Assignment 3 Part 2

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1. a.

LPR^+ = LPRQST, LPR is not a superkey and LPR→Q violate BCNF.
LR^+ = LRST, LR is not a superkey and LR→ST violate BCNF.
M^+ = MLO, M is not a superkey and M→LO violate BCNF.
MR^+ = MRNLOST, MR is not a superkey and MR→N violate BCNF.
All the FDs in W violate BCNF.

b.

- Decompose V using FD LPR→Q, LPR^+ = LPRQST, so this yields to two relations: V_1 = LPQRST and V_2 = MNOLPR
- Project FDs onto V_1 = LPQRST

L	P	Q	R	S	T	closure	FDs
•						L^+ = L	nothing
	•					P^+ = P	nothing
		•				Q^+ = Q	nothing
			•			R^+ = R	nothing
				•		S^+ = S	nothing
					•	T^+ = T	nothing
•	•					LP^+ = LP	nothing
•		•				LQ^+ = LQ	nothing
•			•			LR^+ = LRST	LR→ST, violate BCNF

We must decompose V_1 further.

- Decompose V_1 using LR→ST. This yields two relationships: V_3 = LRST and V_4 = LRPQ.
- Project FDs onto V_3 = LRST

L	R	S	T	closure	FDs
•				L^+ = L	nothing
	•			R^+ = R	nothing
		•		S^+ = S	nothing
			•	T^+ = T	nothing
•	•			LR^+ = LRST	LR→ST, LR is the superkey of V_3
•		•		LS^+ = LS	nothing
•			•	LT^+ = LT	nothing
	•	•		RS^+ = RS	nothing
	•		•	RT^+ = RT	nothing
		•	•	ST^+ = ST	nothing
superset of LR				irrelevant	
•		•	•	LST^+ = LST	nothing

This relation satisfies BCNF.

- Project FDs onto V_4 = LRPQ

L	R	P	Q	closure	FDs
•				$L^+ = L$	nothing
	•			$R^+ = R$	nothing
		•		$S^+ = S$	nothing
			•	$T^+ = T$	nothing
•	•			$LR^+ = LRST$	nothing
•		•		$LP^+ = LP$	nothing
•			•	$LQ^+ = LQ$	nothing
	•	•		$RP^+ = RP$	nothing
	•		•	$RQ^+ = RQ$	nothing
		•	•	$PQ^+ = PQ$	nothing
•	•	•		$LPR^+ = LPRQST$	LPR→Q, LPR is the superkey of V_4
•	•		•	$LRQ^+ = LRQST$	nothing

This relation satisfies BCNF.

- Project FDs onto V_2 = MNOLPR

M	N	O	L	P	R	closure	FDs
•						$M^+ = MLO$	M→LO, violate BCNF

We must decompose V_2 further.

- Decompose V_2 using M→LO. This yields two relationships: V_5 = MLO and V_6 = MNPR.
- Project FDs onto V_5 = MLO

M	L	O	closure	FDs
•			$M^+ = MLO$	M→LO, M is the superkey of V_5
	•		$L^+ = L$	nothing
		•	$O^+ = O$	nothing
superset of M			irrelevant	
	•	•	$LO^+ = LO$	nothing

This relation satisfies BCNF.

- Project FDs onto V_6 = MNPR

M	N	P	R	closure	FDs
•				$M^+ = MLO$	nothing
	•			$N^+ = N$	nothing
		•		$P^+ = P$	nothing
			•	$R^+ = R$	nothing
•	•			$MN^+ = MNLO$	nothing
•		•		$MP^+ = MPLO$	nothing
•			•	$MR^+ = MRNLOST$	MR→N, violate BCNF

We must decompose V_6 further.

- Decompose V_6 using MR→N. This yields two relationships: V_7 = MRN and V_8 = MRP.

- Project FDs onto V_7 = MRN

M	R	N	closure	FDs
•			M^+ = MLO	nothing
	•		R^+ = R	nothing
		•	N^+ = N	nothing
•	•		MR^+ = MRNLOST	MR→N, MR is the superkey of V_7
•		•	MN^+ = MNLO	nothing
	•	•	RN^+ = RN	nothing

  This relation satisfies BCNF.
- Project FDs onto V_8 = MRP

M	R	P	closure	FDs
•			M^+ = MLO	nothing
	•		R^+ = R	nothing
		•	P^+ = P	nothing
•	•		MR^+ = MRLONST	nothing
•		•	MP^+ = MPLO	nothing
	•	•	RP^+ = RP	nothing

  This relation satisfies BCNF.
- Final decomposition:
  a) V_5 = LMO with FD: M→LO
  b) V_4 = LPQR with FD: LPR→Q
  c) V_3 = LRST with FD: LR→ST.
  d) V_7 = MNR with FD: MR→N
  e) V_8 = MPR with no FDs.

2. a.

Step 1: Split the RHSs to get our initial set of FDs.

1) AB→C
2) AB→D
3) ACDE→B
4) ACDE→F
5) B→A
6) B→C
7) B→D
8) CD→A
9) CD→F
10) CDE→F
11) CDE→G
12) EB→D

Step 2: Try to minimize the LHS. (Same order as in step 1.)

1) A^+ = A, B^+ = BACDF, we can reduce LHS of FD AB→C to **B→C**.
2) A^+ = A, B^+ = BACDF, we can reduce LHS of FD AB→D to **B→D**.

3) Since no singleton LHS of ACDE yields anything, we need only consider two or more attributes. $AC^+ = AC$, $AD^+ = AD$, $AE^+ = AE$, $CD^+ = CDAF$, $CE^+ = CE$, $DE^+ = DE$, $ACD^+ = ACDF$, $ACE^+ = ACE$, $ADE^+ = ADE$, $CDE^+ = CDEAFBG$. So we can reduce the FD ACDE→B to **CDE→B**.
4) Since no singleton LHS of ACDE yields anything, we need only consider two or more attributes. $AC^+ = AC$, $AD^+ = AD$, $AE^+ = AE$, $CD^+ = CDAF$, $CE^+ = CE$, $DE^+ = DE$. So we can reduce the FD ACDE→F to **CD→F**.
5) LHS only has one attribute, cannot reduce the FD **B→A**.
6) LHS only has one attribute, cannot reduce the FD **B→C**.
7) LHS only has one attribute, cannot reduce the FD **B→D**.
8) $C^+ = C$, $D^+ = D$, we cannot reduce LHS of the FD **CD→A**.
9) $C^+ = C$, $D^+ = D$, we cannot reduce LHS of the FD **CD→F**.
10) Since no singleton LHS of CDE yields anything, $CD^+ = CDAF$, $CE^+ = CE$, $DE^+ = DE$. So we can reduce the FD CDE→F to **CD→F**.
11) Since no singleton LHS of CDE yields anything, $CD^+ = CDAF$, $CE^+ = CE$, $DE^+ = DE$. We cannot reduce LHS of the FD **CDE→G**.
12) $E^+ = E$, $B^+ = BACDF$. We can reduce the FD EB→D to **B→D**.

So our new set of FDs T_2:

1) B→C
2) B→D
3) CDE→B
4) CD→F
5) B→A
6) CD→A
7) CDE→G

Step 3: Try to eliminate each FD:

a) Without $T_2$1) $B^+ = ABD$, we need $T_2$1).
b) Without $T_2$2) $B^+ = ABC$, we need $T_2$2).
c) Without $T_2$3) $CDE^+ = ACDEFG$, we need $T_2$3).
d) Without $T_2$4) $CD^+ = ACDEG$, we need $T_2$4).
e) Without $T_2$5) $B^+ = BCDAF$, we do not need to keep $T_2$5).
f) Without $T_2$6) and $T_2$5) $CD^+ = CDF$, we need $T_2$6).
g) Without $T_2$7) and $T_2$5) $CDE^+ = ABCDEF$, we need $T_2$7)

So, the final minimum basis for T is

T = {B→C, B→D, CD→A, CD→F, CDE→B, CDE→G}

b.
From the minimum basis we made in part a:

	Only in LHS	Only in RHS	On both side	Not on FDs	Note
A		•			In no key
B			•		To be analyzed
C			•		To be analyzed
D			•		To be analyzed
E	•				In every key
F		•			In no key
G		•			In no key
H				•	In every key

From the table above, we know that E and H must be in every key and we have to check for BCD.
BEH^+ = ABCDEFGH. BEH is a key. All supersets of BEH are supersets.
CEH^+ = CEH.
DEH^+ = DEH.
$CDEH^+$ = ABCDEFGH. CDEH is a key. All supersets of CDEH are supersets.
Therefore, {BEH, CDEH} are the keys.

c.
First step: combine the RHS.
B→C and B→D becomes B→CD.
CD→A and CD→F become CD→AF.
CDE→B and CDE→G becomes CDE→BG.
So, the FDs are: **B→CD, CD→AF, CDE→BG**.

Step 2: Make relations base on the FDs.
R_1(BCD), R_2(ACDF), R_3(BCDEG)
because BEH and CDEH are keys, we should add a relation that contains the keys.
R_4(BEH)
So, the final relations are R_1(BCD), R_2(ACDF), R_3(BCDEG), R_4(BEH)

d.
R_1, R_2, R_3 are formed by FDs, the LHS of the FDs are the superkey of them. R_4 is the key of the whole relation. However, there may other FDs that violate BCNF so the schema allows redundancy. For instance, project B→CD and CD→AF in R_3, B^+ = ABCDF, B is not a superkey of R_3. Therefore, the schema allow redundancy.