北京航空航天大學
BEIHANG UNIVERSITY

# ForICM 2024

## Forum on Intelligent Computer Mathematics

# Program


**Beihang University**

**Beijing, China, October 12-13, 2024**

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## Welcome

The Forum for Intelligent Computer Mathematics (ForICM 2024) aims to bring together experts and dedicated researchers in the field of computer mathematics, in particular those engaged in symbolic computation and automated reasoning, to review the latest developments and current trends of research and to examine the state of the art on the interaction and the impact of modern methodologies and techniques of machine-learning-driven artificial intelligence with/on classical computer mathematics. It will also serve to highlight the intelligent aspect of computer mathematics and provide an opportunity for participants to exchange new ideas and findings and present recent work, fostering fruitful research collaborations.

Specific topics for the forum include, but are not limited to:

- *Mechanization theory, algorithms, and applications in mathematics*
- *Symbolic computation, and the hybridization of symbolic and numerical computation*
- *Mathematical foundations of artificial intelligence, automated reasoning, and machine proofing*
- *Applications of computer mathematics in information security, program verification, robotics, and numerical control systems*
- *Mathematical methods in high-tech fields such as computer graphics, image processing, and pattern recognition*
- *Intelligent mathematical software that utilizes AI techniques*

For more information, please see the website https://jyangmath.github.io/ForICM.htm

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## Scientific Committee

**Xiao-Shan Gao** (AMSS, Chinese Academy of Sciences, China)

**Dongming Wang** (Beihang University, China and CNRS, France)

**Bican Xia** (Peking University, China)

**Lihong Zhi** (AMSS, Chinese Academy of Sciences, China)

## Organizing Committee

**Bo Huang** (Beihang University, China)

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**Jing Yang** (Guangxi Minzu University, China)

**Lihong Zhi** (AMSS, Chinese Academy of Sciences, China)

## Local Arrangements:

**Zijia Li** (AMSS, Chinese Academy of Sciences, China)

**Chenqi Mou** (Beihang University, China)

**Xiaoxian Tang** (Beihang University, China)

**Location:** Room D215, New Main Building, Beihang University

## Hosted by:

School of Mathematical Sciences and School of Artificial Intelligence, Beihang University

MOE Key Laboratory of Mathematics, Informatics and Behavioral Semantics

Beijing Advanced Innovation Center for Future Blockchain and Privacy Computing

## Co-organized by:

Academy of Mathematics and Systems Science

Peking University

Guangxi Minzu University

北京航空航天大學
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## Saturday, October 12th, 2024

| Time | |
|---|---|
| 8:30 | ***Registration*** |
| 9:00 - 9:10 | ***Welcome*** |
| 9:10 - 10:10 | **Session 1: Plenary talk 1** **Chair: Dingkang Wang**<br>**Naijun Zhan -** Synthesizing (Differential) Invariants by Reduction Non-Convex Programming to SDP |
| 10:10 - 10:40 | ***Break and Group Photo*** |
| | **Session 2: Chair: Jinsan Cheng** |
| 10:40 - 11:10 | **Xiaohong Jia -** Theory and Applications of Moving Curves and Moving Surfaces in Geometric Modeling |
| 11:10 - 11:40 | **Tuo Leng -** Towards the Human-like Deductive Reasoning and Beyond |
| 11:40 - 12:10 | **Dongchen Jiang -** Formal Representation of Mathematical Texts |
| 12:10 - 14:00 | ***Lunch Break*** |
| 14:00 - 15:00 | **Session 3: Plenary talk 2** **Chair: Jing Yang**<br>**Hoon Hong -** Parametrization of Univariate Polynomials with a Given Real Root Structure |
| 15:00 - 15:30 | ***Break*** |
| | **Session 4: Chair: Laigang Guo** |
| 15:30 - 16:00 | **Nan Li -** Improving MVG Algorithms through Algebraic Lens |
| 16:00 - 16:30 | **Changpeng Shao -** Testing Quantum Satisfiability |
| 16:30 - 17:00 | **Tao Zheng -** Isolating Complex Roots of a Mixed Trigonometric-Polynomial |
| 17:00 - 17:30 | **Ruijuan Jing -** Counting the Integer Points of Parametric Polytopes |
| 18:00 - | ***Reception Dinner*** |

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## Sunday, October 13th, 2024

| Time | |
|---|---|
| 9:00 - 10:00 | **Session 5: Plenary talk 3 Chair: Lihong Zhi**<br>**Xiaoshan Gao -** Mathematical Theory for Adversarial Deep Learning |
| 10:00 - 10:30 | ***Break*** |
| | **Session 6: Chair: Ruyong Feng** |
| 10:30 - 11:00 | **Haokun Li -** Methods for Solving the Satisfiability of Quantifier-Free Nonlinear Formulas |
| 11:00 - 11:30 | **Liyong Shen -** Real-time Tool Path Planning Using Deep Learning for Subtractive Manufacturing |
| 11:30 - 12:00 | **Wenyuan Wu -** Structural Analysis by Generalized Embedding Method for Integro-differential-algebraic Equations |
| 12:00 - 14:00 | ***Lunch Break*** |
| | **Session 7: Chair: Wei Li** |
| 14:00 - 14:30 | **Jiaqi Meng -** A Generalization of Habicht's Theorem for Subresultants of Several Univariate Polynomials |
| 14:30 - 15:00 | **Qiuye Song -** Reduced Gröbner Bases of Schubert Determinantal Ideals |
| 15:00 - 15:30 | **Linpeng Wang -** Decomposition of Polynomial Ideals into Triangular Regular Sequences |
| 15:30 - 16:00 | ***Break*** |
| | **Session 8: Chair: Jie Wang** |
| 16:00 - 16:30 | **Yue Jiao -** Multistability of Small Zero-One Reaction Networks |
| 16:30 - 17:00 | **Zhaoxing Qi -** Complexity Analysis of Triangular Decomposition over F_2 with Strongly Chordal Graphs |
| 17:00 - 17:30 | **Weifeng Shang -** Puzzle Ideals for Grassmannians |
| 17:30 - 17:40 | ***Closing*** |

# Plenary Talks

## Mathematical Theory for Adversarial Deep Learning

**Naijun Zhan** (Peking University)

**Abstract:** Deep neural network (DNN) is the central tool for the current breakthrough of artificial intelligence. However, the existence of adversarial samples makes the DNN vulnerable in safety-critical applications. Although many effective methods to defend adversaries haven been proposed, it is shown that adversaries seem inevitable. In this talk, I will present some progresses on the mathematical theories for adversarial deep learning, including the existence of robust DNNs, achieving optimal robust accuracies with Stackelberg games, the information-theoretically safe bias classifier against adversarial attacks.

## Parametrization of Univariate Polynomials with a Given Real Root Structure

**Hoon Hong** (North Carolina State University)

**Abstract:** Given a set, finding a suitable representation for it is a fundamental problem. There are two standard ways to represent a set: implicit and parametric. Implicit representations are good for checking whether an object is in the set or not, and parametric representations are good for exploring all the objects within the set.

The set in question is a fundamental set in real algebra, namely the set of the univariate polynomials which have a specified number of real roots within a given interval. There have been several works on the implicit representation of this set (such as Descartes, Budan-Fourier, Sturm, Sturm-Habicht, Hermite quadratic forms, etc). However, as far as we are aware, there have been no attempts for a parametrization.

In this work we provide a parametrization of this set. Furthermore, our parametrization has desirable properties. Specifically, the map is an almost bijective polynomial with a simple

domain (the positive orthant). We achieve this via three steps. First, we almost bijectively parametrize polynomials with no positive real roots. Then we carry out a rational change of coordinates to transform the positive real line into the specified interval. Lastly, we multiply by factors to create the specified number of real roots in the given interval. The final two steps occurring without breaking almost bijectivity.

This is a joint work of Hoon Hong and Ezra Nance.

# Mathematical Theory for Adversarial Deep Learning

**Xiaoshan Gao** (AMSS, Chinese Academy of Sciences)

**Abstract:** Deep neural network (DNN) is the central tool for the current breakthrough of artificial intelligence. However, the existence of adversarial samples makes the DNN vulnerable in safety-critical applications. Although many effective methods to defend adversaries haven been proposed, it is shown that adversaries seem inevitable. In this talk, I will present some progresses on the mathematical theories for adversarial deep learning, including the existence of robust DNNs, achieving optimal robust accuracies with Stackelberg games, the information-theoretically safe bias classifier against adversarial attacks.

# Short Talks

## Theory and Applications of Moving Curves and Moving Surfaces in Geometric Modeling

**Xiaohong Jia** (AMSS, Chinese Academy of Sciences)

**Abstract:** Moving curves and moving surfaces serve as a bridge between the parametric forms and implicit forms of rational curves and surfaces. Moving curves and moving surfaces are closely related to the Syzygy module theory in algebra. Over the past thirty years, moving curves and moving surfaces have successfully been applied to many geometric modeling problems such as implicitization, intersection and singularity computation, point inversion, etc. We shall review the developments of the technique of moving curves and moving surfaces, and discuss the possible future directions in their applications.

## Formal Representation of Mathematical Texts

**Dongchen Jiang** (Beijing Forestry University)

**Abstract:** Existing large language models for mathematics are usually trained by using natural languages, but the diversity of natural language expressions challenges models in capturing the logical connections among mathematical concepts. To obtain more accurate models, a formal description of a large amount of mathematical knowledge is preferred. Thus, we propose an approach that can automatically convert mathematical texts into predicate-logic based descriptions while keeping the original logical connections. A mathematical text is first parsed into a syntax tree by Chomsky's generative grammar, and then it is converted into a standardized form of predicate-logic by subordinate clause analyzing and sentence component analyzing. Experiment result shows that 471 of 500 sentences from the textbook *CALCULUS* by JAMES STEWART are correctly converted. By adding relevant lemmas and automatic reasoning tools, this approach can be used in automatic verification of mathematical text; and

it can also be used to formalize existing mathematical texts for mathematical large language model training.

## Towards the Human-like Deductive Reasoning and Beyond

**Tuo Leng** (Shanghai University)

**Abstract:** We will demonstrate a novel and consistent formal system for plane geometry, FormalGeo. This system plays a vital role as the bridge between symbolism and connectionism. With FormalGeo, we have enabled the integration with modern AI techniques, achieving human-like deductive automated reasoning without human supervision. Moreover, under this formalized framework, AI can now provide deductive solutions to IMO-level plane geometry problems as naturally as processing other natural languages. These proofs are readable, traceable, and verifiable. Additionally, we have constructed high-quality plane geometry problem datasets that conform to this formal system: FormalGeo7k (v1 & v2) and FormalGeo18k.

Furthermore, we have developed several plane geometry automated reasoning assistance tools and solvers integrated with modern AI models. These include the theorem sequence predictor FGeo-TP, which leverages LLMs; the new embedding method called FGeo-HyperGNet based on hypergraph theory, and the automatic solver FGeo-DRL, which utilizes deep reinforcement learning. The application of reinforcement learning and Monte Carlo Tree Search methods to deductive automated solving and proving in plane geometry is the first success attempt in this field based on our literature investigation. We will also talk about the auto-parser of FormalGeo.

## Improving MVG Algorithms through Algebraic Lens

**Nan Li** (Shenzhen University)

**Abstract:** Multiple View Geometry (MVG) is the algorithmic foundation of many computer vision problems. Motivated by the natural relationship between geometry and algebra, there

arises an active research field, Algebraic Vision, which intends to improve and analyze MVG algorithms through algebraic lens. In this talk, we will introduce some algebraic basics of MVG, and share our recent works on improving some model-based or learning-based MVG algorithms via linear/nonlinear algebraic techniques, including natural image stitching, rolling shutter geometry and monocular depth/pose estimation.

## Testing Quantum Satisfiability

**Changpeng Shao** (AMSS, Chinese Academy of Sciences)

**Abstract:** Quantum k-SAT (the problem of determining whether a k-local Hamiltonian is frustration-free) is known to be QMA_1-complete for k >= 3, and hence likely hard for quantum computers to solve. Building on a classical result of Alon and Shapira, we show that quantum k-SAT can be solved in randomised polynomial time given the `property testing' promise that the instance is either satisfiable (by any state) or far from satisfiable by a product state; by `far from satisfiable by a product state' we mean that \epsilon n^k constraints must be removed before a product state solution exists, for some fixed \epsilon > 0. The proof has two steps: we first show that for a satisfiable instance of quantum k-SAT, most subproblems on a constant number of qubits are satisfiable by a product state. We then show that for an instance of quantum k-SAT which is far from satisfiable by a product state, most subproblems are unsatisfiable by a product state. Given the promise, quantum k-SAT may therefore be solved by checking satisfiability by a product state on randomly chosen subsystems of constant size.

## Isolating Complex Roots of a Mixed Trigonometric-Polynomial

**Tao Zheng** (Xidian University)

**Abstract:** The problem of isolating roots of elementary transcendental functions (containing, e.g., trigonometric functions, exponential functions or logarithmic functions) has varies applications to scientific computing in engineering and economy, while the problem itself is of theoretical importance. In this report, an algorithm isolating complex roots of a given mixed trigonometric-polynomial (MTP) with rational coefficients is introduced. The algorithm artificially divides those complex roots into three parts: the periodic roots, the semi-periodic roots and the aperiodic roots, which generalizes the algorithm that isolating the real roots of an MTP. Algebraic methods (like resultant) and analytic properties of functions are combined in the algorithm, so that it returns periodic isolating rectangles and the multiplicity of the complex roots in them. Finally, the approach introduced in this report is also applicable to isolating real roots of transcendental equations of certain form with 2 variables, which is more general than isolating the complex roots of an MTP.

## Counting the Integer Points of Parametric Polytopes

**Ruijuan Jing** (Jiangsu University)

**Abstract:** In this presentation, I will discuss Barvinok's algorithm for integer point counting and its extension from non-parametric to parametric polytopes. I will also demonstrate a Maple implementation of this extension, highlighting a comprehensive framework that supports the solving of parametric systems.

## Methods for Solving the Satisfiability of Quantifier-Free Nonlinear Formulas

**Haokun Li** (Fermat Labs, Huawei)

**Abstract:** The satisfiability problem of nonlinear formulas is not only a hot topic in theoretical research but also a core issue in program verification. This report focuses on methods for solving quantifier-free nonlinear formulas, including optimizing the projection sequence of Cylindrical Algebraic Decomposition (CAD) using the chordal structure of polynomials, more effectively integrating with Conflict-Driven Clause Learning (CDCL) strategies via the sample-cell projection operator, and quickly finding solutions using local search algorithms.

The report will also introduce methods for solving transcendental equations and mixed trigonometric polynomials.

## Real-time Tool Path Planning Using Deep Learning for Subtractive Manufacturing

**Liyong Shen** (University of Chinese Academy of Sciences)

**Abstract:** We introduce an innovative research in the field of subtractive manufacturing, where we employ deep learning for real-time tool path planning. Our work commences with a comprehensive discussion on the evolution of tool path planning methodologies, setting the stage for our introduction of a novel approach. We present an adaptive iso-scallop height method for tool path generation, which we believe is particularly adept at integrating with learning algorithms. Additionally, we delve into the architecture of the B-spline surface reparameterization network and the design of corresponding loss functions. We conclude with a detailed exposition of our experimental results, which underscore the significant advantages of our method in CNC tool path planning.

## Structural Analysis by Generalized Embedding Method for Integro-differential-algebraic Equations

**Wenyuan Wu** (CIGIT, Chinese Academy of Sciences)

**Abstract:** Structural analysis is essential for understanding the characteristics of integro differential algebraic equations (idaes) before numerical analysis. The Σ-method, utilizing the signature matrix, effectively analyzes differential-algebraic equations (daes) and extends to idaes. However, challenges arise when the signature matrix becomes undefined or overestimated due to derivatives appearing in integral part. Additionally, when a singular Jacobian matrix is yielded after applying the Σ-method, existing conversion methods may fail to ensure termination. This paper addresses these issues by splitting an idae into two parts, redefining the signature matrix for both, and introducing a new degree-of-freedom measure. Furthermore, a point-based detection method corrects signature matrix overestimation. Finally, an embedding method based on our previous work regularizes nonlinear idaes with singular Jacobian matrices. When coupled with the collocation method, it can effectively solve idae numerically with high accuracy.

## A Generalization of Habicht's Theorem for Subresultants of Several Univariate Polynomials

**Jiaqi Meng** (Guangxi Minzu University)

**Abstract:** Subresultant of two univariate polynomials are one of the most classic and ubiquitous objects in computational algebra and algebraic geometry. In 1948, Habicht discovered and proved interesting relationships among subresultants. Those relationships were found to be useful for both structural understanding and efficient computation.

Often one needs to consider several (possibly more than two) polynomials. It is rather straightforward to generalize the notion of subresultants to several polynomials. However, it is not obvious (in fact, quite challenging) to generalize the Habicht's result to several polynomials. The main contribution of this talk is to provide such a generalization.

## Reduced Gröbner Bases of Schubert Determinantal Ideals

**Qiuye Song** (Beihang University)

**Abstract:** Schubert determinantal ideals are an important class of polynomial ideals that have attracted significant attention in algebraic combinatorics, representation theory, and computational algebra in the last two decades. The Schubert determinantal ideal of a permutation is naturally associated with the matrix Schubert variety and is closely related to the double Schubert polynomial of that permutation.

In particular, with Fulton generators identified as Gröbner bases of Schubert determinantal ideals w.r.t. any anti-diagonal term order, minimal Gröbner bases for such ideals

were also studied, where the authors introduced the notion of elusive minors and proved that they form minimal Gröbner bases of Schubert determinantal ideals. Furthermore, for Schubert determinantal ideals, while all the elusive minors form the reduced Gröbner bases when the defining permutations are vexillary, in the non-vexillary case we derived an explicit formula for computing the reduced Gröbner basis from elusive minors which avoids all algebraic operations. Based on this explicit formula, we developed an algorithm named RedGBSchubert to compute the reduced Gröbner bases of Schubert determinantal ideals, where this new algorithm outperforms the built-in function InterReduce in Maple for computing the reduced gröbner bases for complicated Schubert determinantal ideals in terms of computational efficiency.

## Decomposition of Polynomial Ideals into Triangular Regular Sequences

**Linpeng Wang** (Beihang University)

**Abstract:** The present work presents a new algebro-geometric approach that enables one to decompose any polynomial ideal into finitely many triangular regular sequences of polynomials such that certain implicit relations between the Hilbert polynomials and explicit relations between the sets of zeros of the ideals generated by the regular sequences are preserved. The decomposition algorithms make use of the properties and computations of W-characteristic sets of polynomial ideals and perform simultaneous sum-and-quotient operation, a key technique that is used implicitly in the recursive process of computing Hilbert polynomials. This study elaborates and reveals inherent connections between some commonly used concepts in the algorithmic theories of triangular sets, Gr\"obner bases, and Hilbert polynomials.

## Multistability of Small Zero-One Reaction Networks

**Yue Jiao** (Beihang University)

**Abstract:** Multistability of zero-one networks is a key dynamics feature enabling decision-making in cells. Since multistability (or, nondegenerate multistationarity) can be lifted from a "subnetwork" to large networks, we aim to explore the multistability problem of small zero-one networks.

We prove that any zero-one network with a one-dimensional stoichiometric subspace admits at most one (stable) positive steady state, and we completely classify all the one-dimensional zero-one networks according to if they indeed admits a (stable) positive steady state or not.

Also, we prove that any two-dimensional zero-one network with up to three species either admits only degenerate positive steady states, or admits at most one (stable) positive steady state. Moreover, using the tools of computational algebraic geometry, we provide a systematical way for detecting the smallest zero-one networks that admit nondegenerate multistationarity/multistability. We show that the smallest zero-one networks that admit nondegenerate multistationarity contain three species and five reactions, and the smallest zero-one networks that admit multistability contain three species and six reactions.

# Complexity Analysis of Triangular Decomposition over F_2 with Strongly Chordal Graphs

**Zhaoxing Qi** (Beihang University)

**Abstract:** In this talk, we will present the main content of a paper that I co-authored with Chenqi Mou. In this paper, we first introduce a new vertex order of graphs called the substrong elimination ordering based on maximal cliques of the graphs and prove that such an ordering can fully characterize strongly chordal graphs. By using this ordering we propose a new strategy for selecting polynomials for computation in algorithms for triangular decomposition over F_2. Then we show that when this ordering is used as the variable order for triangular decomposition of a polynomial set whose associated graph is strongly chordal, the variables of any polynomial occurring in the decomposition are contained in certain maximal cliques, which gives a uniform description of the structural changes in the decomposition when combined with a bounded treewidth. Consequently, we

prove that for any input set of l polynomials in n variables with a strongly chordal associated graph of treewidth m, the complexity for triangular decomposition over F_2 with the proposed selection strategy is O(4^mln (ml/(n-1))^(n-1)), smaller than the original O(l^n) when m<<n.

## Puzzle Ideals for Grassmannians

**Weifeng Shang** (Beihang University)

**Abstract:** Puzzles are a versatile combinatorial tool to interpret the Littlewood-Richardson coefficients for Grassmannians. In this talk, we will propose the concept of puzzle ideals whose varieties one-one correspond to the tilings of puzzles and present an algebraic framework to construct the puzzle ideals which works with the Knutson-Tao-Woodward puzzle and its T-equivariant and K-theoretic variants for Grassmannians. For puzzles for which one side is free, we propose the side-free puzzle ideals whose varieties one-one correspond to the tilings of side-free puzzles, and the elimination ideals of the side-free puzzle ideals contain all the information of the structure constants for Grassmannians with respect to the free side. Besides the underlying algebraic importance of the introduction of these puzzle ideals is the computational feasibility to find all the tilings of the puzzles for Grassmannians by solving the defining polynomial systems, demonstrated with illustrative puzzles via computation of Gröbner bases.

This work is a collaborative effort with Prof. Chenqi Mou.