Problem Set 4 - Graphs

Problem B

This question just took a little bit of playing around with graphs on paper to try and figure out. We want to find a way to split edges into simple paths, such that each edge only ends up on one path, and that the paths all share at least one common vertex. I think I came to the solution by drawing out a bunch of graphs, and trying to count the degree of each of the edges - trying to find a pattern in graphs where we are able to create a decomposition, and graphs where we aren't.

Ultimately, the observation that I came to, was that in order for there to be a decomposition, every node has to have degree 2 or less (i.e. two outgoing/incoming edges), except for one, which can have any degree of node.

Everytime there is a node with degree 3 or more, we are looking at a place where the graph diverges. This is ultimately fine, if there is only one location where this happens, as that would just be the vertex shared across the simple paths. But if there are multiple, in order to have all simple paths share a vertex, we would need to start duplicating edges across the simple paths.

From this observation, I was able to determine whether or not a decomposition exists for any given graph. In order to actually assign the paths, all we need to do is identify all the leaf nodes. If there is only one vertex in the graph that has a degree that is greater than 2, we can use that one as the shared vertex in all the paths, and since we know from above that it is the only divergence point, we can draw paths from that vertex to all the leaves to get our decomposition :)