



# **Alleviating Over-Smoothing in LightGCN for Improved Graph Representation Learning in Recommender Systems**

ST5188 Final Project Presentation (Project Group 14)

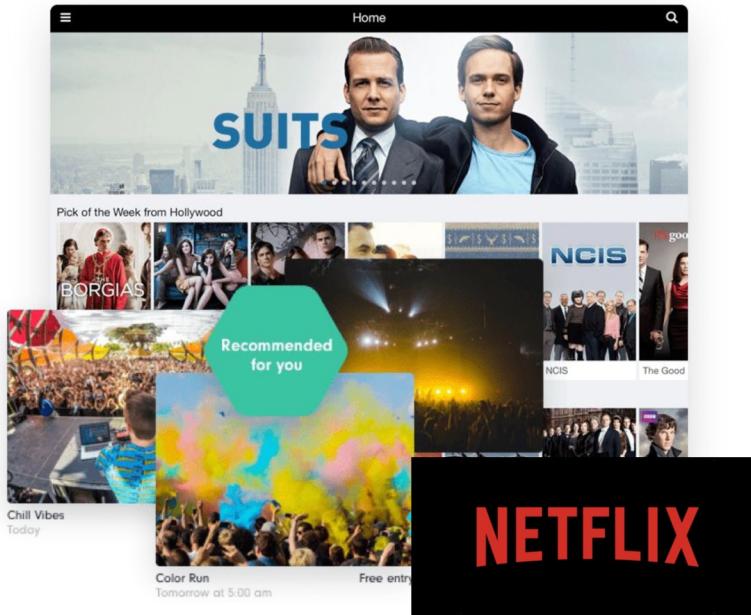
*Yeo Ming Jie, Jonathan*

*Chen Pu*

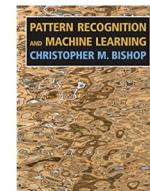
*Shi Qin*

# Recommender Systems: Quick Intro

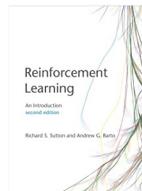
- Recommendation is everywhere - but how does it work?



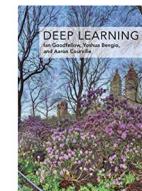
More items to explore



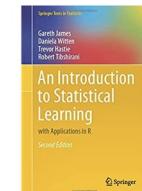
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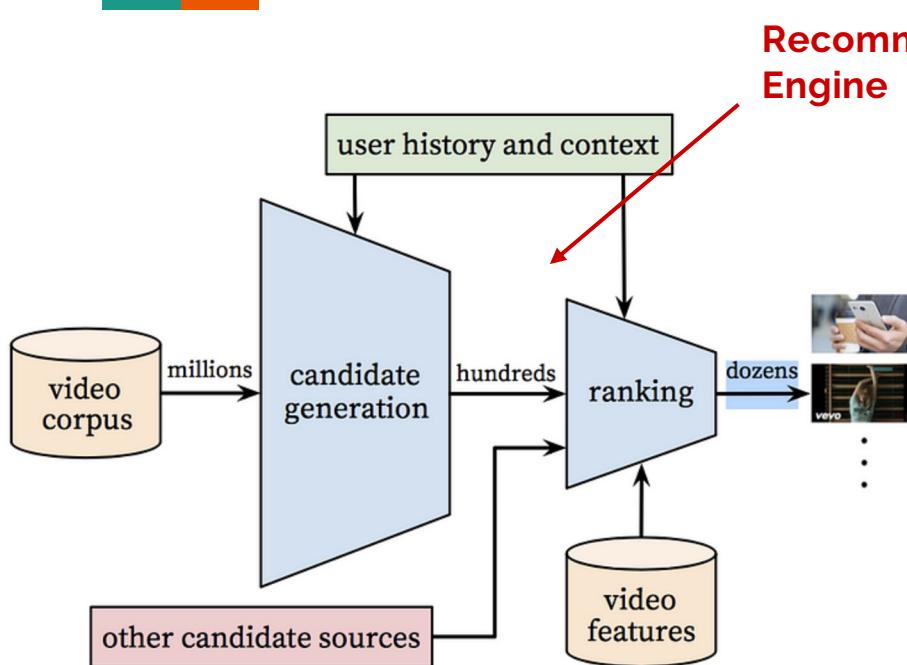
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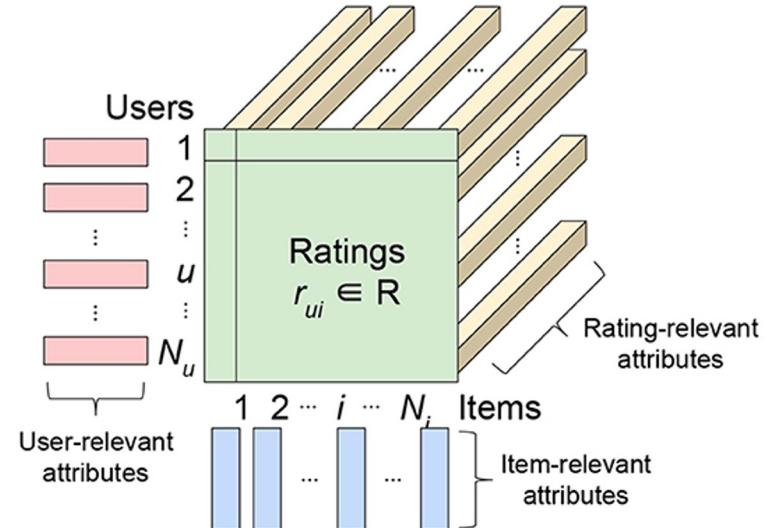
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# Recommender Systems: Quick Intro

- Recommendation is everywhere - but how does it work?



Recommendation Engine

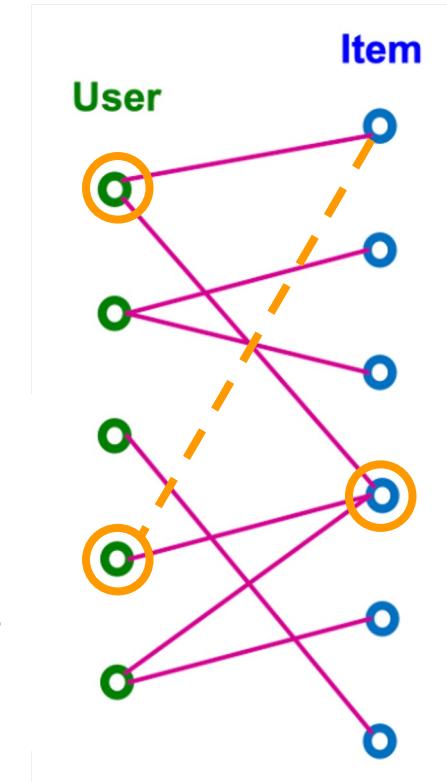


Frame as a representation learning problem

# Recommender Systems as Graphs

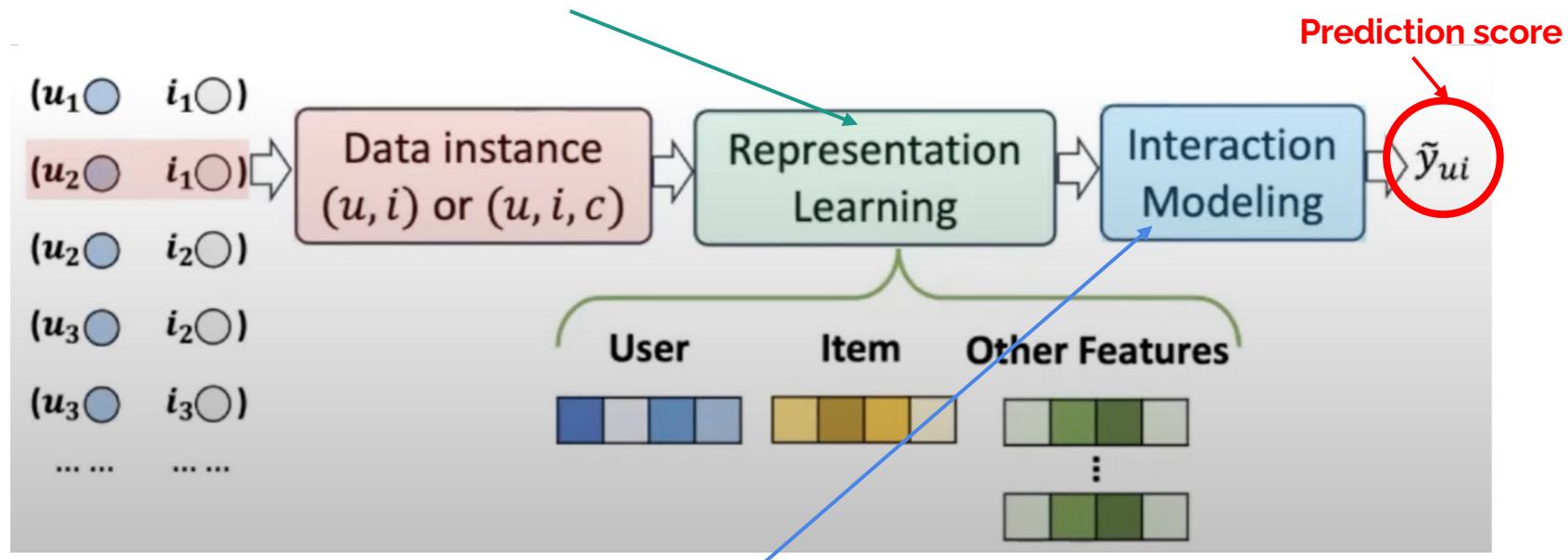
Collaborative Filtering (CF): Behaviourally similar users have similar item preferences

- Recommender systems can be naturally modelled as a **bipartite graph**
  - Nodes: Users and Items
  - Edges: User-item interactions
- Goal: Recommend items users might like
  - Cast as **link prediction** problem i.e. Given past edges, predict new user-item interaction edges



# Recommendation as Representation Learning Task

**Representation Learning:** Generate representations for each input feature



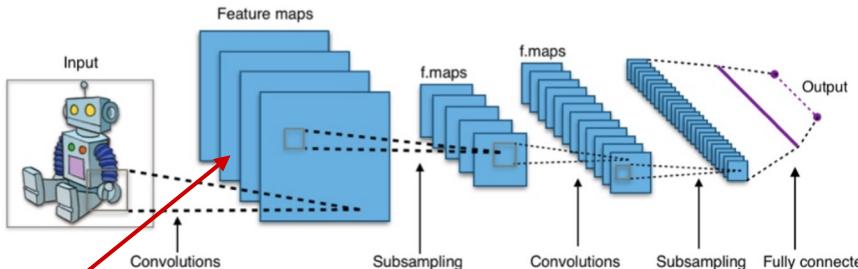
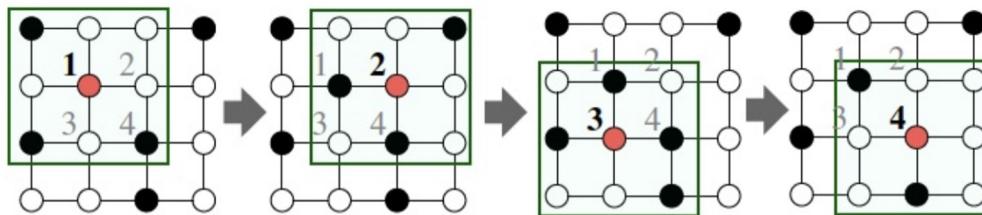
**Interaction Modelling:** Perform prediction based on user-item interactions

Use a Neural Network that can operate on graphs to perform  
Representation Learning

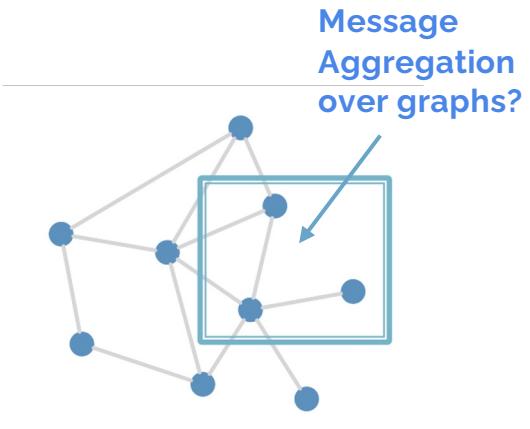
# Graph Neural Networks (GNNs)

- Intuitively, GNNs can be viewed as a generalization of CNNs

CNNs: Convolution as message aggregation over a 2D grid



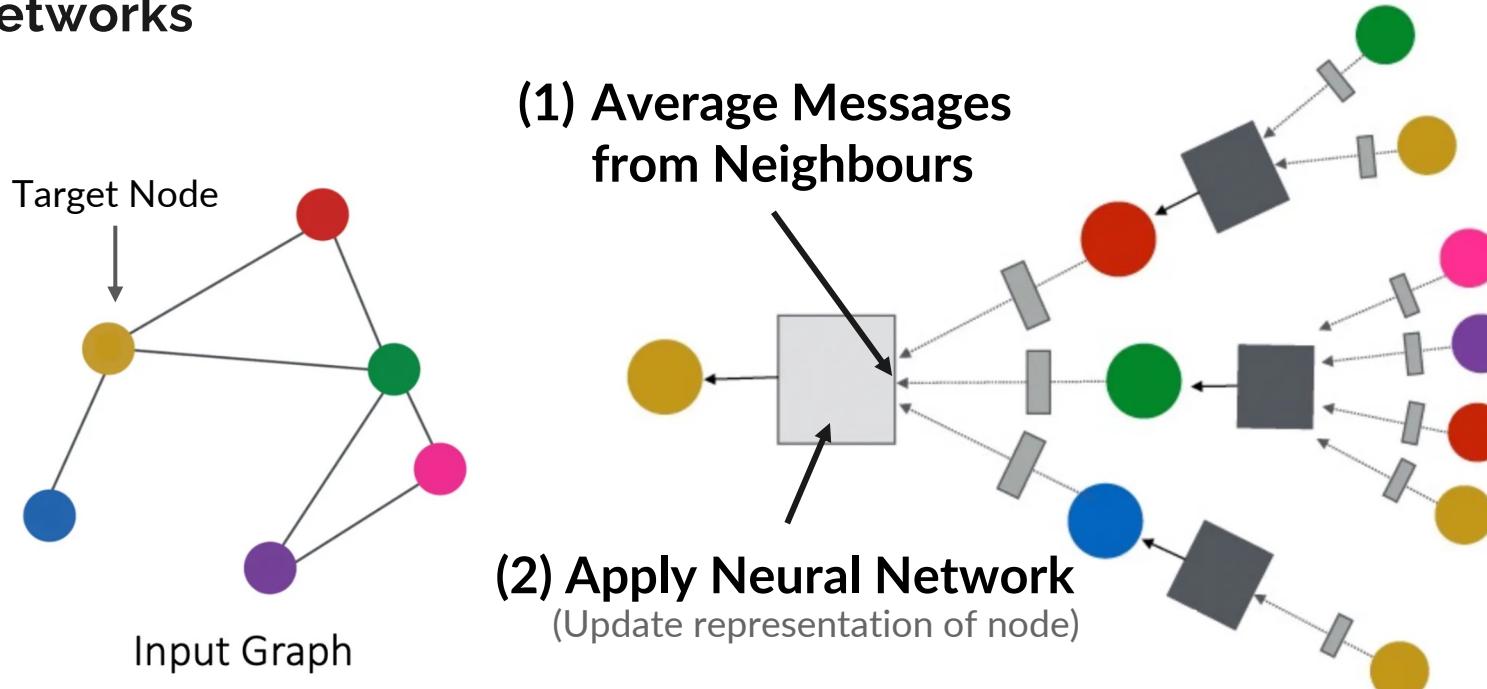
Feature Learning



GNNs consist of multiple permutation invariant operations

# GNNs: Message Aggregation Framework

- Nodes aggregate information from their neighbours using neural networks



**GNN Layer = Message + Aggregation**

# LightGCN Model for Recommendation

*[He et al. 2020]*

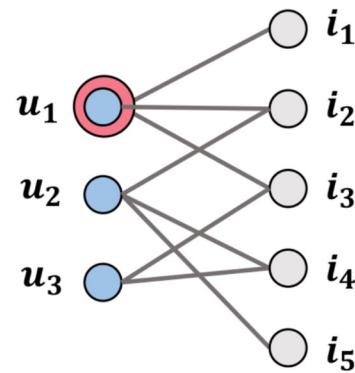
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# LightGCN Overview

- Revisiting CF via **Higher-Order Connectivity**

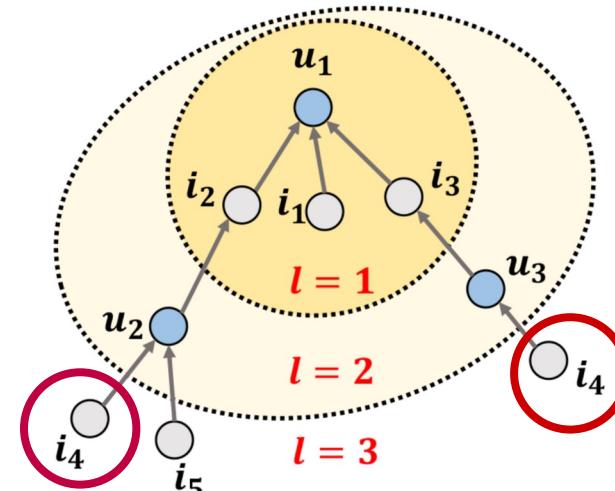
( Similar users → Similar item preferences )

Why might  $u_1$  like  $i_4$ ?



User-Item Interaction Graph

Obtain the collaborative signal via Message Passing



High-order Connectivity for  $u_1$

Higher-Order Connectivity as a natural way to **encode collaborative signal** in the interaction graph structure

# LightGCN: Embedding Propagation Layers

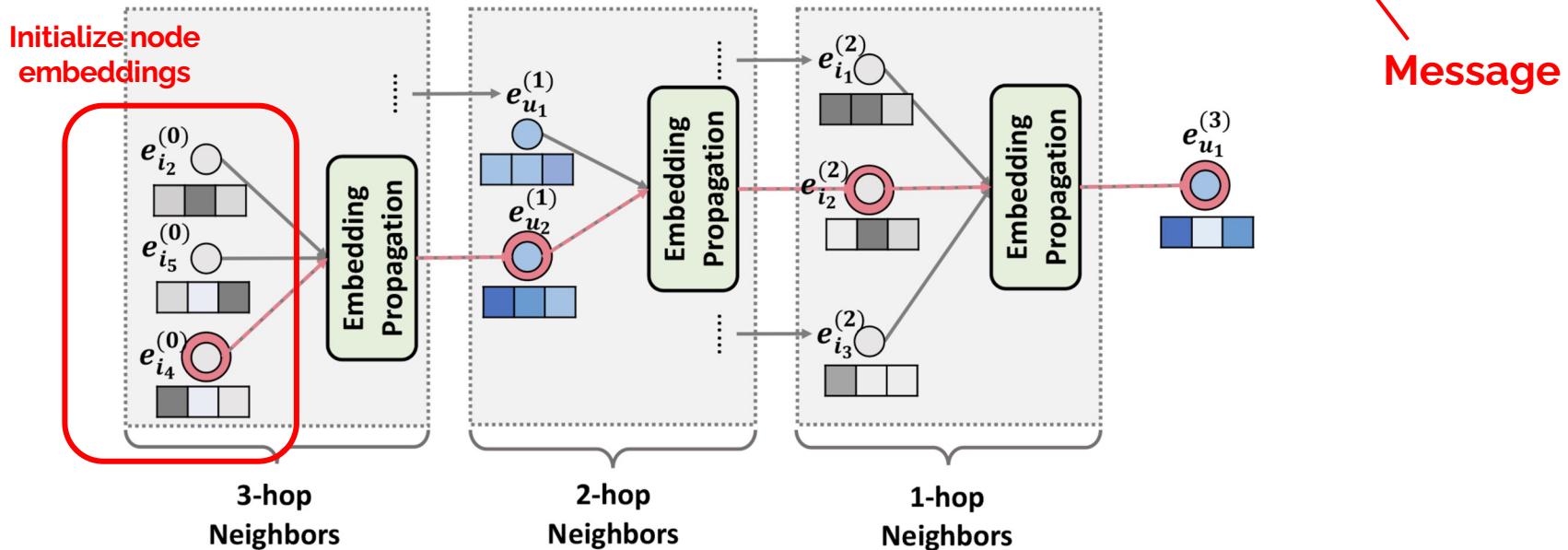
- Propagation Rule:

Aggregating  
over node  
neighbours

$$\mathbf{e}_u^{(k+1)} = \sum_{i \in \mathcal{N}_u} \frac{1}{\sqrt{|\mathcal{N}_u|} \sqrt{|\mathcal{N}_i|}} \mathbf{e}_i^{(k)},$$
$$\mathbf{e}_i^{(k+1)} = \sum_{u \in \mathcal{N}_i} \frac{1}{\sqrt{|\mathcal{N}_i|} \sqrt{|\mathcal{N}_u|}} \mathbf{e}_u^{(k)}.$$

Message

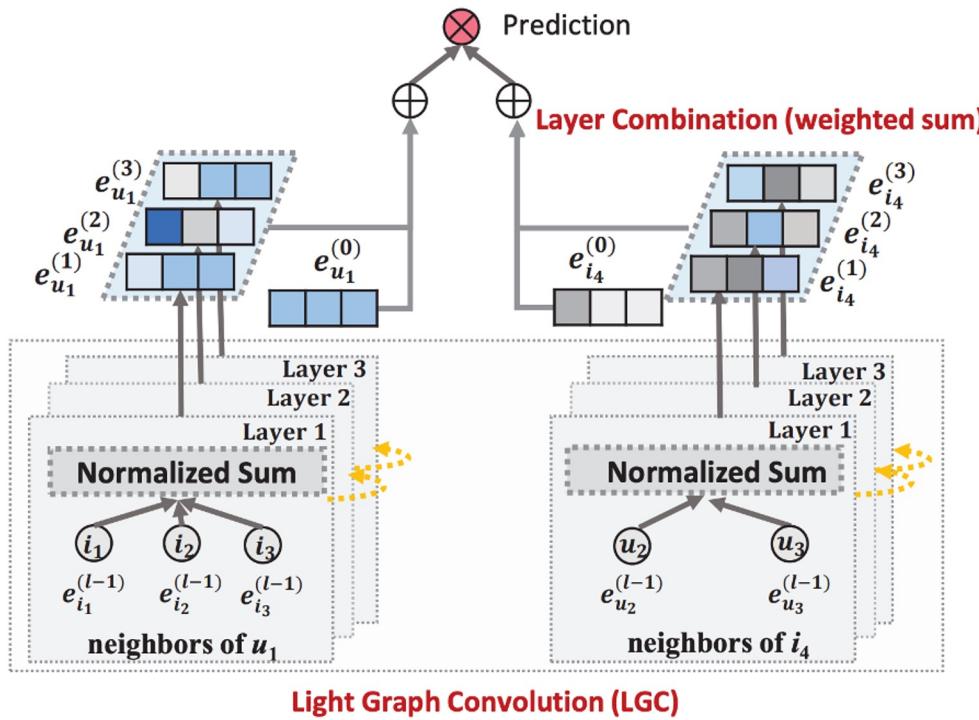
Illustration of Higher-order Propagation



# LightGCN: Model Prediction

$$\alpha_k = \frac{1}{K + 1}$$

Weighted linear combination of embeddings at each layer



- Final User Embedding:

$$\mathbf{e}_u = \sum_{k=0}^K \alpha_k \mathbf{e}_u^{(k)}$$

- Final Item Embedding:

$$\mathbf{e}_i = \sum_{k=0}^K \alpha_k \mathbf{e}_i^{(k)}$$

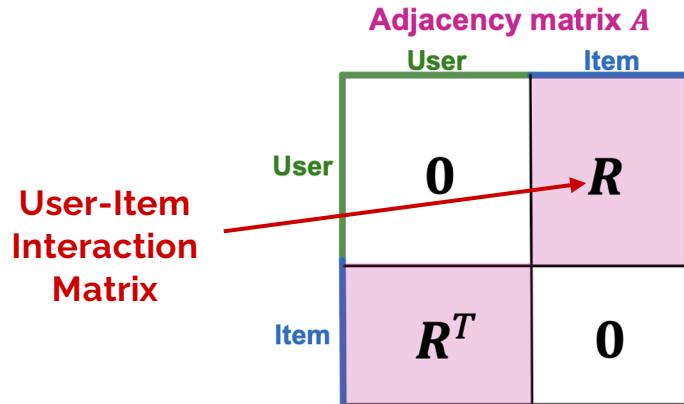
- Prediction: (Taking inner product)

$$\hat{y}_{ui} = \mathbf{e}_u^T \mathbf{e}_i$$

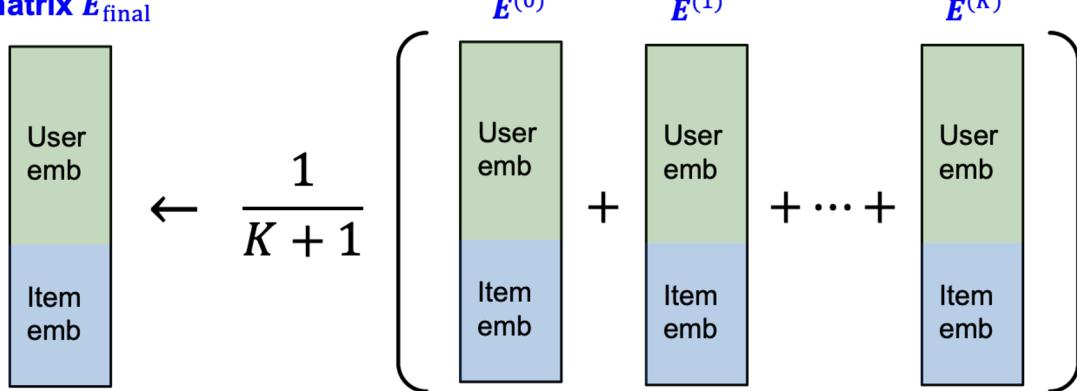
# LightGCN: Matrix Formulation

- Propagation Rule (Matrix Form):

$$\mathbf{E}^{(k+1)} = \underbrace{\left(\mathbf{D}^{-\frac{1}{2}} \mathbf{A} \mathbf{D}^{-\frac{1}{2}}\right)}_{\text{Symmetrically normalized matrix } \tilde{\mathbf{A}}} \mathbf{E}^{(k)}$$



Embedding matrix  $\mathbf{E}_{\text{final}}$



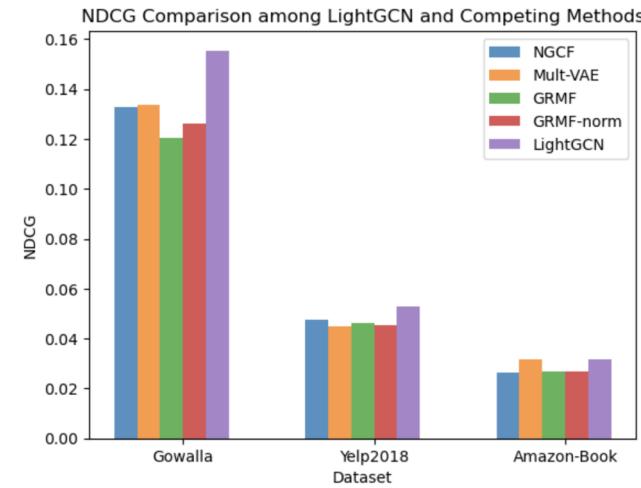
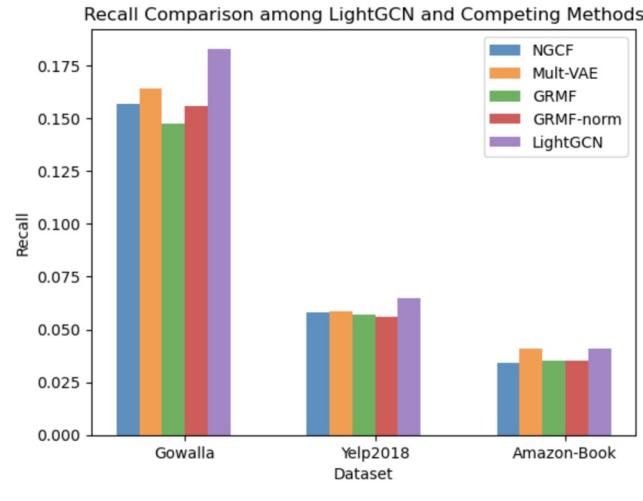
Bipartite Graph Structure

Final Embedding Matrix (Expansion)

$$\begin{aligned}\mathbf{E} &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \mathbf{E}^{(1)} + \alpha_2 \mathbf{E}^{(2)} + \dots + \alpha_K \mathbf{E}^{(K)} \\ &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \tilde{\mathbf{A}} \mathbf{E}^{(0)} + \alpha_2 \tilde{\mathbf{A}}^2 \mathbf{E}^{(0)} + \dots + \alpha_K \tilde{\mathbf{A}}^K \mathbf{E}^{(0)}\end{aligned}$$

# LightGCN: Successes and Limitations

LightGCN demonstrates strong performance over traditional GCNs on recommendation evaluation metrics.



Propagation Rule is simple, easily adaptable to various other tasks e.g. social recommendation systems (SocialLGN)

# LightGCN: Successes and Limitations

LightGCN demonstrates strong performance over traditional GCNs on

- **High Computational Costs → Poor Scalability**
  - Passing the full graph through each propagation layer
  - Core component of neighborhood aggregation requires repeated multiplication of an adjacency matrix
  - e.g. 3 layer LightGCN requiring more than 700 epochs for convergence.
- **Over-Smoothing Problem**
  - Empirically, observed that LightGCN performance degrades after 2-3 layers.

tasks e.g. social recommendation systems (SocialLGN)

# The Over-Smoothing Problem

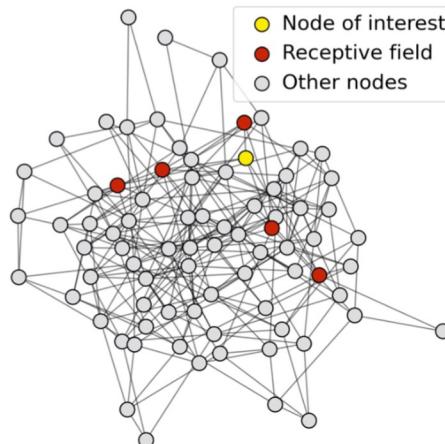
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# Over-Smoothing in GNNs (In General)

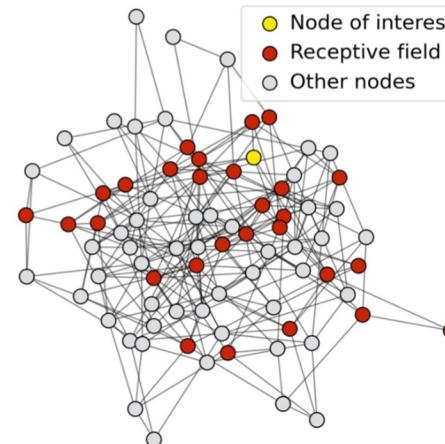
- Stacking too many GNN layers → All node embeddings converge to the same value

In a K-layer GNN, each node has a receptive field of K-hop neighbourhood

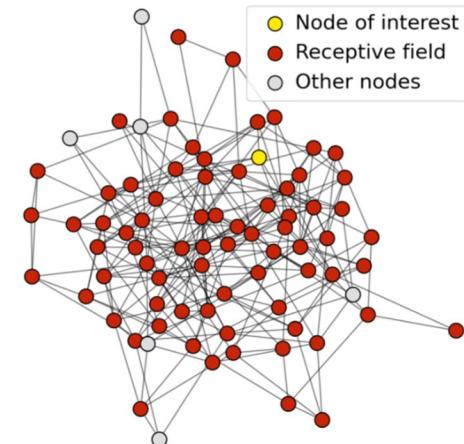
Receptive field for  
1-layer GNN



Receptive field for  
2-layer GNN



Receptive field for  
3-layer GNN



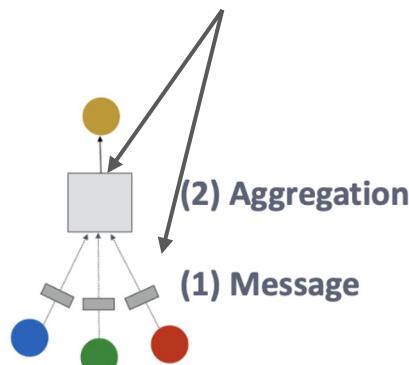
Receptive Field Overlap: Shared Neighbours Grows Rapidly

# Traditional Approaches to Over-smoothing Problem

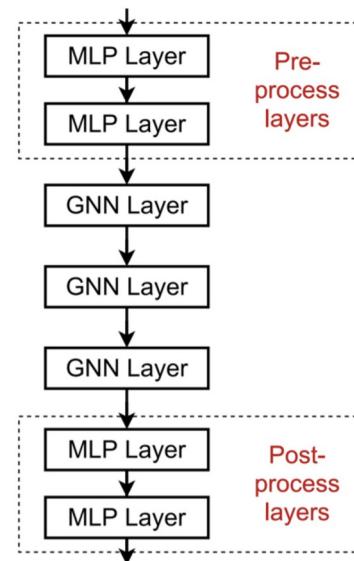
- Main Lesson: Adding more GNN layers does not always help!

**Solution: Make a shallow GNN more expressive**

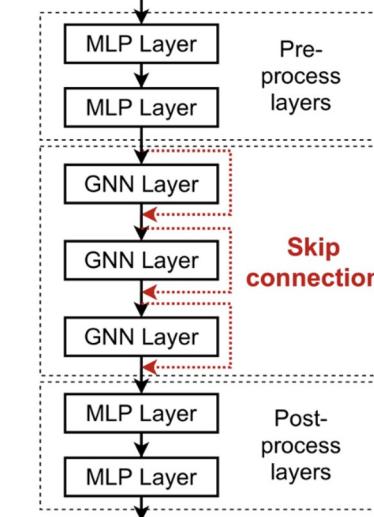
Each box (aggregation/transformation)  
could become a deep neural network



Increase expressive power  
within each GNN layer



Add non-message passing  
layers



Adding skip connections

# **Proposed Solutions**

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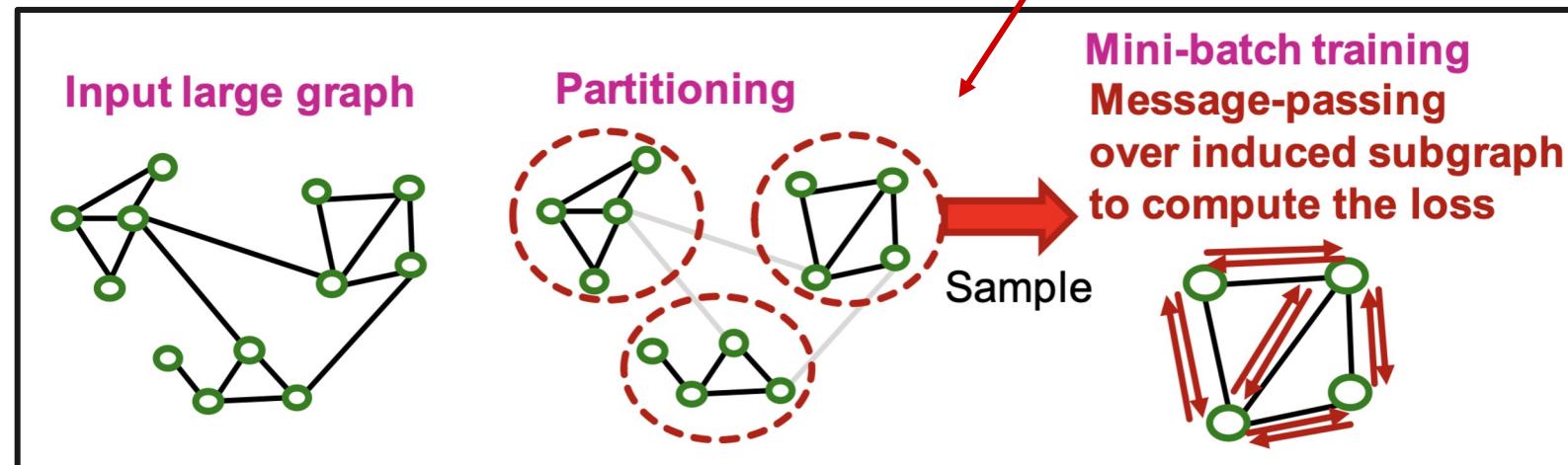
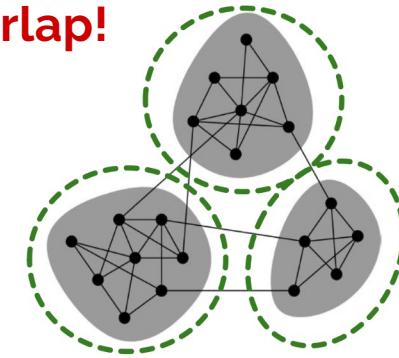
# Solution 1: Cluster-Based Sampling (Chiang et al, 2019)

- In full-batch GNN, all node embeddings are updated together using embeddings of the previous layer

Real-world graphs exhibit community structure

**Key Insight:** Sample a community as a subgraph.  
Each subgraph retains essential connectivity  
patterns of the original graph

Minimize Receptive  
Field Overlap!

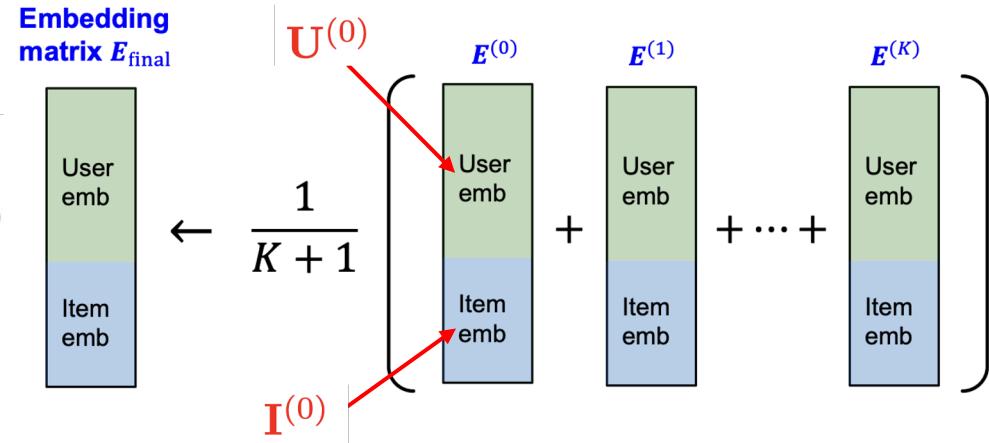


# Solution 2: Collapsing User-Item Representation

## Final Embedding Matrix (Expansion)

$$\begin{aligned} \mathbf{E} &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \mathbf{E}^{(1)} + \alpha_2 \mathbf{E}^{(2)} + \dots + \alpha_K \mathbf{E}^{(K)} \\ &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \tilde{\mathbf{A}} \mathbf{E}^{(0)} + \alpha_2 \tilde{\mathbf{A}}^2 \mathbf{E}^{(0)} + \dots + \alpha_K \tilde{\mathbf{A}}^K \mathbf{E}^{(0)} \end{aligned}$$

Consider 4-layer LGCN, let  $\alpha_k = 1$  for all k



## User-Item Embedding Matrices (By Block Matrix Multiplication)

$$\mathbf{U} = \mathbf{U}^{(0)} + \tilde{\mathbf{R}} \mathbf{I}^{(0)} + \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \mathbf{U}^{(0)} + \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \mathbf{I}^{(0)} + \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \mathbf{U}^{(0)}$$

$$\mathbf{I} = \mathbf{I}^{(0)} + \tilde{\mathbf{R}}^T \mathbf{U}^{(0)} + \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \mathbf{I}^{(0)} + \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \mathbf{U}^{(0)} + \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \tilde{\mathbf{R}}^T \tilde{\mathbf{R}} \mathbf{I}^{(0)}$$

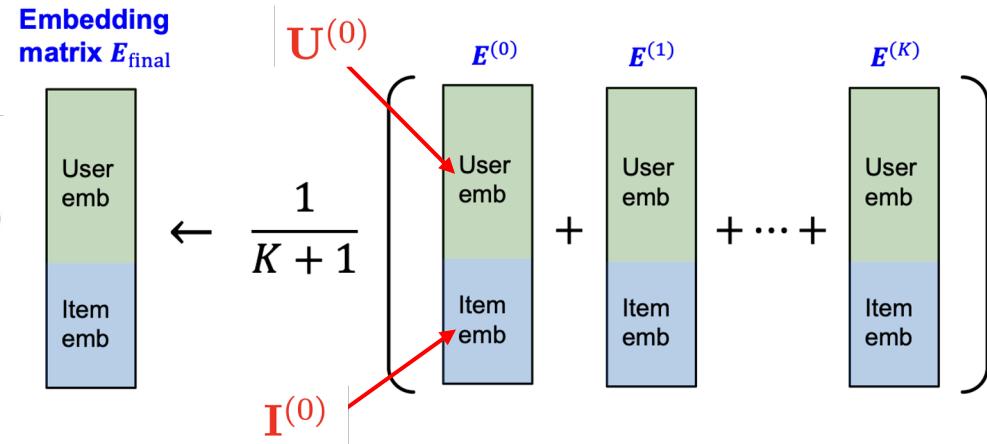
Set  $\mathbf{U}^{(0)} = \tilde{\mathbf{R}} \mathbf{I}^{(0)}$

# Solution 2: Collapsing User-Item Representation

## Final Embedding Matrix (Expansion)

$$\begin{aligned} \mathbf{E} &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \mathbf{E}^{(1)} + \alpha_2 \mathbf{E}^{(2)} + \dots + \alpha_K \mathbf{E}^{(K)} \\ &= \alpha_0 \mathbf{E}^{(0)} + \alpha_1 \tilde{\mathbf{A}} \mathbf{E}^{(0)} + \alpha_2 \tilde{\mathbf{A}}^2 \mathbf{E}^{(0)} + \dots + \alpha_K \tilde{\mathbf{A}}^K \mathbf{E}^{(0)} \end{aligned}$$

Consider 4-layer LGCN, let  $\alpha_k = 1$  for all k



## User-Item Embedding Matrices (SimpleLGN)

Step 1:

$$\mathbf{U} = 2\mathbf{R}\mathbf{I}^{(0)} + 2\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)} + \mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}$$

$$\mathbf{I} = \mathbf{I}^{(0)} + 2\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)} + 2\mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}$$

Note that the 4-Layer LGCN has reduced to 3-Layers of Propagation!

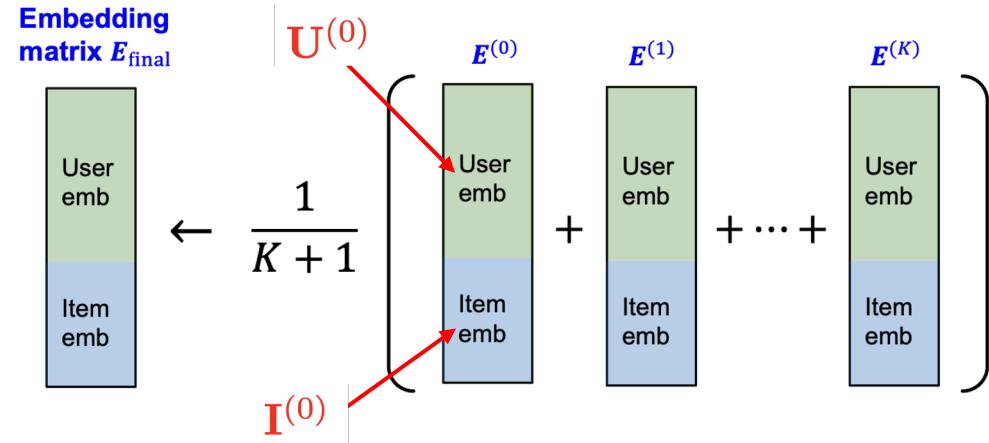
Final User Embedding matrix can be expressed fully in terms of the Item Embedding matrix!

# Solution 2: Collapsing User-Item Representation

## Embedding Matrix (Update)

$$\mathbf{U} = \underline{2\mathbf{R}\mathbf{I}^{(0)}} + \underline{2\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}} + \underline{\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}}$$
$$\mathbf{I} = \underline{\mathbf{I}^{(0)}} + \underline{2\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}} + \underline{2\mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}}$$

Step 2: Consider more simple LGCN,  
let  $\alpha_k = \beta_k = 1$  or  $\mathbf{U}(k) = \mathbf{R}\mathbf{I}(k)$  for all  $k$



## User-Item Embedding Matrices (Weight Sharing SimpleLGN)

Step 3:  
(Trainable)

$$\mathbf{U} = \alpha_0 \mathbf{R}\mathbf{I}^{(0)} + \alpha_1 \mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)} + \alpha_2 \mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)} = \mathbf{R}\mathbf{I}$$

$$\mathbf{I} = \alpha_0 \mathbf{I}^{(0)} + \alpha_1 \mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)} + \alpha_2 \mathbf{R}^T\mathbf{R}\mathbf{R}^T\mathbf{R}\mathbf{I}^{(0)}$$

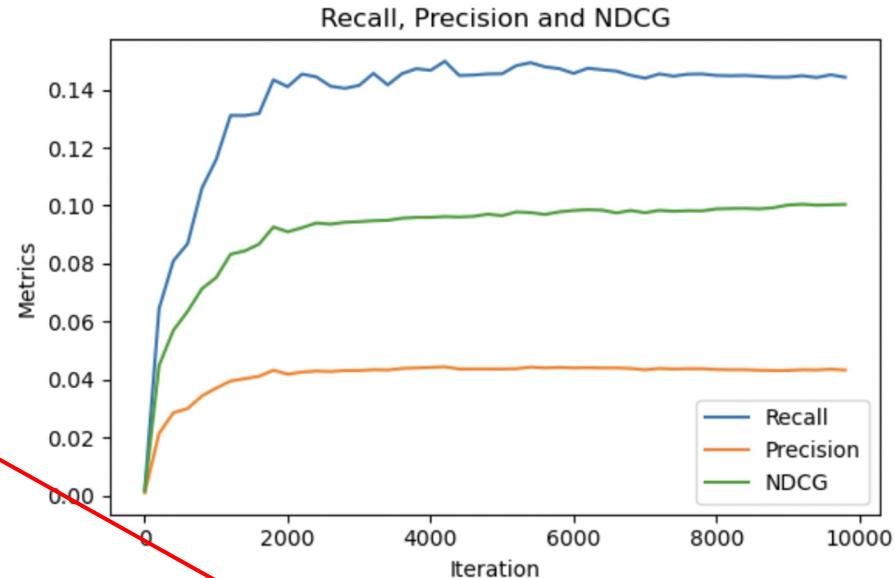
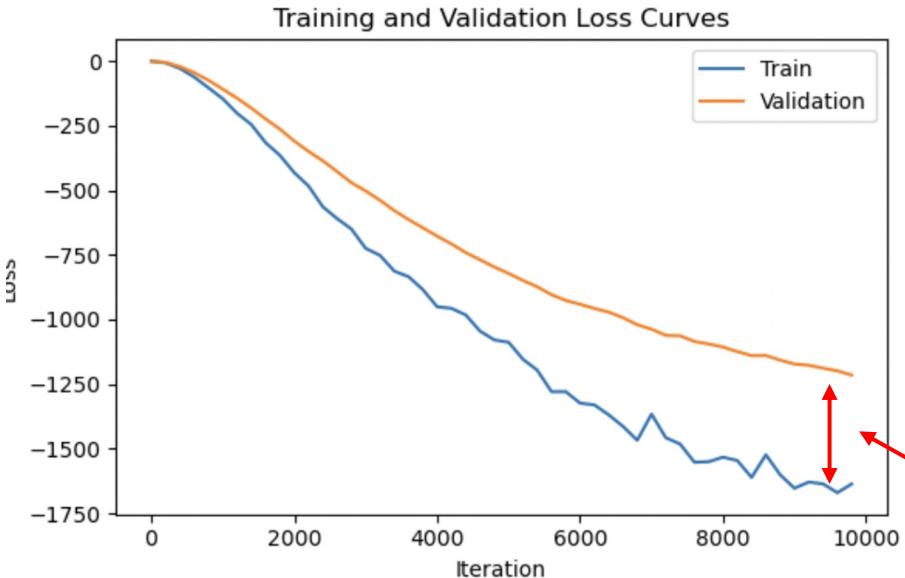
If we make the coefficients of user embedding same as the item embedding, then the final user embedding can be expressed as  $\mathbf{U} = \mathbf{R}\mathbf{I}$

# Experimental Results: Cluster-Based Sampling

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# Experiment 1: Original LightGCN Model

- Ran on own implementation of LightGCN model - MovieLens100K



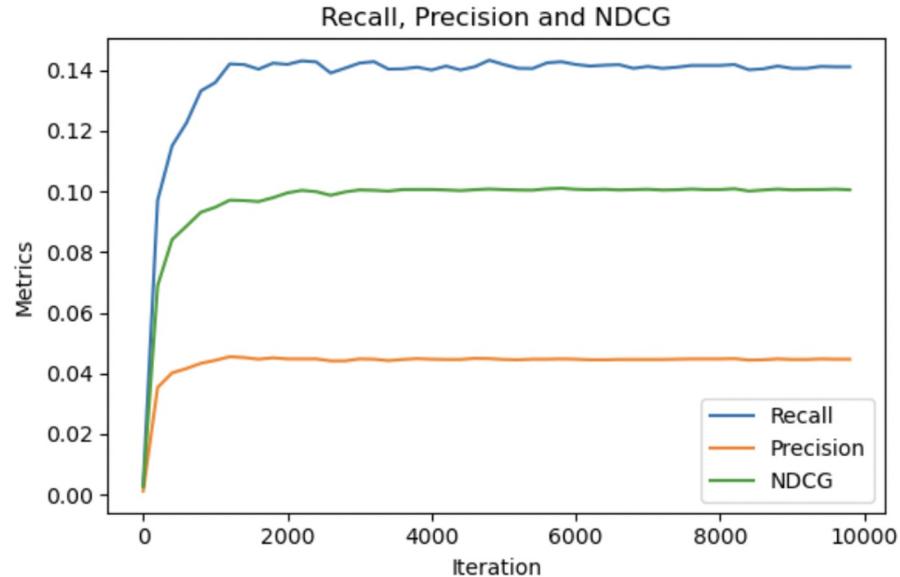
Training Time: 11 mins 47 secs

Overfitting observed

Note: 80 / 10 / 10  
Training-Validation-Test Split

# Experiment 1: Cluster-Based Sampling

Interesting result: Validation curve lies below training curve



Training Time: 6 mins 00 secs (Halved Training Time)

# Experiment 1: Cluster-Based Sampling

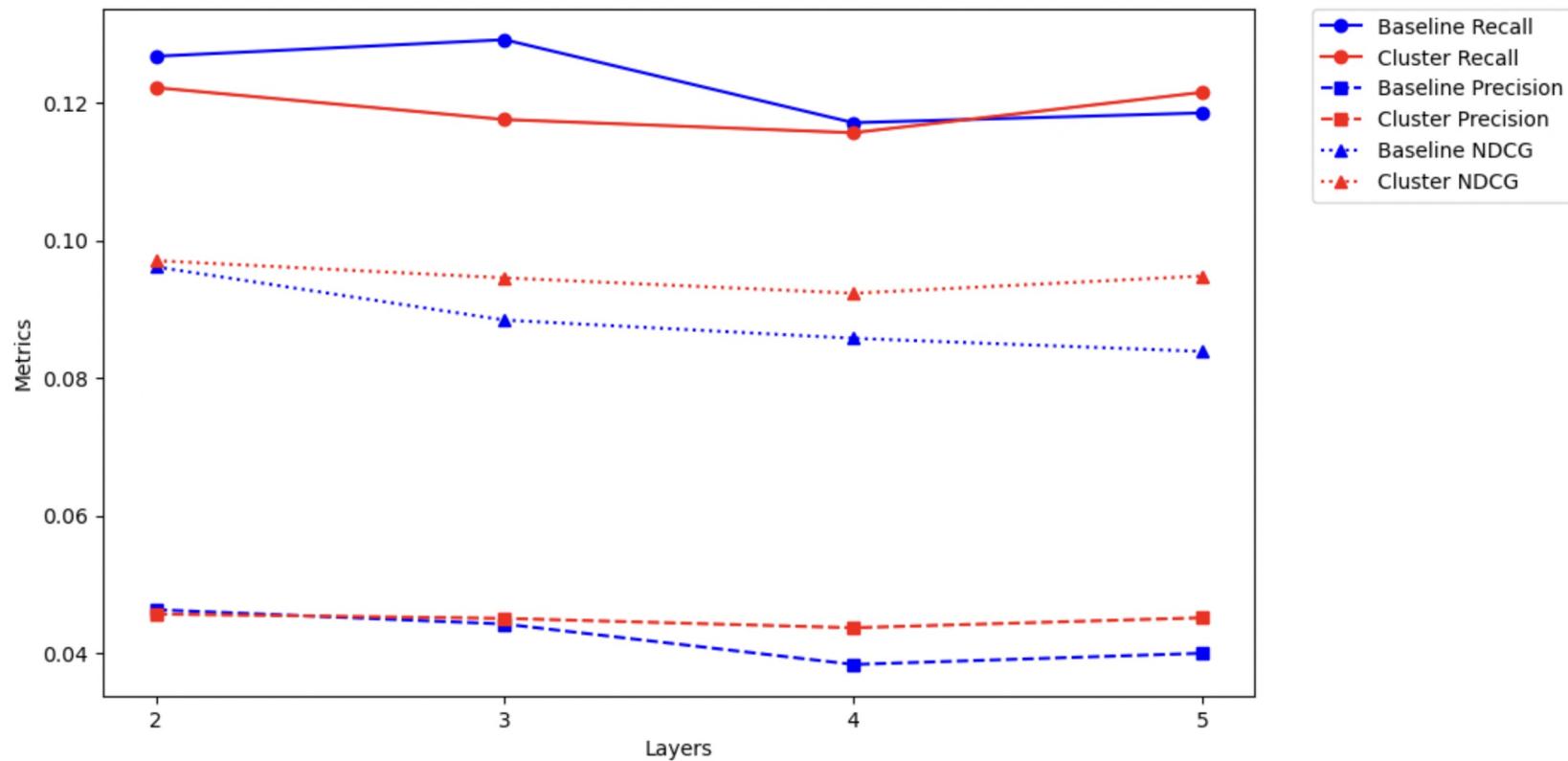
- Training Time + Results over Test Data:

Cluster-LightGCN has better Precision@20 and NDCG@20 scores over Baseline

Model	Layers	Training Time (s)	Recall@20	Precision@20	NDCG@20
Baseline LightGCN	3	707.76	0.12919	0.04420	0.08841
Cluster-LightGCN	3	360.55	0.11758	0.04502	0.09455
Baseline LightGCN	4	774.28	0.11712	0.03832	0.08576
Cluster-LightGCN	4	380.65	0.11566	0.04366	0.09231

Training time is halved!

# Experiment 1: Cluster-Based Sampling



# Summary of Results

1. **Training Time:** **Cluster-LightGCN models** consistently have **faster training** times compared to their corresponding Baseline LightGCN models.
1. **Recall@20:** **Baseline performs better** at lower layers, but **Cluster-LightGCN** has **more consistent trend**
1. **Precision@20:** **Cluster-LightGCN models** have slightly **higher precision** values compared to Baseline LightGCN models across all layer configurations
1. **NDCG@20:** **Cluster-LightGCN models** have slightly **higher NDCG values** compared to Baseline LightGCN models across all layer configurations

Cluster-LightGCN Model offers significantly faster training times and consistent, better recommendation quality compared to Baseline LightGCN

# Experimental Results: Collapsing User-Item Representation

---

# Experiment 2: Collapsing User-Item Representation

Step1: Set  $U(o) = RI(o)$

Best performance

Ran on modified original source code of LightGCN  
on Gowalla benchmark dataset

	Recall@20	Precision@20	NDCG@20
4-L LightGCN	0.1794	0.05486	0.1526
3-L SimpleLGN	0.174	0.0516	0.1435

Step 2: Sharing weight (as fixed parameters)

3-L share-weight SimpleLGN	0.1688	0.04963	0.1398
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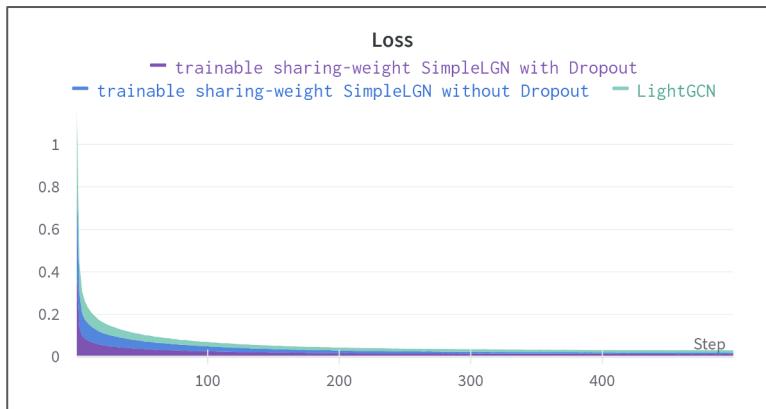
Moderate  
across all  
metrics

Step 3: Sharing weight (as trainable parameters)

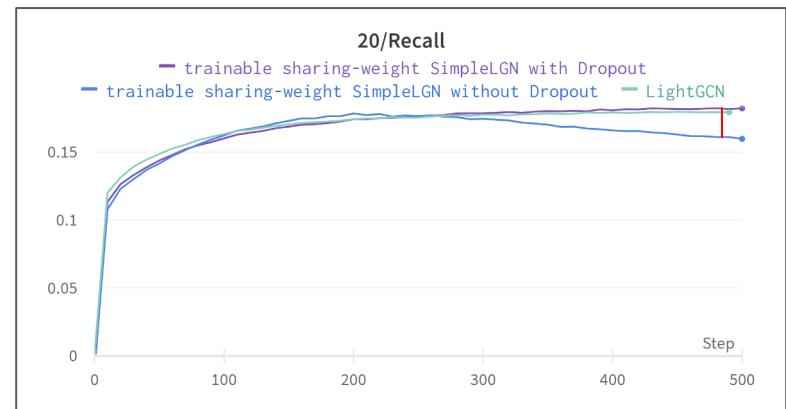
3-L trainable share-weight SimpleLGN	0.1822	0.0536	0.1476
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# Experiment 2: Collapsing User-Item Representation

## Overfitting in Trainable Sharing-weight SimpleLGN model



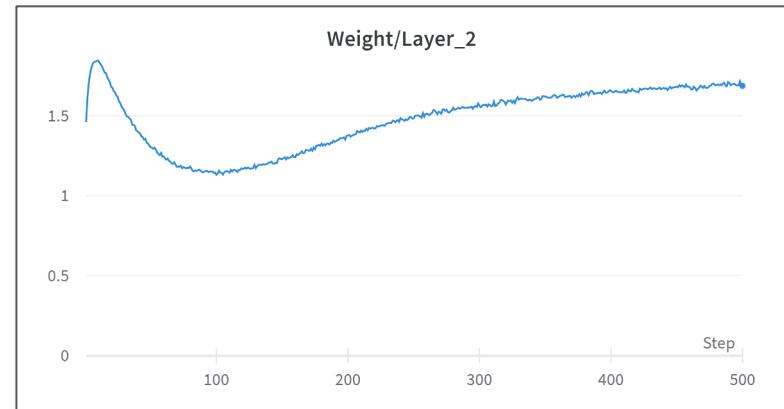
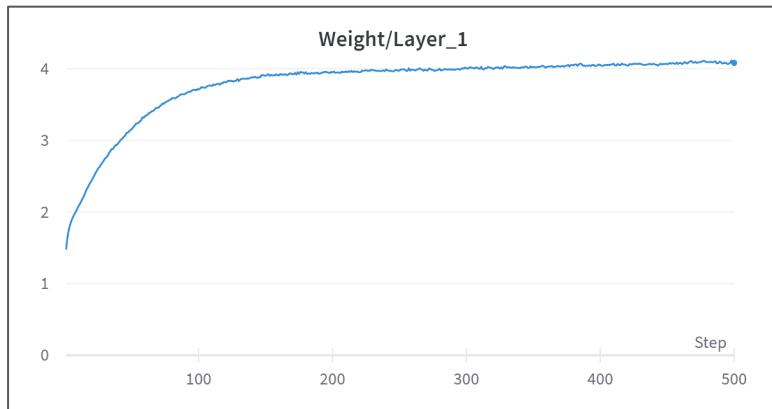
Converging faster



Recall@20: 0.1598 —> 0.1822

# Experiment 2: Collapsing User-Item Representation

## Trainable Sharing-Weights



The weights/coefficients are converged to  
 $\alpha_1=4, \alpha_2=1.7$  (fixed  $\alpha_0=1$ )

# Improving Training

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# Tuned-MixGCF Overview

## Tuned-MixGCF : A Refined Negative Sampling Model based on MixGCF

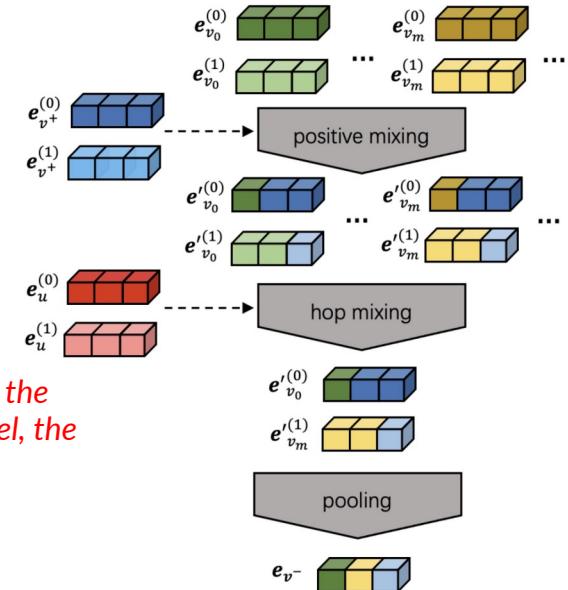
- **Phase 1: Positive Mixing**

- Inject positive information of the target item into the  $k$  negative samples
- The injection ratio of positive information is **trainable**

- **Phase 2: Hop Mixing**

*which is also the biggest difference between the original and refined model – in original model, the injection ratio is randomly selected*

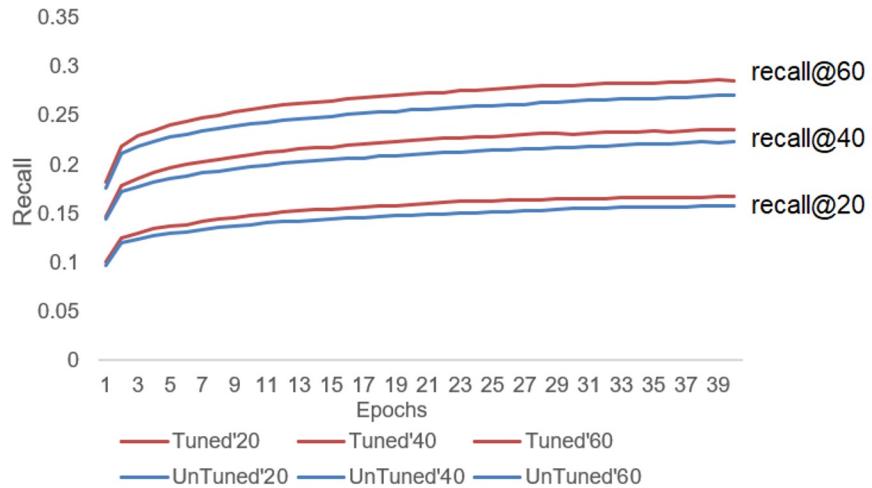
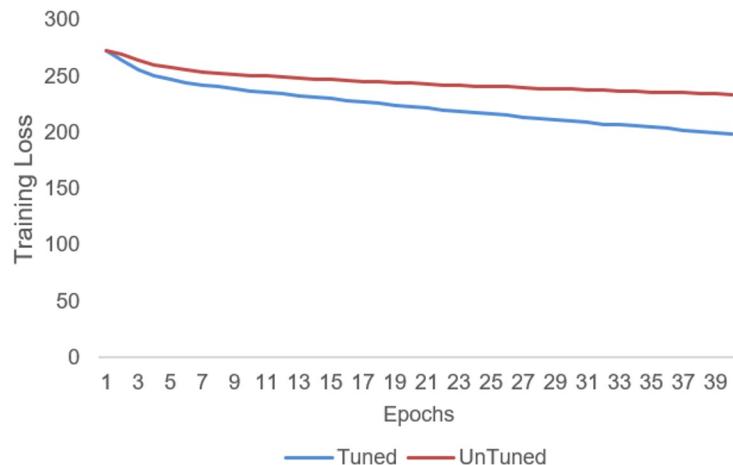
- For each layer  $l$  in LightGCN model, return  $K$  negative samples based on the Highest Product Score principle



\* To alleviate the overfitting problem and improve the robustness of the model, we also add a regularization term in the final BPR loss function.

# Experiment 3: Effectiveness of Tuned model based on LGN

Comparison: training results between Tuned-MixGCF and original MCGF



\* Dataset: Gowalla, we only display 40 epochs because the test recall converges in about 30 epochs.

# Experiment 3: Effectiveness of Tuned model based on LGN

Comparison: training results between Tuned-MixGCF and original LGN

	Model	Recall@20	Precision@20	NDCG@20	Training time
Dataset - Gowalla	4-L LightGCN	0.1794	0.05486	0.1526	22h
	4-L Tuned MGCF	0.1823	0.05621	0.1545	41min
Dataset - Yelp2018	4-L LightGCN	0.0649	...	0.0530	...
	4-L Tuned MGCF	0.0707	0.0318	0.0581	1h

Large improvement on both  
training result and training time!

# Experiment 4: SimpleLGN + Tuned-MixGCF

## Yelp2018 dataset

	Recall@20	Precision@20	NDCG@20
3-L LightGCN	0.0628	0.0281	0.0514
3-L LightGCN+Tuned-MGCF	0.0696	0.0313	0.0569
2-L SimpleLGN+Tuned-MGCF	0.0711	0.0320	0.0585
2-L share-weight SimpleLGN+Tuned-MGCF	0.07055	0.0318	0.0583
2-L trainable share-weight SimpleLGN+Tuned-MGCF	0.0717	0.0323	0.0589

+  
14.2%↑

# Summary of Achievements/Notable Challenges

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# In Conclusion

## Summary of Achievements:

1. Demonstrated the effectiveness of Cluster-Based Sampling Approach in reducing training time and alleviating over-smoothing.
1. Introduced a novel approach of Collapsing the User-Item Representation in the LightGCN expansion, to obtain a more simplified form.
1. Tuned MixGCF Training Method can be used to boost both training speed and performance easily.

# In Conclusion

## Challenges and Further Works:

- 1. Integration of Cluster-Based Sampling on original source code for LightGCN model, deploy and experiment on Gowalla, Yelp benchmark datasets**
  - We were unable to integrate the Cluster approach in the limited research time, as this required extensive modification of the data preprocessing pipeline and mini-batch training pipeline.
- 1. Exploring the joint effects of combining Cluster-Based Sampling with SimpleLGN (Collapsing user-item representation approach), Tuned-MixGCF**

# Thank you!

