

report

name

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1 A

$$u_{tt} = c^2 u_{xx}$$

$$\frac{U_n^{j+1} - 2U_n^j + U_n^{j-1}}{k^2} = c^2 \frac{(U_{n+1}^j - 2U_n^j + U_{n-1}^j)}{h^2} \tag{1}$$

$$\begin{aligned} T_{(h,k)} &= \left(\frac{\delta_t^2}{k} - \frac{\delta_x^2}{h}\right) - \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right) \\ Tu(x,t) &= \frac{1}{12}u_{tttt}(x,t)k^2 - \frac{1}{12}u_{xxxx}(x,t)h^2 + O(k^4 + h^4) \end{aligned}$$

$$\text{let } q = \frac{ck}{h}$$

$$\begin{aligned} u(x,k) &= u(x,0) + ku_t(x,0) + \frac{1}{2}q^2u_{tt}(x,0) + O(k^3) \\ &= u^0(x) + kv^0(x) + \frac{1}{2}q^2(u^0(x+h) - 2u^0(x) + u^0(x-h)) + O(k^3 + k^2h^2) \end{aligned}$$

$$Tu(x,k) = O(k^3 + k^2h)$$

$$u_t = 0 \text{ when } t = 0$$

$$U_n^1 = U_n^0 + \frac{1}{2}q^2(U_{n+1}^0 - 2U_n^0 + U_{n-1}^0)$$

1.1 (a)

$$cu_x = +u_t \quad x = -2$$

$$cu_x = -u_t \quad x = 2$$

$$q(U_1^j - U_{-1}^j) = U_0^{j+1} - U_0^{j-1}$$

$$q(U_{N+1}^j - U_{N-1}^j) = U_N^{j+1} - U_N^{j-1}$$

$$U_{-1}^j = U_1^j - \frac{1}{q}(U_0^{j+1} - U_0^{j-1})$$

$$U_{N+1}^j = U_{N-1}^j + \frac{1}{q}(U_N^{j+1} - U_N^{j-1})$$

$$\frac{U_0^{j+1} - U_0^j + U_0^{j-1}}{k^2} = c^2 \frac{2(U_1^j - U_0^j) - (U_0^{j+1} - U_0^{j-1})/q}{h^2}$$

$$\frac{U_N^{j+1} - 2U_N^j + U_N^{j-1}}{k^2} = c^2 \frac{2(U_{N-1}^j - U_N^j) + (U_N^{j+1} - U_N^{j-1})/q}{h^2}$$

$$U_0^{j+1} = \frac{1}{1+q}[2U_0^j - (1-q)U_0^{j-1} + 2q^2(U_1^j - U_0^j)]$$

$$U_N^{j+1} = \frac{1}{1-q}[2U_N^{j+1} - (1-q)U_N^{j-1} + 2q^2(U_{N-1}^j - U_N^j)]$$

1.2 (b)

$$u(-2,t) = u(2,t) = 0$$

$$U_0^m = U_N^m = 0$$

2 B

formula

$$u_{tt} = \nabla^2 u = u_{xx} + u_{yy}$$

$$\frac{U_{mn}^{j+1} - 2U_{mn}^j + U_{mn}^{j-1}}{k^2} = \frac{1}{h_x^2} \delta_x^2 U_{mn}^j + \frac{1}{h_y^2} \delta_y^2 U_{mn}^j$$

2.1 (a)

$$y=2$$

$$-u_{yt} = u_{tt} - \frac{1}{2}u_{xx}$$

$$-\left(\frac{U_{m,N}^{j+1} - U_{m,N-1}^{j+1}}{h_y} - \frac{U_{m,N}^j - U_{m,N-1}^j}{h_y}\right)/k = \frac{U_{m,N}^{k+1} - 2U_{m,N}^k + U_{m,N-1}^k}{k^2} - \frac{1}{2} \frac{U_{m+1,N}^j - 2U_{m,N}^j + U_{m-1,N}^j}{h_y^2}$$

$$U_{m,N}^{j+1} = \left(\frac{U_{m,N-1} + U_{m,N}^j - U_{m,N-1}^j}{kh_y} + \frac{2U_{m,N}^j - U_{m,N}^{j-1}}{k^2} + \frac{U_{m+1,N}^j - 2U_{m,N}^j + U_{m-1,N}^j}{2h_x^2}\right)/\left(\frac{1}{k^2} + \frac{1}{h_y^2}\right)$$

$x = -2$ and $y > 0$

$$u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$\left(\frac{U_{1,n}^{j+1} - U_{0,n}^{j+1}}{h_x} - \frac{U_{1,n}^j - U_{0,n}^j}{h_x}\right)/k = \frac{U_{0,n}^{j+1} - 2U_{0,n}^j + U_{0,n}^{j-1}}{k^2} - \frac{U_{0,n+1}^j - 2U_{0,n}^j + U_{0,n-1}^j}{2h_y^2}$$

$$U_{0,n}^{j+1} = \frac{U_{1,n}^{j+1} - U_{1,n}^j + U_{0,n}^j}{kh_x} + \frac{2U_{0,n}^j - U_{0,n}^{j-1}}{k^2} + \frac{U_{0,n+1}^j - 2U_{0,n}^j + U_{0,n-1}^j}{2h_y^2}$$

$x = 2$ and $y > 0$

$$-u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$-\left(\frac{U_{M,n}^{j+1} - U_{M-1,n}^{j+1}}{h_x} - \frac{U_{M,n}^j - U_{M-1,n}^j}{h_x}\right)/k = \frac{U_{M,n}^{j+1} - 2U_{M,n}^j + U_{M,n}^{j-1}}{k^2} - \frac{U_{M,n+1}^j - 2U_{M,n}^j + U_{M,n-1}^j}{2h_y^2}$$

$$U_{M,n}^{j+1} = \left(\frac{U_{M-1,n}^{j+1} + U_{M,n}^j - U_{M-1,n}^j}{kh_x} + \frac{2U_{M,n}^j}{k^2} + \frac{U_{M,n+1}^j - 2U_{M,n}^j + U_{M,n-1}^j}{2h_y^2}\right)/\left(\frac{1}{k^2} + \frac{1}{kh_x}\right)$$

2.2 (b)