**Assignment 3 Part 2**

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1. **a.**

LPR+ = LPRQST, LPR is not a superkey and LPR→Q violate BCNF.

LR+ = LRST, LR is not a superkey and LR→ST violate BCNF.

M+ = MLO, M is not a superkey and M→LO violate BCNF.

MR+ = MRNLOST, MR is not a superkey and MR→N violate BCNF.

All the FDs in W violate BCNF.

**b.**

* Decompose V using FD LPR→Q, LPR+ = LPRQST, so this yields to two relations: V1 = LPQRST and V2 = MNOLPR
* Project FDs onto V1 = LPQRST

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| L | P | Q | R | S | T | closure | FDs |
| • |  |  |  |  |  | L+ = L | nothing |
|  | • |  |  |  |  | P+ = P | nothing |
|  |  | • |  |  |  | Q+ = Q | nothing |
|  |  |  | • |  |  | R+ = R | nothing |
|  |  |  |  | • |  | S+ = S | nothing |
|  |  |  |  |  | • | T+ = T | nothing |
| • | • |  |  |  |  | LP+ = LP | nothing |
| • |  | • |  |  |  | LQ+ = LQ | nothing |
| • |  |  | • |  |  | LR+ = LRST | LR→ST, violate BCNF |

We must decompose V1 further.

* Decompose V1 using LR→ST. This yields two relationships: V3 = LRST and V4 = LRPQ.
* Project FDs onto V3 = LRST

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| L | R | S | T | closure | FDs |
| • |  |  |  | L+ = L | nothing |
|  | • |  |  | R+ = R | nothing |
|  |  | • |  | S+ = S | nothing |
|  |  |  | • | T+ = T | nothing |
| • | • |  |  | LR+ = LRST | LR→ST, LR is the superkey of V3 |
| • |  | • |  | LS+ = LS | nothing |
| • |  |  | • | LT+ = LT | nothing |
|  | • | • |  | RS+ = RS | nothing |
|  | • |  | • | RT+ = RT | nothing |
|  |  | • | • | ST+ = ST | nothing |
| superset of LR | | | | irrelevant |  |
| • |  | • | • | LST+ = LST | nothing |

This relation satisfies BCNF.

* Project FDs onto V4 = LRPQ

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| L | R | P | Q | closure | FDs |
| • |  |  |  | L+ = L | nothing |
|  | • |  |  | R+ = R | nothing |
|  |  | • |  | S+ = S | nothing |
|  |  |  | • | T+ = T | nothing |
| • | • |  |  | LR+ = LRST | nothing |
| • |  | • |  | LP+ = LP | nothing |
| • |  |  | • | LQ+ = LQ | nothing |
|  | • | • |  | RP+ = RP | nothing |
|  | • |  | • | RQ+ = RQ | nothing |
|  |  | • | • | PQ+ = PQ | nothing |
| • | • | • |  | LPR+ = LPRQST | LPR→Q, LPR is the superkey of V4 |
| • | • |  | • | LRQ+ = LRQST | nothing |

This relation satisfies BCNF.

* Project FDs onto V2 = MNOLPR

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| M | N | O | L | P | R | closure | FDs |
| • |  |  |  |  |  | M+ = MLO | M→LO, violate BCNF |

We must decompose V2 further.

* Decompose V2 using M→LO. This yields two relationships: V5 = MLO and V6 = MNPR.
* Project FDs onto V5 = MLO

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| M | L | O | closure | FDs |
| • |  |  | M+ = MLO | M→LO, M is the superkey of V5 |
|  | • |  | L+ = L | nothing |
|  |  | • | O+ = O | nothing |
| superset of M | | | irrelevant |  |
|  | • | • | LO+ = LO | nothing |

This relation satisfies BCNF.

* Project FDs onto V6 = MNPR

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| M | N | P | R | closure | FDs |
| • |  |  |  | M+ = MLO | nothing |
|  | • |  |  | N+ = N | nothing |
|  |  | • |  | P+ = P | nothing |
|  |  |  | • | R+ = R | nothing |
| • | • |  |  | MN+ = MNLO | nothing |
| • |  | • |  | MP+ = MPLO | nothing |
| • |  |  | • | MR+ = MRNLOST | MR→N, violate BCNF |

We must decompose V6 further.

* Decompose V6 using MR→N. This yields two relationships: V7 = MRN and V8 = MRP.
* Project FDs onto V7 = MRN

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| M | R | N | closure | FDs |
| • |  |  | M+ = MLO | nothing |
|  | • |  | R+ = R | nothing |
|  |  | • | N+ = N | nothing |
| • | • |  | MR+ = MRNLOST | MR→N, MR is the superkey of V7 |
| • |  | • | MN+ = MNLO | nothing |
|  | • | • | RN+ = RN | nothing |

This relation satisfies BCNF.

* Project FDs onto V8 = MRP

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| M | R | P | closure | FDs |
| • |  |  | M+ = MLO | nothing |
|  | • |  | R+ = R | nothing |
|  |  | • | P+ = P | nothing |
| • | • |  | MR+ = MRLONST | nothing |
| • |  | • | MP+ = MPLO | nothing |
|  | • | • | RP+ = RP | nothing |

This relation satisfies BCNF.

* Final decomposition:

1. V5 = LMO with FD: M→LO
2. V4 = LPQR with FD: LPR→Q
3. V3 = LRST with FD: LR→ST.
4. V7 = MNR with FD: MR→N
5. V8 = MPR with no FDs.
6. **a.**

Step 1: Split the RHSs to get our initial set of FDs.

1. AB→C
2. AB→D
3. ACDE→B
4. ACDE→F
5. B→A
6. B→C
7. B→D
8. CD→A
9. CD→F
10. CDE→F
11. CDE→G
12. EB→D

Step 2: Try to minimize the LHS. (Same order as in step 1.)

1. A+ = A, B+ = BACDF, we can reduce LHS of FD AB→C to **B→C**.
2. A+ = A, B+ = BACDF, we can reduce LHS of FD AB→D to **B→D**.
3. Since no singleton LHS of ACDE yields anything, we need only consider two or more attributes. AC+ = AC, AD+ = AD, AE+ = AE, CD+ = CDAF, CE+ = CE, DE+ = DE, ACD+ = ACDF, ACE+ = ACE, ADE+ = ADE, CDE+ = CDEAFBG. So we can reduce the FD ACDE→B to **CDE→B**.
4. Since no singleton LHS of ACDE yields anything, we need only consider two or more attributes. AC+ = AC, AD+ = AD, AE+ = AE, CD+ = CDAF, CE+ = CE, DE+ = DE. So we can reduce the FD ACDE→F to **CD→F**.
5. LHS only has one attribute, cannot reduce the FD **B→A**.
6. LHS only has one attribute, cannot reduce the FD **B→C**.
7. LHS only has one attribute, cannot reduce the FD **B→D**.
8. C+ = C, D+ = D, we cannot reduce LHS of the FD **CD→A**.
9. C+ = C, D+ = D, we cannot reduce LHS of the FD **CD→F**.
10. Since no singleton LHS of CDE yields anything, CD+ = CDAF, CE+ = CE, DE+ = DE. So we can reduce the FD CDE→F to **CD→F**.
11. Since no singleton LHS of CDE yields anything, CD+ = CDAF, CE+ = CE, DE+ = DE. We cannot reduce LHS of the FD **CDE→G**.
12. E+ = E, B+ = BACDF. We can reduce the FD EB→D to **B→D**.

So our new set of FDs T2:

1. B→C
2. B→D
3. CDE→B
4. CD→F
5. B→A
6. CD→A
7. CDE→G

Step 3: Try to eliminate each FD:

1. Without T21) B+ = ABD, we need T21).
2. Without T22) B+ = ABC, we need T22).
3. Without T23) CDE+ = ACDEFG, we need T23).
4. Without T24) CD+ = ACDEG, we need T24).
5. Without T25) B+ = BCDAF, we do not need to keep T25).
6. Without T26) and T25) CD+ = CDF, we need T26).
7. Without T27) and T25) CDE+ = ABCDEF, we need T27)

So, the final minimum basis for T is

T = {B→C, B→D, CD→A, CD→F, CDE→B, CDE→G}

**b.**

From the minimum basis we made in part a:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Only in LHS | Only in RHS | On both side | Not on FDs | Note |
| A |  | • |  |  | In no key |
| B |  |  | • |  | To be analyzed |
| C |  |  | • |  | To be analyzed |
| D |  |  | • |  | To be analyzed |
| E | • |  |  |  | In every key |
| F |  | • |  |  | In no key |
| G |  | • |  |  | In no key |
| H |  |  |  | • | In every key |

From the table above, we know that E and H must be in every key and we have to check for BCD.

BEH+ = ABCDEFGH. BEH is a key. All supersets of BEH are supersets.

CEH+ = CEH.

DEH+ = DEH.

CDEH+ = ABCDEFGH. CDEH is a key. All supersets of CDEH are supersets.

Therefore, {BEH, CDEH} are the keys.

**c.**

First step: combine the RHS.

B→C and B→D becomes B→CD.

CD→A and CD→F become CD→AF.

CDE→B and CDE→G becomes CDE→BG.

So, the FDs are: **B→CD, CD→AF, CDE→BG**.

Step 2: Make relations base on the FDs.

R1(BCD), R2(ACDF), R3(BCDEG)

because BEH and CDEH are keys, we should add a relation that contains the keys.

R4(BEH)

So, the final relations are R1(BCD), R2(ACDF), R3(BCDEG), R4(BEH)

**d.**

R1, R2, R3 are formed by FDs, the LHS of the FDs are the superkey of them. R4 is the key of the whole relation. However, there may other FDs that violate BCNF so the schema allows redundancy. For instance, project B→CD and CD→AF in R3, B+ = ABCDF, B is not a superkey of R3. Therefore, the schema allow redundancy.