1 三状态环流大偏差的 rate function:

三状态环流大偏差的 rate function:

$$
\begin{aligned}
I(\nu) = & \sum_{i\in I}\left(-\nu^i \log\frac{\nu^i}{w^i} + (\nu^i - \nu_i)\log(\nu^i - \nu_i)\right) \\
& - (\tilde{\nu} - \sum_{i\in I}\nu_i)\log(\tilde{\nu} - \sum_{i\in I}\nu_i) + \sum_{t\in C_\infty}\nu_t \log\nu_t \\
& - (\nu_1\log w_1 + \nu_2\log w_2 + \nu_3\log w_3) \\
& - (\nu_{12} + \nu_{123})\log(w_{12} + w_{123}) \\
& - (\nu_{13} + \nu_{132})\log(w_{13} + w_{132}) \\
& - (\nu_{12} + \nu_{132})\log(w_{12} + w_{132}) \\
& - (\nu_{23} + \nu_{123})\log(w_{23} + w_{123}) \\
& - (\nu_{13} + \nu_{123})\log(w_{13} + w_{123}) \\
& - (\nu_{23} + \nu_{132})\log(w_{23} + w_{132})
\end{aligned}
$$

其中 $I = \{1,2,3\}$ 是马氏链的状态空间。$\mathcal{C}_\infty = \{(1),(2),(3),(1,2),(2,3),(1,3),(1,2,3),$ 表示所有可能出现环的集合。ν_c 表示环 c 出现的频率，w_c 表示环 c 的环流。

且 $\nu^i = \sum_{J_{c_s}(i)=1}\nu_{c_s}$，例如 $\nu^1 = \nu_1 + \nu_{12} + \nu_{13} + \nu_{123} + \nu_{132}$。$\tilde{\nu} = \nu_1 + \nu_2 + \nu_3 + \nu_{12} + \nu_{13} + \nu_{23} + \nu_{123} + \nu_{132}$。

w^i, w_i 表示类似的含义。

2 $p_{13}=0$ 的情形

rate function:

$$
\begin{aligned}
I(\nu) &= -(\nu^1 \log(\frac{\nu^1}{w^1}) + \nu^2 \log(\frac{\nu^2}{w^2}) + \nu^3 \log(\frac{\nu^3}{w^3})) \\
&= \nu_1 \log(\frac{\nu_1}{w_1}) + \nu_2 \log(\frac{\nu_2}{w_2}) + \nu_3 \log(\frac{\nu_3}{w_3}) \\
&= (\nu_{12}+\nu 123) \log(\frac{\nu_{12}+\nu 123}{w_{12}+w123}) + (\nu_{23}+\nu 123) \log(\frac{\nu_{23}+\nu 123}{w_{23}+w123}) \\
&+ \nu_{12} \log(\frac{\nu_{12}}{w_{12}}) + \nu_{23} \log(\frac{\nu_{23}}{w_{23}}) + \nu_{123} \log(\frac{\nu_{123}}{w_{123}})
\end{aligned}
$$

即

$$
\begin{aligned}
I(\nu) &= \sum_{i,j\in I} \left(\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} \nu_c \right) \log(\frac{\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} w_c \sum_{c\in\mathcal{C}_\infty, J_c(i)=1} \nu_c}{\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} \nu_c \sum_{c\in\mathcal{C}_\infty, J_c(i)=1} w_c}) \\
&= \sum_{i,j\in I} \left(\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} \nu_c \right) \log(\frac{\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} w_c}{\sum_{c\in\mathcal{C}_\infty, J_c(i,j)=1} \nu_c} / \frac{\sum_{c\in\mathcal{C}_\infty, J_c(i)=1} w_c}{\sum_{c\in\mathcal{C}_\infty, J_c(i)=1} \nu_c})
\end{aligned}
$$

3 $p_{ii} = 0$ 的情形

rate function:

$$
\begin{aligned}
I(\nu) = & \sum_{i\in I}(\nu^i - \nu_i)\log(w^i - w_i) - (\tilde{\nu} - \sum_{i\in I}\nu^i)\log(\tilde{\nu} - \sum_{i\in I}\nu^i) \\
& + \nu_{12}\log\nu_{12} + \nu_{23}\log\nu_{23} + \nu_{13}\log\nu_{13} + \nu_{123}\log\nu_{123} + \nu_{132}\log\nu_{132} \\
& - (\nu_1\log w_1 + \nu_2\log w_2 + \nu_3\log w_3) \\
& - (\nu_{12} + \nu_{123})\log(w_{12} + w_{123}) \\
& - (\nu_{13} + \nu_{132})\log(w_{13} + w_{132}) \\
& - (\nu_{12} + \nu_{132})\log(w_{12} + w_{132}) \\
& - (\nu_{23} + \nu_{123})\log(w_{23} + w_{123}) \\
& - (\nu_{13} + \nu_{123})\log(w_{13} + w_{123}) \\
& - (\nu_{23} + \nu_{132})\log(w_{23} + w_{132})
\end{aligned}
$$