report

name

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1 A

$$u_{tt} = c^2 u_{xx}$$

$$\frac{U_n^{j+1} - 2U_n^j + U_n^{j-1}}{k^2} = c^2 \frac{(U_{n+1}^j - 2U_n^j + U_{n-1}^j)}{h^2} \tag{1}$$

$$\begin{aligned} T_{(h,k)} &= \left(\frac{\delta_t^2}{k} - \frac{\delta_x^2}{h}\right) - \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right) \\ Tu(x,t) &= \frac{1}{12} u_{tttt}(x,t)k^2 - \frac{1}{12} u_{xxxx}(x,t)h^2 + O(k^4 + h^4) \end{aligned}$$

let $q = \frac{ck}{h}$

$$\begin{aligned} u(x,k) &= u(x,0) + ku_t(x,0) + \frac{1}{2} q^2 u_{tt}(x,0) + O(k^3) \\ &= u^0(x) + kv^0(x) + \frac{1}{2} q^2 (u^0(x+h) - 2u^0(x) + u^0(x-h)) + O(k^3 + k^2h^2) \end{aligned}$$

$Tu(x,k) = O(k^3 + k^2 h)$

$u_t = 0$ when $t = 0$

$$U_n^1 = U_n^0 + \frac{1}{2} q^2 (U_{n+1}^0 - 2U_n^0 + U_{n-1}^0)$$

1.1 (a)

$$\begin{aligned} cu_x &= +u_t \quad x = -2 \\ cu_x &= -u_t \quad x = 2 \end{aligned}$$

$$
\begin{aligned}
q(U_1^j - U_{-1}^j) &= U_0^{j+1} - U_0^{j-1} \\
q(U_{N+1}^j - U_{N-1}^j) &= U_N^{j+1} - U_N^{j-1}
\end{aligned}
$$

$$
\begin{aligned}
U_{-1}^j &= U_1^j - \frac{1}{q}(U_0^{j+1} - U_0^{j-1}) \\
U_{N+1}^j &= U_{N-1}^j + \frac{1}{q}(U_N^{j+1} - U_N^{j-1})
\end{aligned}
$$

$$
\begin{aligned}
\frac{U_0^{j+1} - 2U_0^j + U_0^{j-1}}{k^2} &= c^2 \frac{2(U_1^j - U_0^j) - (U_0^{j+1} - U_0^{j-1})/q}{h^2} \\
\frac{U_N^{j+1} - 2U_N^j + U_N^{j-1}}{k^2} &= c^2 \frac{2(U_{N-1}^j - U_N^j) + (U_N^{j+1} - U_N^{j-1})/q}{h^2}
\end{aligned}
$$

$$
Tu(0,t) = \frac{c}{3}(chu_{xxx}(0,t) - \frac{k}{h}u_{ttt}(0,t))Tu(N,t) = \frac{c}{3}(-chu_{xxx}(0,t) + \frac{k}{h}u_{ttt}(0,t))
$$

$$
\begin{aligned}
U_0^{j+1} &= \frac{1}{1+q}[2U_0^j - (1-q)U_0^{j-1} + 2q^2(U_1^j - U_0^j)] \\
U_N^{j+1} &= \frac{1}{1-q}[2U_N^{j+1} - (1-q)U_N^{j-1} + 2q^2(U_{N-1}^j - U_N^j)]
\end{aligned}
$$

1.2 (b)

$$
\begin{aligned}
u(-2,t) = u(2,t) &= 0 \\
U_0^m = U_N^m &= 0
\end{aligned}
$$

2 B

formula

$$
u_{tt} = \nabla^2 u = u_{xx} + u_{yy}
$$

$$
\frac{U_{mn}^{j+1} - 2U_{mn}^j + U_{mn}^{j-1}}{k^2} = \frac{1}{h_x^2}\delta_x^2 U_{mn}^j + \frac{1}{h_y^2}\delta_y^2 U_{mn}^j
$$

2.1 (a)

$y = 2$

$$-u_{yt} = u_{tt} - \frac{1}{2}u_{xx}$$

$$-\Big(\frac{U^{j+1}_{m,N} - U^{j+1}_{m,N-1}}{h_y} - \frac{U^{j}_{m,N} - U^{j}_{m,N-1}}{h_y}\Big)/k = \frac{U^{k+1}_{m,N} - 2U^{k}_{m,N} + U_{m,N-1}k}{k^2} - \frac{1}{2}\frac{U^{j}_{m+1,N} - 2U^{j}_{m,N} + U^{j}_{m-1,N}}{h_y^2}$$

$$U^{j+1}_{m,N} = \Big(\frac{U_{m,N-1} + U^{j}_{m,N} - U^{j}_{m,N-1}}{kh_y} + \frac{2U^{j}_{m,N} - U^{j-1}_{m,N}}{k^2} + \frac{U^{j}_{m+1,N} - 2U^{j}_{m,N} + U^{j}_{m-1,N}}{2h_x^2}\Big)/(\frac{1}{k^2} + \frac{1}{h_y^2})$$

$x = -2$ and $y > 0$

$$u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$\Big(\frac{U^{j+1}_{1,n} - U^{j+1}_{0,n}}{h_x} - \frac{U^{j}_{1,n} - U^{j}_{0,n}}{h_x}\Big)/k = \frac{U^{j+1}_{0,n} - 2U^{j}_{0,n} + U^{j-1}_{0,n}}{k^2} - \frac{U^{j}_{0,n+1} - 2U^{j}_{0,n} + U^{j}_{0,n-1}}{2h_y^2}$$

$$U^{j+1}_{0,n} = \frac{U^{j+1}_{1,n} - U^{j}_{1,n} + U^{j}_{0,n}}{kh_x} + \frac{2U^{j}_{0,n} - U^{j-1}_{0,n}}{k^2} + \frac{U^{j}_{0,n+1} - 2U^{j}_{0,n} + U^{j}_{0,n-1}}{2h_y^2}$$

$x = 2$ and $y > 0$

$$-u_{xt} = u_{tt} - \frac{1}{2}u_{yy}$$

$$-\Big(\frac{U^{j+1}_{M,n} - U^{j+1}_{M-1,n}}{h_x} - \frac{U^{j}_{M,n} - U^{j}_{M-1,n}}{h_x}\Big)/k = \frac{U^{j+1}_{M,n} - 2U^{j}_{M,n} + U^{j-1}_{M,n}}{k^2} - \frac{U^{j}_{M,n+1} - 2U^{j}_{M,n} + U^{j}_{M,n-1}}{2h_y^2}$$

$$U^{j+1}_{M,n} = \Big(\frac{U^{j+1}_{M-1,n} + U^{j}_{M,n} - U^{j}_{M-1,n}}{kh_x} + \frac{2U^{j}_{M,n}}{k^2} + \frac{U^{j}_{M,n+1} - 2U^{j}_{M,n} + U^{j}_{M,n-1}}{2h_y^2}\Big)/(\frac{1}{k^2} + \frac{1}{kh_x})$$

2.2 (b)