Problem Set 1 - General

<u>Question D</u>

This question was a pretty hard one to wrap my head around at first. I think the key thing that got me over the line was trying to view it from the lens of minimising the total number of marks on each day. The total marks (ti) on any given day is equal to the number of marks above the water on that day (mi), + 1, + the number of marks below the water on that day (di). So if we were to minimise the total number of marks across all days, we minimise the total number of marks below the water.

For any one of the i'th days, we want to minimise the total number of marks - and notice that the total number of marks on any given day i, must be greater than the max of:

- Total marks from prev day,
- Number of marks above the water on the current day.

So, on a first pass, we can calculate the minimum possible value of total marks on each day, and then compute the number of marks below the water on each of these days. However, we also note that the total number of marks can only increase by at most one per day, (i.e. we can't make a jump from 1 mark to 3 marks from day 2 to 3), so we need each day to have enough marks to account for the number of marks above the water on a future day.

So to add this further minimum bound to the number of marks needed on each day, we can iterate right to left, keeping a counter that starts at the required total marks for the last day and decreases by 1 as we move backward. At each step, we take the maximum of this counter and the current total marks to ensure the requirements for all future days are met. We assign this value to the current day's total marks.

While doing all of this, we keep track of the marks below the water with res, and the formula total marks = mi + 1 + di. In summary, the first past sets a lower bound for ti, and the second pass ensures that it satisfies the future constraints for ti.