

1 三状态环流大偏差的 rate function:

三状态环流大偏差的 rate function:

$$\begin{aligned} I(\nu) = & \sum_{i \in I} \left(-\nu^i \log \frac{\nu^i}{w^i} + (\nu^i - \nu_i) \log(\nu^i - \nu_i) \right) \\ & - (\tilde{\nu} - \sum_{i \in I} \nu_i) \log(\tilde{\nu} - \sum_{i \in I} \nu_i) + \sum_{t \in C_\infty} \nu_t \log \nu_t \\ & - (\nu_1 \log w_1 + \nu_2 \log w_2 + \nu_3 \log w_3) \\ & - (\nu_{12} + \nu_{123}) \log(w_{12} + w_{123}) \\ & - (\nu_{13} + \nu_{132}) \log(w_{13} + w_{132}) \\ & - (\nu_{12} + \nu_{132}) \log(w_{12} + w_{132}) \\ & - (\nu_{23} + \nu_{123}) \log(w_{23} + w_{123}) \\ & - (\nu_{13} + \nu_{123}) \log(w_{13} + w_{123}) \\ & - (\nu_{23} + \nu_{132}) \log(w_{23} + w_{132}) \end{aligned}$$

其中 $I = \{1, 2, 3\}$ 是马氏链的状态空间。 $\mathcal{C}_\infty = \{(1), (2), (3), (1, 2), (2, 3), (1, 3), (1, 2, 3)\}$, 表示所有可能出现环的集合。 ν_c 表示环 c 出现的频率, w_c 表示环 c 的环流。

且 $\nu^i = \sum_{J_{c_s}(i)=1} \nu_{c_s}$, 例如 $\nu^1 = \nu_1 + \nu_{12} + \nu_{13} + \nu_{123} + \nu_{132}$ 。 $\tilde{\nu} = \nu_1 + \nu_2 + \nu_3 + \nu_{12} + \nu_{13} + \nu_{23} + \nu_{123} + \nu_{132}$ 。
 w^i, w_i 表示类似的含义。

2 $p_{13} = 0$ 的情形

rate function:

$$\begin{aligned}
 I(\nu) &= -(\nu^1 \log(\frac{\nu^1}{w^1}) + \nu^2 \log(\frac{\nu^2}{w^2}) + \nu^3 \log(\frac{\nu^3}{w^3})) \\
 &= \nu_1 \log(\frac{\nu_1}{w_1}) + \nu_2 \log(\frac{\nu_2}{w_2}) + \nu_3 \log(\frac{\nu_3}{w_3}) \\
 &= (\nu_{12} + \nu_{123}) \log(\frac{\nu_{12} + \nu_{123}}{w_{12} + w_{123}}) + (\nu_{23} + \nu_{123}) \log(\frac{\nu_{23} + \nu_{123}}{w_{23} + w_{123}}) \\
 &\quad + \nu_{12} \log(\frac{\nu_{12}}{w_{12}}) + \nu_{23} \log(\frac{\nu_{23}}{w_{23}}) + \nu_{123} \log(\frac{\nu_{123}}{w_{123}})
 \end{aligned}$$

即

$$\begin{aligned}
 I(\nu) &= \sum_{i,j \in I} \left(\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} \nu_c \right) \log \left(\frac{\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} w_c \sum_{c \in \mathcal{C}_\infty, J_c(i)=1} \nu_c}{\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} \nu_c \sum_{c \in \mathcal{C}_\infty, J_c(i)=1} w_c} \right) \\
 &= \sum_{i,j \in I} \left(\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} \nu_c \right) \log \left(\frac{\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} w_c}{\sum_{c \in \mathcal{C}_\infty, J_c(i,j)=1} \nu_c} / \frac{\sum_{c \in \mathcal{C}_\infty, J_c(i)=1} w_c}{\sum_{c \in \mathcal{C}_\infty, J_c(i)=1} \nu_c} \right)
 \end{aligned}$$

3 $p_{ii} = 0$ 的情形

rate function:

$$\begin{aligned} I(\nu) = & \sum_{i \in I} (\nu^i - \nu_i) \log(w^i - w_i) - (\tilde{\nu} - \sum_{i \in I} \nu^i) \log(\tilde{\nu} - \sum_{i \in I} \nu^i) \\ & + \nu_{12} \log \nu_{12} + \nu_{23} \log \nu_{23} + \nu_{13} \log \nu_{13} + \nu_{123} \log \nu_{123} + \nu_{132} \log \nu_{132} \\ & - (\nu_1 \log w_1 + \nu_2 \log w_2 + \nu_3 \log w_3) \\ & - (\nu_{12} + \nu_{123}) \log(w_{12} + w_{123}) \\ & - (\nu_{13} + \nu_{132}) \log(w_{13} + w_{132}) \\ & - (\nu_{12} + \nu_{132}) \log(w_{12} + w_{132}) \\ & - (\nu_{23} + \nu_{123}) \log(w_{23} + w_{123}) \\ & - (\nu_{13} + \nu_{123}) \log(w_{13} + w_{123}) \\ & - (\nu_{23} + \nu_{132}) \log(w_{23} + w_{132}) \end{aligned}$$