# report

name

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## 1 A

$$u_{tt} = c^2 u_{xx}$$

$$\frac{U_n^{j+1} - 2U_n^j + U_n^{j-1}}{k^2} = c^2 \frac{(U_{n+1}^j - 2U_n^j + U_{n-1}^j)}{h^2} \tag{1}$$

$$T_{(h,k)} = \left(\frac{\delta_t^2}{k} - \frac{\delta_x^2}{h}\right) - \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)$$
$$Tu(x,t) = \frac{1}{12}u_{tttt}(x,t)k^2 - \frac{1}{12}u_{xxxx}(x,t)h^2 + O(k^4 + h^4)$$

let $q = \frac{ck}{h}$

$$\begin{aligned} u(x,k) &= u(x,0) + ku_t(x,0) + \frac{1}{2}q^2 u_{tt}(x,0) + O(k^3) \\ &= u^0(x) + kv^0(x) + \frac{1}{2}q^2(u^0(x+h) - 2u^0(x) + u^0(x-h)) + O(k^3 + k^2h^2) \end{aligned}$$

$Tu(x,k) = O(k^3 + k^2 h)$

$u_t = 0$ when $t = 0$

$$U_n^1 = U_n^0 + \frac{1}{2}q^2(U_{n+1}^0 - 2U_n^0 + U_{n-1}^0)$$

### 1.1 (a)

$$\begin{aligned} cu_x &= +u_t \quad x = -2 \\ cu_x &= -u_t \quad x = 2 \end{aligned}$$

$$q(U_1^j - U_{-1}^j) = U_0^{j+1} - U_0^{j-1}$$
$$q(U_{N+1}^j - U_{N-1}^j) = U_N^{j+1} - U_N^{j-1}$$

$$U_{-1}^j = U_1^j - \frac{1}{q}(U_0^{j+1} - U_0^{j-1})$$
$$U_{N+1}^j = U_{N-1}^j + \frac{1}{q}(U_N^{j+1} - U_N^{j-1})$$

$$\frac{U_0^{j+1} - 2U_0^j + U_0^{j-1}}{k^2} = c^2 \frac{2(U_1^j - U_0^j) - (U_0^{j+1} - U_0^{j-1})/q}{h^2}$$
$$\frac{U_N^{j+1} - 2U_N^j + U_N^{j-1}}{k^2} = c^2 \frac{2(U_{N-1}^j - U_N^j) + (U_N^{j+1} - U_N^{j-1})/q}{h^2}$$

$$Tu(0,t) = \frac{c}{3}(chu_{xxx}(0,t) - \frac{k}{h}u_{ttt}(0,t))Tu(N,t) = \frac{c}{3}(-chu_{xxx}(0,t) + \frac{k}{h}u_{ttt}(0,t))$$

$$U_0^{j+1} = \frac{1}{1+q}[2U_0^j - (1-q)U_0^{j-1} + 2q^2(U_1^j - U_0^j)]$$
$$U_N^{j+1} = \frac{1}{1-q}[2U_N^{j+1} - (1-q)U_N^{j-1} + 2q^2(U_{N-1}^j - U_N^j)]$$

### 1.2 (b)

$$u(-2,t) = u(2,t) = 0$$
$$U_0^m = U_N^m = 0$$

## 2 B

formula

$$u_{tt} = \nabla^2 u = u_{xx} + u_{yy}$$

$$\frac{U_{mn}^{j+1} - 2U_{mn}^j + U_{mn}^{j-1}}{k^2} = \frac{1}{h_x^2}\delta_x^2 U_{mn}^j + \frac{1}{h_y^2}\delta_y^2 U_{mn}^j$$

$$T_{(h_x,h_y,k)} = \left(\frac{\delta_t^2}{k} - \frac{\delta_x^2}{h_x} - \frac{\delta_y^2}{h_y}\right) - \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)$$
$$Tu(x,y,t) = \frac{1}{12}u_{tttt}(x,y,t)k^2 - \frac{1}{12}u_{xxxx}(x,y,t)h_x^2 - \frac{1}{12}u_{yyyy}(x,y,t)h_y^2 + O(k^4 + h^4)$$

## 2.1 (a)

$y = 2$

$$-u_{yt} = u_{tt} - \frac{1}{2}u_{xx}$$

$$-\Big(\frac{U^{j+1}_{m,N} - U^{j+1}_{m,N-1}}{h_y} - \frac{U^j_{m,N} - U^j_{m,N-1}}{h_y}\Big)/k = \frac{U^{j+1}_{m,N} - 2U^j_{m,N} + U^{j-1}_{m,N-1}}{k^2} - \frac{U^j_{m+1,N} - 2U^j_{m,N} + U^j_{m-1,N}}{2h_x^2}$$
$$U^{j+1}_{m,N} = \Big(\frac{U_{m,N-1} + U^j_{m,N} - U^j_{m,N-1}}{kh_y} + \frac{2U^j_{m,N} - U^{j-1}_{m,N}}{k^2} + \frac{U^j_{m+1,N} - 2U^j_{m,N} + U^j_{m-1,N}}{2h_x^2}\Big)/(\frac{1}{k^2} + \frac{1}{kh_y})$$

$x = -2$ and $y > 0$

$$u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$\Big(\frac{U^{j+1}_{1,n} - U^{j+1}_{0,n}}{h_x} - \frac{U^j_{1,n} - U^j_{0,n}}{h_x}\Big)/k = \frac{U^{j+1}_{0,n} - 2U^j_{0,n} + U^{j-1}_{0,n}}{k^2} - \frac{U^j_{0,n+1} - 2U^j_{0,n} + U^j_{0,n-1}}{2h_y^2}$$
$$U^{j+1}_{0,n} = \Big(\frac{U^{j+1}_{1,n} - U^j_{1,n} + U^j_{0,n}}{kh_x} + \frac{2U^j_{0,n} - U^{j-1}_{0,n}}{k^2} + \frac{U^j_{0,n+1} - 2U^j_{0,n} + U^j_{0,n-1}}{2h_y^2}\Big)/(\frac{1}{k^2} + \frac{1}{kh_x})$$

$x = 2$ and $y > 0$

$$-u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$-\Big(\frac{U^{j+1}_{M,n} - U^{j+1}_{M-1,n}}{h_x} - \frac{U^j_{M,n} - U^j_{M-1,n}}{h_x}\Big)/k = \frac{U^{j+1}_{M,n} - 2U^j_{M,n} + U^{j-1}_{M,n}}{k^2} - \frac{U^j_{M,n+1} - 2U^j_{M,n} + U^j_{M,n-1}}{2h_y^2}$$
$$U^{j+1}_{M,n} = \Big(\frac{U^{j+1}_{M-1,n} + U^j_{M,n} - U^j_{M-1,n}}{kh_x} + \frac{2U^j_{M,n}}{k^2} + \frac{U^j_{M,n+1} - 2U^j_{M,n} + U^j_{M,n-1}}{2h_y^2}\Big)/(\frac{1}{k^2} + \frac{1}{kh_x})$$

## 2.2 (b)