Exercise 1: Population Growth and Numerical Differentiation
Consider the U.S. population for years 1810-2010 listed below:

Calendar Year	Years since 1900, t	Population in millions, $N(t)$
1810	-90	7.24
1820	-80	9.64
1830	-70	12.87
1840	-60	17.07
1850	-50	23.19
1860	-40	31.44
1870	-30	38.56
1880	-20	50.19
1890	-10	62.98
1900	0	76.21
1910	10	92.23
1920	20	106.02
1930	30	123.20
1940	40	132.16
1950	50	151.33
1960	60	179.32
1970	70	203.30
1980	80	226.54
1990	90	248.71
2000	100	281.42
2010	110	307.75

```
N = [ 7.24; 9.64; 12.87; 17.07; 23.19; 31.44; 38.56; 50.19; 62.98; ...
      76.21; 92.23; 106.02; 123.20; 132.16; 151.33; 179.32; 203.30; ...
      226.54; 248.71; 281.42; 307.75];
```

Estimate the growth rate $\frac{dN}{dt}$ (millions/year) for years 1810, 1820, ... 2010 using finite differences of *second order* accuracy. Save

$$\left[\frac{dN}{dt}(t=1810), \frac{dN}{dt}(t=1820), \dots, \frac{dN}{dt}(t=2010)\right]^T$$

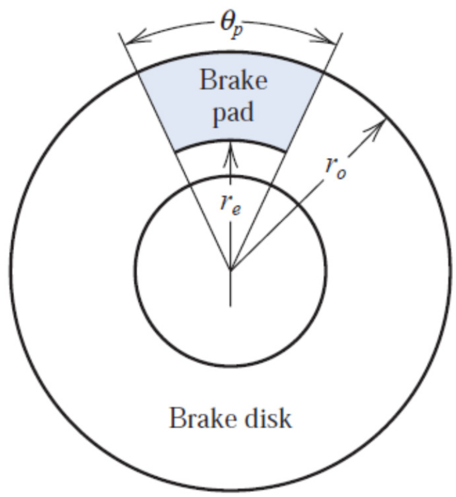
as a column vector in **A1.dat**.
Hint: Second-order forward/backward differences must be used at the end points.

Exercise 2: Brake Pads and Numerical Integration

To simulate the thermal characteristics of disk brakes, D. A. Secrets and R. W. Hornbeck needed to approximate numerically the area-averaged lining temperature, $\overline{T}$ of the brake pad defined by:

$$\overline{T} = \frac{\int_{r_\ell}^{r_0} rT(r)\theta_p \, dr}{\int_{r_\ell}^{r_0} r\theta_p \, dr}$$

where r_ℓ represents the radius at which the pad-disk contact begins, r_0 represents the outside radius of the pad-disk contact, θ_p represents the angle subtended by the sector brake pads, and $T(r)$ is the temperature at each point of the pad, obtained numerically from analyzing the heat equation. Suppose $r_\ell = 0.308$ ft, $r_0 = 0.478$ ft, and $\theta_p = 0.7051$ radians, and the temperatures at equidistant points on the disk are listed in the table below.



r (ft)	T (°F)
.308	640
.325	794
.342	885
.359	943
.376	1034
.393	1064
.410	1114
.427	1152
.444	1204
.461	1222
.478	1239

(a) Calculate $\overline{T}$ using the composite Simpson rule for both the integral in the numerator and the denominator. Save the numerator as **A2.dat**, the denominator as **A3.dat**, and $\overline{T}$ as **A4.dat**.

(b) Calculate $\overline{T}$ using Matlab's function `trapz` for both the integral in the numerator and the denominator. Save the numerator as **A5.dat**, the denominator as **A6.dat**, and $\overline{T}$ as **A7.dat**.