

Group 2 ME5413 Homework 3: Planning

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1 Pre-Processing of the Map

The map given has some paths whose width is only 0.2m, which are also inaccessible areas for us, so we pre-process the map to make these narrow paths whose widths are less than 0.3m impassible.

2 Task 1

For task 1, we mainly use the A* algorithm to plan paths for different start and end points in the map. The planned paths shown in the map, total traveled distance, the number of visited nodes, and total run time with different start and end points are stored in tables.

2.1 A* Algorithm

A* algorithm is a graph traversal and path search algorithm, which selects the path that minimizes

$$f(n) = g(n) + h(n) \quad (1)$$

where n is the next node on the path, $g(n)$ is the cost of the path from the start node to n, and $h(n)$ is a heuristic function that estimates the cost of the cheapest path from n to the goal.

In this task, we apply three different heuristic functions (Manhattan distance, Euclidean distance, and Chebyshev distance) and get different path-planning results.

2.1.1 Heuristic Function: Manhattan Distance

Manhattan distance, d_M , between two vectors $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ in an $n - dimensional$ vector space is defined as

$$d_M(\mathbf{p}, \mathbf{q}) := \sum_{i=1}^n |p_i - q_i| \quad (2)$$

The indexes of figures showing planned paths in the map with different start and end points are stored in [Table 1](#), total traveled distances of different start and end points are shown in [Table 2](#), numbers of visited nodes with different start and end points are shown in [Table 3](#), and total run times of different start and end points are shown in [Table 4](#).

Table 1: Figures of Planned Paths in Original and Gray Scaled Floor Plane

From To \	start	snacks	store	movie	food
start	---	Figure 8	Figure 9	Figure 16	Figure 17
snacks	Figure 1	---	Figure 10	Figure 15	Figure 18
store	Figure 2	Figure 7	---	Figure 14	Figure 19
movie	Figure 3	Figure 6	Figure 11	---	Figure 20
food	Figure 4	Figure 5	Figure 12	Figure 13	---

Table 2: Total Travelled Distance (meter)

From To \	start	snacks	store	movie	food
start	0.0	151.13	163.28	182.58	232.15
snacks	142.85	0.0	118.06	106.91	133.13
store	161.60	122.44	0.0	218.48	131.06
movie	207.44	139.46	248.82	0.0	117.34
food	230.97	133.78	114.40	117.25	0.0

Table 3: Number of Visited Nodes

From To \	start	snacks	store	movie	food
start	0.0	8805	4983	2334	15852
snacks	5796	0.0	597	1018	24618
store	8306	541	0.0	1686	1555
movie	4164	3022	4081	0.0	1110
food	12548	9825	709	870	0.0

Table 4: Total Run Time (second)

From To \	start	snacks	store	movie	food
start	\diagup	0.4	0.4	0.1	1.0
snacks	0.3	\diagup	0.1	0.0	0.9
store	0.5	0.0	\diagup	0.1	0.1
movie	0.3	0.2	0.4	\diagup	0.0
food	0.9	0.4	0.0	0.1	\diagup

2.1.2 Heuristic Function: Euclidean Distance

Euclidean distance, d_E , between two vectors $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ in an $n - dimensional$ vector space is defined as

$$d_E(\mathbf{p}, \mathbf{q}) := \sum_{i=1}^n (p_i - q_i)^2 \quad (3)$$

The indexes of figures showing planned paths in the map with different start and end points are stored in [Table 5](#), total traveled distances of different start and end points are shown in [Table 6](#), numbers of visited nodes with different start and end points are shown in [Table 7](#), and total run times of different start and end points are shown in [Table 8](#).

Table 5: Planned Paths in Original and gray scaled floor plane

From To \	start	snacks	store	movie	food
start	\diagup	Figure 28	Figure 29	Figure 36	Figure 37
snacks	Figure 21	\diagup	Figure 30	Figure 35	Figure 38
store	Figure 22	Figure 27	\diagup	Figure 34	Figure 39
movie	Figure 23	Figure 26	Figure 31	\diagup	Figure 40
food	Figure 24	Figure 25	Figure 32	Figure 33	\diagup

Table 6: Total Travelled Distance

From To \	start	snacks	store	movie	food
start	0.0	155.78	167.66	183.67	238.73
snacks	144.15	0.0	117.53	108.87	139.81
store	162.40	121.62	0.0	237.25	127.55
movie	181.65	126.17	253.83	0.0	120.02
food	238.01	132.80	114.97	205.88	0.0

Table 7: Number of Visited Nodes

From To \	start	snacks	store	movie	food
start	0.0	7177	975	2030	7713
snacks	5608	0.0	569	738	23574
store	723	523	0.0	1004	819
movie	2142	1664	2189	0.0	749
food	6276	6609	534	7467	0.0

Table 8: Total Run Time

From To \	start	snacks	store	movie	food
start	0.0	0.3	0.1	0.3	0.9
snacks	0.4	0.0	0.0	0.2	1.3
store	0.0	0.0	0.0	0.1	0.4
movie	0.1	0.2	0.3	0.0	0.1
food	0.7	0.5	0.1	0.6	0.0

2.1.3 Heuristic Function: Chebyshev Distance

Chebyshev distance, d_C , between two vectors $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\mathbf{q} = (q_1, q_2, \dots, q_n)$ in an n -dimensional vector space is defined as

$$d_C(\mathbf{p}, \mathbf{q}) := \max_i(|p_i - q_i|) \quad (4)$$

The indexes of figures showing planned paths in the map with different start and end points are stored in [Table 9](#), total traveled distances of different start and end points are shown in [Table 10](#), numbers of visited nodes with different start and end points are shown in [Table 11](#), and total run times of different start and end points are shown in [Table 12](#).

Table 9: Planned Paths in Original and gray scaled floor plane

From To \	start	snacks	store	movie	food
start	\	Figure 48	Figure 49	Figure 56	Figure 57
snacks	Figure 41	\	Figure 50	Figure 55	Figure 58
store	Figure 42	Figure 47	\	Figure 54	Figure 59
movie	Figure 43	Figure 46	Figure 51	\	Figure 60
food	Figure 44	Figure 45	Figure 52	Figure 53	\

Table 10: Total Travelled Distance

From To \	start	snacks	store	movie	food
start	0.0	143.83	158.09	178.20	230.49
snacks	146.07	0.0	125.67	109.37	170.30
store	168.98	114.17	0.0	223.54	112.71
movie	185.64	109.72	226.17	0.0	124.98
food	235.97	168.12	112.80	121.38	0.0

Table 11: Number of Visited Nodes

From To \	start	snacks	store	movie	food
start	0.0	4484	702	881	6411
snacks	5702	0.0	20653	747	26052
store	6790	589	0.0	1335	517
movie	16055	517	1753	0.0	1231
food	7120	9351	532	12909	0.0

Table 12: Total Run Time

From To \	start	snacks	store	movie	food
start	0.0	0.2	0.0	0.1	0.3
snacks	0.3	0.0	1.4	0.0	1.2
store	0.3	0.0	0.0	0.1	0.0
movie	0.7	0.0	0.2	0.0	0.1
food	0.8	0.5	0.0	0.6	0.0

2.2 Summary

Because a constraint has been given, the width required for a person is at least 0.3m, before using A* algorithm to plan the paths, we pre-process the map to ensure that the planned paths do not pass through paths with a width of less than 0.3m.

We apply three different heuristic functions to the A* algorithm, and we found that different heuristic functions can give us different planned paths the shortest distances are different for different starting and ending points, and the shortest distances between each pair of locations are shown in [Table 13](#).

Table 13: Summary of the Shortest Distances (meter) Between Each Pair of Locations

From To \	start	snacks	store	movie	food
start	0.0	143.83	158.09	178.20	230.49
snacks	142.85	0.0	117.53	106.91	133.13
store	161.60	114.17	0.0	218.48	112.71
movie	181.65	109.72	226.17	0.0	117.34
food	230.97	132.80	112.80	117.25	0.0

3 Task3

In this section we need to complete: from the starting point (the escalator), find the optimal route to visit all four locations, and return to the endpoint (the escalator). This is a Travelling Salesman Problem (TSP), which can be described as a traveling businessman who has to visit n cities and has to choose the path he wants to take, with the restriction that he can only visit each city once and has to return to the city from which he started, requiring the return of the shortest path.

3.1 Establish the math model

In order to solve this TSP problem, the following aspects were figured out.

1. Objective: The task requires minimizing the total distance through all locations.
2. Variables: Distance between locations and different traversal schemes.
3. Binding conditions: Each location can only be visited once and must eventually return to the starting point.

After that, to be solved by a computer, the above text needs to be converted into a mathematical formula. d_{ij} means the distance between location i and location j. Introduce x to indicate a one-way passage between two locations.

$$x_{ij} = \begin{cases} 1, & \text{denotes the route containing the route from location i to city j} \\ 0, & \text{denotes that the route from location i to location j is not included} \end{cases} \quad (5)$$

In order to avoid the formation of a sub-tour (meaning the existence of multiple pathways), we must add sub-tour elimination constraints.

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1, \forall S \subseteq V, 1 < |S| < n \quad (6)$$

According to the constrained condition, the other constraints equation can be expressed in the following form. V represents the set of all positions.

$$\min z = \sum_{i=1}^n \sum_{j=1}^n d_{ij} x_{ij} \quad (7)$$

3.2 Algorithms

Based on the previous formula, researchers have proposed different solutions. The algorithms used in this experiment are described below.

3.2.1 Exhaustive Search

The algorithm will traverse all possible paths and select the shortest of them to return as the optimal path. It has a time complexity of $O(n!)$, which makes it difficult to use in practical problems, but in this experiment, it was chosen for comparison with other algorithms because of the limited number of variables (only 24 possible cases).

3.2.2 Ant Colony Optimization

The colony algorithm is based on a scenario where ants are looking for food. One ant has found food, which it then needs to take back to the colony. For this one ant, it obviously does not know where to go. It is possible that this ant will choose a random route. This route is likely to be a long one. However, the ant leaves a mark, or pheromone, along the way. If this ant continues to carry food constantly, or if there are many other ants carrying it together. They will always take the quicker route to and from the food if they are lucky. The better the path the ant chooses, the more round trips it makes in the same amount of time, and the more pheromones it leaves on the path. The ants then always find that there are paths that have more pheromones and these are the better routes. By repeating this process over and over again, they eventually find a definite route, and this route is the optimal one. In other words, by increasing or decreasing the amount of pheromone in the line, the ants are able to guide other ants in their path selection.

The advantage of this algorithm is that it is able to overcome the problem of locally optimal solutions and is well adapted for large-scale, complex optimization problems. At the same time, the ACO algorithm has good scalability and parallelism and can handle the actions of multiple ants in parallel. However, the disadvantage of the ACO algorithm is that it is slow to converge and requires a large number of iterations to obtain the optimal solution.

3.3 Summary

According to the below [Table 14](#), both algorithms chose the same path, and according to the [Figure 61](#) shown in Appendix both algorithms did not pass through a path smaller than 0.3m, so the path is passable. The shortest path is the same as the optimal path given by the ACO, so it can be assumed that the ACO also finds the shortest path, which is 651.38 m. It can also be seen that the exhaustive search is two orders of magnitude more time-consuming than the ACO, so in practice, it is not recommended to use the exhaustive search.

Table 14: Comparison of Exhaustive Search and Ant Colony Optimization

	Exhaustive Search	Ant Colony Optimization
Best path	start → snacks → store → movie → food → start	
Shortest distance	651.38m	651.38m
Run time	40507ms(Avg:1687 for each path)	132.416ms

4 Appendix

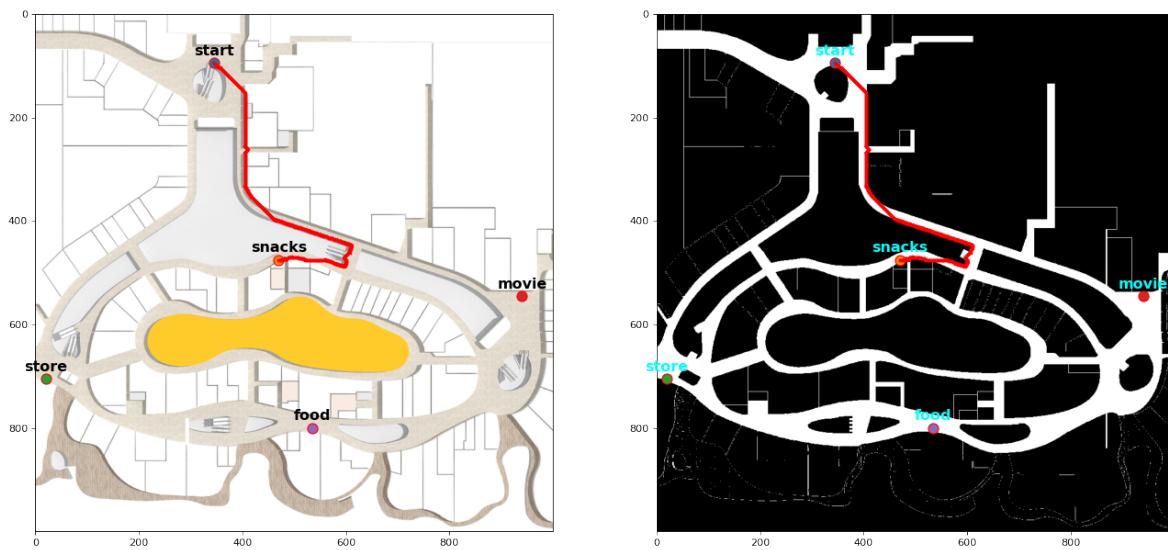


Figure 1: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to snacks.

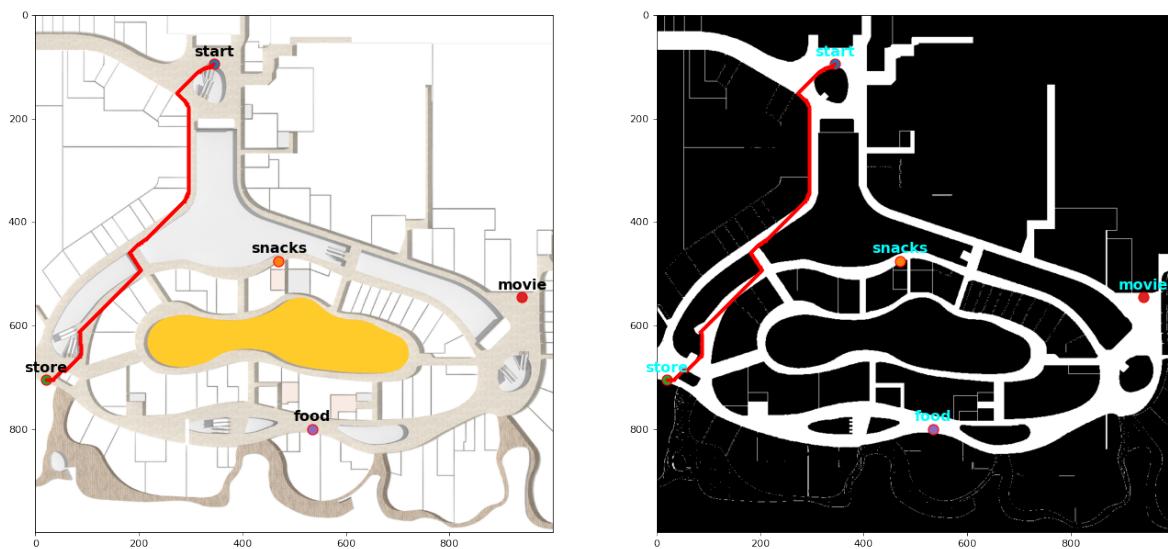


Figure 2: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to store.



Figure 3: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to movie.

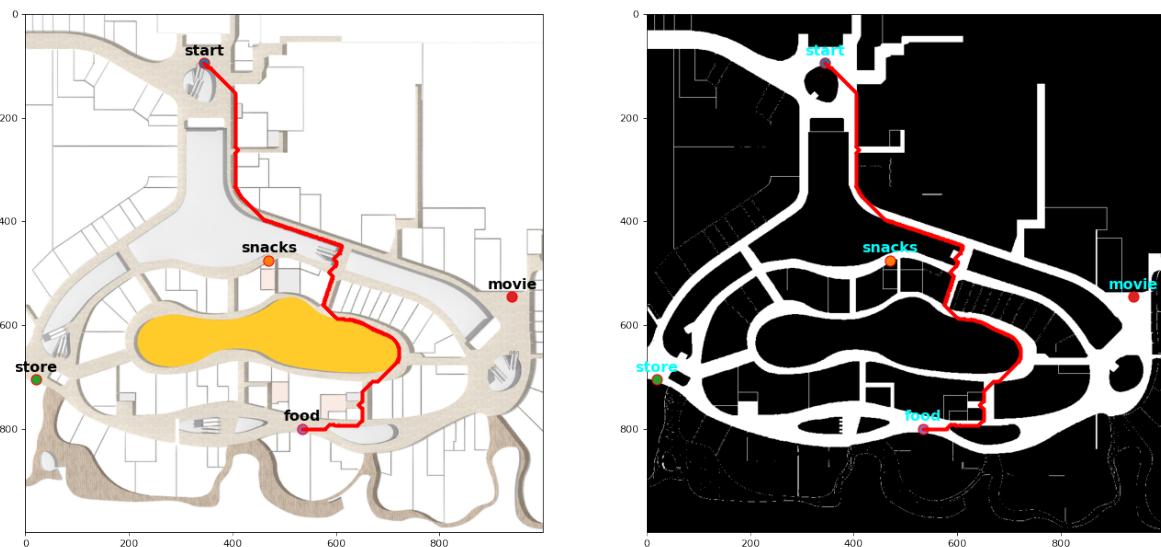


Figure 4: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to food.

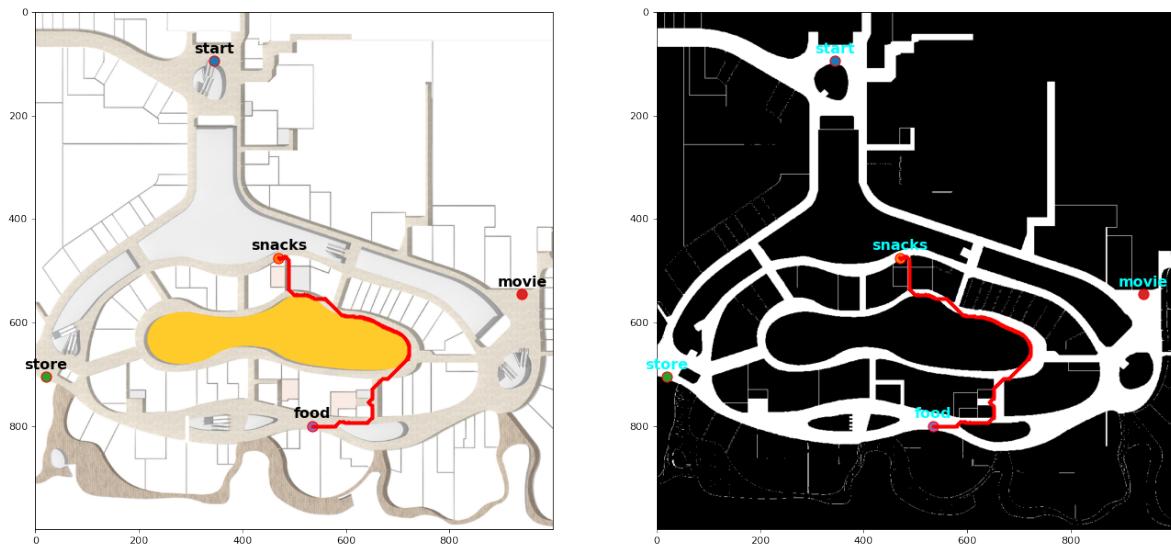


Figure 5: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to food.

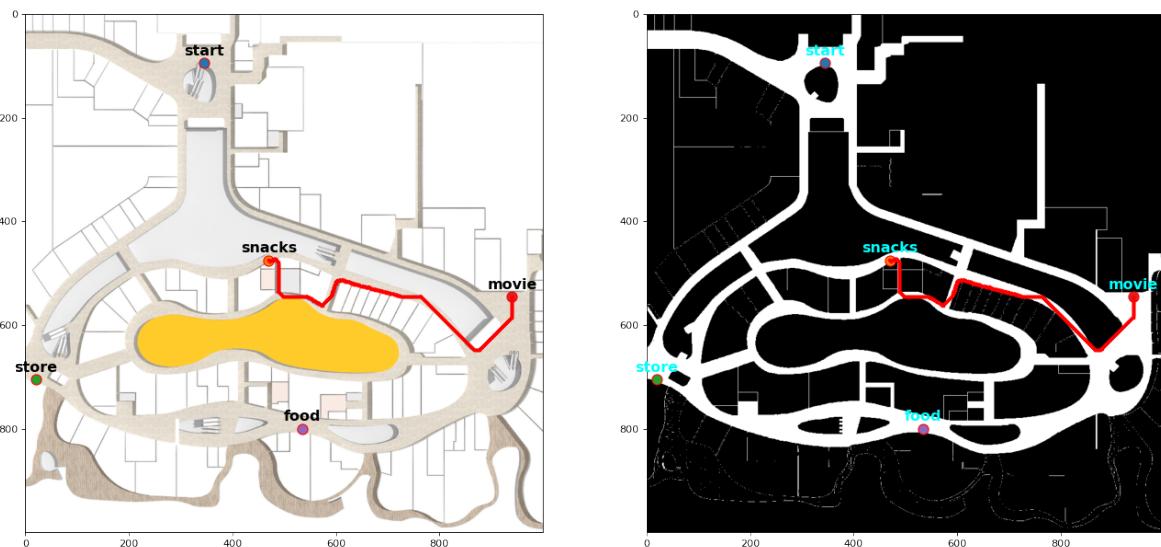


Figure 6: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to movie.

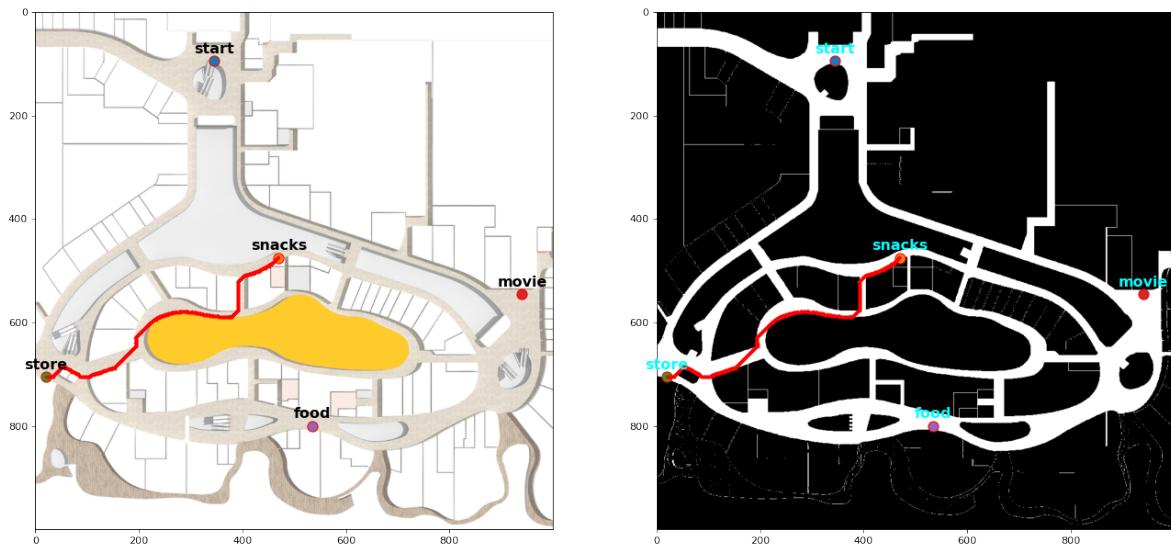


Figure 7: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to store.

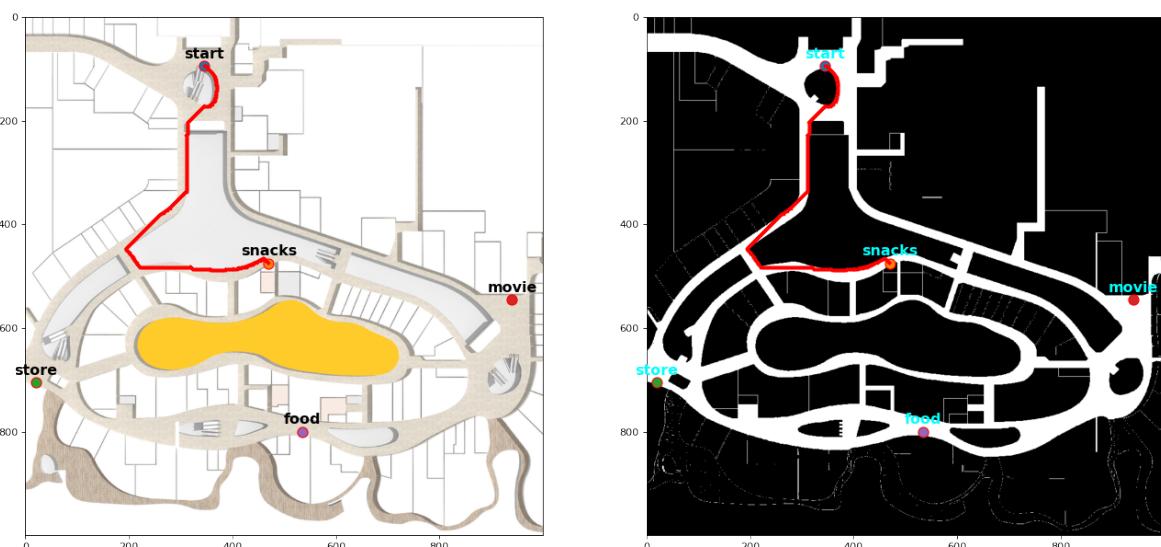


Figure 8: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to start.

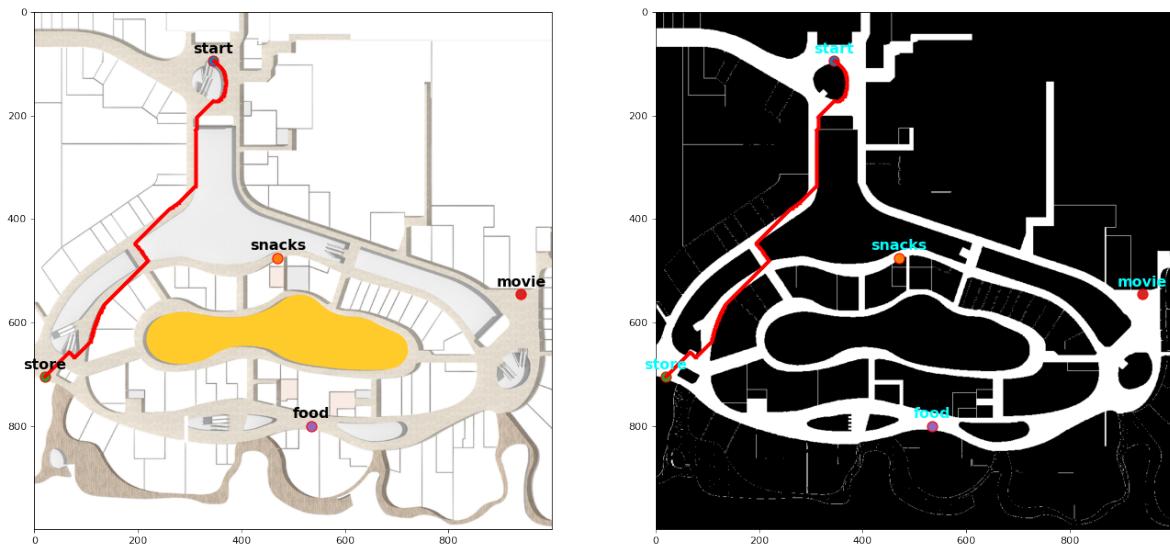


Figure 9: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to start.

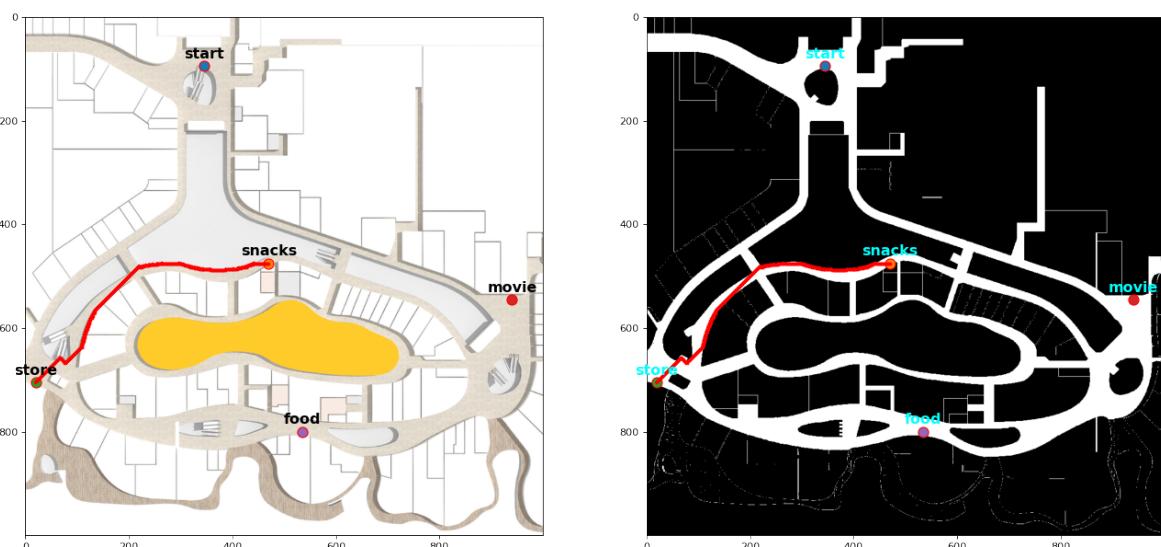


Figure 10: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to snacks.

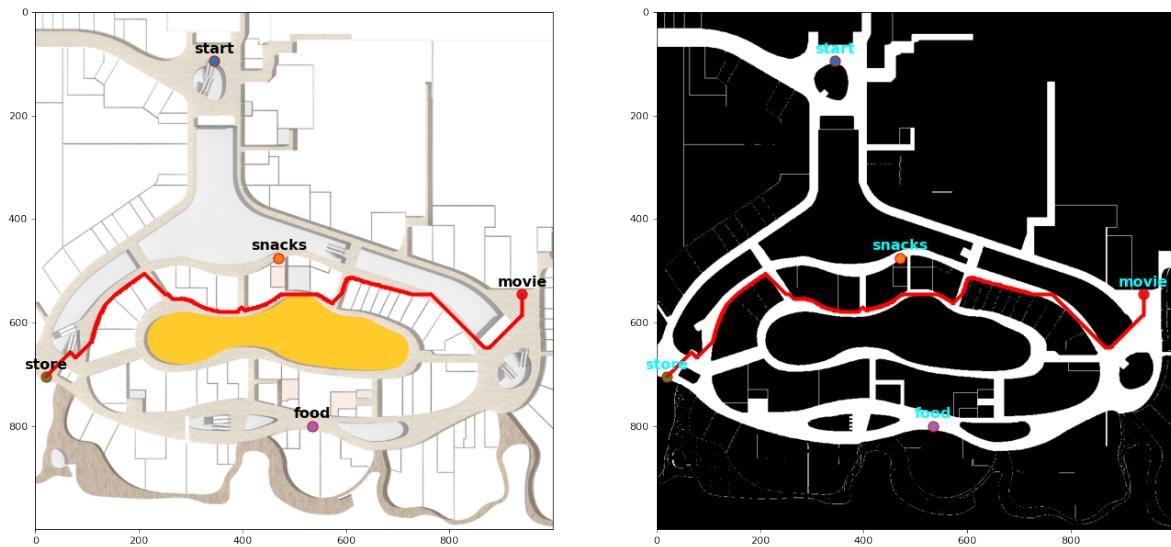


Figure 11: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to movie.

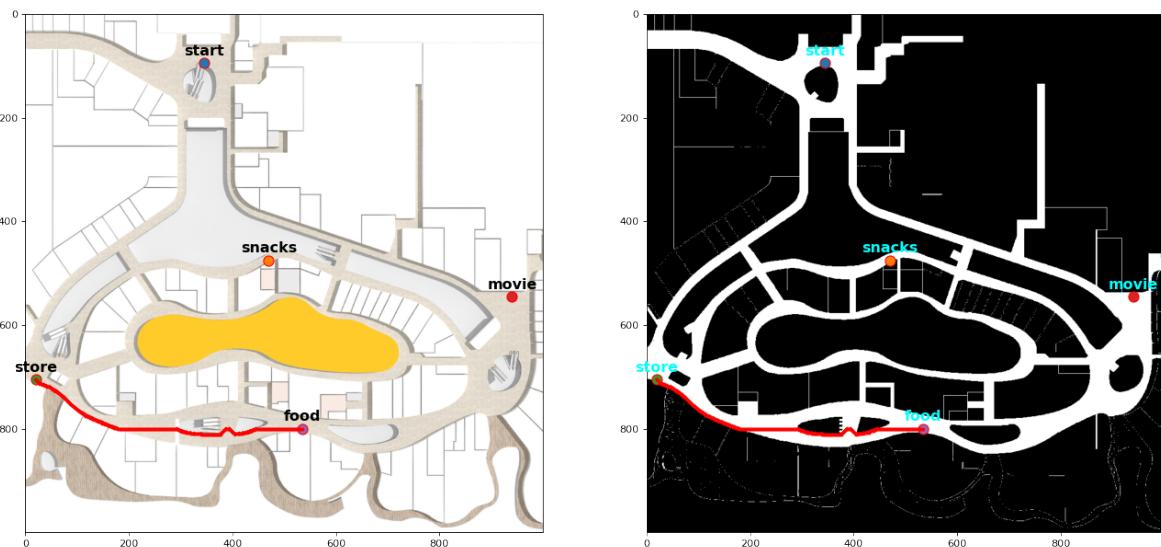


Figure 12: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to food.

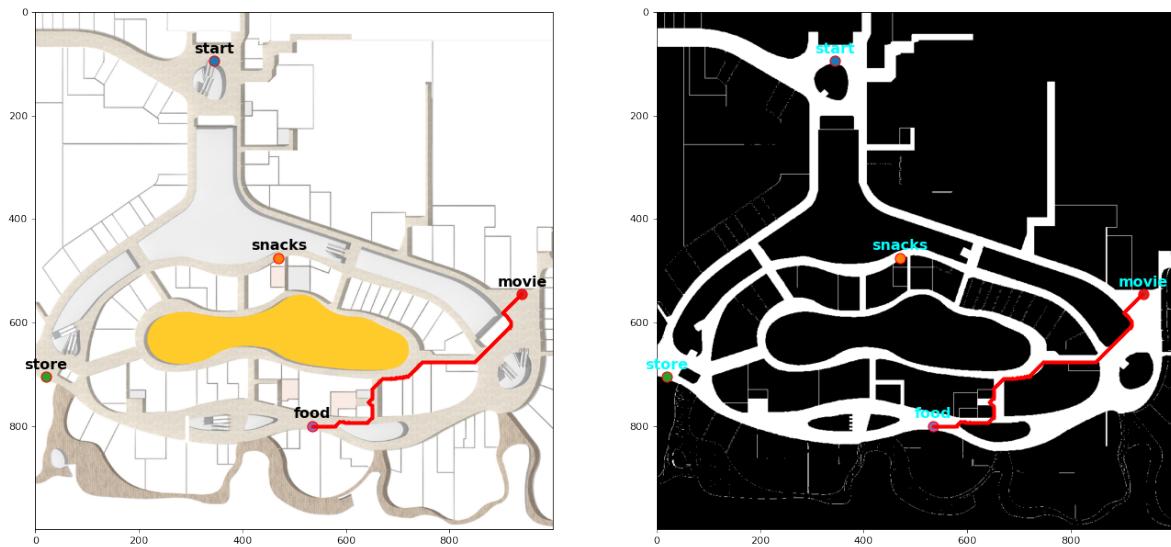


Figure 13: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to food.

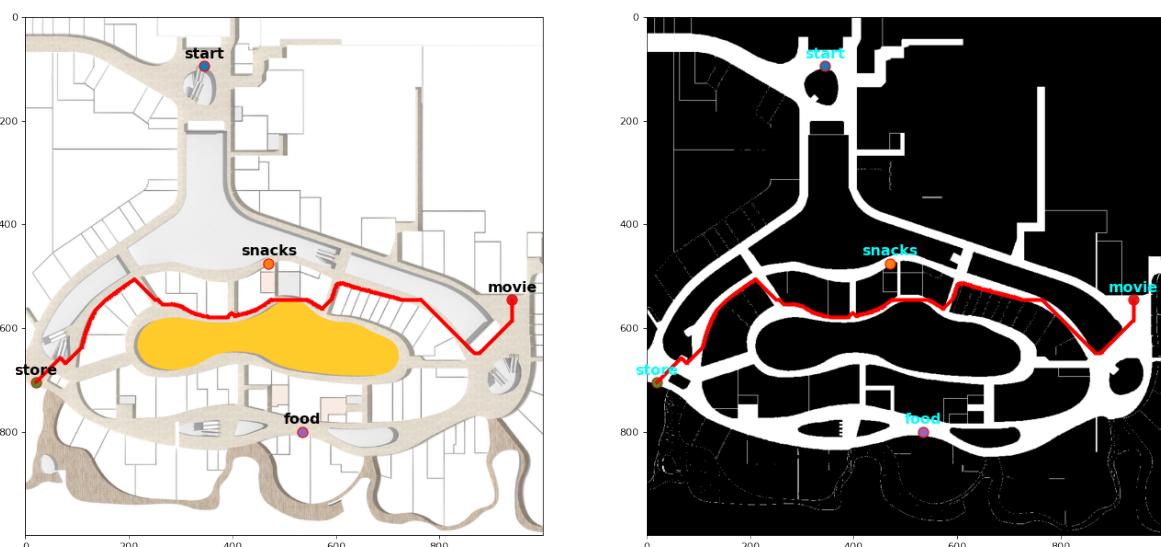


Figure 14: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to store.

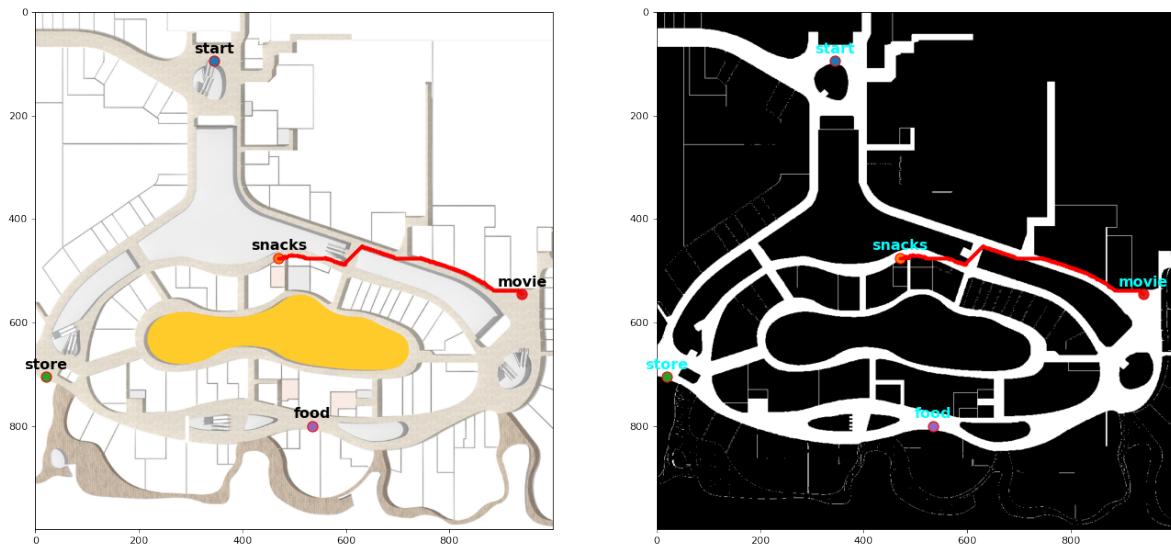


Figure 15: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to snacks.

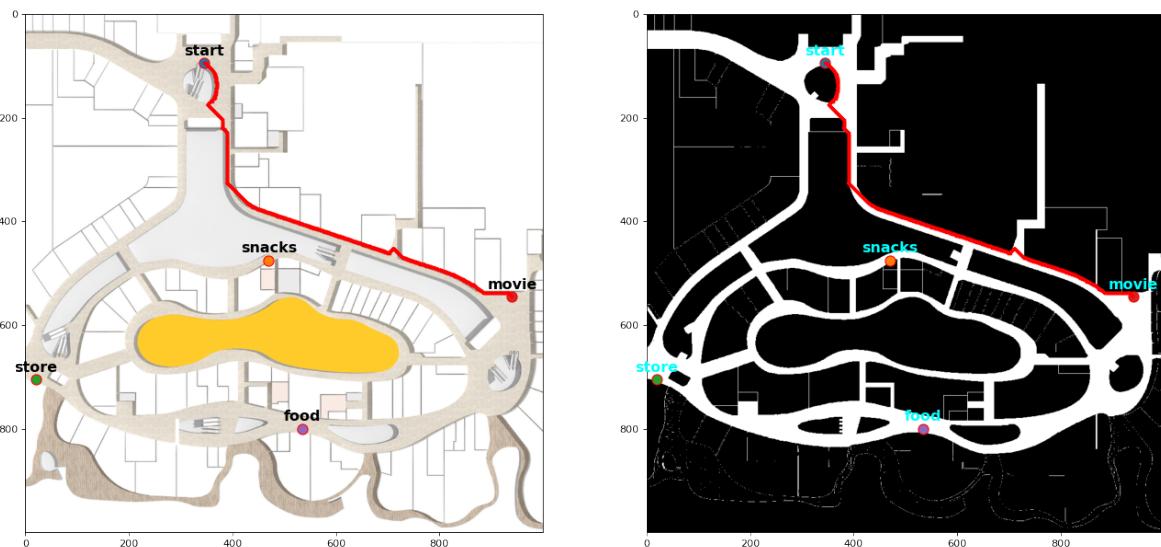


Figure 16: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to start.

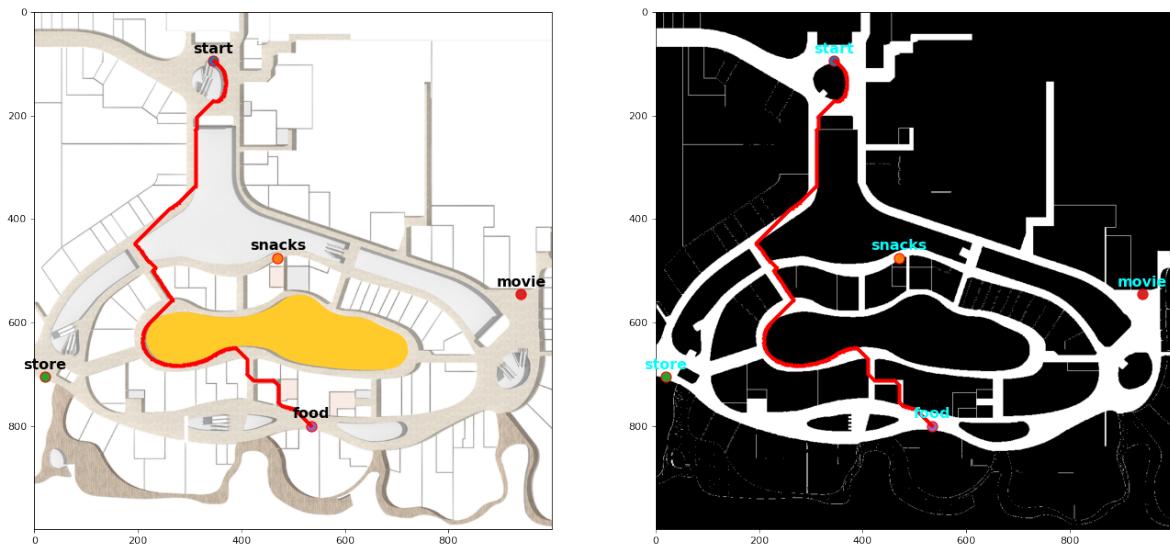


Figure 17: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to start.

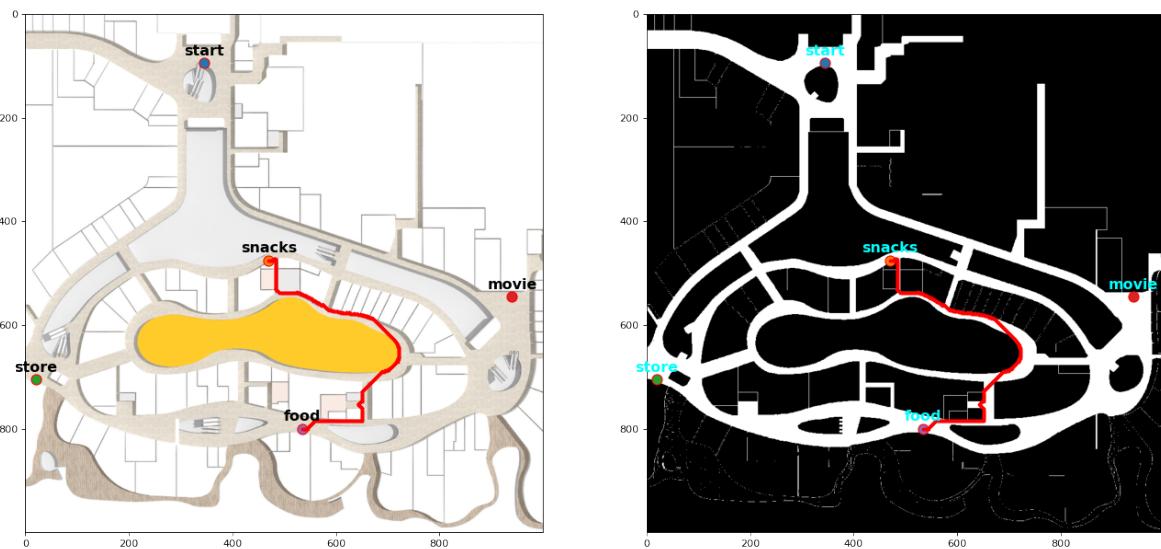


Figure 18: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to snacks.

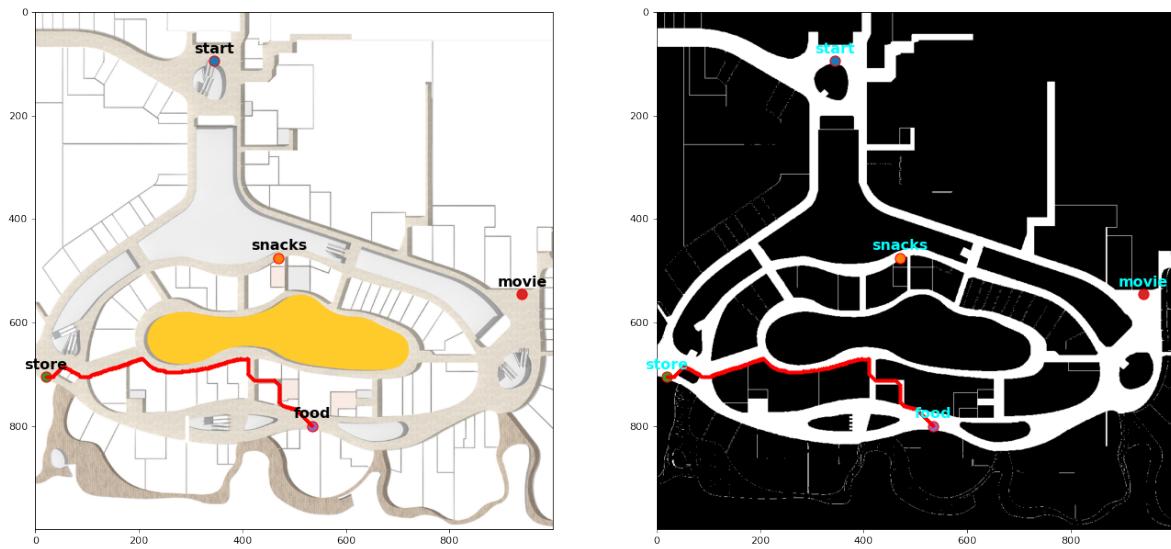


Figure 19: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to store.

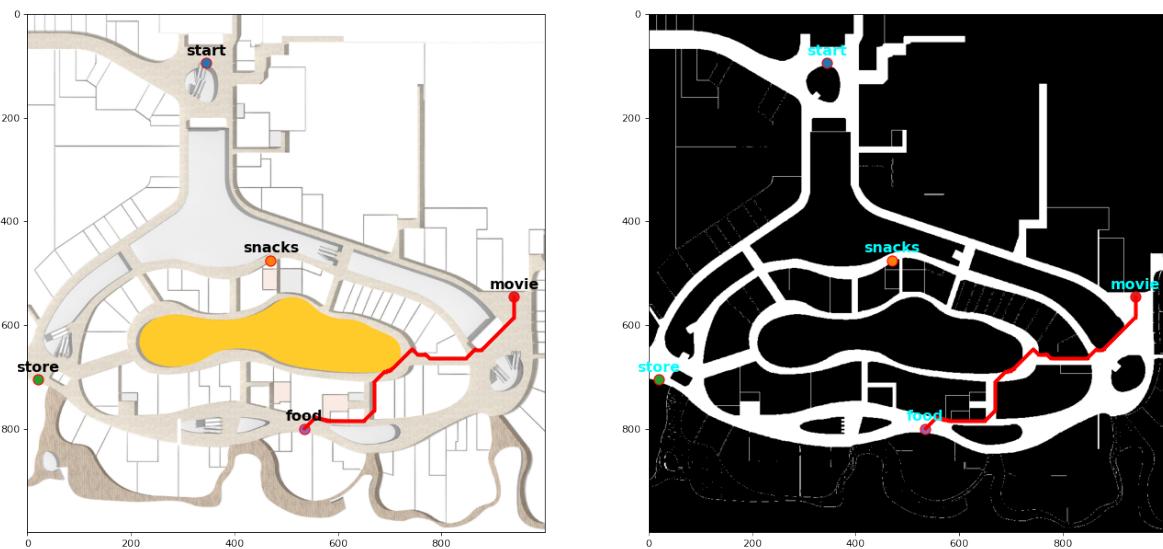


Figure 20: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to movie.

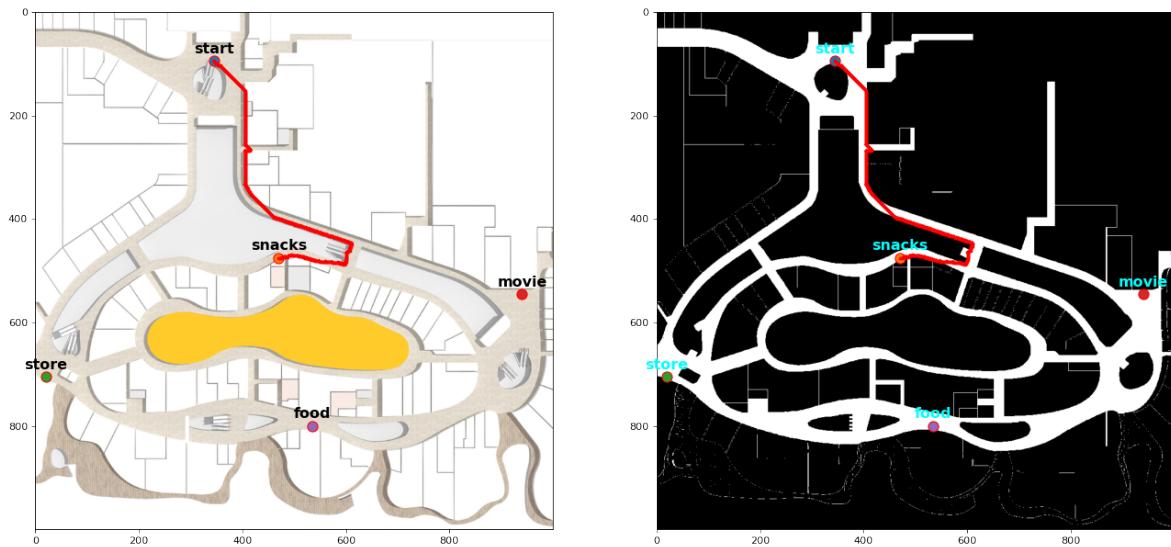


Figure 21: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to snacks.

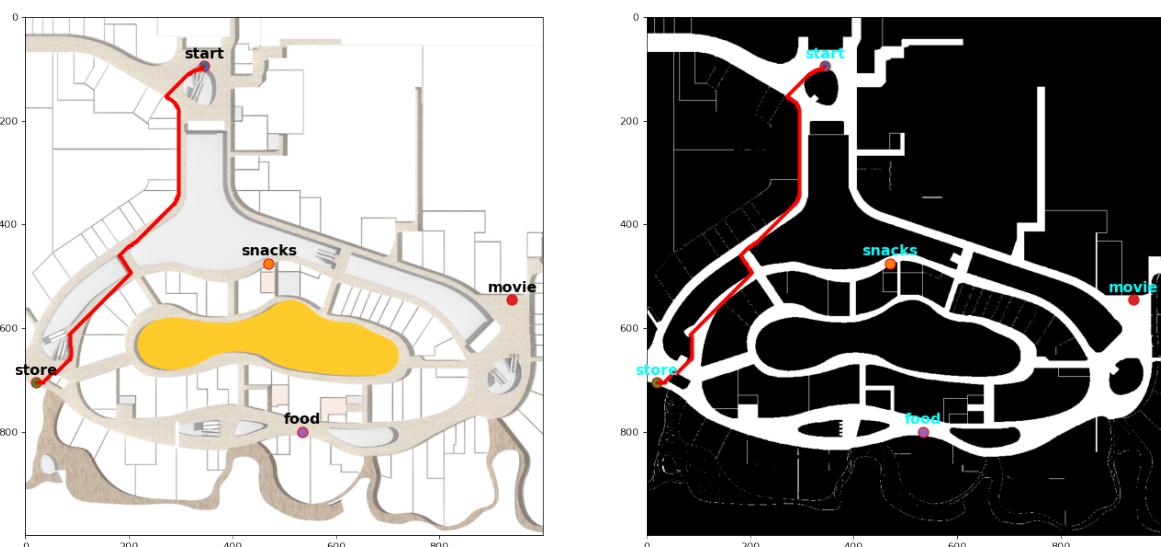


Figure 22: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to store.

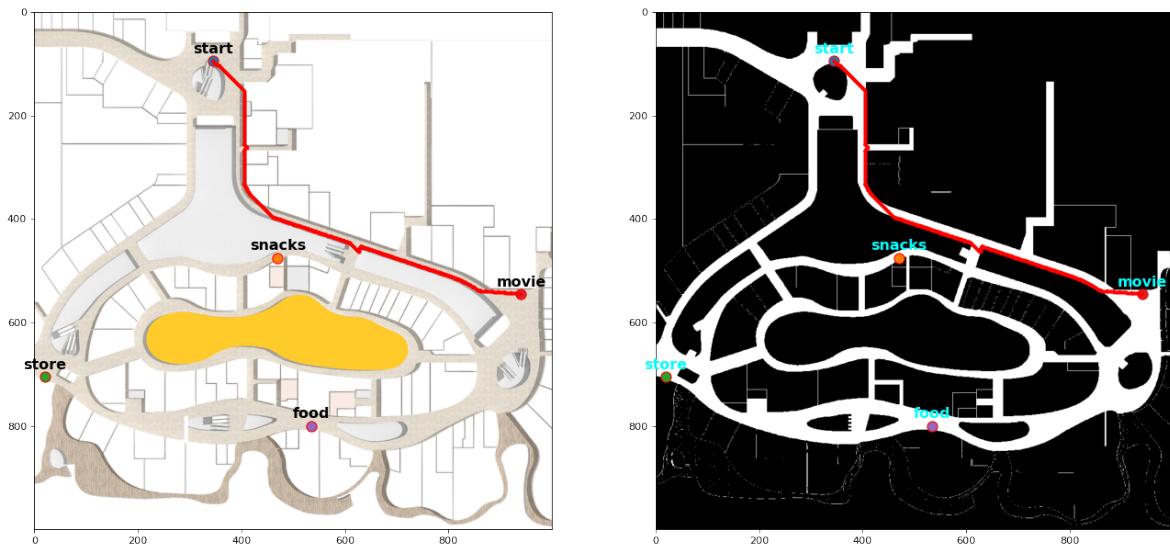


Figure 23: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to movie.

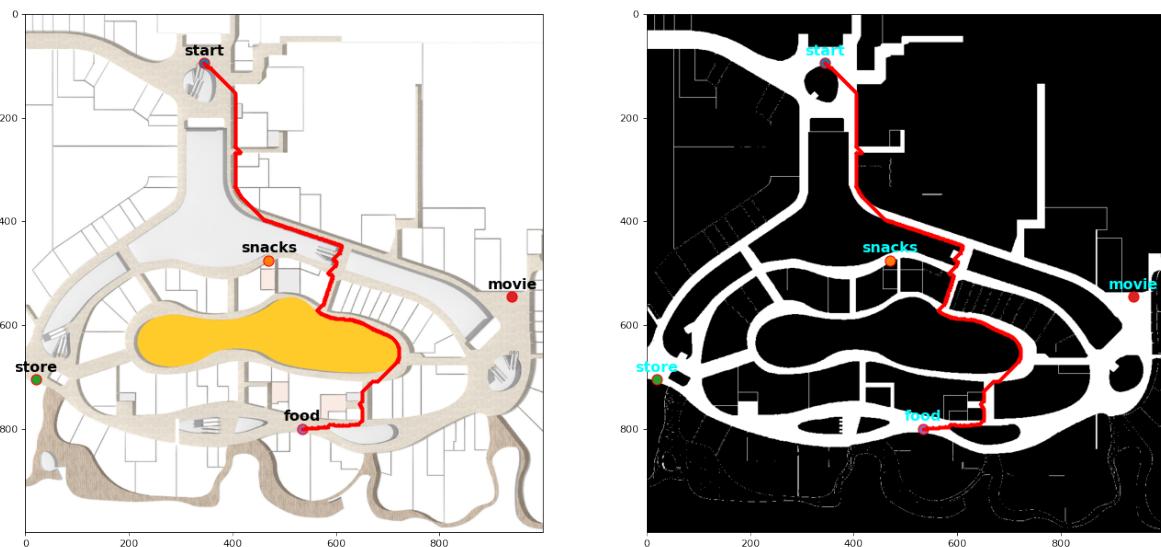


Figure 24: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to food.

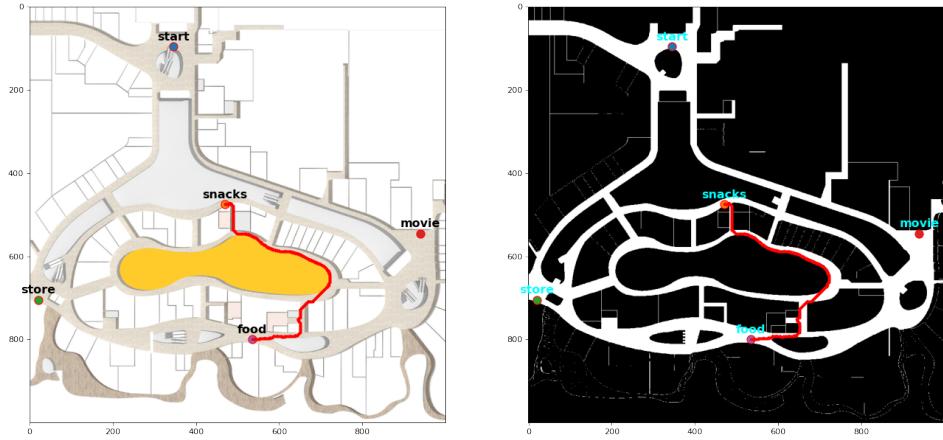


Figure 25: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to food.

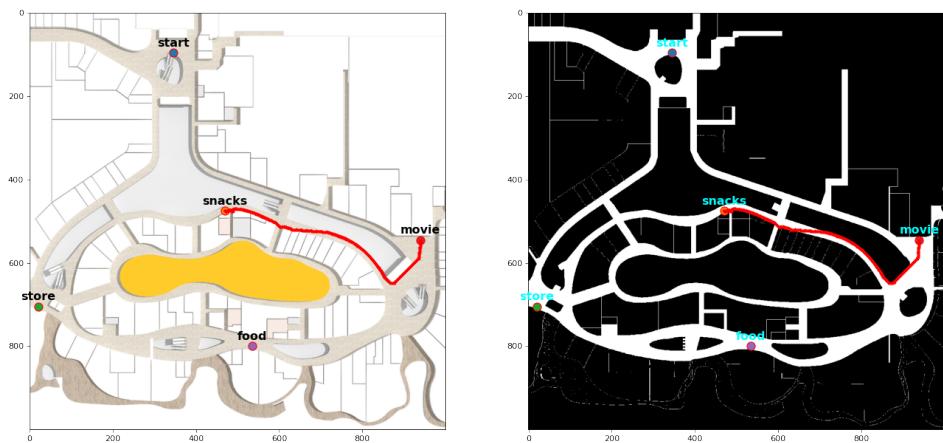


Figure 26: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to movie.

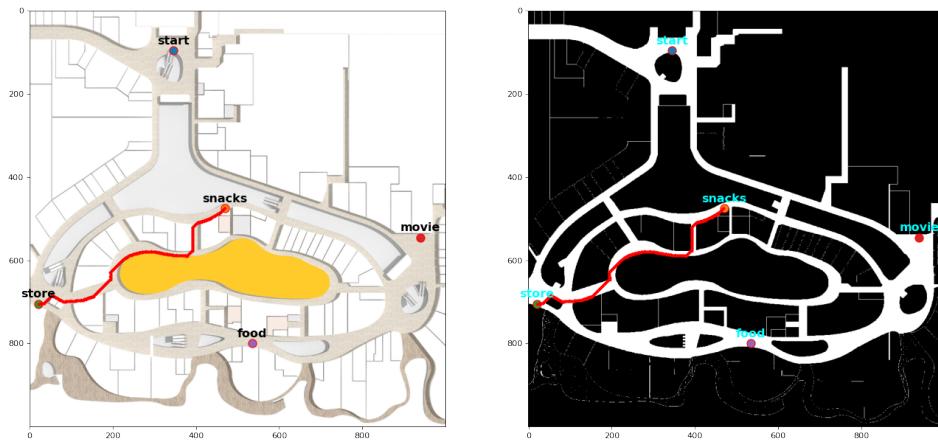


Figure 27: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to store.

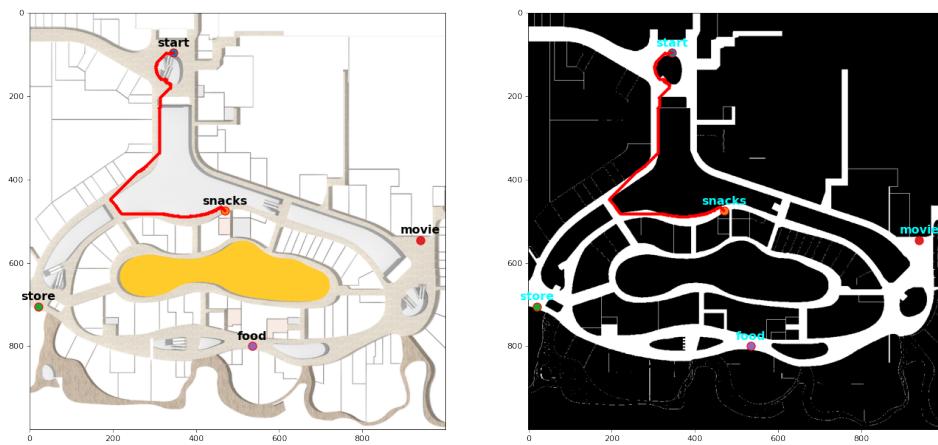


Figure 28: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to start.

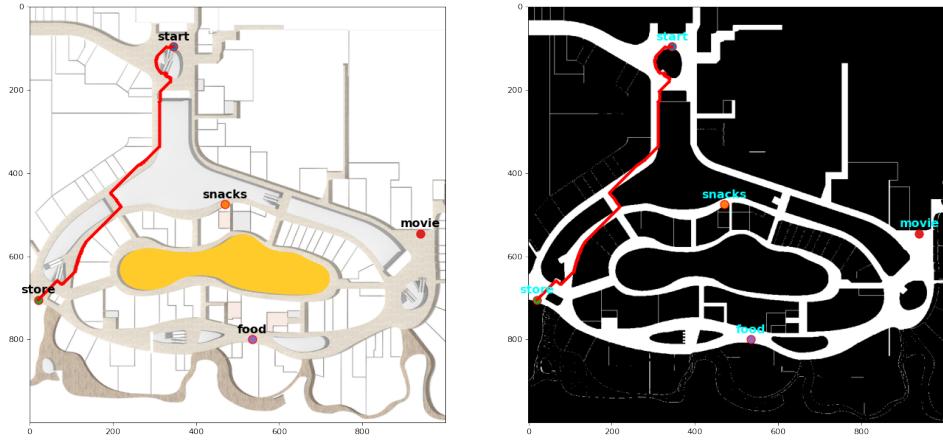


Figure 29: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to start.

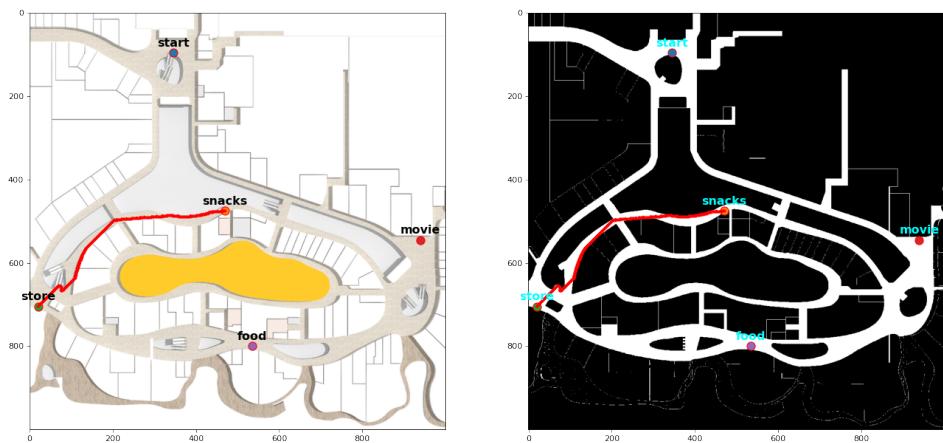


Figure 30: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to snacks.

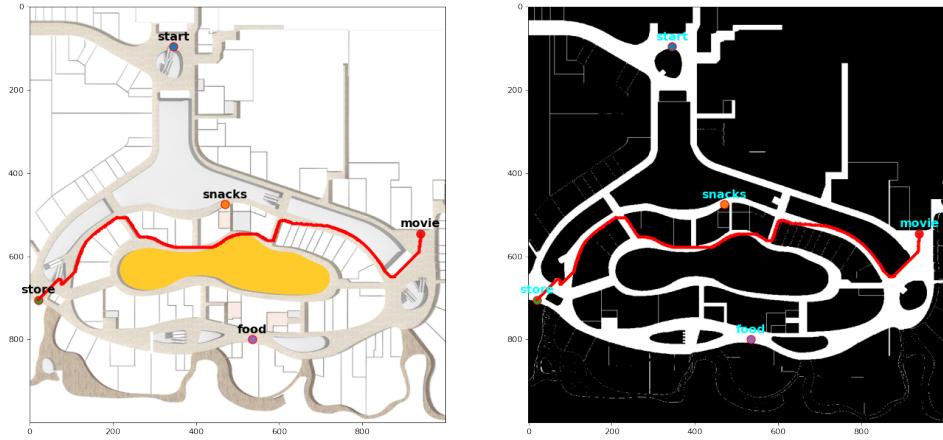


Figure 31: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to movie.

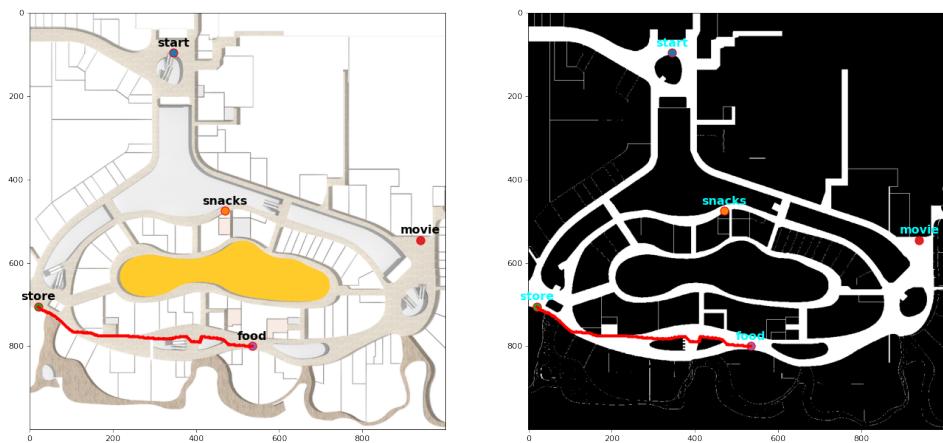


Figure 32: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to food.

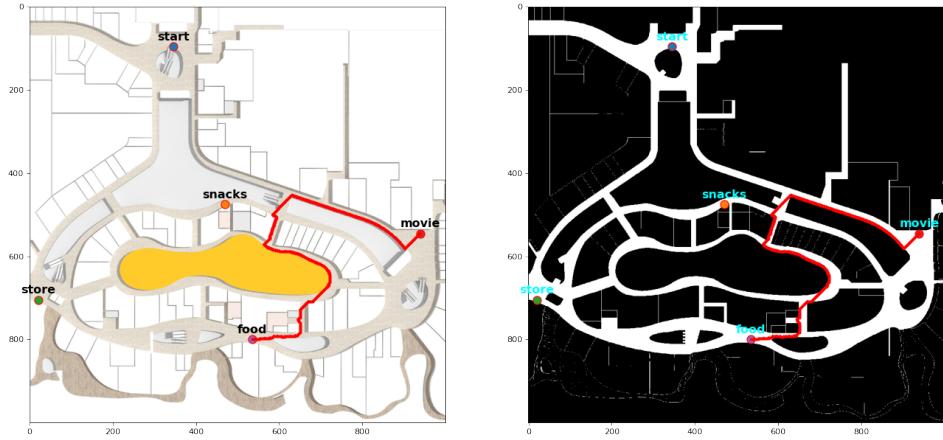


Figure 33: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to food.

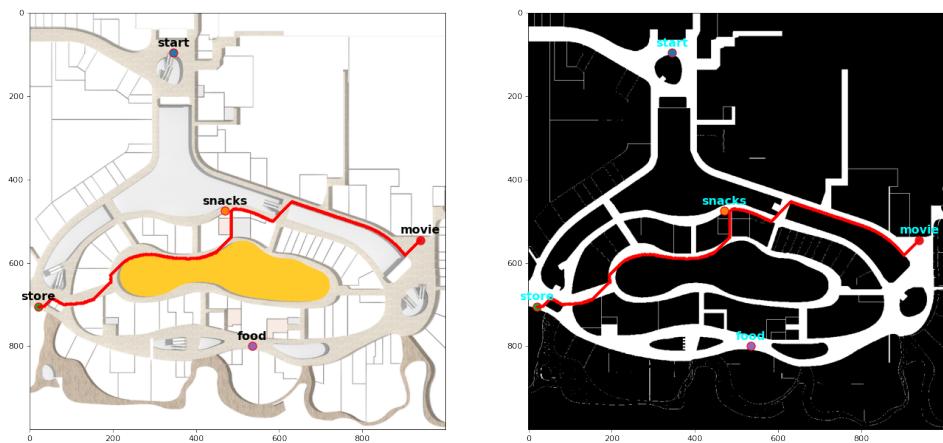


Figure 34: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to store.

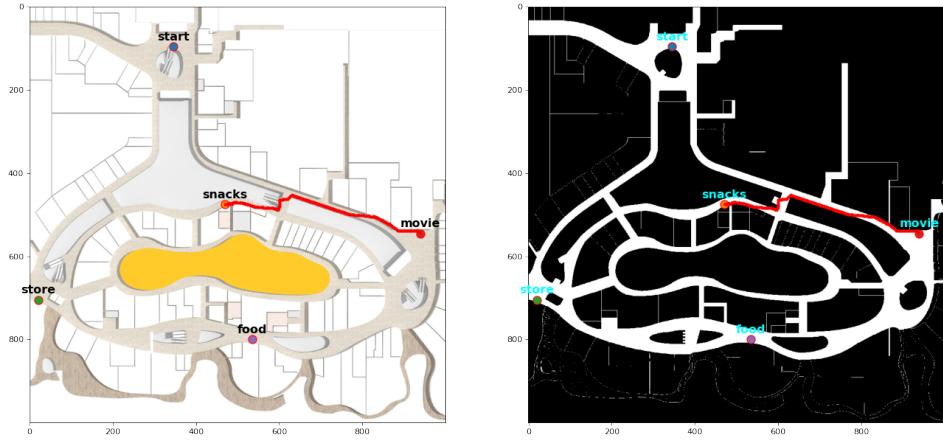


Figure 35: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to snacks.

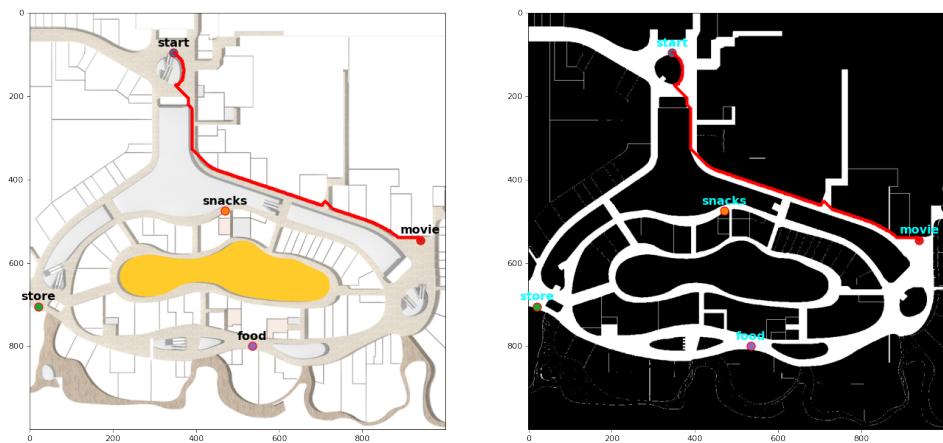


Figure 36: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to start.

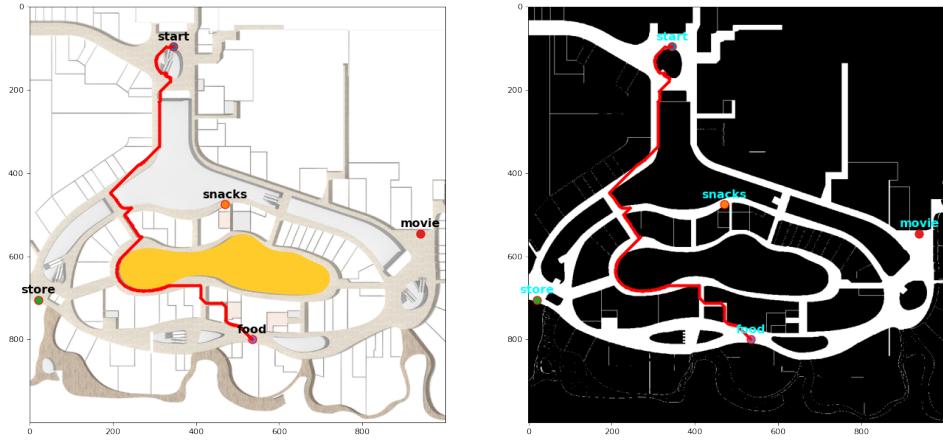


Figure 37: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to start.

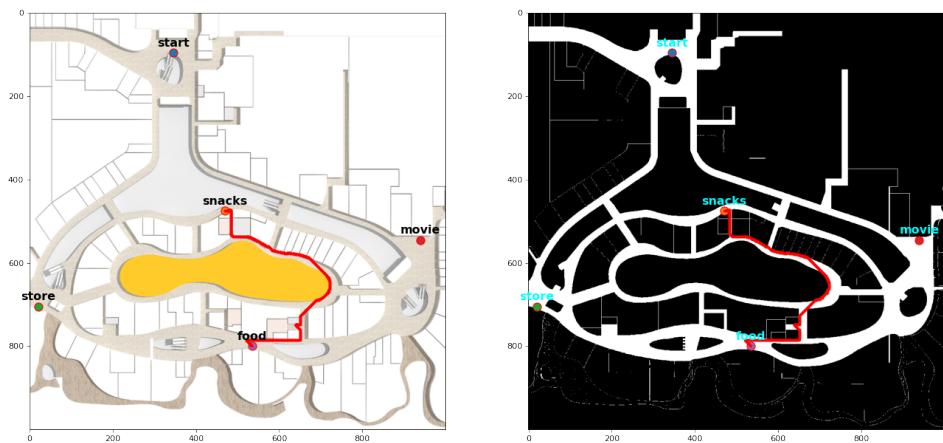


Figure 38: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to snacks.

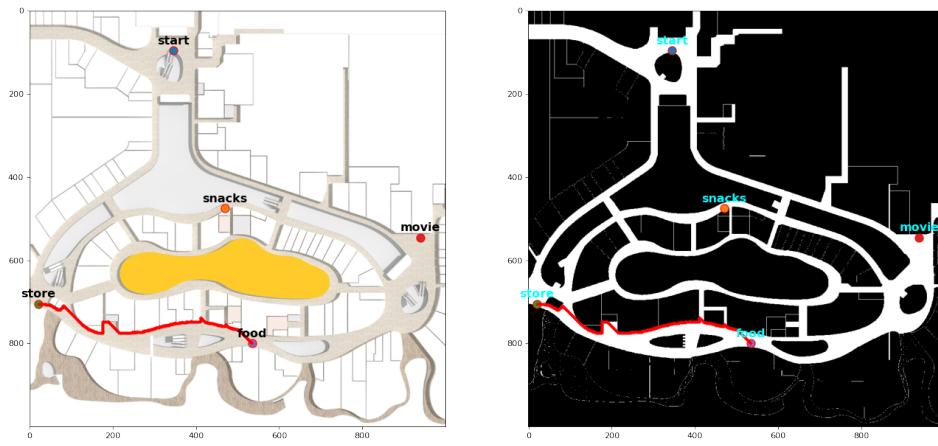


Figure 39: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to store.

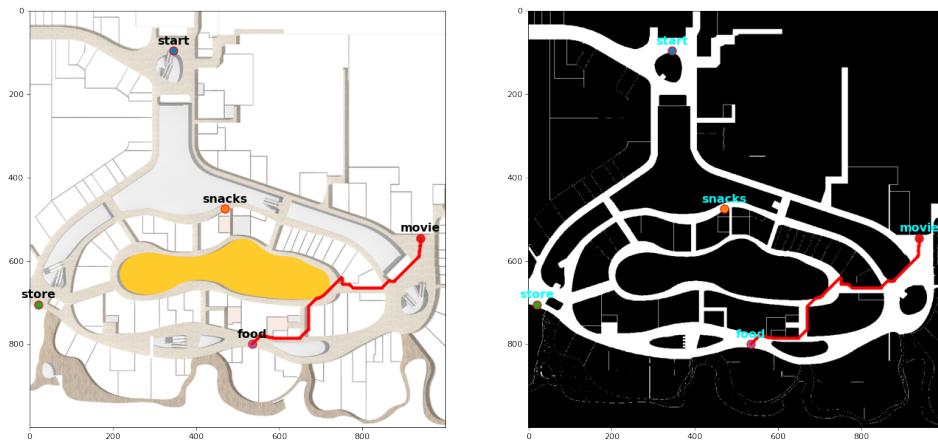


Figure 40: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to movie.

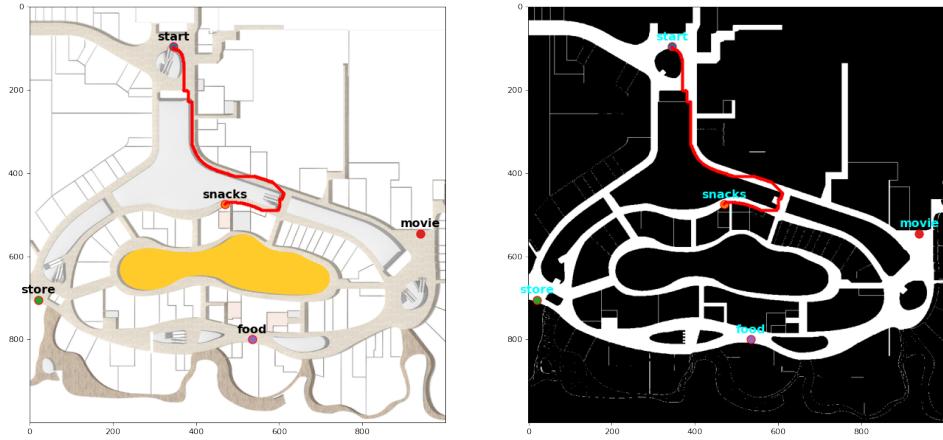


Figure 41: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to snacks.

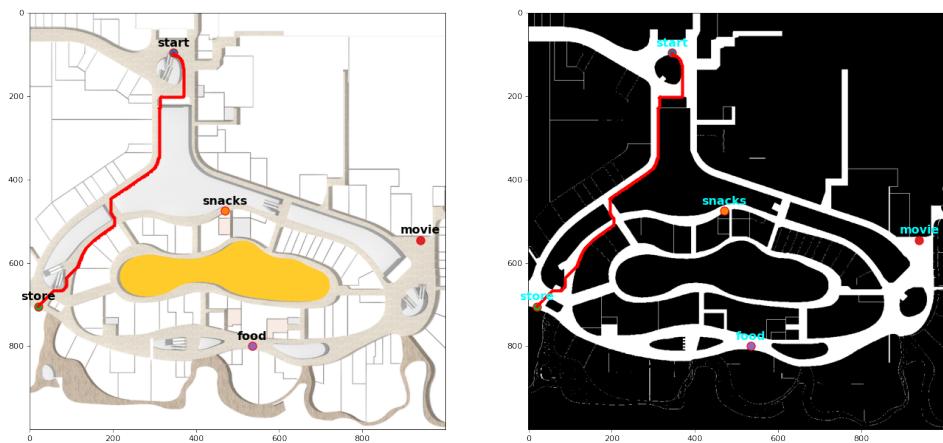


Figure 42: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to store.

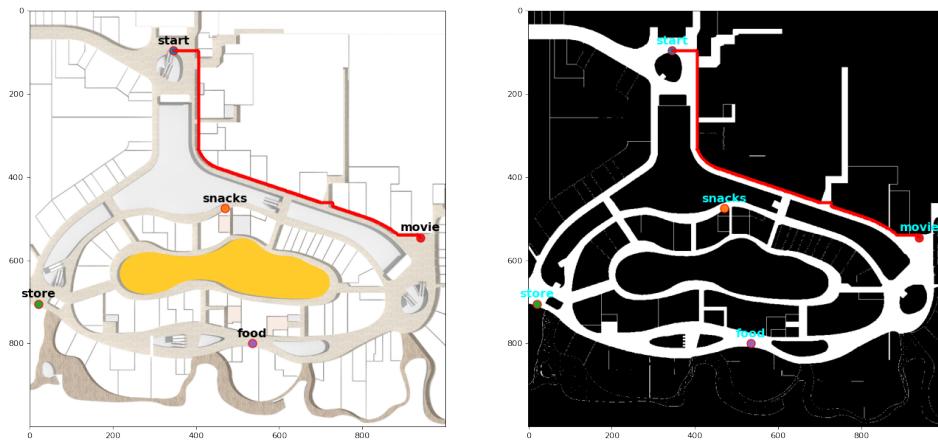


Figure 43: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to movie.

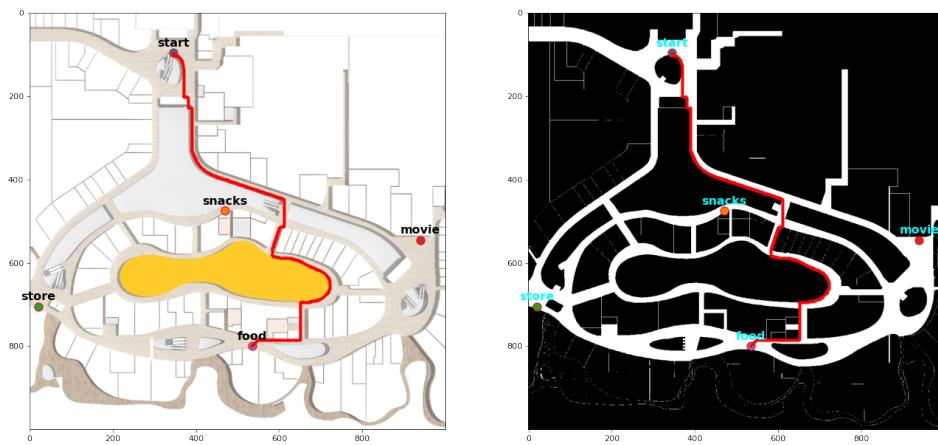


Figure 44: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from start to food.

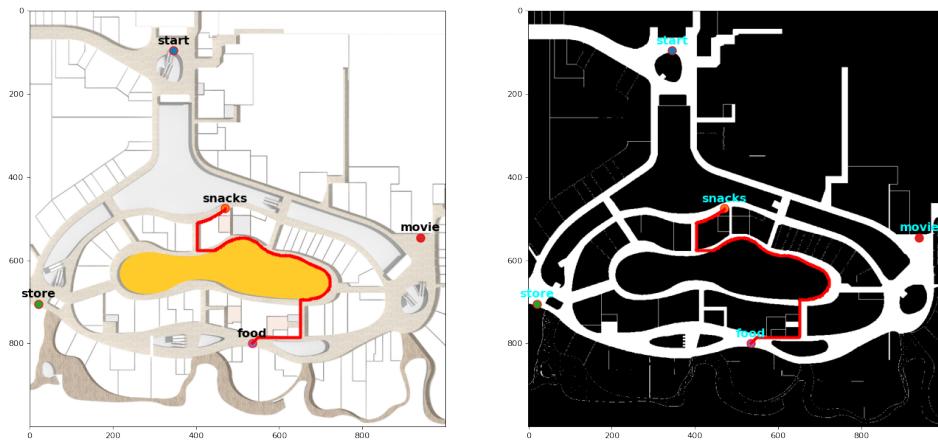


Figure 45: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to food.

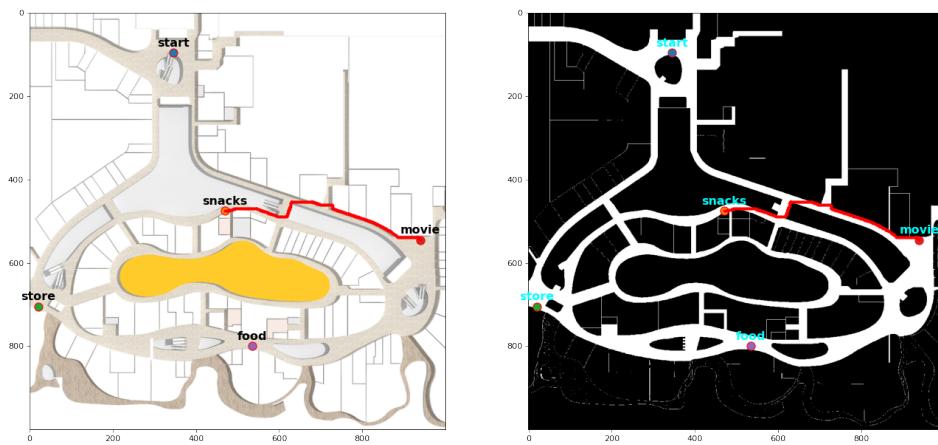


Figure 46: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to movie.

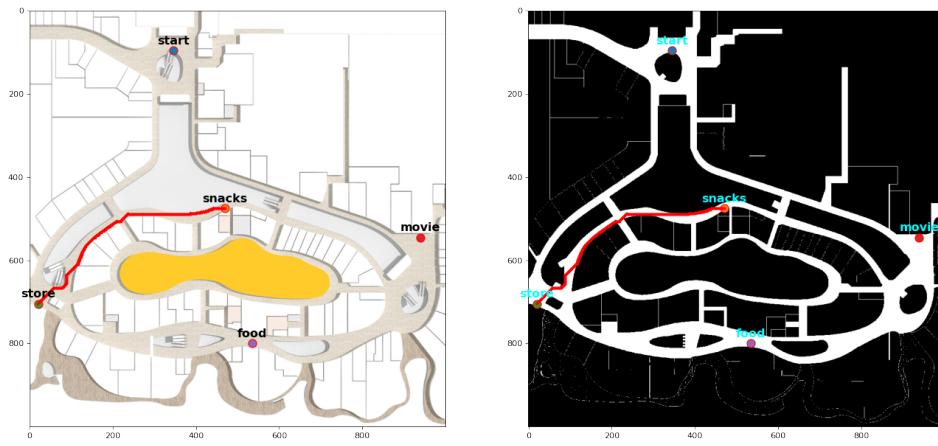


Figure 47: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to store.

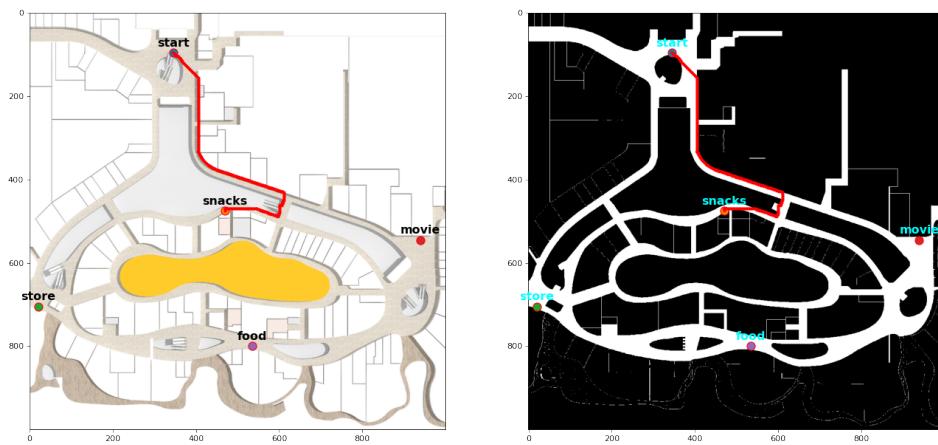


Figure 48: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from snacks to start.

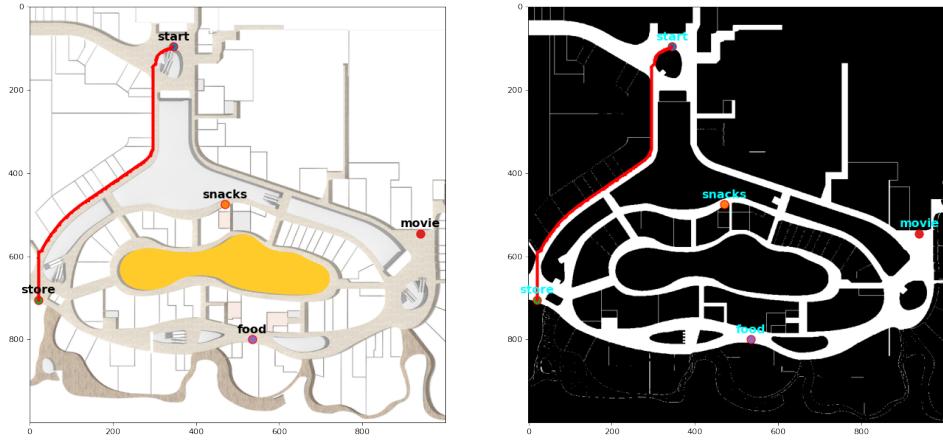


Figure 49: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to start.

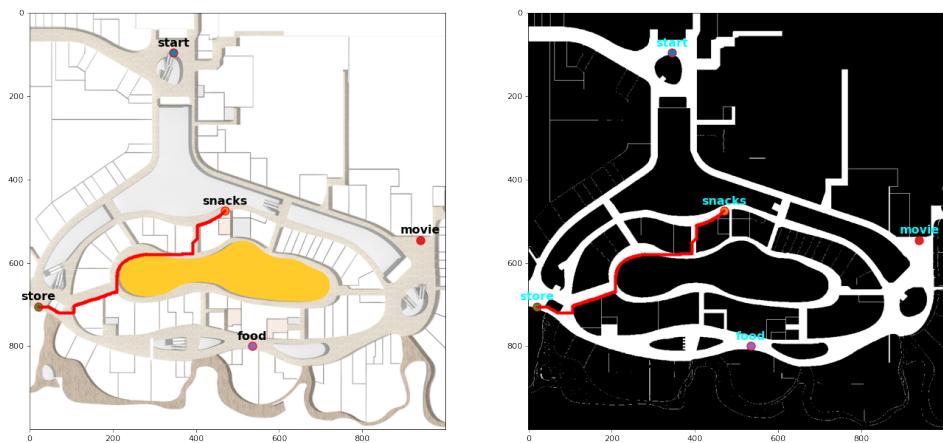


Figure 50: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to snacks.



Figure 51: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to movie.

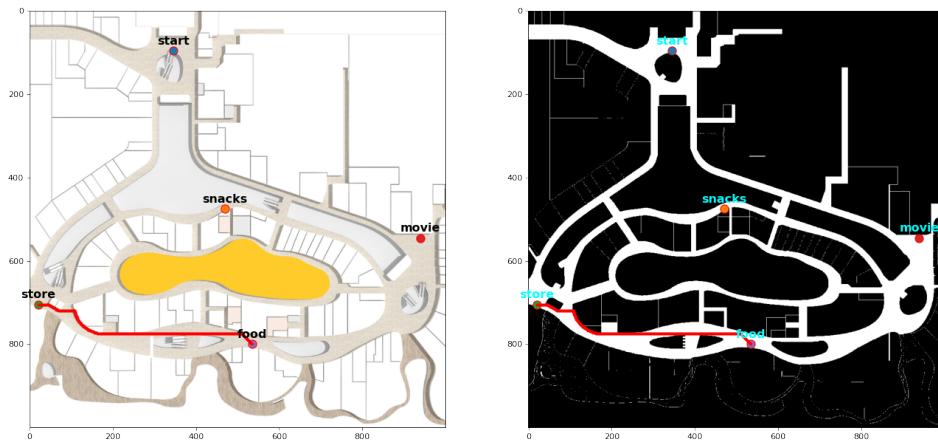


Figure 52: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from store to food.

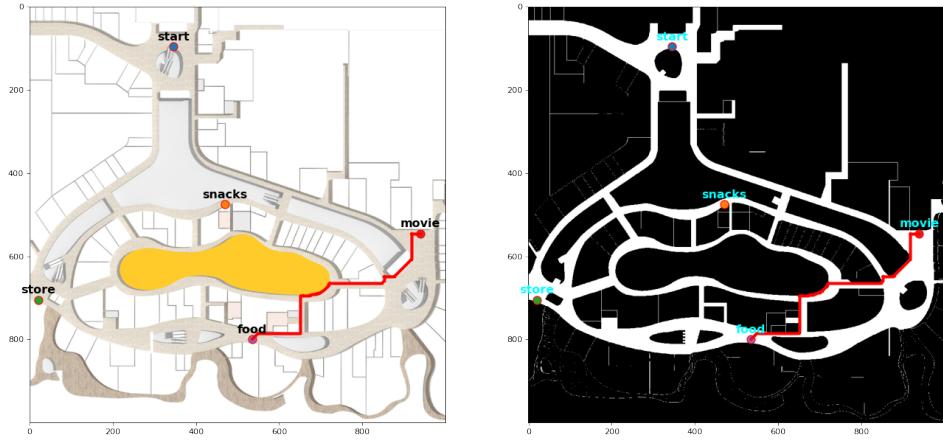


Figure 53: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to food.

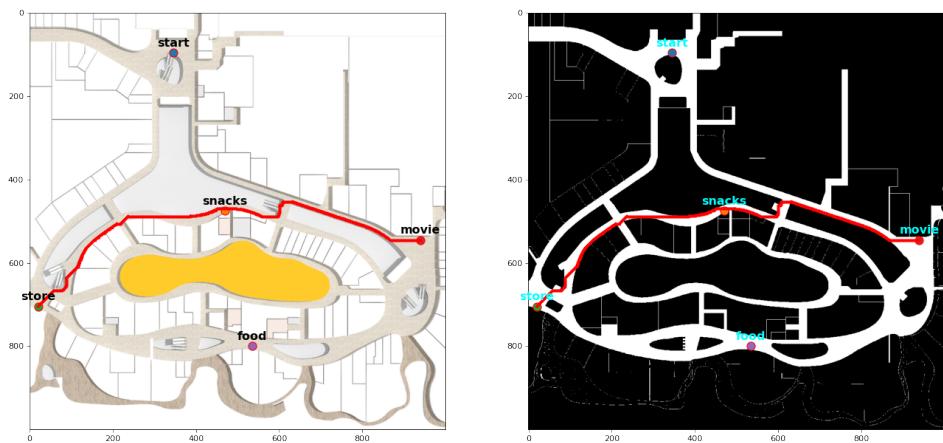


Figure 54: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to store.

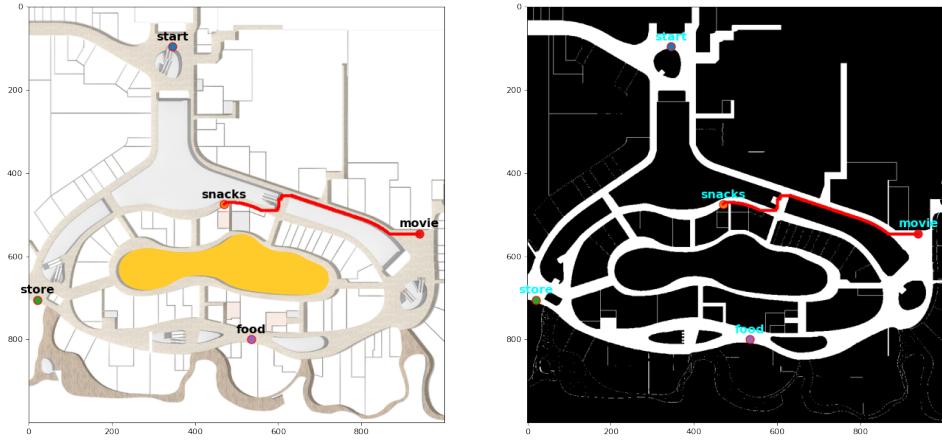


Figure 55: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to snacks.

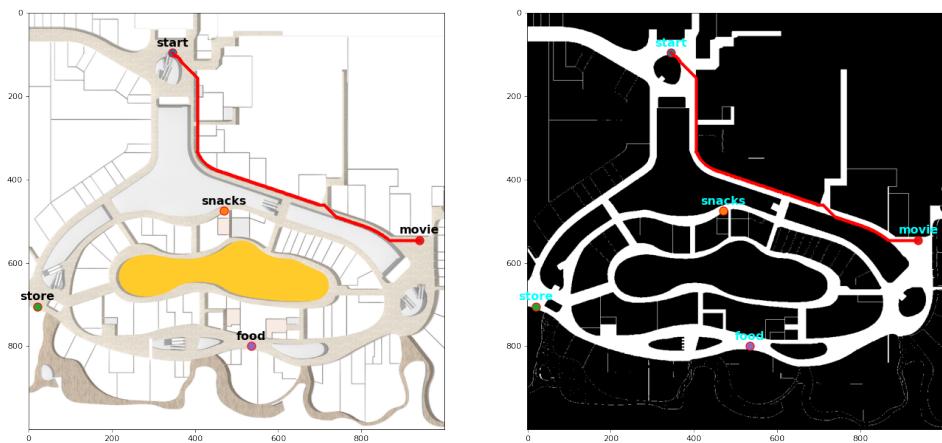


Figure 56: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from movie to start.

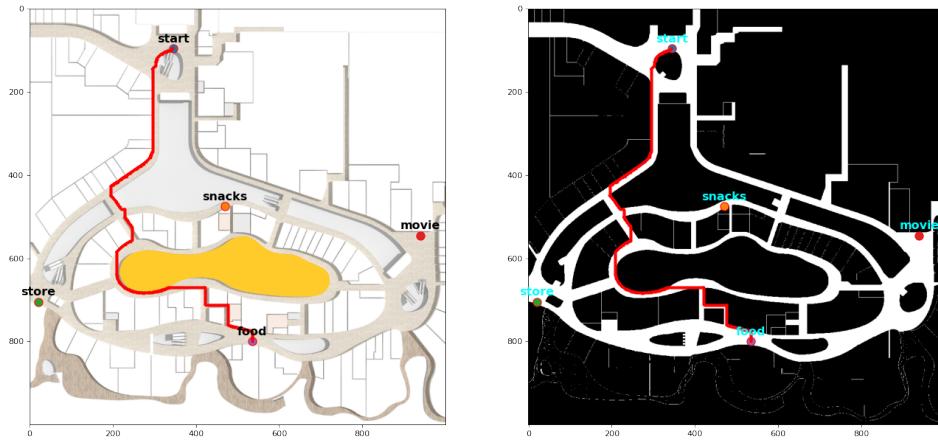


Figure 57: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to start.

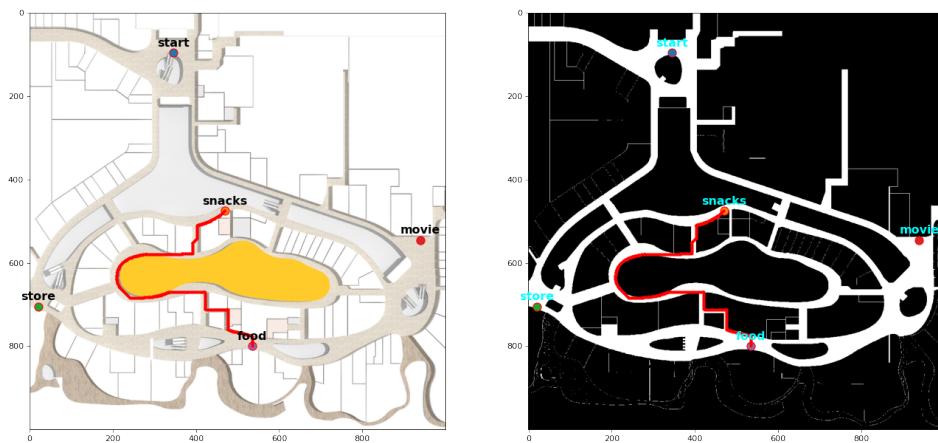


Figure 58: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to snacks.

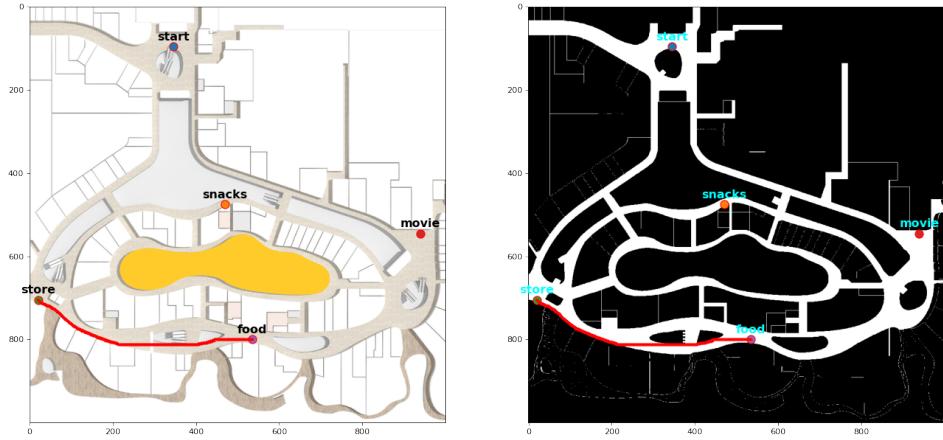


Figure 59: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to store.



Figure 60: Original floor plane (left) and gray scaled version (right) with the planned path (red line) from food to movie.

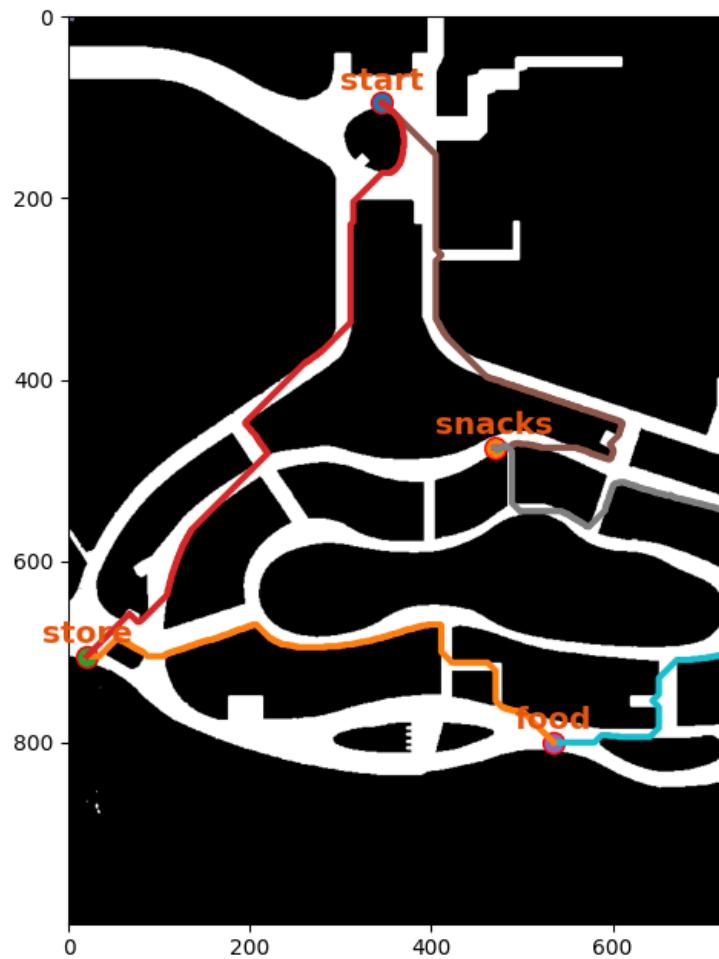


Figure 61: The best path.