System Simulation on 1-D Mask

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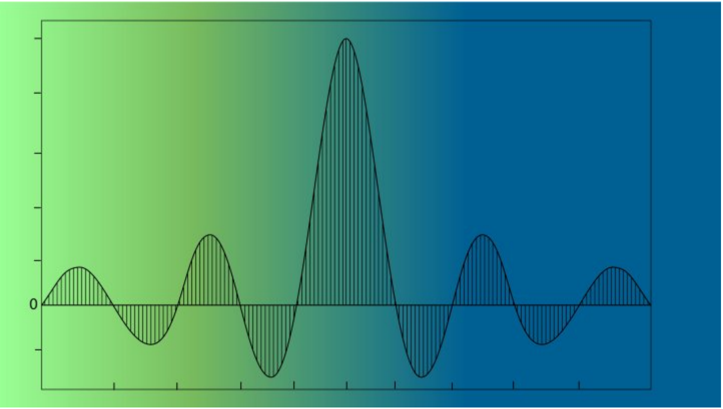
Outline

Introduction

The Nature of Curiosity Deepens our Horizons



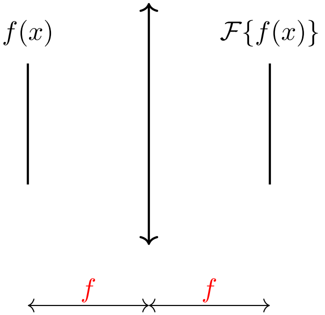
Fourier Transform (1/1)

1-D Fourier Transform pair is shown below

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g(k)\, e^{jkx}\, \mathrm{d}k \longleftrightarrow g(k) = \int_{-\infty}^{+\infty} f(x)\, e^{-jkx}\, \mathrm{d}x$$

Below shows a system that has a relationship with Fourier Transform pairs.

Condensed Lens



With this arrangement, we will have

$$\text{Output} = \mathcal{F}\{f(x)\} = \mathcal{F}\{\text{Input}\}$$

and the notation $\mathcal{F}$ represents

$$\mathcal{F} : f(x) \longrightarrow g(k)$$

Sampling Theorem (1/5)

Assume we have a continuous time signal $x(t)$ and

$$\mathcal{F} : x(t) \longleftrightarrow X(j\omega)$$

Typically, we will use an impulse train $s(t)$ to discretize the signals that

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad \text{where} \quad T_s \text{ is the sampling period}$$

Hence, we can obtain $x[n]$ from $x(t)$ and $s(t)$ that

$$x[n] = x(t)s(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

With sifting property, we rewrite it as

$$x[n] = x(nT_s)$$

Sampling Theorem (2/5)

Since $s(t)$ is periodic, we can find its FS coefficient

$$\mathcal{FS} : s(t) = \sum_{k=-\infty}^{\infty} S[k]e^{jk\omega_0 t} \longleftrightarrow S[k] = \frac{1}{T}$$

and shifting property that

$$\mathcal{F} : e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega - \omega_0)$$

Thus, we can get $S(j\omega)$ that

$$S(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

Hence, for signal $x(t)s(t)$, we could have the complete form that

$$\mathcal{F} : x(t)s(t) \longleftrightarrow \frac{1}{2\pi} X(j\omega) * S(jw)$$

Consider a signal of $x(t)$ having this $X(j\omega)$ and impulse train $s(t)$.

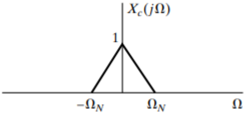


Figure: $X(j\omega)$ of signal $x(t)$

It is bandlimited because

$$X(j\omega) = 0 \quad |\omega| \geq \omega_N$$

It is better to sample bandlimited signals.

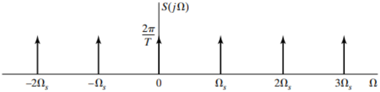


Figure: $S(j\omega)$ of signal $s(t)$

We can choose the sampling period T_s.

$$\omega_s = \frac{2\pi}{T_s}$$

Sampling Theorem (4/5)

Consider $x_s(t) = x(t)s(t)$ with different sampling period.

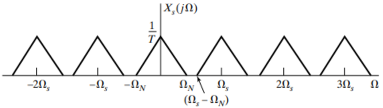


Figure: $X_s(j\omega)$ of signal $x_s(t)$

The sampling period is quite small because

$$\omega_s > \omega_N$$

which ensures a intact pattern of $X(j\omega)$.

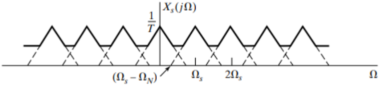


Figure: $X_s(j\omega)$ of signal $x_s(t)$

The sampling period is quite large because

$$\omega_s < \omega_N$$

which shows a mixed pattern of $X(j\omega)$.

The importance of the sampling period T_s is now shown above.

Sampling Theorem (5/5)

Generally, a mixed pattern for a bandlimited signal is called aliasing.

Nyquist-Shannon Sampling Theorem

Let $x_c(t)$ be a bandlimited signals with

$$X_c(j\omega) \approx 0 \quad \text{for } |\omega| \geq \omega_N$$

Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT_s)$ where

$$\omega_s = \frac{2\pi}{T_s} \geq 2\omega_N$$

Also, it is also referred to as

$$\omega_N \longrightarrow \text{Nyquist frequency}$$
$$2\omega_N \longrightarrow \text{Nyquist rate}$$

To prevent aliasing, it is required to sample at Nyquist rate where

$$T_s < \frac{\pi}{\omega_N} \Longrightarrow f_s > \frac{\omega_N}{\pi}$$

Discrete Fourier Transform (1/3)

During simulation, we still can't get a continuous frequency response. Recall that

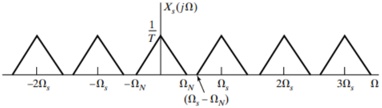


Figure: $X_s(j\omega)$ of signal $x_s(t)$

Observation :

- Input : discrete
- Output : continuous

Problem occurs with continuous signals.

Is there any method to solve this problem ? DFT.

Discrete Fourier Transform (2/3)

Discrete Fourier Transform is used for finite discrete signals.

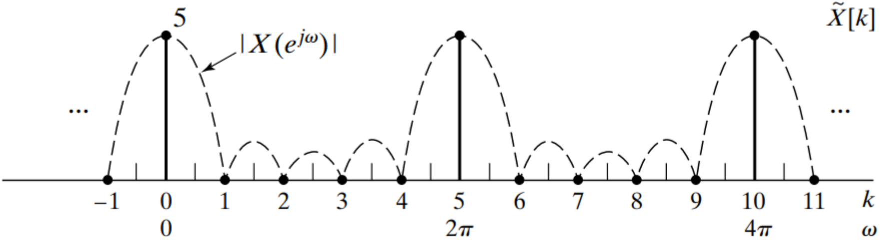


Figure: Example for relationship between $X(e^{j\omega})$ and $X[k]$

The picture shown above easily depicts $X[k]$ just samples from $X(e^{j\omega})$.

Discrete Fourier Transform (3/3)

What are the benefits using DFT ?

- It provides a discrete structure for computers to store.
- It makes the number of sampling from input and output the same.

It will be helpful with Nyquist-Shannon Sampling Theorem.

- Find the appropriate sampling frequency at Nyquist rate.
- Divide this valid region to the number of input.

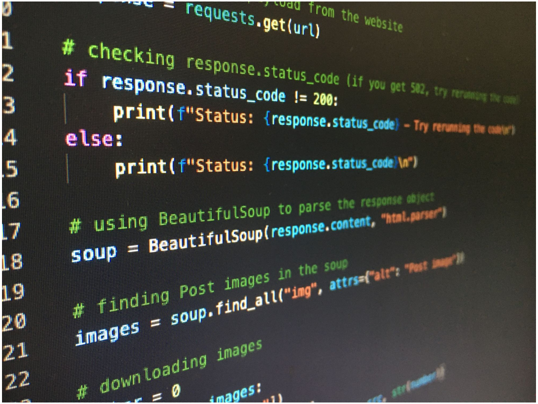
Hence, it is easy to get the value of every discrete point in frequency space.

$$\text{frequency of } X[k] = \frac{k}{N} \cdot \omega_s$$

because the valid region based on ω_s.

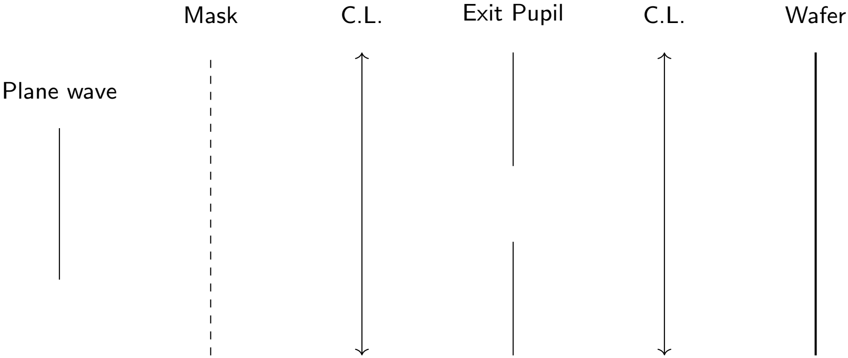
Simulation

Putting Everything into Practice Prevents Cheap Talks



Simulation Systems (1/4)

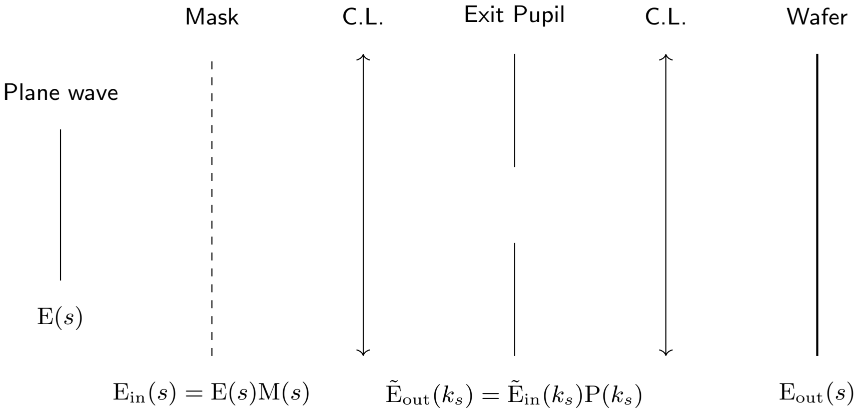
Below shows the structure of our simulation system.



Note that C.L. means condensed lens.

Simulation Systems (2/4)

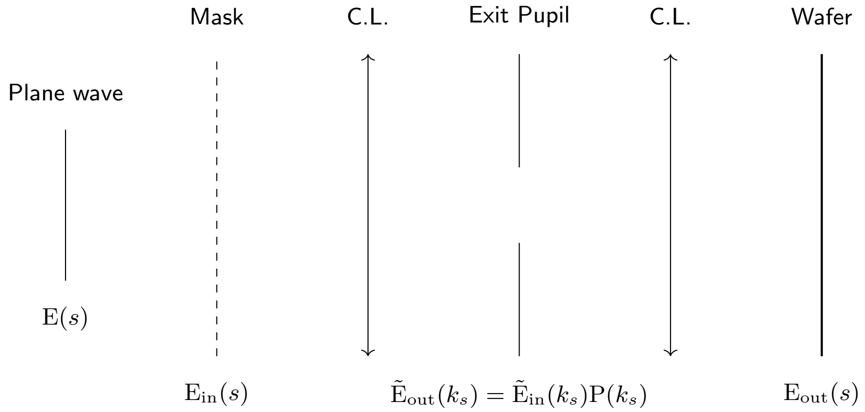
We assume the focal length of both C.L. are f, which Fraunhofer diffraction is FT.



The states are shown above respectively.

Simulation Systems (3/4)

Let's take a closer look at all the states.



All relationships between $\mathrm{E}_{\{.\}}$ and $\tilde{\mathrm{E}}_{\{.\}}$ are FT pairs.

Simulation Systems (4/4)

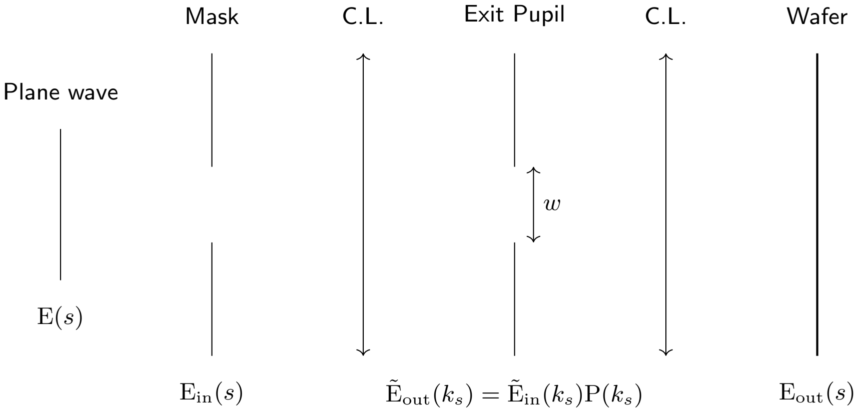
Simulations are under these conditions.

- Numerical Aperature $\mathrm{N.A.} = 0.86$
- $\lambda = 248\ \ nm$
- The wave number $k = \dfrac{2\pi}{\lambda}$
- The width of Exit pupil $w = \dfrac{\mathrm{N.A.}}{\lambda}$
- Wafer Intensity Threshold = 0.5 (Normalized)

These conditions will be pointed out when we see the structure of the system.

Single Slit

Take an easy look at the system using a single slit as mask.



If we consider a plane wave traveling at an angle of 0, we set $\mathrm{E}_{\mathrm{in}}(s)$ as a constant.

Simulation Results - Single Slit (2/11)

Since there is a rect is at "Mask", we expect a sinc before "Exit Pupil".

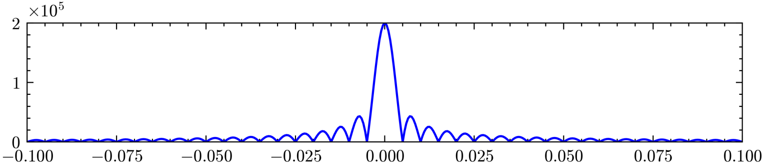


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

Also, at "Exit Pupil", the distribution will be affected by the size of it.

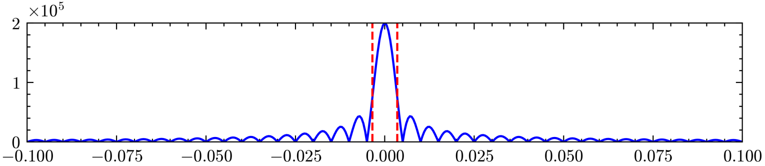


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ and Cut-off Region

Hence, the distribution at "Exit Pupil" is shown below.

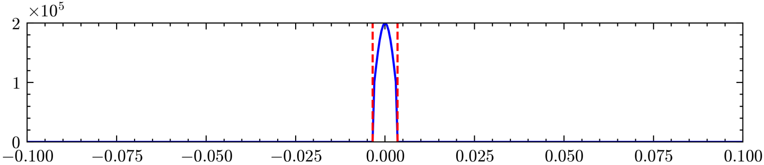


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ after Exit Pupil

With the last condensed lens, the aerial image is just an inverse Fourier Transform.

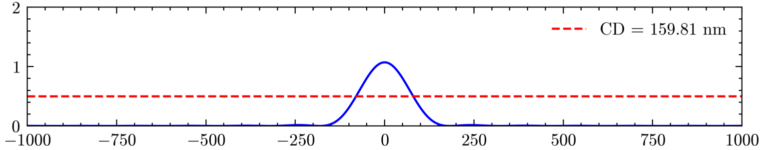
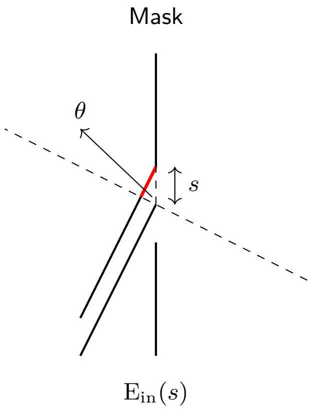


Figure: Magnitude Distribution on the Wafer

Now consider a plane wave traveling at an angle of θ with respect to the x-axis.



The plot shows that

- position s leads to path difference

For convenience, it is good to choose that

- constant at the center because $s = 0$
- others multiplied by a factor $e^{jks\sin\theta}$, ex. the red part

Let's see the results when traveling at angle $20°$.

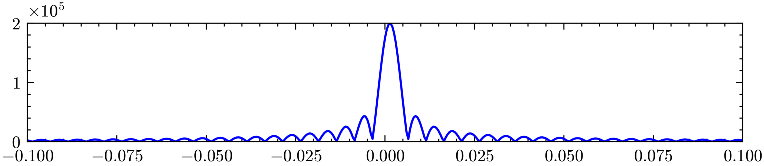


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

The pattern seems to have a shifting compared to one traveling at angle $0°$.

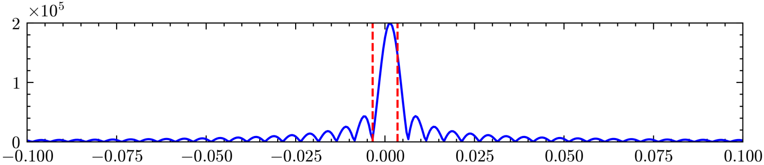


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ and Cut-off Region

Simulation Results - Single Slit (6/11)

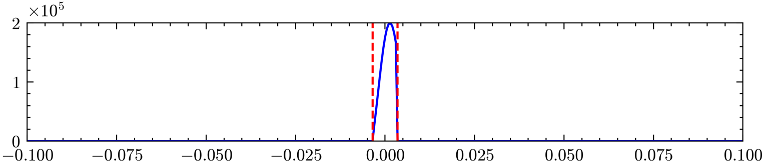


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ after Exit Pupil

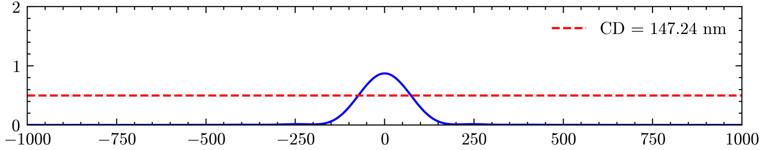


Figure: Magnitude Distribution on the Wafer

Let's see the results when traveling at angle $40°$.

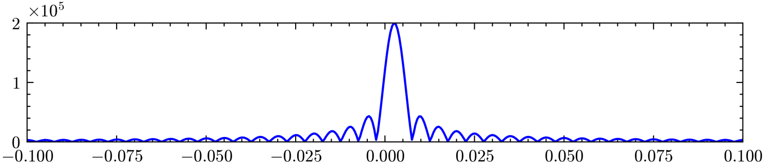


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

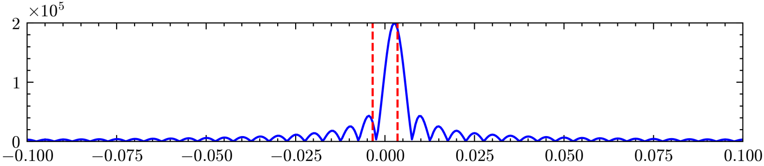


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$ and Cut-off Region

Simulation Results - Single Slit (8/11)

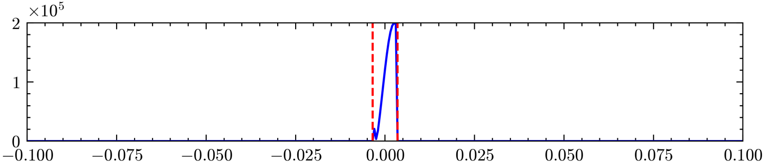


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E_{in}}(s)\}$ after Exit Pupil

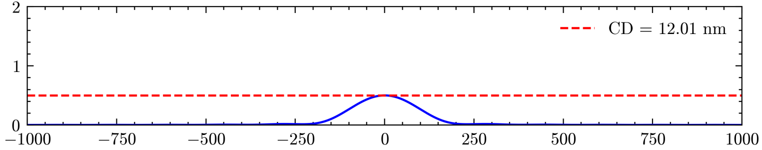


Figure: Magnitude Distribution on the Wafer

Let's see the results when traveling at angle $60°$.

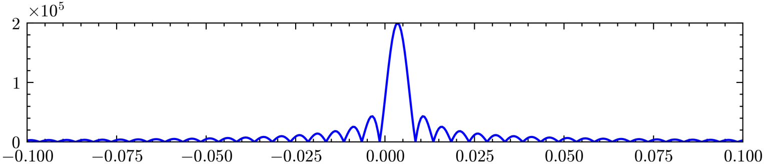


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

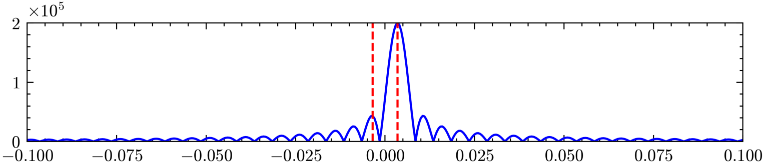


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$ and Cut-off Region

Simulation Results - Single Slit (10/11)

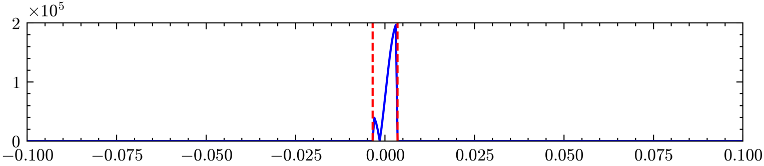


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E_{in}}(s)\}$ after Exit Pupil

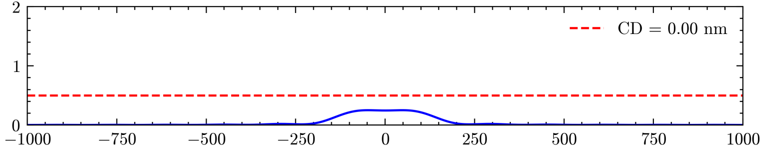


Figure: Magnitude Distribution on the Wafer

From these results, we can observe that when the pattern shifts too much

- Dominant intensity parts are lost.
- Measured $\mathrm{C.D.}$ are decreased.

Thus, let's change the width to see what happens when traveling at an angle 0.

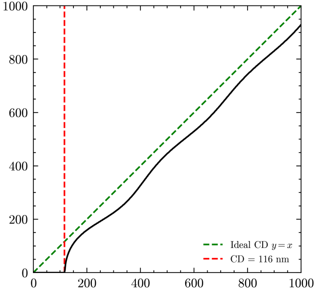


Figure: $\mathrm{C.D.}$ v.s. Measured $\mathrm{C.D.}$

We can see that

- Measured $\mathrm{C.D.}$ is getting closed to $\mathrm{C.D.}$
- No patterns under approximately $116\ nm$

Dense Line

Simulation Results - Dense Line (1/10)

Discuss the signal traveling at an angle of 0, which the distribution is periodic.

$$x(t) = \mathrm{rect}(t) * s_p(t) \longleftrightarrow X(j\omega) = \tilde{\mathrm{rect}}(j\omega)S(j\omega)$$

Thus, $X(j\omega)$ can be regarded as sampling from a $\tilde{\mathrm{rect}}(j\omega)$ which is a sinc.

By Nyquist Theorem, it is possible to find

$$X(j\omega) \approx 0 \Longrightarrow f_s = \frac{\omega_N}{\pi} \neq \frac{1}{p}$$

Also, we can expect a impulse graph when we start simulating.

Simulation Results - Dense Line (2/10)

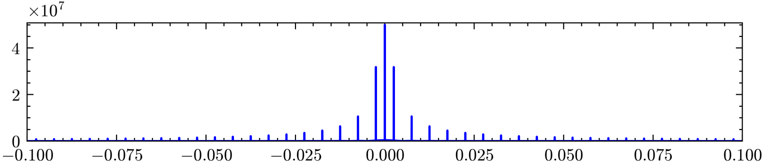


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

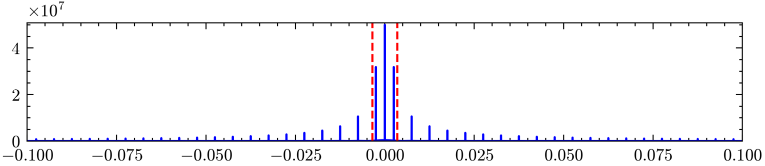


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

Simulation Results - Dense Line (3/10)

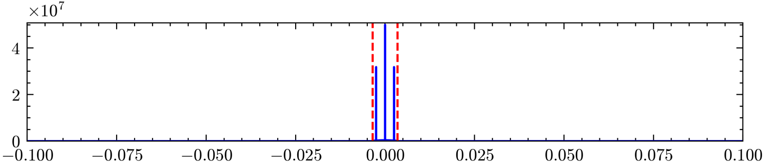


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ after Exit Pupil

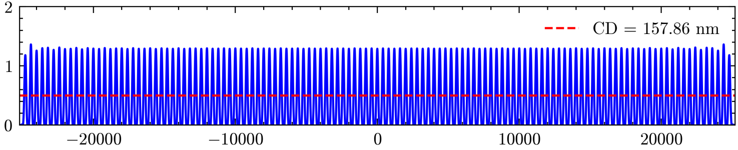


Figure: Magnitude Distribution on the Wafer

Simulation Results - Dense Line (4/10)

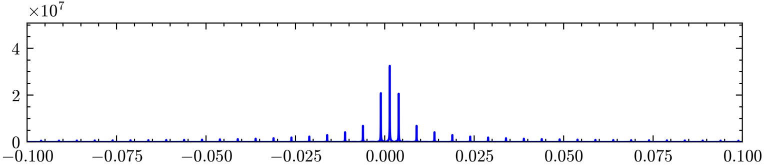


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

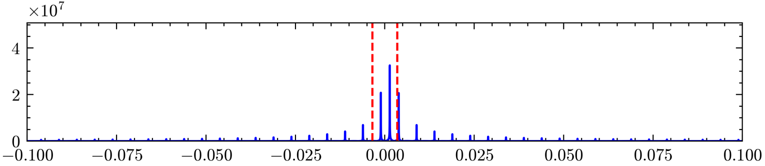


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$

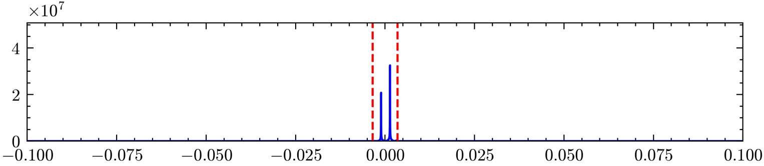


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ after Exit Pupil

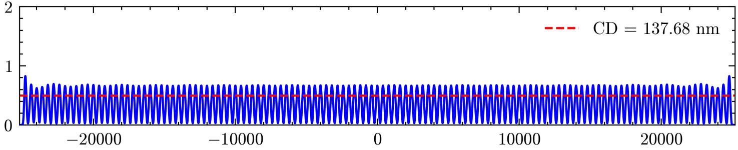


Figure: Magnitude Distribution on the Wafer

Simulation Results - Dense Line (6/10)

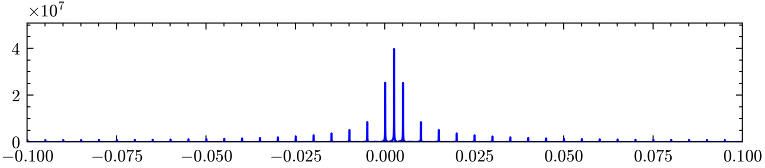


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

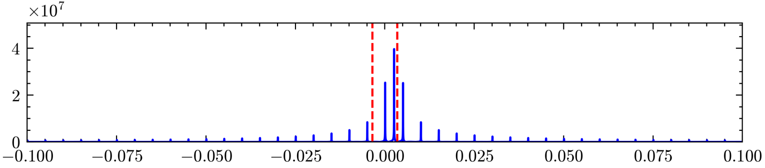


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

Simulation Results - Dense Line (7/10)

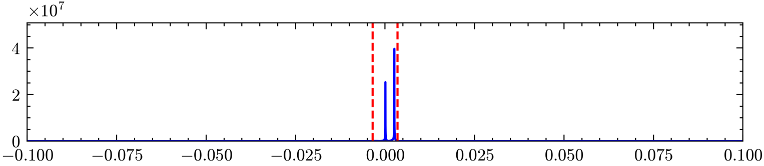


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E}_{\mathrm{in}}(s)\}$ after Exit Pupil

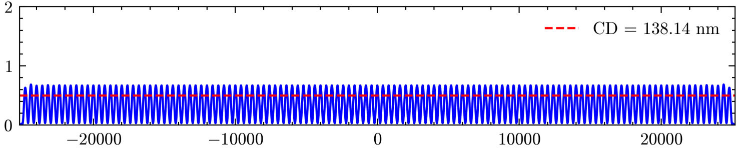


Figure: Magnitude Distribution on the Wafer

Simulation Results - Dense Line (8/10)

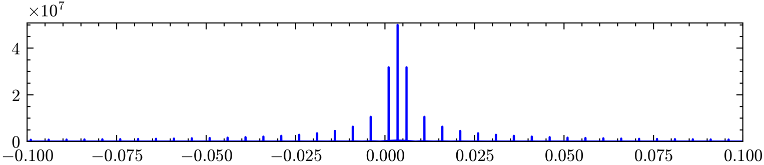


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

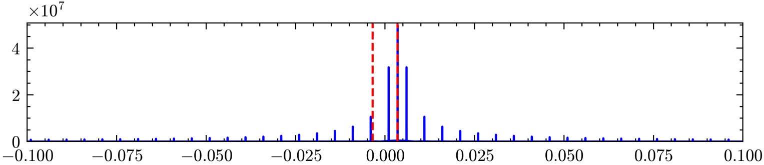


Figure: Absolute Value of Fourier Transform $\mathcal{F}\{\mathrm{E_{in}}(s)\}$

Simulation Results - Dense Line (9/10)

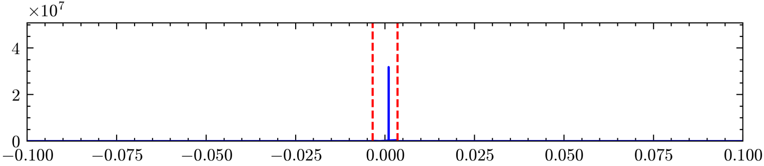


Figure: Absolute Value of Cut-off $\mathcal{F}\{\mathrm{E_{in}}(s)\}$ after Exit Pupil

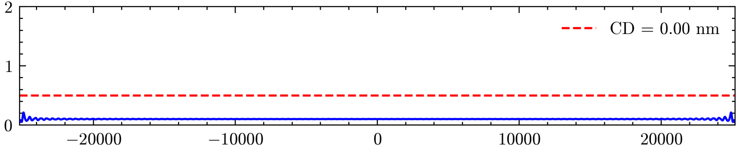


Figure: Magnitude Distribution on the Wafer

Simulation Results - Dense Line (10/10)

From these results, we can observe that

- The Fourier Transform patterns truly comprise impulses.
- At least 2 impulses to have pattern on Wafer.

Hence, it is important to let $\mathrm{C.D.}$ have enough width based on the observations.

Rayleigh Criterion

The Rayleigh Criterion equation is

$$\mathrm{C.D.} = k_1 \frac{\lambda}{\mathrm{N.A.}}$$

where

$$\mathrm{C.D.} \text{ is critical dimension}$$
$$\lambda \text{ is the wavelength of light}$$
$$\mathrm{N.A} \text{ is the numerical aperture}$$

Physical limit of k_1 is 0.25.

Simulation Results - Dense Line (11/)

From these results, we can observe that when the pattern shifts too much

- Dominant intensity parts are lost.
- Measured $\mathrm{C.D.}$ are decreased.

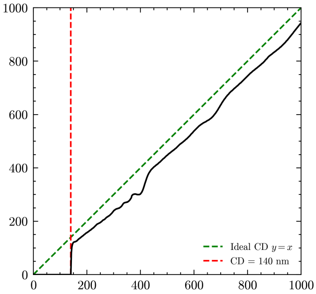
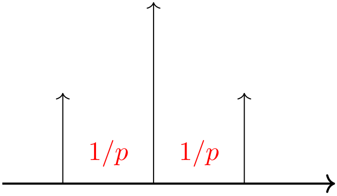


Figure: $\mathrm{C.D.}$ v.s. Measured $\mathrm{C.D.}$

Also, if we just change the width $(\mathrm{C.D.})$

- distance between impulses gets closer
- pattern shows when features are enough

As the distance between each order is $1/p$, the sampling frequency of $s_p(t)$.



For the dominant part, we have

$$\frac{1}{p} = \frac{1}{2\,\mathrm{C.D.}} = \frac{\mathrm{N.A.}}{\lambda}$$

Thus, it can be found that

$$k_1 = 0.5$$

Future Work

Discovering new ideas is a boost for the future.

Future Work

As we complete 1-D simulation, it helps me get the structure of the system.

- How a point source affects the aerial image.
- What single slits patterns and dense line patterns are.

However, it is an obstacle for me to simulate from source to image.

- Refer to books to know how a lens affects in a system.
- It is required to know 2-D Sampling.

Last, computing such a big matrix is a big problem.

- Find the minimum sampling rate to compute less.
- Learn how to use parallel computing.

Reference

1. F. Pedrotti, Introduction to Optics, 3rd Edition
   - Chapter 21 Fourier Optics
2. Oppenheim, Discrete-Time Signal Processing, 3rd Edition
   - Chapter 4 Sampling of Continuous-Time Signals
   - Chapter 8 The Discrete Fourier Transform