Q. Calculate the Eigen value and Eigen vector for the following matrix.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Ans:

$$[A - \lambda I]x = 0$$

$$[A - \lambda I] = \begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = 0$$

$$-\lambda(-3-\lambda) - (-2) = 0$$

$$3\lambda + \lambda^2 + 2 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda+2)(\lambda+1) = 0$$

$$\lambda = -2, -1$$

For $\lambda = -2$

$$[A - \lambda I] = [A - (-2) I]$$

$$= [A + 2I]$$

$$\therefore \quad [A+2I]\, x = 0$$

$$\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$2x_1 + x_2 = 0 \quad \longrightarrow (1)$$

$$x_2 = -2x_1$$

$$-2x_1 - x_2 = 0 \quad \longrightarrow (2)$$

$$\Rightarrow \quad 2x_1 + x_2 = 0$$

Eq^n (1) & (2) are dependent

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

For $\lambda = -1$

$$[A - \lambda I] = [A - (-1)I]$$

$$= [A + I]$$

$$\therefore \quad [A + I]X = 0$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$x_1 + x_2 = 0 \longrightarrow (1)$$

$$\Rightarrow \quad x_1 = -x_2$$

$$-2x_1 - 2x_2 = 0 \longrightarrow (2)$$

Eq^n (1) & (2) are dependent

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Q2) Find all eigenvalues and corresponding eigenvectors for the matrix A.

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Ans:

$$(A-\lambda I)x = 0$$

$$A-\lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$|A-\lambda I| = 0$$

$$(2-\lambda)\left[(-5-\lambda)(3-\lambda)\right] - {}^{-}3(2)(3-\lambda) = 0$$

$$(3-\lambda)\left[(2-\lambda)(-5-\lambda)+6\right] = 0$$

$$(3-\lambda)\left[-10-2\lambda+5\lambda+\lambda^2+6\right] = 0$$

$$(3-\lambda)\left[\lambda^2+3\lambda-4\right] = 0$$

$$(3-\lambda)(\lambda-1)(\lambda+4) = 0$$

$$\lambda = 1, 3, -4$$

For $\lambda=1$

$$[A-\lambda I]x = 0$$

$$[A-I]x = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 - 3x_2 = 0 \Rightarrow x_1 = 3x_2$$

$$2x_1 - 6x_2 = 0 \Rightarrow x_1 = 3x_2$$

$$2x_3 = 0 \Rightarrow x_3 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda=3$

$$[A-\lambda I]x = 0$$
$$[A-3I]x = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-x_1 - 3x_2 = 0 \rightarrow ①$$
$$2x_1 - 8x_2 = 0 \rightarrow ②$$
$$x_1 = 0$$
$$x_2 = 0$$

x_3 can be any value.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$-x_1 - 3x_2 = 0$$
$$2x_1 - 8x_2 = 0$$

Elimination

$$-2x_1 - 6x_2 = 0$$
$$-2x_1 + 8x_2 = 0$$
$$2x_2 = 0$$
$$x_2 = 0$$

Substitute $x_2=0$ in eq^n ①

$$-x_1 = 0$$
$$\Rightarrow x_1 = 0$$

For $\lambda=4$

$$[A-\lambda I]x = 0$$
$$[A+4I]x = 0$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left.\begin{aligned} 6x_1 - 3x_2 &= 0 \Rightarrow x_2 = 2x_1 \\ 2x_1 - x_2 &= 0 \end{aligned}\right\} \text{Eq}^n \text{ ① \& ② are dependent}$$

$$7x_3 = 0 \Rightarrow x_3 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

Q3) Calculate the inverse of the matrix $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

$$|A| = 2(12+10) - 4(28-5) + {}^{-}6(-14-3)$$

$$= 2 \times 22 - 4 \times 23 - 6 \times {}^{-}17$$

$$= 44 - 92 + 102$$

$$= \underline{\underline{54}}$$

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix} = (12+10) = 22$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 7 & 5 \\ 1 & 4 \end{vmatrix} = -(28-5) = {}^{-}23$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 7 & 3 \\ 1 & -2 \end{vmatrix} = (-14-3) = {}^{-}17$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 4 & -6 \\ -2 & 4 \end{vmatrix} = -(16-12) = {}^{-}4$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -6 \\ 1 & 4 \end{vmatrix} = 8+6 = 14$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix} = {}^{-}(-4-4) = 8$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 4 & -6 \\ 3 & 5 \end{vmatrix} = 20+18 = 38$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -6 \\ 7 & 5 \end{vmatrix} = -(10+42) = -52$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 4 \\ 7 & 3 \end{vmatrix} = 6-28 = -22$$

$$\text{Adj}A = \begin{bmatrix} 22 & -4 & 38 \\ -23 & 14 & -52 \\ -17 & 8 & -22 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}A$$

$$= \frac{1}{54} \begin{bmatrix} 22 & -4 & 38 \\ -23 & 14 & -52 \\ -17 & 8 & -22 \end{bmatrix}$$

$$= \begin{bmatrix} 11/27 & -2/27 & 19/27 \\ -23/54 & 7/27 & -26/27 \\ -17/54 & 4/27 & -11/27 \end{bmatrix}$$

Q4) Calculate the inverse of matrix $\begin{bmatrix} 4 & 3 & 8 \\ 6 & 2 & 5 \\ 1 & 5 & 9 \end{bmatrix}$

$$|A| = 4(18-25) - 3(54-5) + 8(30-2)$$

$$= 4 \times -7 - 3 \times 49 + 8 \times 28$$

$$= -28 - 147 + 224$$

$$= 49$$

$$C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 5 \\ 5 & 9 \end{vmatrix} = (18-25) = -7$$

$$C_{12} = (-1)^{1+2} \begin{vmatrix} 6 & 5 \\ 1 & 9 \end{vmatrix} = -(54-5) = -49$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 6 & 2 \\ 1 & 5 \end{vmatrix} = 28$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 5 & 9 \end{vmatrix} = -(27-40) = 13$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 4 & 8 \\ 1 & 9 \end{vmatrix} = 36-8 = 28$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 4 & 3 \\ 1 & 5 \end{vmatrix} = -(20-3) = -17$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 8 \\ 2 & 5 \end{vmatrix} = (15-16) = -1$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 4 & 8 \\ 6 & 5 \end{vmatrix} = -(20-48) = 28$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = 8-18 = -10$$

$$\text{Adj } A = \begin{bmatrix} -7 & 13 & -1 \\ -49 & 28 & 28 \\ 28 & -17 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$= \frac{1}{49} \begin{bmatrix} -7 & 13 & -1 \\ -49 & 28 & 28 \\ 28 & -17 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} -1/7 & 13/49 & -1/49 \\ -1 & 4/7 & 4/7 \\ 4/7 & -17/49 & -10/49 \end{bmatrix}$$