4 Relations among Events

We use Ω to denote sample space and capitals to denote events (sets of sample points). The fact that a sample point x is contained in the event A is denoted by $x \in A$. Thus $x \in \Omega$ for every sample point x. We write $A = B$ only if the two events consist of exactly the same points.

Definations

1. We shall use the notation $A = 0$ to express that the event A contains no sample points. The zero must be interpreted in **a symbolic sense and not as the numeral**.

2. The event consisting of all points not contained in the event A will be called the *complementary event* (or *negation*) and will be denoted by A'. In particular $\Omega' = 0$.

3. With any two events A and B we can associate two new events defined by the conditions "*both A and B occur*" and "*either A or B or both occur*".These events will be denoted by AB and $A \cup B$.